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12TH MATHS MODEL PAPER 8 Date : 16-Nov-19

12th Standard

Maths

Reg.No. : | | | | | | |

Exam Time : 03:00:00 Hrs Total Marks : 90

PART – I 20 x 1 = 20

ANSWER ALL THE QUESTIONS

1) If A = $\begin{bmatrix} 1 & \tan\frac{\theta}{2} \\ -\tan\frac{\theta}{2} & 1 \end{bmatrix}$ and AB = I , then B =

(a) $\left(\cos^2\frac{\theta}{2}\right)A$ (b) $\left(\cos^2\frac{\theta}{2}\right)A^T$ (c) $\left(\cos^2\theta\right)I$ (d) $(\text{Sin}^2\frac{\theta}{2})$A

2) If A is a non-singular matrix then IA^{-1}|= ______

(a) $\left|\frac{1}{A^2}\right|$ (b) $\frac{1}{|A^2|}$ (c) $\left|\frac{1}{A}\right|$ (d) $\frac{1}{|A|}$

3) The principal argument of $(\sin 40°+i\cos 40°)^5$ is

(a) −110° (b) −70° (c) 70° (d) 110°

4) If $(2+\sqrt{3})^{x^2-2x+1}+(2-\sqrt{3})^{x^2-2x-1}=\frac{2}{2-\sqrt{3}}$ then x=

(a) 0,2 (b) 0,1 (c) 0,3 (d) 0, √3

5) If |x| ≤ 1, then $2\tan^{-1}x-\sin^{-1}\frac{2x}{1+x^2}$ is equal to

(a) $\tan^{-1}x$ (b) $\sin^{-1}x$ (c) 0 (d) π

6) If $tan^{-1}\left(\frac{x+1}{x-1}\right)+tan^{-1}\left(\frac{x-1}{x}\right)=tan^{-1}(-7)$ then x Is

(a) 0 (b) -2 (c) 1 (d) 2

7) The locus of a point whose distance from (-2,0) is $\frac{2}{3}$ times its distance from the line x = $\frac{-9}{2}$ is

(a) a parabola (b) a hyperbola (c) an ellipse (d) a circle

8) If t_1 and t_2 are the extremities of any focal chord of $y^2 = 4ax$ then t_1t_2 is ______________

(a) -1 (b) 0 (c) ±1 (d) $\frac{1}{2}$

9) The coordinates of the point where the line $\vec{r} = (6\hat{i}-\hat{j}-3\hat{k})+t(\hat{i}+4\hat{j})$ meets the plane $\vec{r} = (\hat{i}+\hat{j}-\hat{k})$ = 3 are

(a) (2,1,0) (b) (7,1,7) (c) (1,2,6) (d) (5,1,1)

10) If $\left|\vec{a}\times\vec{b}\right| = \vec{a}.\vec{b}$, then the angle between the vector $\vec{a}$ and $\vec{b}$ is _____________

(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{3}$ (c) $\frac{\pi}{6}$ (d) $\frac{\pi}{2}$

11) One of the closest points on the curve $x^2 - y^2$.= 4 to the point (6, 0) is

(a) (2,0) (b) $(\sqrt{5}, 1)$ (c) $(3, \sqrt{5})$ (d) $(\sqrt{13}, -\sqrt{3})$

12) If u = $(x-y)^4+(y-z)^4+(z-x)^4$ then $\sum\frac{\partial u}{\partial x}$ =

(a) 4 (b) 1 (c) 0 (d) -4

13) For any value of n∈Z, $\int_0^{\pi} ecos^{2x}cos^3[(2n+1)x]$ is

(a) $\frac{\pi}{2}$ (b) π (c) 0 (d) 2

14) The area enclosed by the curve y = $\frac{x^2}{2}$, the x - axis and the lines x = 1, x = 3 is

(a) 4 (b) $8\frac{2}{3}$ (c) 13 (d) $4\frac{1}{3}$

15) The slope at any point of a curve y = f (x) is given by $\frac{dy}{dx} = 3x^2$ and it passes through (-1,1). Then the equation of the curve is

(a) $y=x^3+2$ (b) $y=3x^2+4$ (c) $y=3x^4+4$ (d) $y=3x^2+5$

16) The I.F. of $(1+y^2)dx=(\tan^{-1}$-t-x)dy is ________.

