RAVI MATHS TUITION CENTER, CHENNAI – 82. PH - 8056206308

12TH MATHS MODEL PAPER 7 Date : 29-Nov-19

12th Standard

Maths

Reg.No. : | | | | | | |

Exam Time : 03:00:00 Hrs Total Marks : 90

PART – I 20 x 1 = 20

ANSWER ALL THE QUESTIONS.

1) If A = $\begin{bmatrix} 2 & 3 \\ 5 & -2 \end{bmatrix}$ be such that λA^{-1} =A, then λ is

(a) 17 (b) 14 (c) 19 (d) 21

2) If A is a non-singular matrix then $|A^{-1}|$= ______

(a) $\left|\frac{1}{A^2}\right|$ (b) $\frac{1}{|A^2|}$ (c) $\left|\frac{1}{A}\right|$ (d) $\frac{1}{|A|}$

3) If (1+i)(1+2i)(1+3i)...(1+ni)=x+iy, then $2 \cdot 5 \cdot 10 ... \left(1+n^2\right)$ is

(a) 1 (b) i (c) x^2+y^2 (d) $1+n^2$

4) The polynomial x^3-kx^2+9x has three real zeros if and only if, k satisfies

(a) |k|≤6 (b) k=0 (c) |k| > 6 (d) |k|≥6

5) If $\sin^{-1}\frac{x}{5} + cosec^{-1}\frac{5}{4} = \frac{\pi}{2}$, then the value of x is

(a) 4 (b) 5 (c) 2 (d) 3

6) Area of the greatest rectangle inscribed in the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1.$ is

(a) 2ab (b) ab (c) $\sqrt{ab}$ (d) $\frac{a}{b}$

7) The volume of the parallelepiped with its edges represented by the vectors $\hat{i}+\hat{j}, \hat{i}+2\hat{j}, \hat{i}+\hat{j}+\pi\hat{k}$ is

(a) $\frac{\pi}{2}$ (b) $\frac{\pi}{3}$ (c) π (d) $\frac{\pi}{4}$

8) If the volume of the parallelepiped with $\vec{a}\times\vec{b}, \vec{b}\times\vec{c}, \vec{c}\times\vec{a}$ as coterminous edges is 8 cubic units, then the volume of the parallelepiped with $(\vec{a}\times\vec{b})\times(\vec{b}\times\vec{c}), (\vec{b}\times\vec{c})\times(\vec{c}\times\vec{a})$ and $(\vec{c}\times\vec{a})\times(\vec{a}\times\vec{b})$ as coterminous edges is,

(a) 8 cubic units (b) 512 cubic units (c) 64 cubic units (d) 24 cubic units

9) A balloon rises straight up at 10 m/s. An observer is 40 m away from the spot where the balloon left the ground. Find the rate of change of the balloon's angle of elevation in radian per second when the balloon is 30 metres above the ground.

(a) $\frac{3}{25} radians/sec$ (b) $\frac{4}{25} radians/sec$ (c) $\frac{1}{5} radians/sec$ (d) $\frac{1}{3} radians/sec$

10) The number given by the Rolle's theorem for the functlon x-3-3x2,x∈[0,3] is

(a) 1 (b) $\sqrt{2}$ (c) $\frac{3}{2}$ (d) 2

11) If we measure the side of a cube to be 4 cm with an error of 0.1 cm, then the error in our calculation of the volume is

(a) 0.4 cu.cm (b) 0.45 cu.cm (c) 2 cu.cm (d) 4.8 cu.cm

12) If (x,y, z) = xy +yz +zx, then f_x - f_z is equal to

(a) z - x (b) y - z (c) x - z (d) y - x

13) The value of $\frac{(n+2)}{(n)} = 90$ then n is

(a) 10 (b) 5 (c) 8 (d) 9

14)

For any value of n∈Z, $\int_0^{\pi} e^{cos^{2}x} cos^{3}[(2n+1)x]$ is

(a) $\frac{\pi}{2}$ (b) π (c) 0 (d) 2

15) The solution of $\frac{dy}{dx}$ + p(x)y=0 is

(a) $y = ce^{\int pdx}$ (b) $y = ce^{-\int pdx}$ (c) $x = ce^{-\int pdx}$ (d) $xce^{\int pdx}$

