RAVI MATHS TUITION CENTER ,GKM COLONY, CHENNAI- 82. PH: 8056206308

Application of Differential Calculus 1 MARK Date : 18-Oct-19

12th Standard

Maths

Reg.No. :

Exam Time : 00:20:00 Hrs Total Marks : 20

20 x 1 = 20

1) The volume of a sphere is increasing in volume at the rate of 3 πcm^3 sec. The rate of change of its radius when radius is $\frac{1}{2}$ cm

(a) 3 cm/s (b) 2 cm/s (c) 1 cm/s (d) $\frac{1}{2} cm/s$

2) A balloon rises straight up at 10 m/s. An observer is 40 m away from the spot where the balloon left the ground. Find the rate of change of the balloon's angle of elevation in radian per second when the balloon is 30 metres above the ground.

(a) $\frac{3}{25} radians/sec$ (b) $\frac{4}{25} radians/sec$ (c) $\frac{1}{5} radians/sec$ (d) $\frac{1}{3} radians/sec$

3) The position of a particle moving along a horizontal line of any time t is given by set) = $3t^2$ -2t- 8. The time at which the particle is at rest is

(a) t= 0 (b) $t = \frac{1}{3}$ (c) t =1 (d) t = 3

4) A stone is thrown up vertically. The height it reaches at time t seconds is given by x = 80t -$16t^2$. The stone reaches the maximum height in time t seconds is given by

(a) 2 (b) 2.5 (c) 3 (d) 3.5

5) Find the point on the curve 6y = .x^3 + 2 at which y-coordinate changes 8 times as fast as x-coordinate is

(a) (4,11) (b) (4,-11) (c) (-4,11) (d) (-4,-11)

6) The abscissa of the point on the curve $f(x) = \sqrt{8-2x}$ at which the slope of the tangent is -0.25 ?

(a) -8 (b) -4 (c) -2 (d) 0

7) The slope of the line normal to the curve f(x) = 2cos 4x at $x = \frac{\pi}{12}$

(a) $-4\sqrt{3}$ (b) -4 (c) $\frac{\sqrt{3}}{12}$ (d) $4\sqrt{3}$

8) The tangent to the curve y^2 - xy + 9 = 0 is vertical when

(a) y = 0 (b) $y = \pm\sqrt{3}$ (c) $y = \frac{1}{2}$ (d) $y = \pm 3$

9) Angle between y^2 = x and.x^2= y at the origin is

(a) $tan^{-1}\frac{3}{4}$ (b) $tan^{-1}(\frac{4}{3})$ (c) $\frac{\pi}{2}$ (d) $\frac{\pi}{4}$

10) The value of the limit $\lim_{x\to 0} (cotx - \frac{1}{x})$

(a) 0 (b) 1 (c) 2 (d) ∞

11) The function $\sin^4 x + \cos^4 X$ is increasing in the interval

(a) $[\frac{5\pi}{8}, \frac{3\pi}{4}]$ (b) $[\frac{\pi}{2}, \frac{5\pi}{8}]$ (c) $[\frac{\pi}{4}, \frac{\pi}{2}]$ (d) $[0, \frac{\pi}{4}]$

12) The number given by the Rolle's theorem for the functlon x-3-3x2,x∈[0,3] is

(a) 1 (b) $\sqrt{2}$ (c) $\frac{3}{2}$ (d) 2

13) The number given by the Mean value theorem for the function $\frac{1}{x}$,x∈[1,9] is

(a) 2 (b) 2.5 (c) 3 (d) 3.5

14) The minimum value ofthe function |3-x|+9 is

(a) 0 (b) 3 (c) 6 (d) 9

15) The maximum slope of the tangent to the curve y = t:r sin x, x ∈ [0, 2π] is at

(a) $x = \frac{\pi}{4}$ (b) $x = \frac{\pi}{2}$ (c) $x = \pi$ (d) $x = \frac{3\pi}{2}$

16) The maximum value of the function $x^2 e^{-2x}$,

(a) $\frac{1}{e}$ (b) $\frac{1}{2e}$ (c) $\frac{1}{e^2}$ (d) $\frac{4}{e^4}$

17) One of the closest points on the curve $x^2 - y^2$.= 4 to the point (6, 0) is

(a) (2,0) (b) (Ã$\overline{5}$, 1) (c) (3, Ã$\overline{5}$) (d) (Ã$\overline{13}$, −Ã$\overline{3}$)

18) The maximum value of the product of two positive numbers, when their sum' of the squares is 200, is

(a) 100 (b) 25Ã$\overline{7}$ (c) 28 (d) 24Ã$\overline{14}$

19) The curve y= $ax^4 + bx^2$ with ab > 0

(a) has, no horizontal tangent (b) is concave up (c) isconeavedown (d) has no points of inflection

20) The point of inflection of the curve y = $(x - 1)^3$ is

(a) (0,0) (b) (0,1) (c) (1,0) (d) (1,1)

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RAVI MATHS TUITION CENTER ,GKM COLONY, CHENNAI- 82. PH: 8056206308

Application of Differential Calculus 2 MARKS SET 1 Date : 18-Oct-19

12th Standard

Maths

Reg.No. : | | | | | | |

Exam Time : 01:00:00 Hrs Total Marks : 50

25 x 2 = 50

1) A point moves along a straight line in such a way that after t seconds its distance from the origin is $s = 2t^2 + 3t$ metres
Find the average velocity of the points between t = 3 and t = 6 seconds.

2) A camera is accidentally knocked off an edge of a cliff 400 ft high. The camera falls a distance of $s = 16t^2$ in t seconds
What is the instantaneous velocity of the camera when it hits the ground?

3) If the mass m(x) (in kilograms) of a thin rod of length x (in metres) is given by, $m(x) = \sqrt{3x}$ then what is the rate of change of mass with respect to the length when it is x = 3 and x = 27 metres.

