

**REDUCED Random Variable and Mathematical Expectation**

12th Standard

Business Maths

**REDUCED PORTION CHAPTER WISE STUDY MATERIALS**

65 x 1 = 65

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- 1) Value which is obtained by multiplying possible values of random variable with probability of occurrence and is equal to weighted average is called  
(a) Discrete value (b) Weighted value (c) Expected value (d) Cumulative value
- 2) Demand of products per day for three days are 21, 19, 22 units and their respective probabilities are  $0.29, 0.40, 0.35$ . Profit per unit is  $0.50$  paisa then expected profits for three days are  
(a) 21, 19, 22 (b) 21.5, 19.5, 22.5 (c) 0.29, 0.40, 0.35 (d) 3.045, 3.8, 3.85
- 3) Probability which explains x is equal to or less than particular value is classified as  
(a) discrete probability (b) cumulative probability (c) marginal probability (d) continuous probability
- 4) Given  $E(X) = 5$  and  $E(Y) = -2$ , then  $E(X - Y)$  is  
(a) 3 (b) 5 (c) 7 (d) -2
- 5) A variable that can assume any possible value between two points is called  
(a) discrete random variable (b) continuous random variable (c) discrete sample space (d) random variable
- 6) A formula or equation used to represent the probability distribution of a continuous random variable is called  
(a) probability distribution function (b) density function (c) probability density function (d) mathematical expectation
- 7) If X is a discrete random variable and  $p(x)$  is the probability of X, then the expected value of this random variable is equal to  
(a)  $\sum f(x)$  (b)  $\sum [x + f(x)]$  (c)  $\sum f(x) + x$  (d)  $\sum xp(x)$
- 8) Which of the following is not possible in probability distribution?  
(a)  $\sum p(x) \geq 0$  (b)  $\sum p(x) = 1$  (c)  $\sum xp(x) = 2$  (d)  $p(x) = -0.5$
- 9) If c is a constant, then  $E(c)$  is  
(a) 0 (b) 1 (c)  $c f(c)$  (d) c
- 10) A discrete probability distribution may be represented by  
(a) table (b) graph (c) mathematical equation (d) all of these
- 11) A probability density function may be represented by:  
(a) table (b) graph (c) mathematical equation (d) both (b) and (c)
- 12) If c is a constant in a continuous probability distribution, then  $p(x = c)$  is always equal to  
(a) zero (b) one (c) negative (d) does not exist
- 13)  $E[X - E(X)]$  is equal to  
(a)  $E(X)$  (b)  $V(X)$  (c) 0 (d)  $E(X) - X$
- 14)  $E[X - E(X)]^2$  is  
(a)  $E(X)$  (b)  $E(X^2)$  (c)  $V(X)$  (d) S.D(X)

- 15) If the random variable takes negative values, then the negative values will have  
 (a) positive probabilities (b) negative probabilities (c) constant probabilities (d) difficult to tell
- 16) If we have  $f(x)=2x$ ,  $0 \leq x \leq 1$ , then  $f(x)$  is a  
 (a) probability distribution (b) probability density function (c) distribution function (d) continuous random variable
- 17)  $\int_{-\infty}^{\infty} f(x)dx$  is always equal to  
 (a) zero (b) one (c)  $E(X)$  (d)  $f(x)+1$
- 18) A listing of all the outcomes of an experiment and the probability associated with each outcome is called  
 (a) probability distribution (b) probability density function (c) attributes (d) distribution function
- 19) Which one is not an example of random experiment?  
 (a) A coin is tossed and the outcome is either head or a tail  
 (b) A six-sided die is rolled  
 (c) Some number of persons will be admitted to a hospital emergency room during any hour.  
 (d) All medical insurance claims received by a company in a given year.
- 20) A set of numerical values assigned to a sample space is called  
 (a) random sample (b) random variable (c) random numbers (d) random experiment
- 21) A variable which can assume finite or countably infinite number of values is known as  
 (a) continuous (b) discrete (c) qualitative (d) none of them
- 22) The probability function of a random variable is defined as
- |        |     |      |      |      |      |
|--------|-----|------|------|------|------|
| $X=x$  | -1  | -2   | 0    | 1    | 2    |
| $P(x)$ | $k$ | $2k$ | $3k$ | $4k$ | $5k$ |
- Then  $k$  is equal to  
 (a) zero (b)  $\frac{1}{4}$  (c)  $\frac{1}{15}$  (d) one
- 23) If  $p(x)=\frac{1}{10}$ ,  $c=10$ , then  $E(X)$  is  
 (a) zero (b)  $\frac{6}{8}$  (c) 1 (d) -1
- 24) A discrete probability function  $p(x)$  is always  
 (a) non-negative (b) negative (c) one (d) zero
- 25) In a discrete probability distribution the sum of all the probabilities is always equal to  
 (a) zero (b) one (c) minimum (d) maximum
- 26) An expected value of a random variable is equal to it's  
 (a) variance (b) standard deviation (c) mean (d) covariance
- 27) A discrete probability function  $p(x)$  is always non-negative and always lies between  
 (a) 0 and  $\infty$  (b) 0 and 1 (c) -1 and +1 (d)  $-\infty$  and  $+\infty$
- 28) The probability density function  $p(x)$  cannot exceed  
 (a) zero (b) one (c) mean (d) infinity
- 29) The height of persons in a country is a random variable of the type  
 (a) discrete random (b) continuous random (c) both (a) (d) neither (a)

