Application of Differential Calculus 1 MARK

Maths

The volume of a sphere is increasing in volume at the rate of 3 π cm³ sec. The rate of change of its radius when radius is $\frac{1}{2}$ cm

2) A balloon rises straight up at 10 m/s. An observer is 40 m away from the spot where the balloon left the ground. Find the rate of

change of the balloon's angle of elevation in radian per second when the balloon is 30 metres above the ground.

3) The position of a particle moving along a horizontal line of any time t is given by set) = $3t^2$ -2t-8. The time at which the

(c) 1 cm/s

(c) $\frac{1}{5}$ radians/sec

(c) t=1

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Total Marks: 20 $20 \times 1 = 20$

Reg.No.:

(d) $\frac{1}{2}$ cm/s

(d) $\frac{1}{3}$ radians/sec

(d) t = 3

12th Standard

(b) 2 cm/s

(b) $t = \frac{1}{3}$

(b) $\frac{4}{25}$ radians/sec

Exam Time: 00:20:00 Hrs

(a) $\frac{3}{25}$ radians/sec

particle is at rest is

(a) t=0

(a) 3 cm/s

4)	A stone is thrown	up vertically. The height it reaches at t	time t seconds is given by x	$= 80t - 16t^2$. The sto	ne reaches the
	maximum height i	n time t seconds is given by			
	(a) 2	(b) 2.5	(c) 3	(d) 3.5	
5)	Find the point on t	he curve $6y = .x^3 + 2$ at which y-coord	dinate changes 8 times as fa	st as x-coordinate is	;
	(a) (4,11)	(b) (4,-11)	(c) (-4,11)	(d) (-4,-11	.)
6)	The abscissa of the	e point on the curve $f(x) = \tilde{A} \bar{8} - \bar{2}x$	at which the slope of the t	tangent is -0.25?	
	(a) -8	(b) -4	(c) -2	(d) ()
7)	The slope of the lin	ne normal to the curve $f(x) = 2\cos 4x$	at $x = \frac{\pi}{12}$		
	(a) $-4\tilde{A}\overline{3}$	(b) -4	(c) $\frac{\tilde{A}\overline{3}}{12}$	(d) $4\tilde{A}\overline{3}$	•
8)	The tangent to the	curve y^2 - $xy + 9 = 0$ is vertical when			
	(a) $y = 0$	(b) $y = \pm \tilde{A} \overline{3}$	(c) $y = \frac{1}{2}$	(d) $y = \pm \frac{1}{2}$	±3
9)	Angle between y ²	= x and x^2 = y at the origin is			
	(a) $tan^{-1}\frac{3}{4}$	(b) $tan^{-1}(\frac{4}{3})$		(c) $\frac{\pi}{2}$	(d) $\frac{\pi}{4}$
10)	The value of the li	$\min_{x \to 0} \lim_{c \to 0} (\cot c - \frac{1}{x})$			
	(a) 0	(b) 1	(c) 2	(d) ∞	
11)	The function sin ⁴	$x + \cos^4 X$ is increasing in the interval			
	(a) $\left[\frac{5\pi}{8}, \frac{3\pi}{4}\right]$	(b) $\left[\frac{\pi}{2}, \frac{5\pi}{8}\right]$	(c) $\left[\frac{\pi}{4}, \frac{\pi}{2}\right]$	(d)	$[0,\frac{\pi}{4}]$
12)	The number given	by the Rolle's theorem for the function	n x-3-3x2,x∈[0,3] is		
	(a) 1	(b) Ã 2	(c) $\frac{3}{2}$	(d)	2
13)	The number given	by the Mean value theorem for the fu	nction $\frac{1}{x}$, $x \in [1,9]$ is		
	(a) 2	(b) 2.5	(c) 3	(d) 3.5	

	(a) 0	(b) 3	(c) 6	(d) 9
15)	The maximum slope of the	tangent to the cu	rve $y = t:r \sin x, x \in [0, 2\pi]$ is at	
	(a) $x = \frac{\pi}{4}$	(b) $x = \frac{\pi}{2}$	(c) $x = \pi$	$(d) x = \frac{3\pi}{2}$
16)	The maximum value of the	function $x^2 e^{-2x}$,		
	(a) $\frac{1}{}$	(b) $\frac{1}{2e}$	(c) $\frac{1}{e^2}$	(d) $\frac{4}{e^4}$
	е	2e	e^2	e^4
17)	One of the closest points on	the curve $x^2 - y^2$	2 = 4 to the point $(6, 0)$ is	
	(a) $(2,0)$ (b) (A)	(5,1)	(c) $(3, \tilde{A} \overline{5})$	(d) $(\tilde{A} \overline{13}, -\tilde{A} \overline{3})$
18)	The maximum value of the	product of two p	ositive numbers, when their sum' o	f the squares is 200, is
	(a) 100	(b) $25\tilde{A}\overline{7}$	(c) 28	(d) $24\tilde{A}\overline{14}$
19)	The curve $y=ax^4 + bx^2$ with	ab > 0		
	(a) has, no horizontal tangent	t (b) is	s concave up (c) isconeavedov	vn (d) has no points of inflection
20)	The point of inflection of th	the curve $y = (x - 1)$	$(1)^3$ is	
	(a) (0,0)	(b) (0,1)	(c) (1,0)	(d) (1,1)
	***	*****	*******	*****

14) The minimum value of the function |3-x|+9 is

Application of Differential Calculus 2 MARKS SET 1

12th Standard

Maths

Reg.No. : Total Marks : 50

 $25 \times 2 = 50$

Date: 18-Oct-19

Exam Time : 01:00:00 Hrs

- 1) A point moves along a straight line in such a way that after t seconds its distance from the origin is $s = 2t^2 + 3t$ metres Find the average velocity of the points between t = 3 and t = 6 seconds.
- 2) A camera is accidentally knocked off an edge of a cliff 400 ft high. The camera falls a distance of $s = 16t^2$ in t seconds What is the instantaneous velocity of the camera when it hits the ground?
- 3) If the mass m(x) (in kilograms) of a thin rod of length x (in metres) is given by, $m(x) = \sqrt{3}x$ then what is the rate of change of mass with respect to the length when it is x = 3 and x = 27 metres.
- 4) A ladder 17 metre long is leaning against the wall. The base of the ladder is pulled away from the wall at a rate of 5 m/s. When the base of the ladder is 8 metres from the wall.
 - At what rate, the area of the triangle formed by the ladder, wall, and the floor, is changing?
- 5) Find the slope of the tangent to the curves at the respective given points.

$$y = x^4 + 2x^2 - x$$
 at $x = 1$

6) Find the slope of the tangent to the curves at the respective given points.

$$x = a \cos^3 t$$
, $y = b \sin^3 t$ at $t = \frac{\pi}{2}$

- 7) Find the points on the curve $y = x^3 x^2 + x + 3$ where the normal is parallel to the line x + y = 1729.
- 8) Find the tangent and normal to the following curves at the given points on the curve

$$x = \cos t$$
, $y = 2\sin t^2$ at $t = \frac{\pi}{3}$

- 9) Show that the two curves $x^2 y^2 = r^2$ and $xy = c^2$ where c, r are constants, cut orthogonally
- 10) Find the absolute extrema of the following functions on the given closed interval.