4) A ladder 17 metre long is leaning against the wall. The base of the ladder is pulled away from the wall at a rate of 5 m/s. When the base of the ladder is 8 metres from the wall.
At what rate, the area of the triangle formed by the ladder, wall, and the floor, is changing?

5) Find the slope of the tangent to the curves at the respective given points.
$y = x^4 + 2x^2 - x$ at $x = 1$

6) Find the slope of the tangent to the curves at the respective given points.
$x = a\cos^3 t$, $y = b\sin^3 t$ at $t = \frac{\pi}{2}$

7) Find the points on the curve $y = x^3 - x^2 + x + 3$ where the normal is parallel to the line $x + y = 1729$.

8) Find the tangent and normal to the following curves at the given points on the curve
$x = \cos t$, $y = 2\sin t^2$ at $t = \frac{\pi}{3}$

9) Show that the two curves $x^2 - y^2 = r^2$ and $xy = c^2$ where c, r are constants, cut orthogonally

10) Find the absolute extrema of the following functions on the given closed interval.
$f(x) = 6x^{\frac{3}{4}} - 3x^{\frac{1}{3}}; [-1, 1]$

11) Find the intervals of mono tonicities and hence find the local extremum for the following functions:
$f(x)=2x^3+3x^2-12x$

12) Find the intervals of mono tonicities and hence find the local extremum for the following functions:
$$f(x) = \frac{e^x}{1 - e^x}$$

13) Explain why Lagrange's mean value theorem is not applicable to the following functions in the respective intervals:
$f(x) = \frac{x+1}{x}$, $x \in [-1, 2]$

14) Find intervals of concavity and points of inflexion for the following functions
$f(x) = x(x - 4)^3$

15) Find intervals of concavity and points of inflexion for the following function:
$$f(x) = \frac{1}{2}\left(e^x - e^{-x}\right)$$

16) Find the smallest possible value x^2+y^2 given that $x + y = 10$.

17) A farmer plans to fence a rectangular pasture adjacent to a river. The pasture must contain1,80,000 sq. mtrs in order to provide enough grass for herds. No fencing is needed along the river. What is the length of the I minimum needed fencing material

18)

Write the Maclaurin series expansion of the following function

cos x

19) Write the Maclaurin series expansion of the following function

log(1 - x); $-1 \leq x < 1$

20) Write down the Taylor series expansion, of the function log x about x =1 upto three nonzero terms for x > 0.

21) Find the asymptotes of the following curves :$f(x) = \dfrac{x^2 + 6x - 4}{3x - 6}$

22) Evaluate the following limit, if necessary use l'Hôpital Rule

$lim_{x \to 0} \dfrac{1 - cosx}{x^2}$

23) Evaluate the following limit, if necessary use l'Hôpital Rule

$lim_{x \to 1^+} \left(\dfrac{2}{x^2 - 1} - \dfrac{x}{x - 1} \right)$

24) Sketch the graphs of the followingfunction $y = \dfrac{x^3}{24} - logx$

25) If an initial amount A_0 of money is invested at an interest rate r compounded n times a year, the value of the investment after t years is $A = A_0\left(1 + \frac{r}{n}\right)^{nt}$. If the interest is compounded continuously, (that is as n $\to\infty$), show that the amount after t years is $A=A_o e^{rt}$.

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RAVI MATHS TUITION CENTER ,GKM COLONY, CHENNAI- 82. PH: 8056206308

Application of Differential Calculus 2 MARKS SET 2 Date : 18-Oct-19

12th Standard

Maths

Reg.No. : | | | | | | |

Exam Time : 01:00:00 Hrs Total Marks : 50

25 x 2 = 50

1) A camera is accidentally knocked off an edge of a cliff 400 ft high. The camera falls a distance of $s = 16t^2$ in t seconds What is the average velocity with which the camera falls during the last 2 seconds?

2) A particle moves along a line according to the law $s(t) = 2t^3 - 9t^2 + 12t - 4$, where $t \geq 0$.
Find the total distance travelled by the particle in the first 4 seconds.

3) A stone is dropped into a pond causing ripples in the form of concentric circles. The radius r of the outer ripple is increasing at a constant rate at 2 cm per second. When the radius is 5 cm find the rate of changing of the total area of the disturbed water?

4) A conical water tank with vertex down of 12 metres height has a radius of 5 metres at the top. If water flows into the tank at a rate 10 cubic m/min, how fast is the depth of the water increases when the water is 8 metres deep?

5) Find the slope of the tangent to the curves at the respective given points.
$y = x^4 + 2x^2 - x$ at $x = 1$

6) Find the points on the curve $y^2 - 4xy = x^2 + 5$ for which the tangent is horizontal.

7) Find the equations of the tangents to the curve $y = \frac{x+1}{x-1}$ which are parallel to the line $x + 2y = 6$.

8) Find the angle between the rectangular hyperbola $xy = 2$ and the parabola $x^2 y + 4 = 0$

9) Find the absolute extrema of the following functions on the given closed interval.

$$f(x) = 2cosx + sin2x; \left[0, \frac{\pi}{2}\right]$$

10) Explain why Rolle's theorem is not applicable to the following functions in the respective intervals.
$f(x) = x - 2logx, x \in [2, 7]$

11) Using the Rolle's theorem, determine the values of x at which the tangent is parallel to the x -axis for the following functions:
$f(x) = \sqrt{x} - \frac{x}{3}, x \in [0, 9]$

12) Using the Lagrange's mean value theorem determine the values of x at which the tangent is parallel to the secant line at the end points of the given interval:
$f(x) = x^3 - 3x + 2, x \in [-2, 2]$

13) A race car driver is racing at 20th km. If his speed never exceeds 150 km/hr, what is the maximum distance he can cover in the next two hours.