(a) $e^{\tan-1}$ y (b) $e^{\tan-1}$ x (c) $\tan^{-1}$ y (d) $\tan^{-1}$x

17) The random variable X has the probability density function $f(x) = \begin{cases} ax+b & 0<x<1 \\ 0 & otherwise \end{cases}$ and $E(X) = \frac{7}{12}$ then a and b are respectively.

(a) 1 and $\frac{1}{2}$ (b) $\frac{1}{2}$ and 1 (c) 2 and 1 (d) 1 and 2

18) Which one is the inverse of the statement (PVq)→(p∧q)?

(a) (p∧q)→(pVq) (b) ¬(pvq)→(p∧q) (c) (¬pv¬q)→(¬p∧¬q) (d) (¬p∧¬q)→(¬pV¬q)

19) The number of commutative binary operations which can be defined on a set containing n elements is

(a) $n^{\frac{n(n+1)}{2}}$ (b) n^{n^2} (c) $n^{\frac{n}{2}}$ (d) n^2

20) A matrix which is obtained from an identity matrix by applying only one elementary transformation is

(1) Identity matrix

(2) Elementary matrix

(3) Square matrix

(4) Equivalent to identify matrix

PART - II

7 x 2 = 14

ANSWER ANY 7 QUESTIONS IN WHICH QUESTION NO. 30 IS COMPULSORY.

21) Find the rank of the following matrices which are in row-echelon form :

$$\begin{bmatrix} 6 & 0 & -9 \\ 0 & 2 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

22) Simplify the following:

$i\, i^2 i^3 \ldots i^{40}$

23) Examine for the rational roots of $x^8-3x+1=0$

24) Prove that $\tan^{-1} x+\tan^{-1} z=\tan^{-1}\left[\frac{x+y+z-xyz}{1-xy-yz-zx}\right]$

25) Obtain the equation of the circle for which (3,4) and (2,-7) are the ends of a diameter.

26) Prove that $(\vec{a}.(\vec{b}\times\vec{c}))\vec{a} = (\vec{a}\times\vec{b})\times(\vec{a}\times\vec{c})$

27) Find the points on the curve $y = x^3 - x^2 + x + 3$ where the normal is parallel to the line x + y =1729.

28) Write the Maclaurin series expansion of the following function

cos x

29) Find the partial derivatives of the following functions at the indicated point

$g(x,y) =3x^2 + y^2 + 5x + 2$, (1,-2)

30)

Evaluate the following:

$\int_0^{\frac{\pi}{2}} \frac{dx}{1+5cos^2x}$

PART - IIi

7 x 3 = 21

ANSWER ANY 7 QUESTIONS IN WHICH QUESTION NO. 40 IS COMPULSORY.

31) In a competitive examination, one mark is awarded for every correct answer while $\frac{1}{4}$ mark is deducted for every wrong answer. A student answered 100 questions and got 80 marks. How many questions did he answer correctly ? (Use Cramer's rule to solve the problem).

32) If |z|= 3, show that $7 \leq |z + 6 - 8i| \leq 13$.

33) It is known that the roots of the equation x^3-6x^2-4x+24=0 are in arithmetic progression. Find its roots.

34) Solve:

$2tan^{-1}x = cos^{-1}\frac{1-a^2}{1+a^2} - cos^{-1}\frac{1-b^2}{1+b^2}, a > 0, b > 0$

35) Find the vertex, focus, equation of directrix and length of the latus rectum of the following:

$y^2 = -8x$

36) Show that the four points (6, -7, 0), (16, -19, -4), (0, 3, -6), (2, -5, 10) lie on a same plane.

37) Evaluate: : $lim_{x\to 0^+}$ x log x.