$$f(x) = 6x^{\frac{3}{4}} - 3x^{\frac{1}{3}}; [-1, 1]$$

11) Find the intervals of mono tonicities and hence find the local extremum for the following functions:

$$f(x)=2x^3+3x^2-12x$$

12) Find the intervals of mono tonicities and hence find the local extremum for the following functions:

$$f(x) = \frac{e^x}{1 - e^x}$$

13) Explain why Lagrange's mean value theorem is not applicable to the following functions in the respective intervals:

$$f(x) = \frac{x+1}{x}, x \in [-1, 2]$$

14) Find intervals of concavity and points of inflexion for the following functions

$$f(x) = x(x - 4)^3$$

15) Find intervals of concavity and points of inflexion for the following function:

$$f(x) = \frac{1}{2} \left(e^x - e^{-x} \right)$$

- 16) Find the smallest possible value x^2+y^2 given that x+y=10.
- 17) A farmer plans to fence a rectangular pasture adjacent to a river. The pasture must contain 1,80,000 sq. mtrs in order to provide enough grass for herds. No fencing is needed along the river. What is the length of the I minimum needed fencing material

Write the Maclaurin series expansion of the following function $\cos x$

- 19) Write the Maclaurin series expansion of the following function log(1 x); $-1 \le x < 1$
- 20) Write down the Taylor series expansion, of the function $\log x$ about x = 1 upto three nonzero terms for x > 0.
- Find the asymptotes of the following curves: $f(x) = \frac{x^2 + 6x 4}{3x 6}$
- 22) Evaluate the following limit, if necessary use l'Hôpital Rule $\lim_{x\to 0} \frac{1-\cos x}{x^2}$
- 23) Evaluate the following limit, if necessary use l'Hôpital Rule

$$\lim x \to 1^+ \left(\frac{2}{x^2 - 1} - \frac{x}{x - 1} \right)$$

- Sketch the graphs of the following function $y = \frac{x^3}{-\log x}$
- 25) If an initial amount A_0 of money is invested at an interest rate r compounded n times a year, the value of the investment after t years is $A = A_0 \left(1 + \frac{r}{n}\right)^{nt}$. If the interest is compounded continuously, (that is as $n \to \infty$), show that the amount after t years is $A = A_0^{\text{ert}}$.

Application of Differential Calculus 2 MARKS SET 2

12th Standard

Maths

Reg.No. : Total Marks : 50

 $25 \times 2 = 50$

Date: 18-Oct-19

1) A camera is accidentally knocked off an edge of a cliff 400 ft high. The camera falls a distance of $s = 16t^2$ in t seconds

- What is the average velocity with which the camera falls during the last 2 seconds?
 A particle moves along a line according to the law s(t) = 2t³ 9t² +12t 4, where t≥ 0.
 Find the total distance travelled by the particle in the first 4 seconds.
- 3) A stone is dropped into a pond causing ripples in the form of concentric circles. The radius r of the outer ripple is increasing at a constant rate at 2 cm per second. When the radius is 5 cm find the rate of changing of the total area of the disturbed water?
- 4) A conical water tank with vertex down of 12 metres height has a radius of 5 metres at the top. If water flows into the tank at a rate 10 cubic m/min, how fast is the depth of the water increases when the water is 8 metres deep?
- 5) Find the slope of the tangent to the curves at the respective given points. $y = x^4 + 2x^2 x$ at x = 1
- 6) Find the points on the curve $y^2 4xy = x^2 + 5$ for which the tangent is horizontal.
- 7) Find the equations of the tangents to the curve $y = \frac{x+1}{x-1}$ which are parallel to the line x + 2y = 6.
- 8) Find the angle between the rectangular hyperbola xy = 2 and the parabola $x^2y + 4 = 0$
- 9) Find the absolute extrema of the following functions on the given closed interval.

$$f(x) = 2\cos x + \sin 2x; \left[0, \frac{\pi}{2}\right]$$

Exam Time: 01:00:00 Hrs

10) Explain why Rolle's theorem is not applicable to the following functions in the respective intervals.

$$f(x) = x - 2logx, x \in [2, 7]$$

11) Using the Rolle's theorem, determine the values of x at which the tangent is parallel to the x -axis for the following functions:

$$f(x) = \sqrt{x} - \frac{x}{3}, x \in [0, 9]$$

12) Using the Lagrange's mean value theorem determine the values of x at which the tangent is parallel to the secant line at the end points of the given interval:

$$f(x) = x^3 - 3x + 2, x \in [-2, 2]$$

- 13) A race car driver is racing at 20th km. If his speed never exceeds 150 km/hr, what is the maximum distance he can cover in the next two hours.
- 14) Does there exist a differentiable function f(x) such that f(0) = -1, f(2) = 4 and $f'(x) \le 2$ for all x. Justify you answer.
- 15) Using mean value theorem prove that for, a > 0, b > 0, $le^{-a} e^{-b}l < la bl$.
- 16) Prove that among all the rectangles of the given perimeter, the square has the maximum area.
- 17) Write the Maclaurin series expansion of the following function sin x
- 18) Write the Maclaurin series expansion of the following function $tan^{-1}(x)$; $-1 \le x \le 1$
- 19) Write the Maclaurin series expansion of the following function: $\cos^2 x$

Find the asymptotes of the following curves $f(x) = \frac{3x}{\sqrt{x^2 + 2}}$

- 21) Expand the polynomial $f(x) = x^2 3x + 2$ in powers of x 1
- 22) Evaluate the following limit, if necessary use l'Hôpital Rule $\lim_{x \to \frac{\pi^{-}}{2} \frac{secx}{tanx}}$

Sketch the graphs of the following function
$$y = \frac{1}{1 + e^{-x}}$$

24) Find the local extrema for the following function using second derivative test:

$$f(x) = -3x^5 + 5x^3$$

Find the local extreme for the following function using second derivative to

25) Find the local extrema for the following function using second derivative test: $f(x) = x^2 e^{-2x}$

Application of Differential Calculus 3 MARK

12th Standard

Maths

Reg.No. : Total Marks : 60

 $20 \times 3 = 60$

Date: 18-Oct-19

Exam Time: 01:00:00 Hrs

1) For the function $f(x) = x^2 \in [0, 2]$ compute the average rate of changes in the subintervals [0,0.5], [0.5,1], [1,1.5], [1.5,2] and the instantaneous rate of changes at the points x = 0.5,1,1.5,2

2) A particle moves so that the distance moved is according to the law $s(t) = s(t) = \frac{t^3}{3} - t^2 + 3$. At what time the velocity and acceleration are zero respectively?

3) A particle moves along a horizontal line such that its position at any time $t \ge 0$ is given by $s(t) = t^3 - t^2 + t + 6.9.1$, where s is measured in metres and t in seconds?