14) Does there exist a differentiable function f(x) such that $f(0) = -1$, $f(2) = 4$ and $f'(x) \leq 2$ for all x. Justify you answer.

15) Using mean value theorem prove that for, $a > 0, b > 0$, $|e^{-a} - e^{-b}| < |a - b|$.

16) Prove that among all the rectangles of the given perimeter, the square has the maximum area.

17) Write the Maclaurin series expansion of the following function
sin x

18) Write the Maclaurin series expansion of the following function
$\tan^{-1}(x); -1 \leq x \leq 1$

19) Write the Maclaurin series expansion of the following function:
$\cos^2 x$

20)

Find the asymptotes of the following curves $f(x) = \frac{3x}{\sqrt{x^2 + 2}}$

21) Expand the polynomial f (x) = x^2 - 3x + 2 in powers of x - 1

22) Evaluate the following limit, if necessary use l'Hôpital Rule

$\lim_{x \to \frac{\pi}{2}^-} \frac{\sec x}{\tan x}$

23) Sketch the graphs of the followingfunction $y = \frac{1}{1 + e^{-x}}$

24) Find the local extrema for the following function using second derivative test:

$f(x) = -3x^5 + 5x^3$

25) Find the local extrema for the following function using second derivative test:

$f(x) = x^2 e^{-2x}$

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RAVI MATHS TUITION CENTER ,GKM COLONY, CHENNAI- 82. PH: 8056206308

Application of Differential Calculus 3 MARK Date : 18-Oct-19

12th Standard

Maths

Reg.No. : | | | | | | |

Exam Time : 01:00:00 Hrs Total Marks : 60

20 x 3 = 60

1) For the function f(x) = $x^2 \in [0, 2]$ compute the average rate of changes in the subintervals [0,0.5], [0.5,1], [1,1.5], [1.5,2] and the instantaneous rate of changes at the points x = 0.5,1,1.5, 2

2) A particle moves so that the distance moved is according to the law s(t) = $s(t) = \frac{t^3}{3} - t^2 + 3$. At what time the velocity and acceleration are zero respectively?

3) A particle moves along a horizontal line such that its position at any time t ≥ 0 is given by s(t) = $t^3 - t^2 + t + 6\ 9\ 1$, where s is measured in metres and t in seconds?

(1) At what time the particle is at rest?

(2) At what time the particle changes direction?

(3) Find the total distance travelled by the particle in the first 2 seconds.

4) Salt is poured from a conveyer belt at a rate of 30 cubic metre per minute forming a conical pile with a circular base whose height and diameter of base are always equal. How fast is the height of the pile increasing when the pile is 10 metre high?

5) Compute the value of 'c' satisfied by the Rolle's theorem for the function f (x) = $x^2 (1 - x)^2$, x ∈ [0.1]

6) Find the values in the interval $(\frac{1}{2}, 2)$ satisfied by the Rolle's theorem for the function $f(x) = x + \frac{1}{x}, x \in [\frac{1}{2}, 2]$

7) Prove using the Rolle's theorem that between any two distinct real zeros of the polynomial $a_n x^n + a_{n-1}x^{n-1} + \ldots + a_1 x + a_0$ there is a zero of the polynomial $na_n x^{n-1} + (n-1)a_{n-1}x^{n-2} + \ldots + a_1$

8) Prove that there is a zero of the polynomial $2x^3 - 9x^2 - 11x + 12$ in the interval (2,7) given that 2 and 7 are the zeros of the polynomial $x^4 - 6x^3 - 11x^2 + 24x + 28$

9) Prove, using mean value theorem, that $|sin\alpha - sin\beta| \le |\alpha - \beta|, \alpha, \beta \in R$

10) A thermometer was taken from a freezer and placed in a boiling water. It took 22 seconds for the thermometer to raise from −10°C to 100°C. Show that the rate of change of temperature at some time t is 5°C per second.

11) Compute the limit $\lim\limits_{x \to 1} (\frac{x^2 - 3x + 2}{x^2 - 4x + 3})$.

12) Evaluate the limit $\lim\limits_{x \to 0} (\frac{sinmx}{x})$

13) $\lim\limits_{\theta \to 0} (\frac{1 - cos\ m\theta}{1 - cos\ n\theta})$ =1, then prove that, $m = \pm n$

14) Evaluate: : $\lim\limits_{x \to 0^+}$ x log x.

15) Using the l'Hôpital Rule prove that, $\lim\limits_{x \to 0^+} (1 + x)^{\frac{1}{x}} = e$

16) Determine the intervals of concavity of the curve f (x) = $(x - 1)^3$. (x − 5), x∈R and, points of inflection if any.

17) Find the local extremum of the function f (x) = x^4 + x 32x

18) Find the asymptotes of the curve $f(x) = \frac{2x^2 - 8}{x^2 - 16}$

19) Sketch the curve y = f (x) = $x^2 - x - 6$.

20) Find the local maximum and minimum of the function $x^2 y^2$ on the line x + y =10

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RAVI MATHS TUITION CENTER ,GKM COLONY, CHENNAI- 82. PH: 8056206308

Application of Differential Calculus 3 MARKS SET 2 Date : 18-Oct-19

12th Standard

Maths

Reg.No. : | | | | | | |

Exam Time : 01:00:00 Hrs Total Marks : 60

20 x 3 = 60

1) A particle is fired straight up from the ground to reach a height of s feet in t seconds, where s(t) =128t −16t^2.
(1) Compute the maximum height of the particle reached.
(2) What is the velocity when the particle hits the ground?

2) If we blow air into a balloon of spherical shape at a rate of 1000^3 cm per second. At what rate the radius of the baloon changes when the radius is 7cm? Also compute the rate at which the surface area changes.