38) Find the volume of a sphere of radius a.

39) Solve $\frac{dy}{dx} + \frac{y^2}{x^2} = \frac{y}{x}$

40) Let A={a+$\sqrt{5}$ b:a,b∈Z}. Check whether the usual multiplication is a binary operation on A.

PART – IV

7 x 5 = 35

ANSWER ALL THE QUESTIONS

41) a) Solve $\frac{dy}{dx} + 2y = e^{-x}$

(OR)

b) Define an operation∗ on Q as follows: a*b=$\left(\frac{a+b}{2}\right)$; a,b ∈Q. Examine the existence of identity and the existence of inverse for the operation * on Q.

42) a) A steel plant is capable of producing x tonnes per day of a low-grade steel and y tonnes per day of a high-grade steel, where $y = \frac{40 - 5x}{10 - x}$ If the fixed market price of low-grade steel is half that of high-grade steel, then what should be optimal productions in low-grade steel and high-grade steel in order to have maximum receipts.

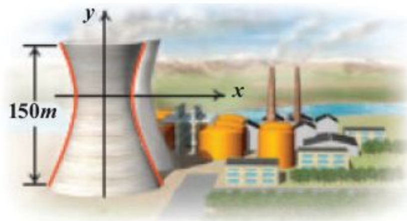
(OR)

b) Using integration find the area of the region bounded by triangle ABC, whose vertices A, B, and C are (−1,1), (3, 2), and (0,5) respectively

43) a) Form the equation whose roots are the squares of the roots of the cubic equation x^3+ax^2+bx+c = 0.

(OR)

b) Cross section of a Nuclear cooling tower is in the shape of a hyperbola with equation $\frac{x^2}{30^2} - \frac{y^2}{44^2} = 1$ The tower is 150m tall and the distance from the top of the tower to the centre of the hyperbola is half the distance from the base of the tower to the centre of the hyperbola. Find the diameter of the top and base of the tower.



44) a) Let $w(x, y) = xy+\frac{e^y}{y^2+1}$ for all $(x, y) \in R^2$. Calculate $\frac{\partial^2 w}{\partial y \partial x}$ and $\frac{\partial^2 w}{\partial x \partial y}$

(OR)

b) The probability density function of random variable X is given by $f(x) = \begin{cases} k & 1 \le x \le 5 \\ 0 & otherwise \end{cases}$ Find

(i) Distribution function

(ii) $P(X < 3)$

(iii) $P(2 < X < 4)$

(iv) $P(3 \le X)$

45) a) Show that $\left(2 + i\sqrt{3}\right)^{10} + \left(2 - i\sqrt{3}\right)^{10}$ is real ii) $\left(\frac{19 + 9i}{5 - 3i}\right)^{15} - \left(\frac{8 + i}{1 + 2i}\right)^{15}$ is purely imaginary.

(OR)

b) If the straight lines $\frac{x-1}{2} = \frac{y+1}{\lambda} = \frac{z}{2}$ and $\frac{x-1}{2} = \frac{y+1}{\lambda} = \frac{z}{\lambda}$ are coplanar, find λ and equations of the planes containing these two lines.

46) a) A bridge has a parabolic arch that is 10m high in the centre and 30m wide at the bottom. Find the height of the arch 6m from the centre, on either sides.

(OR)

b) Find the parametric form of vector equation, and Cartesian equations of the plane passing through the points (2, 2,1), (9,3,6) and perpendicular to the plane $2x + 6y + 6z = 9$

47) a) A boy is walking along the path $y = ax^2 + bx + c$ through the points (−6, 8),(−2, −12) , and (3, 8) . He wants to meet his friend at P(7,60) . Will he meet his friend? (Use Gaussian elimination method.)

(OR)

b) Investigate the values of λ and m the system of linear equations $2x + 3y + 5z = 9$, $7x + 3y - 5z = 8$, $2x + 3y + \lambda z = \mu$, have

(i) no solution

(ii) a unique solution

(iii) an infinite number of solutions.

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