- (1) At what time the particle is at rest?
- (2) At what time the particle changes direction?
- (3) Find the total distance travelled by the particle in the first 2 seconds.
- 4) Salt is poured from a conveyer belt at a rate of 30 cubic metre per minute forming a conical pile with a circular base whose height and diameter of base are always equal. How fast is the height of the pile increasing when the pile is 10 metre high?
- 5) Compute the value of 'c' satisfied by the Rolle's theorem for the function $f(x) = x^2 (1 x)^2$, $x \in [0.1]$
- 6) Find the values in the interval $(\frac{1}{2},2)$ satisfied by the Rolle's theorem for the function $f(x)=x+\frac{1}{x}, x\in [\frac{1}{2},2]$
- 7) Prove using the Rolle's theorem that between any two distinct real zeros of the polynomial $a_n x^n + a_{n-1} x^{n-1} + \ldots + a_1 x + a_0$ there is a zero of the polynomial $na_n x^{n-1} + (n-1)a_{n-1} x^{n-2} + \ldots + a_1$
- 8) Prove that there is a zero of the polynomial $2x^3 9x^2 11x + 12$ in the interval (2,7) given that 2 and 7 are the zeros of the polynomial $x^4 6x^3 11x^2 + 24x + 28$
- 9) Prove, using mean value theorem, that $|sin\alpha sin\beta| \leq |\alpha \beta|, \alpha, \beta \in R$
- 10) A thermometer was taken from a freezer and placed in a boiling water. It took 22 seconds for the thermometer to raise from -10°C to 100°C. Show that the rate of change of temperature at some time t is 5°C per second.
- 11) Compute the limit $\lim_{x\to 1} \left(\frac{x^2-3x+2}{x^2-4x+3}\right)$.
- 12) Evaluate the limit $\lim_{x\to 0} (\frac{sinmx}{x})$
- 13) $\lim_{ heta \to 0} (\frac{1-\cos m heta}{1-\cos n heta})$ =1, then prove that, $m=\pm n$
- 14) Evaluate: $\lim_{x\to 0^+} x \log x$.
- 15) Using the l'Hôpital Rule prove that, $\lim_{x \to 0^+} (1+x)^{\frac{1}{x}} = e$
- 16) Determine the intervals of concavity of the curve $f(x) = (x-1)^3$. (x-5), $x \in \mathbb{R}$ and, points of inflection if any.
- 17) Find the local extremum of the function $f(x) = x^4 + x 32x$
- 18) Find the asymptotes of the curve $f(x) = \frac{2x^2-8}{x^2-16}$
- 19) Sketch the curve $y = f(x) = x^2 x 6$.
- 20) Find the local maximum and minimum of the function $x^2 y^2$ on the line x + y = 10

Application of Differential Calculus 3 MARKS SET 2

12th Standard

Maths

Reg.No.: Total Marks: 60

 $20 \times 3 = 60$

Date: 18-Oct-19

Exam Time: 01:00:00 Hrs

1) A particle is fired straight up from the ground to reach a height of s feet in t seconds, where $s(t) = 128t - 16t^2$.

- (1) Compute the maximum height of the particle reached.
- (2) What is the velocity when the particle hits the ground?
- 2) If we blow air into a balloon of spherical shape at a rate of 1000³ cm per second. At what rate the radius of the baloon changes when the radius is 7cm? Also compute the rate at which the surface area changes.
- 3) A road running north to south crosses a road going east to west at the point P. Car A is driving north along the first road, and car B is driving east along the second road. At a particular time car A 10 kilometres to the north of P and traveling at 80 km/hr, while car B is 15 kilometres to the easst of P and traveling at 100 km/hr. How fast is the distance between the two cars changing?
- Find the values in the interval $(\frac{1}{2}, 2)$ satisfied by the Rolle's theorem for the function $f(x) = x + \frac{1}{x}, x \in [\frac{1}{2}, 2]$
- Compute the value of 'c' satisfied by Rolle's theorem for the function $f(x) = log(\frac{x^2+6}{5x})$ in the interval [2,3]
- 6) A truck travels on a toll road with a speed limit of 80 km/hr. The truck completes a 164 km journey in 2 hours. At the end of the toll road the trucker is issued with a speed violation ticket. Justify this using the Mean Value Theorem.
- 7) Suppose f(x) is a differentiable function for all x with $f'(x) \le 29$ and f(2) = 17. What is the maximum value of f(7)?
- Compute the limit $\lim_{x \to a} (\frac{x^n a^n}{x a})$
- 9) Evaluate the limit $\lim_{x \to 0^+} (\frac{\sin x}{2})$
- 10) Evaluate : $\lim_{x\to 1^-} \left(\frac{\log(1-x)}{\cot(\pi x)}\right)$.
- 11) Evaluate: $\lim_{x\to\infty} \left(\frac{x^2+17x+29}{x^4}\right)$.
- 12) Using the l'Hôpital Rule prove that, $\lim_{x\to 0^+} (1+x)^{\frac{1}{x}} = e^{-\frac{1}{x}}$
- 13) Evaluate: $log(limx \rightarrow \infty(1+2x)) \frac{1}{2log x}$
- 14) Evaluate: $\lim_{x\to 1} x^{\frac{1}{1-x}}$
- 15) Determine the intervals of concavity of the curve $y = 3 + \sin x$.
- 16) Find the local extremum of the function $f(x) = x^4 + x \cdot 32x$
- 17) Sketch the curve $y = f(x) = x^2 x 6$.
- 18) Find the local extrema of the function $f(x) = 4x^6 6x^4$
- 19) Sketch the curve $y = \frac{x^2 3x}{(x-1)}$
- 20) Sketch the graph of the function $y = \frac{3x}{x^2 + 1}$

Application of Differential Calculus 5 MARK SET 1

12th Standard

Maths

Reg.No. : Total Marks : 60

 $12 \times 5 = 60$

Date: 18-Oct-19

Exam Time: 01:30:00 Hrs

1) For what value of x the tangent of the curve $y = x^3 - x^2 + x - 2$ is parallel to the line y = x.

- 2) Find the equation of the tangent and normal to the Lissajous curve given by $x = 2\cos 3t$ and $y = 3\sin 2t$, $t \in \mathbb{R}$
- 3) Find the acute angle between $y = x^2$ and $y = (x 3)^2$.
- 4) If the curves $ax^2+by^2=1$ and $cx^2+dy^2=1$ intersect each other orthogonally then, $\frac{1}{a}-\frac{1}{b}=\frac{1}{c}-\frac{1}{d}$
- 5) Prove that the ellipse $x^2+4y^2=8$ and the hyperbola $x^2-2y^2=4$ intersect orthogonally.
- 6) Expand tan x in ascending powers of x upto 5th power for $\left(-\frac{\pi}{2} < x < \frac{\pi}{2}\right)$
- 7) Find the absolute maximum and absolute minimum values of the function $f(x) = 2x^3 + 3x^2 12x$ on [-3, 2]
- 8) Prove that the function $f(x) = x \sin x$ is increasing on the real line. Also discuss for the existence of local extrema.
- 9) Discuss the monotonicity and local extrema of the function $f(x) = log(1+x) \frac{x}{1+x}, x > -1$ and hence find the domain where, $log(1+x) > \frac{x}{1+x}$
- 10) Find the intervals of monotonicity and local extrema of the function $f(x) = \frac{1}{1+x^2}$
- 11) We have a 12 square unit piece of thin material and want to make an open box by cutting small squares from the corners of our material and folding the sides up. The question is, which cut produces the box of maximum volume?
- 12) A steel plant is capable of producing x tonnes per day of a low-grade steel and y tonnes per day of a high-grade steel, where $y = \frac{40 5x}{10 x}$ If the fixed market price of low-grade steel is half that of high-grade steel, then what should be optimal productions in low-grade steel and high-grade steel in order to have maximum receipts.