3) A road running north to south crosses a road going east to west at the point P. Car A is driving north along the first road, and car B is driving east along the second road. At a particular time car A 10 kilometres to the north of P and traveling at 80 km/hr, while car B is 15 kilometres to the easst of P and traveling at 100 km/hr. How fast is the distance between the two cars changing?

4) Find the values in the interval $(\frac{1}{2}, 2)$ satisfied by the Rolle's theorem for the function $f(x) = x + \frac{1}{x}, x \in [\frac{1}{2}, 2]$

5) Compute the value of 'c' satisfied by Rolle’s theorem for the function $f(x) = log(\frac{x^2+6}{5x})$ in the interval [2,3]

6) A truck travels on a toll road with a speed limit of 80 km/hr. The truck completes a 164 km journey in 2 hours. At the end of the toll road the trucker is issued with a speed violation ticket. Justify this using the Mean Value Theorem.

7) Suppose f(x) is a differentiable function for all x with f'(x) ≤ 29 and f(2) =17. What is the maximum value of f (7)?

8) Compute the limit $lim_{x \to a}(\frac{x^n - a^n}{x-a})$

9) Evaluate the limit $lim_{x \to 0^+}(\frac{\sin x}{x^2})$

10) Evaluate : $lim_{x \to 1^-}(\frac{log(1-x)}{cot(\pi x)})$.

11) Evaluate: $lim_{x \to \infty}(\frac{x^2+17x+29}{x^4})$.

12) Using the l’Hôpital Rule prove that, $lim_{x \to 0^+}(1+x)^{\frac{1}{x}} = e$

13) Evaluate: $log(lim_{x \to \infty}(1+2x)^{\frac{1}{2log\, x}}$

14) Evaluate: $lim_{x \to 1}\, x^{\frac{1}{1-x}}$

15) Determine the intervals of concavity of the curve y = 3+ sin x .

16) Find the local extremum of the function f (x) = x^4 + x 32x

17) Sketch the curve y = f (x) = x^2 − x −6 .

18) Find the local extrema of the function f (x)= 4x^6 −6x^4

19) Sketch the curve $y = \frac{x^2-3x}{(x-1)}$

20) Sketch the graph of the function $y = \frac{3x}{x^2-1}$

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RAVI MATHS TUITION CENTER ,GKM COLONY, CHENNAI- 82. PH: 8056206308

Application of Differential Calculus 5 MARK SET 1 Date : 18-Oct-19

12th Standard

Maths

Reg.No. : | | | | | | |

Exam Time : 01:30:00 Hrs Total Marks : 60

12 x 5 = 60

1) For what value of x the tangent of the curve $y = x^3 - x^2 + x - 2$ is parallel to the line y = x.

2) Find the equation of the tangent and normal to the Lissajous curve given by x = 2cos3t and y = 3sin 2t, $t \in R$

3) Find the acute angle between $y = x^2$ and $y = (x - 3)^2$.

4) If the curves $ax^2+by^2=1$ and $cx^2+dy^2=1$ intersect each other orthogonally then, $\frac{1}{a}-\frac{1}{b}=\frac{1}{c}-\frac{1}{d}$

5) Prove that the ellipse $x^2+4y^2=8$ and the hyperbola $x^2-2y^2=4$ intersect orthogonally.

6) Expand tan x in ascending powers of x upto 5^{th} power for $\left(-\frac{\pi}{2}<x<\frac{\pi}{2}\right)$

7) Find the absolute maximum and absolute minimum values of the function $f(x) = 2x^3 + 3x^2 - 12x$ on [−3, 2]

8) Prove that the function f (x) = x − sin x is increasing on the real line. Also discuss for the existence of local extrema.

9) Discuss the monotonicity and local extrema of the function $f(x)=log(1+x)-\frac{x}{1+x}, x>-1$ and hence find the domain where, $log(1+x)>\frac{x}{1+x}$

10) Find the intervals of monotonicity and local extrema of the function $f(x)=\frac{1}{1+x^2}$

11) We have a 12 square unit piece of thin material and want to make an open box by cutting small squares from the corners of our material and folding the sides up. The question is, which cut produces the box of maximum volume?

12) A steel plant is capable of producing x tonnes per day of a low-grade steel and y tonnes per day of a high-grade steel, where $y=\dfrac{40-5x}{10-x}$ If the fixed market price of low-grade steel is half that of high-grade steel, then what should be optimal productions in low-grade steel and high-grade steel in order to have maximum receipts.

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RAVI MATHS TUITION CENTER ,GKM COLONY, CHENNAI- 82. PH: 8056206308

Application of Differential Calculus 5 MARK SET 2 Date : 18-Oct-19

12th Standard

Maths

Reg.No. : | | | | | | |

Exam Time : 01:15:00 Hrs

Total Marks : 50

10 x 5 = 50

1) Find the equations of tangent and normal to the curve $y = x^2 + 3x - 2$ at the point (1, 2)

2) Find the acute angle between the curves $y = x^2$ and $x = y^2$ at their points of intersection (0,0), (1,1).

3) Find the angle of intersection of the curve y = sin x with the positive x -axis.

4) Expand log(1+ x) as a Maclaurin's series upto 4 non-zero terms for $-1 < x \leq 1$.

5) Write the Taylor series expansion of $\frac{1}{x}$ about x = 2 by finding the first three non-zero terms.

6) Prove that the function $f(x) = x^2 - 2x - 3$ is strictly increasing in $(2, \infty)$

7) Find the absolute extrema of the function f (x) = 3cos x on the closed interval $[0, 2\pi]$

8) Find the intervals of monotonicity and local extrema of the function f (x) = x log x + 3x.

9) Find the intervals of monotonicity and local extrema of the function $f(x) = \frac{x}{1+x^2}$

10) Prove that among all the rectangles of the given area square has the least perimeter.