- variable                      variable                      and (b)                      nor (b)
- 30) The distribution function  $F(x)$  is equal to  
 (a)  $P(X = x)$                       (b)  $P(X \leq x)$                       (c)  $P(X \geq x)$                       (d) all of these

- 31) If  $f(x) = \begin{cases} kx^2 & 0 < x < 3 \\ 0, & elsewhere \end{cases}$  if a p.d.f. then the value of  $k$  is

- (a)  $\frac{1}{3}$                       (b)  $\frac{1}{6}$                       (c)  $\frac{1}{9}$                       (d)  $\frac{1}{12}$

- 32) A random variable  $X$  has the following probability distribution

|          |               |      |      |      |      |               |
|----------|---------------|------|------|------|------|---------------|
| $X$      | 0             | 1    | 2    | 3    | 4    | 5             |
| $P(X=x)$ | $\frac{1}{4}$ | $2a$ | $3a$ | $4a$ | $5a$ | $\frac{1}{4}$ |

Then  $P(1 \leq X \leq 4)$  is

- (a)  $\frac{10}{21}$                       (b)  $\frac{2}{7}$                       (c)  $\frac{1}{14}$                       (d)  $\frac{1}{2}$

- 33) A random variable  $X$  has the following probability mass function

|          |                     |                     |                      |
|----------|---------------------|---------------------|----------------------|
| $X$      | -2                  | 3                   | 1                    |
| $P(X=x)$ | $\frac{\lambda}{6}$ | $\frac{\lambda}{4}$ | $\frac{\lambda}{12}$ |

Then  $\lambda$  is

- (a) 1                      (b) 2                      (c) 3                      (d) 4

- 34)  $X$  is a discrete random variable. Which take values 0, 1, 2 and  $P(X = 0) = \frac{144}{169}$ ,

$P(X=1) = \frac{1}{169}$ , then the value of  $P(X=2)$  is

- (a)  $\frac{145}{169}$                       (b)  $\frac{24}{169}$                       (c)  $\frac{2}{169}$                       (d)  $\frac{143}{169}$

- 35) Given  $E(X+c)=8$  and  $E(X-c)=12$ , then the value of  $c$  is

- (a) -2                      (b) 4                      (c) -4                      (d) 2

- 36)  $X$  is a random variable. Taking the values 3, 4 and 12 with probabilities

$\frac{1}{3}$ ,  $\frac{1}{4}$  and  $\frac{5}{12}$ . Then  $E(X)$  is

- (a) 5                      (b) 7                      (c) 6                      (d) 3

- 37) Variance of the random variable.  $X$  is 4, Its mean is 2. Then  $E(X^2)$  is

- (a) 2                      (b) 4                      (c) 6                      (d) 8

- 38)  $\mu_2=20$ ,  $\mu_2^1=276$  for a discrete random variable.  $X$ . Then the mean of the random variable.  $X$  is

- (a) 16                      (b) 5                      (c) 2                      (d) 1

- 39)  $V(4X+3)$  is

- (a) 7                      (b)  $16V(X)$                       (c)  $4V(X)$                       (d) 19

- 40) If  $E(X) = \frac{1}{2}$ ,  $E(X^2) = \frac{1}{4}$ , then  $V(X)$  is

- (a) 0                      (b)  $\frac{1}{4}$                       (c)  $\frac{1}{2}$                       (d) 1

- 41) If  $f(x)=cx^2$ ,  $0 < x < 2$  is the p.d.f. of  $x$ , then  $c$  is

- (a)  $\frac{1}{3}$                       (b)  $\frac{4}{3}$                       (c)  $\frac{8}{3}$                       (d)  $\frac{3}{8}$

- 42) A probability distribution function is defined by  $F(x) = \begin{cases} 0, & x < 0 \\ 1 - e^{-x}, & x \geq 0 \end{cases}$  The

probability density function is

- (a)                      (b)                      (c)                      (d)  $f(x)=3e^{-3x}$   
 $f(X) = 0 \forall x \in (-\infty, \infty)$

$$f(x) \begin{cases} 3e^{-3x}, & x \geq 0 \\ 0, & x < 0 \end{cases} \quad f(x) \begin{cases} 1 - e^{-3x}, & x \geq 0 \\ 0, & x < 0 \end{cases}$$