Application of Differential Calculus 5 MARK SET 2

12th Standard

	Maths	Reg.No.:				
Exam Time: 01:15:00 Hrs			,	Total N	larks	: 50
				10) x 5 =	= 50

Date: 18-Oct-19

1) Find the equations of tangent and normal to the curve $y = x^2 + 3x - 2$ at the point (1, 2)

- 2) Find the acute angle between the curves $y = x^2$ and $x = y^2$ at their points of intersection (0,0), (1,1).
- 3) Find the angle of intersection of the curve $y = \sin x$ with the positive x -axis.
- 4) Expand $\log(1+x)$ as a Maclaurin's series upto 4 non-zero terms for $-1 < x \le 1$.
- 5) Write the Taylor series expansion of $\frac{1}{x}$ about x = 2 by finding the first three non-zero terms.
- 6) Prove that the function $f(x) = x^2 2x 3$ is strictly increasing in $(2, \infty)$
- 7) Find the absolute extrema of the function $f(x) = 3\cos x$ on the closed interval $[0, 2\pi]$
- 8) Find the intervals of monotonicity and local extrema of the function $f(x) = x \log x + 3x$.
- 9) Find the intervals of monotonicity and local extrema of the function $f(x) = \frac{x}{1+x^2}$
- 10) Prove that among all the rectangles of the given area square has the least perimeter.

Application of Differential Calculus FULL TEST

12th Standard

Maths

Reg.No.:

Total Marks: 90

Date: 18-Oct-19

 $20 \times 1 = 20$

1) The volume of a sphere is increasing in volume at the rate of 3 π cm³ sec. The rate of change of its radius when radius is – cm

- (a) 3 cm/s (b) 2 cm/s (c) 1 cm/s (d) 1 cm/s
- 2) A balloon rises straight up at 10 m/s. An observer is 40 m away from the spot where the balloon left the ground. Find the rate of change of the balloon's angle of elevation in radian per second when the balloon is 30 metres above the ground.
- 3) The position of a particle moving along a horizontal line of any time t is given by set) = $3t^2$ -2t- 8. The time at which the particle is at rest is
- (a) t=0 (b) 1 (c) t=1 (d) t=3
- 4) A stone is thrown up vertically. The height it reaches at time t seconds is given by $x = 80t 16t^2$. The stone reaches the maximum height in time t seconds is given by
- (a) 2
 (b) 2.5
 (c) 3
 (d) 3.5
 5) Find the point on the curve 6y = .x³ + 2 at which y-coordinate changes 8 times as fast as x-coordinate is
- (a) (4,11) (b) (4,-11) (c) (-4,11) (d) (-4,-11)
- 6) The abscissa of the point on the curve $f(x) = \sqrt{8-2x}$ at which the slope of the tangent is -0.25?

 (a) -8

 (b) -4

 (c) -2

 (d) 0
- 7) The slope of the line normal to the curve $f(x) = 2\cos 4x$ at $x = \frac{\pi}{12}$

Exam Time: 02:30:00 Hrs

- (a) $-4\sqrt{3}$ (b) -4 (c) $\sqrt{3}$ (d) $4\sqrt{3}$
- 8) The tangent to the curve y^2 xy + 9 = 0 is vertical when
 (a) y = 0 (b) $y = \pm \sqrt{3}$ (c) 1
 (d) $y = \pm 3$
- 9) Angle between $y^2 = x$ and $x^2 = y$ at the origin is

 (a) $\frac{3}{\tan^{-1} -}$ (b) $\frac{\pi}{\tan^{-1} -}$ (c) π (d) π
- 10) The value of the limit $\lim_{x\to 0} \left(\cot x \frac{1}{x}\right)$ (a) 0 (b) 1 (c) 2 (d) ∞
- 11) The function $\sin^4 x + \cos^4 X$ is increasing in the interval

(a) $\left[\frac{5\pi}{8}, \frac{3\pi}{4}\right]$	(b) $\begin{bmatrix} \frac{\pi}{2}, \frac{5}{8} \\ \frac{1}{2}, \frac{7}{8} \end{bmatrix}$	$\begin{bmatrix} \pi \\ -1 \\ -1 \end{bmatrix} \qquad \begin{bmatrix} c \\ -1 \\ 4 \end{bmatrix} $	$\begin{bmatrix} d \\ 0, \frac{\pi}{4} \end{bmatrix}$	
12) The number giv	en by the Rolle's theorem for	the function x-3-3x2,x \in [0,3] is		
(a) 1	(b) $\sqrt{2}$	(c) 3	(d) 2	
		2		
13)		1		
	ren by the Mean value theorem	m for the function $-\frac{1}{x}$, $x \in [1,9]$ is		
(a) 2	(b) 2.5	(c) 3	(d) 3.5	
14) The minimum v	value of the function 3-x +9 is			
(a) 0	(b) 3	(c) 6	(d) 9	
15) The maximum s	slope of the tangent to the cur	we $y = t$: $r \sin x$, $x \in [0, 2\pi]$ is at		
(a) π	(b) π	(c) $x = \pi$	(d) 3π	
$x = \frac{-}{4}$	$(b) \qquad \frac{\pi}{x} = \frac{1}{2}$		$x = \frac{}{2}$	
			_	
	value of the function $x^2 e^{-2x}$,	(c) 1	(4)	
(a) 1 -	(b) 1	· /	(d) 4 —	
e	2e	$\frac{\overline{e^2}}{e^2}$	e^4	
17) One of the close	est points on the curve $x^2 - y^2$	= 4 to the point $(6, 0)$ is		
(a) (2,0)	(b) $(\sqrt{5}, 1)$	(c) $(3, \sqrt{5})$	(d) $(\sqrt{13}, -\sqrt{3})$	
18) The maximum v	` ′	ositive numbers, when their sum' of	()	
(a) 100	(b) $25\sqrt{7}$	(c) 28	(d) $24\sqrt{14}$	
19) The curve y= ax	•		•	
(a) has, no horiz		concave up (c) isconeavedow	n (d) has no points of inflection	
	lection of the curve $y = (x - 1)$	-		
(a) (0,0)	(b) (0,1)	(c) (1,0)	(d) (1,1)	
ANY 7	(, (, ,		7 x 2	= 14
21) A point moves a			e from the origin is $s = 2t^2 + 3t$ metres	
	aneous velocities at $t = 3$ and			
•	c c	e law $s(t) = 2t^3 - 9t^2 + 12t - 4$, whe	$\text{ are } t \geq 0.$	
	he particle changes direction?			
23) If the volume of	f a cube of side length x is v =	= x^3 . Find the rate of change of the	volume with respect to x when $x = 5$ units.	
24) A ladder 17 met	tre long is leaning against the	wall. The base of the ladder is pull	ed away from the wall at a rate of 5 m/s. When	nen
the base of the l	adder is 8 metres from the wa	all.		
How fast is the	top of the ladder moving dow	n the wall?		
25) Find the tangent	t and normal to the following	curves at the given points on the c	urve	
$y = x^2 - x^4$ at (1)	, 0)			

27) Using the Rolle's theorem, determine the values of x at which the tangent is parallel to the x -axis for the following functions: $x^2 - 2x$

26) Find the absolute extrema of the following functions on the given closed interval.