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RAVI MATHS TUITION CENTER ,GKM COLONY, CHENNAI- 82. PH: 8056206308

Application of Differential Calculus FULL TEST Date : 18-Oct-19

12th Standard

Maths

Reg.No. : | | | | | | |

Exam Time : 02:30:00 Hrs Total Marks : 90

20 x 1 = 20

1) The volume of a sphere is increasing in volume at the rate of 3 πcm^3 sec. The rate of change of its radius when radius is $\frac{1}{2}$ cm

(a) 3 cm/s (b) 2 cm/s (c) 1 cm/s (d) $\frac{1}{2} cm/s$

2) A balloon rises straight up at 10 m/s. An observer is 40 m away from the spot where the balloon left the ground. Find the rate of change of the balloon's angle of elevation in radian per second when the balloon is 30 metres above the ground.

(a) $\frac{3}{25} radians/sec$ (b) $\frac{4}{25} radians/sec$ (c) $\frac{1}{5} radians/sec$ (d) $\frac{1}{3} radians/sec$

3) The position of a particle moving along a horizontal line of any time t is given by set) = $3t^2$ -2t- 8. The time at which the particle is at rest is

(a) t= 0 (b) $t = \frac{1}{3}$ (c) t =1 (d) t = 3

4) A stone is thrown up vertically. The height it reaches at time t seconds is given by x = 80t -$16t^2$. The stone reaches the maximum height in time t seconds is given by

(a) 2 (b) 2.5 (c) 3 (d) 3.5

5) Find the point on the curve 6y = .x^3 + 2 at which y-coordinate changes 8 times as fast as x-coordinate is

(a) (4,11) (b) (4,-11) (c) (-4,11) (d) (-4,-11)

6) The abscissa of the point on the curve $f(x) = \sqrt{8 - 2x}$ at which the slope of the tangent is -0.25 ?

(a) -8 (b) -4 (c) -2 (d) 0

7) The slope of the line normal to the curve f(x) = 2cos 4x at $x = \frac{\pi}{12}$

(a) $-4\sqrt{3}$ (b) -4 (c) $\frac{\sqrt{3}}{12}$ (d) $4\sqrt{3}$

8) The tangent to the curve y^2 - xy + 9 = 0 is vertical when

(a) y = 0 (b) $y = \pm\sqrt{3}$ (c) $y = \frac{1}{2}$ (d) $y = \pm 3$

9) Angle between y^2 = x and.x^2= y at the origin is

(a) $tan^{-1}\frac{3}{4}$ (b) $tan^{-1}\left(\frac{4}{3}\right)$ (c) $\frac{\pi}{2}$ (d) $\frac{\pi}{4}$

10) The value of the limit $lim_{x\to 0}\left(cotx - \frac{1}{x}\right)$

(a) 0 (b) 1 (c) 2 (d) ∞

11) The function $\sin^4 x + \cos^4 X$ is increasing in the interval

(a) $\left[\frac{5\pi}{8}, \frac{3\pi}{4}\right]$ (b) $\left[\frac{\pi}{2}, \frac{5\pi}{8}\right]$ (c) $\left[\frac{\pi}{4}, \frac{\pi}{2}\right]$ (d) $\left[0, \frac{\pi}{4}\right]$

12) The number given by the Rolle's theorem for the functlon x-3-3x2,x∈[0,3] is

(a) 1 (b) $\sqrt{2}$ (c) $\frac{3}{2}$ (d) 2

13) The number given by the Mean value theorem for the function $\frac{1}{x}$,x∈[1,9] is

(a) 2 (b) 2.5 (c) 3 (d) 3.5

14) The minimum value ofthe function |3-x|+9 is

(a) 0 (b) 3 (c) 6 (d) 9

15) The maximum slope of the tangent to the curve y = t:r sin x, x ∈ [0, 2π] is at

(a) $x = \frac{\pi}{4}$ (b) $x = \frac{\pi}{2}$ (c) $x = \pi$ (d) $x = \frac{3\pi}{2}$

16) The maximum value of the function $x^2 e^{-2x}$,

(a) $\frac{1}{e}$ (b) $\frac{1}{2e}$ (c) $\frac{1}{e^2}$ (d) $\frac{4}{e^4}$

17) One of the closest points on the curve $x^2 - y^2$.= 4 to the point (6, 0) is

(a) (2,0) (b) $\left(\sqrt{5}, 1\right)$ (c) $\left(3, \sqrt{5}\right)$ (d) $\left(\sqrt{13}, -\sqrt{3}\right)$

18) The maximum value of the product of two positive numbers, when their sum' of the squares is 200, is

(a) 100 (b) $25\sqrt{7}$ (c) 28 (d) $24\sqrt{14}$

19) The curve y= $ax^4 + bx^2$ with ab > 0

(a) has, no horizontal tangent (b) is concave up (c) isconeavedown (d) has no points of inflection

20) The point of inflection of the curve y = $(x - 1)^3$ is

(a) (0,0) (b) (0,1) (c) (1,0) (d) (1,1)

ANY 7 7 x 2 = 14

21) A point moves along a straight line in such a way that after t seconds its distance from the origin is s = $2t^2 + 3t$ metres

Find the instantaneous velocities at t = 3 and t = 6 seconds.

22) A particle moves along a line according to the law s(t) = $2t^3 - 9t^2 + 12t - 4$, where t ≥ 0.

At what times the particle changes direction?

23) If the volume of a cube of side length x is v = x^3. Find the rate of change of the volume with respect to x when x = 5 units.

24) A ladder 17 metre long is leaning against the wall. The base of the ladder is pulled away from the wall at a rate of 5 m/s. When the base of the ladder is 8 metres from the wall.

How fast is the top of the ladder moving down the wall?