16) The degree of the differential equation y $y(x) = 1 + \frac{dy}{dx} + \frac{1}{1.2}\left(\frac{dy}{dx}\right)^2 + \frac{1}{1.2.3}\left(\frac{dy}{dx}\right)^3 + \ldots$ is

(a) 2 (b) 3 (c) 1 (d) 4

17) The number of arbitrary constants in the general solutions of order n and n +1are respectively

(a) n-1,n (b) n,n+1 (c) n+1,n+2 (d) n+1,n

18) Consider a game where the player tosses a sixsided fair die. H the face that comes up is 6, the player wins Rs.36, otherwise he loses Rs. k^2 , where k is the face that comes up k = {I, 2,3,4, 5}.

The expected amount to win at this game in Rs is

(a) $\frac{19}{6}$ (b) $-\frac{19}{6}$ (c) $\frac{3}{2}$ (d) $-\frac{3}{2}$

19) If a compound statement involves 3 simple statements, then the number of rows in the truth table is

(a) 9 (b) 8 (c) 6 (d) 3

20) Which one of the following is not true?

(a) Negation of a negation of a statement is the statement itself (b) If the last column of the truth table contains only T then it is a tautology. (c) If the last column of its truth table contains only F then it is a contradiction (d) If p and q are any two statements then p↔q is a tautology.

PART - II

7 x 2 = 14

ANSWER ANY 7 QUESTIONS IN WHICH QUESTION NO. 30 IS COMPULSORY.

21) Solve the following system of homogenous equations.

2x + 3y − z = 0, x − y − 2z = 0, 3x + y + 3z = 0

22) Simplify the following

$i^{1948} - i^{-1869}$

23) A 12 metre tall tree was broken into two parts. It was found that the height of the part which was left standing was the cube root of the length of the part that was cut away. Formulate this into a mathematical problem to find the height of the part which was cut away.

24) Find all values of x such that

$-5\pi \le x \le 5\pi$ and cos x -1

25) Find centre and radius of the following circles.

$x^2+y^2+6x-4y+4=0$

26) If $\vec{a}, \vec{b}, \vec{c}$ are three vectors, prove that $[\vec{a}+\vec{c}, \vec{a}+\vec{b}, \vec{a}+\vec{b}+\vec{c}] = [\vec{a}, \vec{b}, \vec{c}]$

27) A camera is accidentally knocked off an edge of a cliff 400 ft high. The camera falls a distance of $s = 16t^2$ in t seconds

What is the instantaneous velocity of the camera when it hits the ground?

28) Prove that among all the rectangles of the given perimeter, the square has the maximum area.

29) Find the local extrema for the following function using second derivative test:

$f(x) = x^2 e^{-2x}$

30) The trunk of a tree has diameter 30 cm. During the following year, the circumference grew 6cm.

What is the percentage increase in area of the tree's cross-section?

PART - III

7 x 3 = 21

ANSWER ANY 7 QUESTIONS IN WHICH QUESTION NO. 30 IS COMPULSORY.

31) Solve the equation $x^3-5x^2-4x+20=0$

32) Prove that

$sin^{-1}(\frac{3}{5}) - cos^{-1}(\frac{12}{13}) = sin^{-1}(\frac{16}{65})$

33) Identify the type of conic and find centre, foci, vertices, and directrices of each of the following :

$\frac{(y-2)^2}{25} \frac{(x+1)^2}{16} = 1$

34) Find the equation of the plane passing through the intersection of the planes 2x+3y−z+7=0 and andx+y−2z+5=0 and is perpendicular to the planex+y−3z−5=0.

35) Find the slant (oblique) asymptote for the function $f(x) = \frac{x^2-6x+7}{x+5}$

36) If $w(x, y, z) = x^2 y + y^2z + z^2x$, x, y, z∈R, бт find the differential dw .

37) Show that $\Gamma(n)=2\int_0^{\infty} e^{-x^2} x^{2n-1}dx$

38) Show that y = a cos(log x) + bsin (log x), x > 0 is a solution of the differential equation x^2 y"+ xy'+y= 0.