$$f(x) = \frac{x^2 - 2x}{x + 2}, x \in [-1, 6]$$

 $f(x) = 2\cos x + \sin 2x; \begin{bmatrix} 0, -\frac{\pi}{2} \end{bmatrix}$

28) Using the Lagrange's mean value theorem determine the values of x at which the tangent is parallel to the secant line at the end points of the given interval:

$$f(x) = x^3 - 3x + 2, x \in [-2, 2]$$

- 29) Find two positive numbers whose product is 20 and their sum is minimum.
- 30) Write the Maclaurin series expansion of the following function

$$log(1 - x); -1 \le x < 1$$

ANY 7 $7 \times 3 = 21$

- 31) A particle is fired straight up from the ground to reach a height of s feet in t seconds, where $s(t) = 128t 16t^2$.
 - (1) Compute the maximum height of the particle reached.
 - (2) What is the velocity when the particle hits the ground?
- 32) Salt is poured from a conveyer belt at a rate of 30 cubic metre per minute forming a conical pile with a circular base whose height and diameter of base are always equal. How fast is the height of the pile increasing when the pile is 10 metre high?
- 33) Prove that there is a zero of the polynomial $2x^3 9x^2 11x + 12$ in the interval (2,7) given that 2 and 7 are the zeros of the polynomial $x^4 6x^3 11x^2 + 24x + 28$
- Compute the limit $\lim_{x\to 1} \left(\frac{x^2-3x+2}{x^2-4x+3}\right)$.
- 35) Evaluate: $\lim_{x\to 0^+} (\frac{1}{x} \frac{1}{e^x 1})$.
- 36) Using the l'Hôpital Rule prove that, $\lim_{x\to 0^+} (1+x)^{\frac{1}{x}} = e^{-\frac{1}{x}}$
- 37) Find the local extremum of the function $f(x) = x^4 + x 32x$
- 38) Find the asymptotes of the curve $f(x) = \frac{2x^2 8}{x^2 16}$
- 39) Find the local maximum and minimum of the function $x^2 y^2$ on the line x + y = 10
- 40) Sketch the curve $y = \frac{x^2 3x}{(x-1)}$

ANY 7 $7 \times 5 = 35$

- 41) For what value of x the tangent of the curve $y = x^3 x^2 + x 2$ is parallel to the line y = x.
- 42) Find the acute angle between $y = x^2$ and $y = (x 3)^2$.
- 43) If the curves $ax^2+by^2=1$ and $cx^2+dy^2=1$ intersect each other orthogonally then, $\frac{1}{a}-\frac{1}{b}=\frac{1}{c}-\frac{1}{d}$
- 44) Expand tan x in ascending powers of x upto 5th power for $\left(-\frac{\pi}{2} < x < \frac{\pi}{2}\right)$
- 45) Find the absolute maximum and absolute minimum values of the function $f(x) = 2x^3 + 3x^2 12x$ on [-3, 2]
- 46) Discuss the monotonicity and local extrema of the function $f(x) = log(1+x) \frac{x}{1+x}$, x > -1 and hence find the domain where, $log(1+x) > \frac{x}{1+x}$
- 47) Find the intervals of monotonicity and local extrema of the function $f(x) = x \log x + 3x$.
- 48) Find the intervals of monotonicity and local extrema of the function $f(x) = \frac{x}{1+x^2}$
- 49) We have a 12 square unit piece of thin material and want to make an open box by cutting small squares from the corners of our material and folding the sides up. The question is, which cut produces the box of maximum volume?
- 50) Prove that among all the rectangles of the given area square has the least perimeter.

RAVI MATHS TUITION CENTER, NEAR VILLIVAKKAM RLY STATION, CHENNAI – 82. WHATSAPP - 8056206308

Application of Differential Calculus

12th Standard

Total Marks: 100

 $20 \times 1 = 20$

Maths

Exam Time: 02:30:00 Hrs

	· .		e of 3 π cm ³ sec. The rate of
change of its radi	us when radius is $\dfrac{1}{2}$ o	cm	
(a) 3 cm/s	(b) 2 cm/s	(c) 1 cm/s	(d) $rac{1}{2}cm/s$
	particle moving along e at which the particle		f any time t is given by set) =
(a) t= 0	(b) $t=rac{1}{3}$	(c) t =1	(d) t = 3
3) Find the point on as x-coordinate is		at which y-coordir	nate changes 8 times as fast
4\	(b) (4,-11)		(d) (-4,-11)
The slope of the li	ine normal to the curv	ve f(x) = 2cos 4x at	$x=rac{\pi}{12}$
(a) $-4\sqrt{3}$	(b) -4	(c) $\frac{\sqrt{3}}{12}$	(d) $4\sqrt{3}$
	= x and. x^2 = y at the	`	
(a) $tan^{-1}rac{3}{4}$	(b) $tan^{-1}\left(rac{1}{2} ight)$	$\left(\frac{4}{3}\right)$	(c) $\frac{\pi}{2}$ (d) $\frac{\pi}{4}$
	x + cos ⁴ X is increasir		7
_, L J	(b) $\left\lfloor \frac{\pi}{2}, \frac{5\pi}{8} \right\rfloor$	(c) $\left[\frac{\pi}{4}, \frac{\pi}{2}\right]$	$\left[\frac{\pi}{2}\right]$ (d) $\left[0, \frac{\pi}{4}\right]$
7) The number giver	n by the Mean value t	heorem for the fun	ction $\frac{1}{x}$,x∈[1,9] is
(a) 2 8) The maximum slo	(b) 2.5 pe of the tangent to t	(c) 3 he curve y = t:r sin	(d) 3.5 x, x ∈ [0, 2π] is at
(a) $x=rac{\pi}{4}$	(b) $x=rac{\pi}{2}$	(c) $x=\pi$	(d) $x=rac{3\pi}{2}$
(a) (2,0) (b)	t points on the curve $(\sqrt{5},1)$ (c)	$(3, \sqrt{5})$ (pint (6, 0) is d) $\left(\sqrt{13},-\sqrt{3} ight)$
10) The curve y= ax ² (a) has, no horizo tangent		ave (c) isconeavedow	(d) has no points of vn inflection