25) Find the tangent and normal to the following curves at the given points on the curve

y = $x^2 - x^4$ at (1, 0)

26) Find the absolute extrema of the following functions on the given closed interval.

$$f(x) = 2cosx + sin2x; \left[0, \frac{\pi}{2}\right]$$

27) Using the Rolle's theorem, determine the values of x at which the tangent is parallel to the x -axis for the following functions:

$$f(x) = \frac{x^2 - 2x}{x+2}, x \in [-1, 6]$$

28) Using the Lagrange's mean value theorem determine the values of x at which the tangent is parallel to the secant line at the end points of the given interval:

$f(x) = x^3 - 3x + 2, x \in [-2, 2]$

29) Find two positive numbers whose product is 20 and their sum is minimum.

30) Write the Maclaurin series expansion of the following function

$\log(1 - x); -1 \le x < 1$

ANY 7 7 x 3 = 21

31) A particle is fired straight up from the ground to reach a height of s feet in t seconds, where $s(t) = 128t - 16t^2$.

(1) Compute the maximum height of the particle reached.

(2) What is the velocity when the particle hits the ground?

32) Salt is poured from a conveyer belt at a rate of 30 cubic metre per minute forming a conical pile with a circular base whose height and diameter of base are always equal. How fast is the height of the pile increasing when the pile is 10 metre high?

33) Prove that there is a zero of the polynomial $2x^3 - 9x^2 - 11x + 12$ in the interval (2,7) given that 2 and 7 are the zeros of the polynomial $x^4 - 6x^3 - 11x^2 + 24x + 28$

34) Compute the limit $\lim_{x\to 1}\left(\frac{x^2-3x+2}{x^2-4x+3}\right)$.

35) Evaluate: $\lim_{x\to 0^+}\left(\frac{1}{x} - \frac{1}{e^x-1}\right)$.

36) Using the l'Hôpital Rule prove that, $\lim_{x\to 0^+}(1 + x)^{\frac{1}{x}} = e$

37) Find the local extremum of the function $f(x) = x^4 + x\ 32x$

38) Find the asymptotes of the curve $f(x) = \frac{2x^2-8}{x^2-16}$

39) Find the local maximum and minimum of the function $x^2\ y^2$ on the line x + y =10

40) Sketch the curve $y = \frac{x^2-3x}{(x-1)}$

ANY 7 7 x 5 = 35

41) For what value of x the tangent of the curve $y = x^3 - x^2 + x - 2$ is parallel to the line y = x.

42) Find the acute angle between $y = x^2$ and $y = (x - 3)^2$.

43) If the curves $ax^2+by^2=1$ and $cx^2+dy^2=1$ intersect each other orthogonally then, $\frac{1}{a} - \frac{1}{b} = \frac{1}{c} - \frac{1}{d}$

44) Expand tan x in ascending powers of x upto 5th power for $\left(-\frac{\pi}{2} < x < \frac{\pi}{2}\right)$

45) Find the absolute maximum and absolute minimum values of the function $f(x) = 2x^3 + 3x^2 - 12x$ on [−3, 2]

46) Discuss the monotonicity and local extrema of the function $f(x) = \log(1 + x) - \frac{x}{1+x}, x > -1$ and hence find the domain where, $\log(1 + x) > \frac{x}{1+x}$

47) Find the intervals of monotonicity and local extrema of the function f (x) = x log x + 3x.

48) Find the intervals of monotonicity and local extrema of the function $f(x) = \frac{x}{1+x^2}$

49) We have a 12 square unit piece of thin material and want to make an open box by cutting small squares from the corners of our material and folding the sides up. The question is, which cut produces the box of maximum volume?

50) Prove that among all the rectangles of the given area square has the least perimeter.

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RAVI MATHS TUITION CENTER, NEAR VILLIVAKKAM RLY STATION, CHENNAI – 82.
WHATSAPP - 8056206308

Application of Differential Calculus

12th Standard

Maths

Exam Time : 02:30:00 Hrs | Total Marks : 100

20 x 1 = 20

1) The volume of a sphere is increasing in volume at the rate of 3 πcm^3 sec. The rate of change of its radius when radius is $\frac{1}{2}$ cm

(a) 3 cm/s (b) 2 cm/s (c) 1 cm/s (d) $\frac{1}{2}cm/s$

2) The position of a particle moving along a horizontal line of any time t is given by set) = $3t^2$ -2t- 8. The time at which the particle is at rest is

(a) t= 0 (b) $t = \frac{1}{3}$ (c) t =1 (d) t = 3

3) Find the point on the curve 6y = $.x^3$ + 2 at which y-coordinate changes 8 times as fast as x-coordinate is

(a) (4,11) (b) (4,-11) (c) (-4,11) (d) (-4,-11)

4) The slope of the line normal to the curve f(x) = 2cos 4x at $x = \frac{\pi}{12}$

(a) $-4\sqrt{3}$ (b) -4 (c) $\frac{\sqrt{3}}{12}$ (d) $4\sqrt{3}$

5) Angle between y^2 = x and.x^2= y at the origin is

(a) $tan^{-1}\frac{3}{4}$ (b) $tan^{-1}\left(\frac{4}{3}\right)$ (c) $\frac{\pi}{2}$ (d) $\frac{\pi}{4}$

6) The function $\sin^4$ x + $\cos^4$X is increasing in the interval

(a) $\left[\frac{5\pi}{8}, \frac{3\pi}{4}\right]$ (b) $\left[\frac{\pi}{2}, \frac{5\pi}{8}\right]$ (c) $\left[\frac{\pi}{4}, \frac{\pi}{2}\right]$ (d) $\left[0, \frac{\pi}{4}\right]$

7) The number given by the Mean value theorem for the function $\frac{1}{x}$,x∈[1,9] is