39) Find the binomial distribution function for each of the following.

(i) Five fair coins are tossed once and X denotes the number of heads.

(ii) A fair die is rolled 10 times and X denotes the number of times 4 appeared.

40)

Let $A = \begin{pmatrix} 1 & 0\,1 & 0 \\ 0 & 1\,0 & 1 \\ 1 & 0\,0 & 1 \end{pmatrix}, B = \begin{pmatrix} 0 & 1\,0 & 1 \\ 1 & 0\,1 & 0 \\ 1 & 0\,0 & 1 \end{pmatrix}, C = \begin{pmatrix} 1 & 1\,0 & 1 \\ 0 & 1\,1 & 0 \\ 1 & 1\,1 & 1 \end{pmatrix}$ be any three boolean matrices of the same type.

Find (A∧B)∨C

PART – IV 7 x 5 = 35

ANSWER ALL THE QUESTIONS.

41) a) A random variable X has the following probability mass function

x	1	2	3	4	5	6
f(x)	k	2k	6k	5k	6k	10k

Find

(i) P(2 < X < 6)

(ii) P(2 ≤ X < 5)

(iii) P(X ≤4)

(iv) P(3 < X)

(OR)

b) Consider p→q : If today is Monday, then 4 + 4 = 8.

Here the component statements p and q are given by,

p: Today is Monday; q: 4 + 4 = 8.

The truth value of p→q is T because the conclusion q is T.

An important point is that p→q should not be treated by actually considering the meanings of p and q in English. Also it is not necessary that p should be related to q at all.

Chapter

42) a) The region enclosed by the circle $x^2 + y^2 = a^2$ is divided into two segments by the line x = h. Find the area of the smaller segment.

(OR)

b) Solve $(1 + x^2)\frac{dy}{dx} = 1 + y^2$

43) a) A family of 3 people went out for dinner in a restaurant. The cost of two dosai, three idlies and two vadais is Rs.150. The cost of the two dosai, two idlies and four vadais is Rs.200. The cost of five dosai, four idlies and two vadais is Rs.250. The family has Rs.350 in hand and they ate 3 dosai and six idlies and six vadais. Will they be able to manage to pay the bill within the amount they had ?

(OR)

b) Investigate for what values of λ and μ the system of linear equations

$x + 2y + z = 7$, $x + y + \lambda z = \mu$, $x + 3y - 5z = 5$ has

(i) no solution

(ii) a unique solution

(iii) an infinite number of solutions

44) a) Let z_1, z_2, and z_3 be complex numbers such that $|z_1| = |z_2| = |z_3| = r > 0$ and $z_1+z_2+z_3 \neq 0$ prove that

$$\left|\frac{z_1z_2 + z_2z_3 + z_3z_1}{z_1 + z_2 + z_3}\right| = r$$

(OR)

b) If $z=x+iy$ and $\arg\left(\frac{z-1}{z+1}\right) = \frac{\pi}{2}$, then show that $x^2+y^2=1$.

45) a) Find the absolute maximum and absolute minimum values of the function $f(x) = 2x^3 + 3x^2 - 12x$ on $[-3, 2]$

(OR)

b) Let $(x, y) = e^{-2y}\cos(2x)$ for all $(x, y) \in R^2$. Prove that u is a harmonic function in R^2.

46) a) A rod of length 1 2 . m moves with its ends always touching the coordinate axes. The locus of a point P on the rod, which is 0 3 . m from the end in contact with x -axis is an ellipse. Find the eccentricity.

(OR)

b) Find the equation of a straight line passing through the point of intersection of the straight lines

$\vec{r} = (\hat{i} + 3\hat{j} - \hat{k}) + t(2\hat{i} + 3\hat{j} + 2\hat{k})$ and $\frac{x-2}{1} = \frac{y-4}{2} = \frac{z+3}{4}$ and perpendicular to both straight lines.

47) a) Solve the following equation: $x^4-10x^3+26x^2-10x+1=0$

(OR)

b) Solve $\tan^{-1}\left(\frac{x-1}{x-2}\right) + \tan^{-1}\left(\frac{x+1}{x+2}\right) = \frac{\pi}{4}$

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