-			e time interval during
which the velocity is negative (a) 2 < t < 5 (b)			(d) t≤2
12) If the rate of increase of s =			
value of x is			,
(a) $\frac{3}{5}$ (b) $\frac{10}{3}$	(c) $\frac{3}{10}$		(d) $\frac{1}{3}$
13) The equation of the tangent	to the curve $y=x^2-4x^2$	+2 at (4,2) is	Ü
(a) $x + 4y + 12 = 0$ (b) $4x$	+ y + 12 = 0 (c) 42	x - y - 14 = 0	(d) $x + 4y - 12 = 0$
14) The least value of a when f			
	` ,	(d	
15) The angle made by any tang (a) obtuse (b) right an	igle (c) acute	e angle	(d) no angle
16) If the curves y = 2e ^x and y =	_	-	
\cdot	(c) 2	` '	
17) The function/(x) = $x^9 + 3x^7 + (a) R$ (b) $(-\infty, 0)$	- 64 is increasing on ₋	 	
		(d) None of the	ese
18) The curve y = e ^x is		lo (d) con	acovo upwordo
(a) convex (b) concave 19) $\lim_{x\to 0}\frac{x}{\tan x}$ is (a) 1 (b) -1	• •	• •	icave upwards
$(a)^{1}$ (b) -1	(c) 0	(d) ∞
20) The statement " If f has a loo			$\sin f(a) = 0$ " is
•	oai chii ciii aiii at o aiii	d II I (C) exists tr	
		. ,	, ,
(a) the extreme value	(b) Fermats'	(c) Law of	(d) Rolle's
		. ,	(d) Rolle's theorem
(a) the extreme value theorem	(b) Fermats' theorem	(c) Law of mean	(d) Rolle's theorem 8 x 2 = 16
(a) the extreme value	(b) Fermats' theorem or an English test. Th	(c) Law of mean	(d) Rolle's theorem 8 x 2 = 16 ords the person
(a) the extreme value theorem 21) A person learnt 100 words for	(b) Fermats' theorem or an English test. Th arning is given by W(t	(c) Law of mean e number of wo 1 = 100×(1- 0.1t	(d) Rolle's theorem $8 \times 2 = 16$ ords the person $truly 2^2$, $0 \le t \le 10$. What is
(a) the extreme value theorem 21) A person learnt 100 words for remembers in t days after lead the rate at which the person for the volume of a cube of significant significant strength.	(b) Fermats' theorem or an English test. Therning is given by W(theorem) forgets the words 2 deceived to the deceived the words 2 deceived to the word	(c) Law of mean e number of wo 1 = 100×(1- 0.1t ays after learnin	(d) Rolle's theorem 8 x 2 = 16 ords the person 0 ² , 0 ≤ t ≤ 10. What is ag?
(a) the extreme value theorem 21) A person learnt 100 words for remembers in t days after lead the rate at which the person for the volume of a cube of side volume with respect to x when	(b) Fermats' theorem or an English test. Therning is given by W(theorem) forgets the words 2 days are to a constant to the series of the constant to the const	(c) Law of mean ne number of wo) =100×(1- 0.1t ays after learnin Find the rate of	(d) Rolle's theorem 8 x 2 = 16 ords the person 0 ² , 0 ≤ t ≤ 10. What is 10 ords the change of the
(a) the extreme value theorem 21) A person learnt 100 words for remembers in t days after lead the rate at which the person for the volume of a cube of side volume with respect to x when 23) Find the slope of the tangen	(b) Fermats' theorem or an English test. The arning is given by W(theorem) forgets the words 2 de de length x is v = x ³ . In x = 5 units. In to the curves at the	(c) Law of mean ne number of wo) =100×(1- 0.1t ays after learnin Find the rate of	(d) Rolle's theorem 8 x 2 = 16 ords the person 0 ² , 0 ≤ t ≤ 10. What is 10 ords the change of the
(a) the extreme value theorem 21) A person learnt 100 words for remembers in t days after lead the rate at which the person for the volume of a cube of six volume with respect to x when the slope of the tangent at a cos ³ t, y = b sin ³ t at t = 24) Suppose f(x) is a differential	(b) Fermats' theorem for an English test. The arning is given by W(the forgets the words 2 does length x is $v = x^3$. For $x = 5$ units. In the curves at the $\frac{\pi}{2}$	(c) Law of mean e number of wo) =100×(1− 0.1t ays after learnin Find the rate of respective give	(d) Rolle's theorem 8 x 2 = 16 ords the person)², 0 ≤ t ≤ 10. What is ag? change of the n points.
(a) the extreme value theorem 21) A person learnt 100 words for remembers in t days after lear the rate at which the person for 22) If the volume of a cube of six volume with respect to x when 23) Find the slope of the tangent x = a cos³ t, y = b sin³ t at t = 24) Suppose f(x) is a differential the maximum value of f (7)?	(b) Fermats' theorem for an English test. The arning is given by W(the forgets the words 2 days are $x = 5$ units. In the curves at the set $\frac{\pi}{2}$ ble function for all x when $x = 5$ units.	(c) Law of mean e number of wo (1 – 0.1t ays after learning find the rate of respective give with f'(x) ≤ 29 and	(d) Rolle's theorem 8 x 2 = 16 ords the person)², 0 ≤ t ≤ 10. What is ag? change of the n points. d f(2) =17. What is
(a) the extreme value theorem 21) A person learnt 100 words for remembers in t days after lead the rate at which the person of the volume with respect to x when 22) If the volume of a cube of six volume with respect to x when 23) Find the slope of the tangent x = a cos³ t, y = b sin³ t at t = 24) Suppose f(x) is a differential the maximum value of f (7)? 25) Explain why Rolle's theorem respective intervals.	(b) Fermats' theorem for an English test. The arning is given by W(the forgets the words 2 days are $x = 5$ units. In the curves at the set $\frac{\pi}{2}$ ble function for all x when $x = 5$ units.	(c) Law of mean e number of wo (1 – 0.1t ays after learning find the rate of respective give with f'(x) ≤ 29 and	(d) Rolle's theorem 8 x 2 = 16 ords the person)², 0 ≤ t ≤ 10. What is ag? change of the n points. d f(2) =17. What is
 (a) the extreme value theorem 21) A person learnt 100 words for remembers in t days after lead the rate at which the person of the volume with respect to x where 22) If the volume of a cube of single volume with respect to x where 23) Find the slope of the tangent x = a cos³ t, y = b sin³ t at t = 24) Suppose f(x) is a differential the maximum value of f (7)? 25) Explain why Rolle's theorem respective intervals. f(x) = tanx, x ∈ [0, π] 	(b) Fermats' theorem for an English test. The arning is given by W(the forgets the words 2 does not be a set to the curves at the set $\frac{\pi}{2}$ ble function for all x we have not applicable to the theorem is not applicable to the set $\frac{\pi}{2}$.	(c) Law of mean e number of wo (1 – 0.1t ays after learning find the rate of respective give with f'(x) ≤ 29 and	(d) Rolle's theorem 8 x 2 = 16 ords the person)², 0 ≤ t ≤ 10. What is ag? change of the n points. d f(2) =17. What is
(a) the extreme value theorem 21) A person learnt 100 words for remembers in t days after lead the rate at which the person of the volume with respect to x when 22) If the volume of a cube of six volume with respect to x when 23) Find the slope of the tangent x = a cos³ t, y = b sin³ t at t = 24) Suppose f(x) is a differential the maximum value of f (7)? 25) Explain why Rolle's theorem respective intervals.	(b) Fermats' theorem for an English test. The arning is given by W(the forgets the words 2 does not be a set to the curves at the set $\frac{\pi}{2}$ ble function for all x we have not applicable to the theorem is not applicable to the set $\frac{\pi}{2}$.	(c) Law of mean e number of wo (1 – 0.1t ays after learning find the rate of respective give with f'(x) ≤ 29 and	(d) Rolle's theorem 8 x 2 = 16 ords the person)², 0 ≤ t ≤ 10. What is ag? change of the n points. d f(2) =17. What is
(a) the extreme value theorem 21) A person learnt 100 words for remembers in t days after lear the rate at which the person of the volume of a cube of six volume with respect to x where 23) Find the slope of the tangent $x = a \cos^3 t$, $y = b \sin^3 t$ at $t = 24$) Suppose $f(x)$ is a differential the maximum value of $f(x)$? 25) Explain why Rolle's theorem respective intervals. $f(x) = tanx, x \in [0, \pi]$ 26) Compute the limit $\lim_{x \to 1} (\frac{x^2}{x^2})$	(b) Fermats' theorem for an English test. The arning is given by W(theorem) for gets the words 2 does be a considered as $x = 5$ units. In the curves at the first tendent in the formula $\frac{\pi}{2}$ ble function for all $x = 5$ when it is not applicable to the curves at the first formula $x = \frac{\pi}{2}$ ble function for all $x = \frac{\pi}$	(c) Law of mean e number of wo (1 - 0.1t) ays after learning the rate of respective give with f'(x) ≤ 29 and the following funds	(d) Rolle's theorem 8 x 2 = 16 ords the person)², 0 ≤ t ≤ 10. What is ag? change of the n points. d f(2) =17. What is
(a) the extreme value theorem 21) A person learnt 100 words for remembers in t days after least the rate at which the person for the volume of a cube of six volume with respect to x when 23) Find the slope of the tangent $x = a \cos^3 t$, $y = b \sin^3 t$ at $t = 24$) Suppose $f(x)$ is a differential the maximum value of $f(x)$? 25) Explain why Rolle's theorem respective intervals. $f(x) = tanx, x \in [0, \pi]$ 26) Compute the limit $\lim_{x \to 1} (\frac{x^2}{x^2})$	(b) Fermats' theorem for an English test. The arning is given by W(theorem) for gets the words 2 does be a considered as $x = 5$ units. In the curves at the first tendent in the formula $\frac{\pi}{2}$ ble function for all $x = 5$ when it is not applicable to the curves at the first formula $x = \frac{\pi}{2}$ ble function for all $x = \frac{\pi}$	(c) Law of mean e number of wo (1 - 0.1t) ays after learning the rate of respective give with f'(x) ≤ 29 and the following funds	(d) Rolle's theorem 8 x 2 = 16 ords the person)², 0 ≤ t ≤ 10. What is ag? change of the n points. d f(2) =17. What is
(a) the extreme value theorem 21) A person learnt 100 words for remembers in t days after lear the rate at which the person of the volume of a cube of six volume with respect to x where 23) Find the slope of the tangent $x = a \cos^3 t$, $y = b \sin^3 t$ at $t = 24$) Suppose $f(x)$ is a differential the maximum value of $f(x)$? 25) Explain why Rolle's theorem respective intervals. $f(x) = tanx, x \in [0, \pi]$ 26) Compute the limit $\lim_{x \to 1} (\frac{x^2}{x^2})$	(b) Fermats' theorem for an English test. The arning is given by W(theorem for gets the words 2 does be a considered as $x = 5$ units. In the form of the curves at the first set of the curves at the first set of the function for all $x = \frac{\pi}{2}$ ble function	(c) Law of mean e number of wo) =100×(1− 0.1t ays after learning find the rate of respective give with f'(x) ≤ 29 and the following functions for the following functin	(d) Rolle's theorem 8 x 2 = 16 ords the person 2, 0 ≤ t ≤ 10. What is ag? change of the n points. d f(2) =17. What is actions in the