(a) 2 (b) 2.5 (c) 3 (d) 3.5

8) The maximum slope of the tangent to the curve y = t:r sin x, x ∈ [0, 2π] is at

(a) $x = \frac{\pi}{4}$ (b) $x = \frac{\pi}{2}$ (c) $x = \pi$ (d) $x = \frac{3\pi}{2}$

9) One of the closest points on the curve x^2 - y^2.= 4 to the point (6, 0) is

(a) (2,0) (b) $(\sqrt{5}, 1)$ (c) $(3, \sqrt{5})$ (d) $(\sqrt{13}, -\sqrt{3})$

10) The curve y= ax^4 + bx^2 with ab > 0

(a) has, no horizontal tangent (b) is concave up (c) isconeavedown (d) has no points of inflection

11) If a particle moves in a straight line according to $s = t^3-6t^2-15t$, the time interval during which the velocity is negative and acceleration is positive is

(a) $2 < t < 5$ (b) $2 \leq t \leq 5$ (c) $t \geq 2$ (d) $t \leq 2$

12) If the rate of increase of $s = x^3-5x^2+5x+8$ is twice the rate of increase of x, then one value of x is

(a) $\frac{3}{5}$ (b) $\frac{10}{3}$ (c) $\frac{3}{10}$ (d) $\frac{1}{3}$

13) The equation of the tangent to the curve $y=x^2-4x+2$ at (4,2) is

(a) $x + 4y + 12 = 0$ (b) $4x + y + 12 = 0$ (c) $4x - y - 14 = 0$ (d) $x + 4y - 12 = 0$

14) The least value of a when f $f(x) = x^2+ax+1$ is increasing on (1, 2) is

(a) -2 (b) 2 (c) 1 (d) -1

15) The angle made by any tangent to the curve $y = x^5 + 8x + 1$ with the X-axis is a

(a) obtuse (b) right angle (c) acute angle (d) no angle

16) If the curves $y = 2e^x$ and $y = ae^{-x}$ intersect orthogonally, then a = _________

(a) $\frac{1}{2}$ (b) $-\frac{1}{2}$ (c) 2 (d) $2e^2$

17) The function/(x) = $x^9 + 3x^7 + 64$ is increasing on ________

(a) R (b) (-∞, 0) (c) (0, ∞) (d) None of these

18) The curve $y = e^x$ is ________

(a) convex (b) concave (c) convex upwards (d) concave upwards

19) $\lim_{x \to 0} \frac{x}{\tan x}$ is _________

(a) 1 (b) -1 (c) 0 (d) ∞

20) The statement " If f has a local extremum at c and if f'(c) exists then f'(c) = 0" is ________

(a) the extreme value theorem (b) Fermats' theorem (c) Law of mean (d) Rolle's theorem

8 x 2 = 16

21) A person learnt 100 words for an English test. The number of words the person remembers in t days after learning is given by $W(t) = 100\times(1-0.1t)^2$, $0 \leq t \leq 10$. What is the rate at which the person forgets the words 2 days after learning?

22) If the volume of a cube of side length x is $v = x^3$. Find the rate of change of the volume with respect to x when x = 5 units.

23) Find the slope of the tangent to the curves at the respective given points.
$x = a \cos^3 t$, $y = b \sin^3 t$ at $t = \frac{\pi}{2}$

24) Suppose f(x) is a differentiable function for all x with $f'(x) \leq 29$ and f(2) =17. What is the maximum value of f (7)?

25) Explain why Rolle's theorem is not applicable to the following functions in the respective intervals.
$f(x) = \tan x, x \in [0, \pi]$

26) Compute the limit $\lim_{x \to 1}\left(\frac{x^2-3x+2}{x^2-4x+3}\right)$.

27) Evaluate the following limit, if necessary use l'Hôpital Rule
$\lim_{x \to \frac{\pi}{2}^-} \frac{\sec x}{\tan x}$

28) Prove that the function $f(x) = x^2 - 2x - 3$ is strictly increasing in $(2, \infty)$

8 x 3 = 24

29) Show that the two curves $x^2 - y^2 = r^2$ and $xy = c^2$ where c, r are constants, cut orthogonally

30) Using the Lagrange's mean value theorem determine the values of x at which the tangent is parallel to the secant line at the end points of the given interval:
$f(x) = (x - 2)(x - 7), x \in [3,11]$

31) A race car driver is racing at 20th km. If his speed never exceeds 150 km/hr, what is the maximum distance he can cover in the next two hours.

32) Does there exist a differentiable function f(x) such that f(0) = -1, f(2) = 4 and $f'(x) \leq 2$ for all x. Justify you answer.

33) Expand sin x in ascending powers x - $\frac{\pi}{4}$ upto three non-zero terms.

34) Evaluate the following limit, if necessary use l'Hôpital Rule
$\lim_{x \to 1^+} \left(\frac{2}{x^2-1} - \frac{x}{x-1} \right)$

35) Find the absolute maximum and absolute minimum values of the function $f(x) = 2x^3 + 3x^2 - 12x$ on [−3, 2]

36) Find the local extremum of the function $f(x) = x^4 + x\ 32x$

8 x 5 = 40

37) A particle moves along a horizontal line such that its position at any time $t \geq 0$ is given by $s(t) = t^3 - t^2 + t + 6\ 9\ 1$, where s is measured in metres and t in seconds?
(1) At what time the particle is at rest?
(2) At what time the particle changes direction?
(3) Find the total distance travelled by the particle in the first 2 seconds.

38) Salt is poured from a conveyer belt at a rate of 30 cubic metre per minute forming a conical pile with a circular base whose height and diameter of base are always equal. How fast is the height of the pile increasing when the pile is 10 metre high?

39) A conical water tank with vertex down of 12 metres height has a radius of 5 metres at the top. If water flows into the tank at a rate 10 cubic m/min, how fast is the depth of the water increases when the water is 8 metres deep?

40) Find the tangent and normal to the following curves at the given points on the curve
$x = \cos t, y = 2\sin t^2$ at $t = \frac{\pi}{3}$

41) Find the smallest possible value x^2+y^2 given that x +y = 10.

42) A rectangular page is to contain 24 cm^2 of print. The margins at the top and bettemof the page are 1.5 cm and the margins at other sides of the page is 1 cm. What should be the dimensions of the page so that the area of the paper used is minimum.

43) Expand log(1+ x) as a Maclaurin's series upto 4 non-zero terms for $-1 < x \leq 1$.

44) Find the local maximum and local minimum values for $f(x)=12x^2-2x^2-x^4$.

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RAVI MATHS TUITION CENTER ,GKM COLONY, CHENNAI- 82. PH: 8056206308

Application of Differential Calculus FULL TEST

12th Standard

Maths

Reg.No. : | | | | | | |

Exam Time : 02:30:00 Hrs

Total Marks : 90

20 x 1 = 20

1) The volume of a sphere is increasing in volume at the rate of 3 πcm^3 sec. The rate of change of its radius when radius is $\frac{1}{2}$ cm

(a) 3 cm/s (b) 2 cm/s (c) 1 cm/s (d) $\frac{1}{2} cm/s$

2) A balloon rises straight up at 10 m/s. An observer is 40 m away from the spot where the balloon left the ground. Find the rate of change of the balloon's angle of elevation in radian per second when the balloon is 30 metres above the ground.

(a) $\frac{3}{25} radians/sec$ (b) $\frac{4}{25} radians/sec$ (c) $\frac{1}{5} radians/sec$ (d) $\frac{1}{3} radians/sec$

3) The position of a particle moving along a horizontal line of any time t is given by set) = $3t^2$ -2t- 8. The time at which the particle is at rest is

(a) t= 0 (b) $t = \frac{1}{3}$ (c) t =1 (d) t = 3

4) A stone is thrown up vertically. The height it reaches at time t seconds is given by x = 80t $-16t^2$. The stone reaches the maximum height in time t seconds is given by

(a) 2 (b) 2.5 (c) 3 (d) 3.5

5) Find the point on the curve 6y = .x^3 + 2 at which y-coordinate changes 8 times as fast as x-coordinate is

(a) (4,11) (b) (4,-11) (c) (-4,11) (d) (-4,-11)

6) The abscissa of the point on the curve $f(x) = \sqrt{8-2x}$ at which the slope of the tangent is -0.25 ?

(a) -8 (b) -4 (c) -2 (d) 0

7) The slope of the line normal to the curve f(x) = 2cos 4x at $x = \frac{\pi}{12}$

(a) $-4\sqrt{3}$ (b) -4 (c) $\frac{\sqrt{3}}{12}$ (d) $4\sqrt{3}$

8) The tangent to the curve y^2 - xy + 9 = 0 is vertical when

(a) y = 0 (b) $y = \pm\sqrt{3}$ (c) $y = \frac{1}{2}$ (d) $y = \pm 3$

9) Angle between y^2 = x and.x^2= y at the origin is

(a) $tan^{-1}\frac{3}{4}$ (b) $tan^{-1}\left(\frac{4}{3}\right)$ (c) $\frac{\pi}{2}$ (d) $\frac{\pi}{4}$

10) The value of the limit $\lim_{x\to 0}\left(cot x - \frac{1}{x}\right)$

(a) 0 (b) 1 (c) 2 (d) ∞

11) The function $\sin^4 x + \cos^4 X$ is increasing in the interval

(a) $\left[\frac{5\pi}{8}, \frac{3\pi}{4}\right]$ (b) $\left[\frac{\pi}{2}, \frac{5\pi}{8}\right]$ (c) $\left[\frac{\pi}{4}, \frac{\pi}{2}\right]$ (d) $\left[0, \frac{\pi}{4}\right]$

12) The number given by the Rolle's theorem for the functlon x^3-3x^2, $x \in [0,3]$ is
(a) 1 (b) $\sqrt{2}$ (c) $\frac{3}{2}$ (d) 2

13) The number given by the Mean value theorem for the function $\frac{1}{x}$,$x \in [1,9]$ is
(a) 2 (b) 2.5 (c) 3 (d) 3.5

14) The minimum value ofthe function |3-x|+9 is
(a) 0 (b) 3 (c) 6 (d) 9

15) The maximum slope of the tangent to the curve y = t:r sin x, x ∈ [0, 2π] is at
(a) $x = \frac{\pi}{4}$ (b) $x = \frac{\pi}{2}$ (c) $x = \pi$ (d) $x = \frac{3\pi}{2}$

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18) The maximum value of the product of two positive numbers, when their sum' of the squares is 200, is
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ANY 7 $7 \times 2 = 14$

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$y = x^2 - x^4$ at (1, 0)

26) Find the absolute extrema of the following functions on the given closed interval.

$$f(x) = 2cosx + sin2x; \left[0, \frac{\pi}{2}\right]$$

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30) Write the Maclaurin series expansion of the following function

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ANY 7 $7 \times 5 = 35$

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44) Expand tan x in ascending powers of x upto 5th power for $\left(-\frac{\pi}{2} < x < \frac{\pi}{2}\right)$

45) Find the absolute maximum and absolute minimum values of the function $f(x) = 2x^3 + 3x^2 - 12x$ on $[-3, 2]$

46) Discuss the monotonicity and local extrema of the function $f(x) = \log(1 + x) - \frac{x}{1+x}, x > -1$ and hence find the domain where, $\log(1 + x) > \frac{x}{1+x}$

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We have a 12 square unit piece of thin material and want to make an open box by cutting small squares from the corners of our material and folding the sides up. The question is, which cut produces the box of maximum volume?

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