29) Show that the two curves x² − y² = r² and xy = c² where c, r are constants, cut orthogonally

- 30) Using the Lagrange's mean value theorem determine the values of x at which the tangent is parallel to the secant line at the end points of the given interval: $f(x) = (x 2)(x 7), x \in [3,11]$
- 31) A race car driver is racing at 20th km. If his speed never exceeds 150 km/hr, what is the maximum distance he can cover in the next two hours.
- 32) Does there exist a differentiable function f(x) such that f(0) = -1, f(2) = 4 and $f'(x) \le 2$ for all x. Justify you answer.
- 33) Expand sin x in ascending powers x $\frac{\pi}{4}$ upto three non-zero terms.
- 34) Evaluate the following limit, if necessary use l'Hôpital Rule

$$\lim_{x o 1^+}\left(rac{2}{x^2-1}-rac{x}{x-1}
ight)$$

- 35) Find the absolute maximum and absolute minimum values of the function $f(x) = 2x^3 + 3x^2 12x$ on [-3, 2]
- 36) Find the local extremum of the function $f(x) = x^4 + x \cdot 32x$

 $8 \times 5 = 40$

- 37) A particle moves along a horizontal line such that its position at any time $t \ge 0$ is given by $s(t) = t^3 t^2 + t + 6.9.1$, where s is measured in metres and t in seconds?
 - (1) At what time the particle is at rest?
 - (2) At what time the particle changes direction?
 - (3) Find the total distance travelled by the particle in the first 2 seconds.
- 38) Salt is poured from a conveyer belt at a rate of 30 cubic metre per minute forming a conical pile with a circular base whose height and diameter of base are always equal. How fast is the height of the pile increasing when the pile is 10 metre high?
- 39) A conical water tank with vertex down of 12 metres height has a radius of 5 metres at the top. If water flows into the tank at a rate 10 cubic m/min, how fast is the depth of the water increases when the water is 8 metres deep?
- 40) Find the tangent and normal to the following curves at the given points on the curve $x = \cos t$, $y = 2\sin t^2$ at $t = \frac{\pi}{3}$
- 41) Find the smallest possible value x^2+y^2 given that x + y = 10.
- 42) A rectangular page is to contain 24 cm² of print. The margins at the top and bettemof the page are 1.5 cm and the margins at other sides of the page is 1 cm. What should be the dimensions of the page so that the area of the paper used is minimum.
- 43) Expand log(1+x) as a Maclaurin's series upto 4 non-zero terms for $-1 < x \le 1$.
- 44) Find the local maximum and local minimum values for $f(x)=12x^2-2x^2-x^4$.

RAVI MATHS TUITION CENTER, GKM COLONY, CHENNAI- 82. PH: 8056206308 **Application of Differential Calculus FULL TEST**

12th Standard Reg.No.: Maths 90 20

			Maths		
Ex	tam Time: 02:30:00 H	rs			Total Marks : 9 20 x 1 = 2
1)	The volume of a sphe	re is increasing in volume	at the rate of 3 π cr	m ³ sec. The rate of	
	when radius is $\frac{1}{2}$ cm	1			
	(a) 3 cm/s		(c) 1 cm/s	(d) $\frac{1}{2} cm/c$	s
2)	•	nt up at 10 m/s. An observ of change of the balloon's e ground.	•	•	
	(a) $\frac{3}{25} radians/sec$	(b) $\frac{4}{25} radians/se$	ec (c) $\frac{1}{5}radi$	ans/sec (d)	$\frac{1}{3} radians/sec$
3)	time at which the part				
	(a) $t=0$	$^{(b)}\ t=\frac{1}{3}$	(c) $t=1$	(d)	t = 3
4)	•	vertically. The height it reacheight in time t seconds in		nds is given by x =	$= 80t - 16t^2.$ The stone
	(a) 2	(b) 2.5	(c) 3	* *	
5)	Find the point on the	curve $6y = .x^3 + 2$ at whic			t as x-coordinate is
	(a) (4,11)	(b) (4,-11)	(c) (-4,11)	(d) (-4	
6)	The abscissa of the po	point on the curve $f(x) = $ (b) -4	$\sqrt{8-2x}$ at wl	nich the slope of th	te tangent is -0.25?
	(a) -8	(b) -4	(c) -2	(d) 0
7)	The slope of the line i	normal to the curve $f(x) =$			
	(a) $-4\sqrt{3}$	(b) -4	(c) $\frac{\sqrt{3}}{12}$	(d) 4 ₁	$\sqrt{3}$
8)	The tangent to the cur	$ve y^2 - xy + 9 = 0 is vertice$	cal when		
		$y=\pm\sqrt{3}$	(c) $y = \frac{1}{2}$	(d) $y =$	± 3
9)	Angle between $y^2 = x$	and $x^2 = y$ at the origin is			
	(a) $tan^{-1}\frac{3}{4}$	(b) $tan^{-1}\left(rac{4}{3} ight)$		(c) $\frac{\pi}{2}$	(d) $\frac{\pi}{4}$
10) The value of the limit	t $\lim_{x o 0}\left(cotx-rac{1}{x} ight)$			
	(a) 0	(b) 1	(c) 2	(d) ∞	

11) The function $\sin^4 x + \cos^4 X$ is increasing in the interval

The number gi	ven by the Mean value theore	em for the function $\frac{1}{x}$, x	€[1,9] is	
(a) 2	(b) 2.5	(c) 3	(d) 3.5	
` '	value of the function 3-x +9 i	S	` '	
(a) 0	(b) 3	(c) 6	(d) 9	
15) The maximum	slope of the tangent to the cu	rve $y = t$: $r \sin x$, $x \in [0, 1]$	2π] is at	
$^{(a)}\ x=\frac{\pi}{4}$		(c) $x=\pi$	$^{\rm (d)} \ \ x = \frac{3\pi}{2}$	
16) The maximum	value of the function $x^2 e^{-2x}$,			
(a) 1	(b) $\frac{1}{2e}$	(c) $\frac{1}{e^2}$	(d) $\frac{4}{e^4}$	
\overline{e}	$\overline{2e}$	$\overline{e^2}$	$\overline{e^4}$	
17) One of the clos	sest points on the curve x^2 - y^2	2 .= 4 to the point (6, 0) is	S	
(a) (2,0)	(b) $(\sqrt{5}, 1)$ (c)	$\left(3,\sqrt{5}\right) \tag{6}$	d) $\left(\sqrt{13}, -\sqrt{3}\right)$	
18) The maximum (a) 100	value of the product of two p (b) $25\sqrt{7}$	oositive numbers, when t (c) 28	heir sum' of the squares is (d) $24\sqrt{14}$	200, is
19) The curve $y=a$	$ax^4 + bx^2$ with $ab > 0$			
(a) has, no hor	izontal tangent (b) is conca	ave up (c) isconeavedo	wn (d) has no points of	inflection
20) The point of in	iflection of the curve $y = (x - $	$(1)^3$ is		
(a) (0,0)	(b) (0,1)	(c) (1,0)	(d) (1,1)	
ANY 7				$7 \times 2 = 14$
	along a straight line in such a	way that after t seconds	s its distance from the orig	$\sin is s = 2t^2$
+ 3t metres				
	taneous velocities at $t = 3$ and			
=	es along a line according to the particle changes direction?		$2t - 4$, where $t \ge 0$.	
23) If the volume of	of a cube of side length x is v	$= x^3$. Find the rate of cha	ange of the volume with re	espect to x
when $x = 5$ unit				•
24) A ladder 17 me	etre long is leaning against the	e wall. The base of the la	dder is pulled away from	the wall at
a rate of 5 m/s.	When the base of the ladder i	s 8 metres from the wall		
How fast is the	top of the ladder moving dov	vn the wall?		
25) Find the tanger	nt and normal to the following	g curves at the given point	nts on the curve	
$y = x^2 - x^4$ at (1)	1,0)			
26) Find the absolu	ate extrema of the following f	unctions on the given cle	osed interval.	
f(x)=2cosx	$+ \ sin2x; \left[0,rac{\pi}{2} ight]$			
27)	LJ			
,				

(c)

12) The $\left[\frac{\sqrt{2}\pi}{8}, \frac{\sqrt{2}\pi}{4}\right]$ ven by the Rolle's theorem or the function x-3- $\left[\frac{\sqrt{2}}{4}, \frac{\sqrt{2}\pi}{2}\right]$ (d) $\left[\frac{1}{2}, \frac{\pi}{4}\right]$

(d)

(a)

(b)

Using the Rolle's theorem, determine the values of x at which the tangent is parallel to the x -axis for the following functions:

$$f(x) = rac{z^2 - 2x}{x + 2}, x \in [-1, 6]$$

28) Using the Lagrange's mean value theorem determine the values of x at which the tangent is parallel to the secant line at the end points of the given interval:

$$f(x) = x^3 - 3x + 2, x \in [-2, 2]$$

- 29) Find two positive numbers whose product is 20 and their sum is minimum.
- 30) Write the Maclaurin series expansion of the following function log(1 x); $-1 \le x < 1$

ANY 7 $7 \times 3 = 21$

- 31) A particle is fired straight up from the ground to reach a height of s feet in t seconds, where $s(t) = 128t 16t^2$.
 - (1) Compute the maximum height of the particle reached.
 - (2) What is the velocity when the particle hits the ground?
- 32) Salt is poured from a conveyer belt at a rate of 30 cubic metre per minute forming a conical pile with a circular base whose height and diameter of base are always equal. How fast is the height of the pile increasing when the pile is 10 metre high?
- 33) Prove that there is a zero of the polynomial $2x^3 9x^2 11x + 12$ in the interval (2,7) given that 2 and 7 are the zeros of the polynomial $x^4 6x^3 11x^2 + 24x + 28$
- 34) Compute the limit $\lim_{x\to 1} \left(\frac{x^2-3x+2}{x^2-4x+3}\right)$
- 35) Evaluate: $\lim_{x\to 0^+} \left(\frac{1}{x} \frac{1}{e^x-1}\right)$
- 36) Using the l'Hôpital Rule prove that, $\lim_{x \to 0^+} (1+x)^{\frac{1}{x}} = e$
- 37) Find the local extremum of the function $f(x) = x^4 + x \cdot 32x$
- 38) Find the asymptotes of the curve $f(x) = \frac{2x^2-8}{x^2-16}$
- 39) Find the local maximum and minimum of the function $x^2 y^2$ on the line x + y = 10
- 40) Sketch the curve $y = \frac{x^2 3x}{(x-1)}$

ANY 7 $7 \times 5 = 35$

- 41) For what value of x the tangent of the curve $y = x^3 x^2 + x 2$ is parallel to the line y = x.
- 42) Find the acute angle between $y = x^2$ and $y = (x 3)^2$.
- 43) If the curves $ax^2+by^2=1$ and $cx^2+dy^2=1$ intersect each other orthogonally then, $\frac{1}{a}-\frac{1}{b}=\frac{1}{c}-\frac{1}{d}$
- 44) Expand tan x in ascending powers of x upto 5th power for $\left(-\frac{\pi}{2} < x < \frac{\pi}{2}\right)$
- 45) Find the absolute maximum and absolute minimum values of the function $f(x) = 2x^3 + 3x^2 12x$ on [-3, 2]
- 46) Discuss the monotonicity and local extrema of the function $f(x) = log(1+x) \frac{x}{1+x}, x > -1$ and hence find the domain where, $log(1+x) > \frac{x}{1+x}$
- 47) Find the intervals of monotonicity and local extrema of the function $f(x) = x \log x + 3x$.
- 48) Find the intervals of monotonicity and local extrema of the function $f(x) = \frac{x}{1+x^2}$
- 49)

We have a 12 square unit piece of thin material and want to make an open box by cutting small squares from the corners of our material and folding the sides up. The question is, which cut produces the box of maximum volume?

50) Prove that among all the rectangles of the given area square has the least perimeter.