- 43) If  $f(x) = \begin{cases} \frac{4}{x}, & 1 < x < e^3 \\ 0, & \text{otherwise} \end{cases}$  is a p.d.f. of a continuous random variable. X

then  $P(X \geq e)$

- (a)  $\frac{2}{3}$  (b)  $\frac{3}{2}$  (c)  $\frac{1}{6}$  (d)  $\frac{1}{8}$

- 44) The random variable. X has variance 4 and  $E(X^2)=8$ . Then the mean of X is

- (a)  $2\sqrt{3}$  (b) 4 (c) 2 (d)  $\sqrt{2}$

- 45) If the p.d.f of a continuous random variable. X is

$$f(x) = \begin{cases} \frac{x}{2}, & 0 < x < 2 \\ 0, & \text{elsewhere} \end{cases} \quad \text{then } E(3X^2 - 2X) =$$

- (a)  $\frac{2}{3}$  (b)  $\frac{4}{3}$  (c)  $\frac{10}{3}$  (d)  $\frac{7}{3}$

- 46) If  $F(x)$  is the probability distribution function, then  $F(-\infty)$  is \_\_\_\_\_.

- (a) 1 (b) 2 (c)  $\infty$  (d) 0

- 47) If  $F(x)$  is the probability distribution function, then  $F(\infty)$  is \_\_\_\_\_.

- (a) 1 (b) 2 (c)  $\infty$  (d) 0

- 48) If  $f(x) = kx(1-x)$ ,  $0 < x < 1$  is a p.d.f. then the value of k is \_\_\_\_\_.

- (a)  $\frac{1}{5}$  (b)  $\frac{2}{5}$  (c)  $\frac{3}{5}$  (d) 6

- 49) Two coins are tossed simultaneously. The values of a, b, c in the probability distribution

|             |   |   |   |
|-------------|---|---|---|
| Np of heads | 0 | 1 | 2 |
| Probability | a | b | c |

are

- (a)  $\frac{1}{3}, \frac{3}{4}, 0$  (b)  $0, \frac{3}{4}, \frac{1}{4}$  (c)  $\frac{3}{2}, \frac{1}{2}, \frac{1}{4}$  (d)  $\frac{1}{4}, \frac{2}{4}, \frac{1}{4}$

- 50) The following table is the probability distribution of random variable X.

|      |     |    |   |     |    |     |
|------|-----|----|---|-----|----|-----|
| X    | 1   | 2  | 3 | 4   | 5  | 6   |
| P(X) | 0.1 | 2k | k | 0.2 | 3k | 0.1 |

The value of k is \_\_\_\_\_.

- (a) 0.2 (b) 0.1 (c) 0.3 (d) 0.4

- 51) If X is a discrete random variable. then  $P(X \geq a) =$  \_\_\_\_\_.

- (a)  $P(X < a)$  (b)  $1 - P(X \leq a)$  (c)  $1 - P(X < a)$  (d) 0

- 52) If X is a continuous random variable. then  $P(X \geq a) =$  \_\_\_\_\_.

- (a)  $P(X < a)$  (b)  $1 - P(X > a)$  (c)  $P(X > a)$  (d)  $1 - P(X \leq a-1)$

- 53) If X is a discrete random variable., then which of the following is correct?

- (a)  $0 \leq F(x) \leq 1$  (b)  $F(-\infty) = 0, F(\infty) \leq 1$  (c)  $P(X = X_n) = F(X_n) - F(X_{n-1})$  (d)  $F(x)$  is a constant function

- 54) Which of the following are correct?

(i)  $E(aX+b) = a E(X) + b$

(ii)  $\mu_2 = \mu_2^1 - (\mu_1^1)^2$

(iii)  $\mu^2 = \text{variance}$

(iv)  $V(aX + b) = a^2 V(x)$

- (a) all (b) i, ii and iii (c) ii and iii (d) i and iv

55) A discrete random variable.  $X$  has the probability mass function  $p(x)$ , then \_\_\_\_\_ is true.

- (a)  $0 \leq P(X) \leq 1$  (b)  $P(X) \geq 0$  (c)  $P(X) \leq 1$  (d)  $0 < P(X) < 1$

56) If  $F(x)$  is the probability distribution function, then  $F(-\infty)$  is \_\_\_\_\_.

- (a) 1 (b) 2 (c)  $\infty$  (d) 0

57) If  $F(x)$  is the probability distribution function, then  $F(\infty)$  is \_\_\_\_\_.

- (a) 1 (b) 2 (c)  $\infty$  (d) 0

58) If  $f(x) = kx(1-x)$ ,  $0 < x < 1$  is a p.d.f. then the value of  $k$  is \_\_\_\_\_.

- (a)  $\frac{1}{5}$  (b)  $\frac{2}{5}$  (c)  $\frac{3}{5}$  (d) 6

59) Two coins are tossed simultaneously. The values of  $a, b, c$  in the probability distribution

|             |   |   |   |
|-------------|---|---|---|
| Np of heads | 0 | 1 | 2 |
| Probability | a | b | c |

are \_\_\_\_\_

- (a)  $\frac{1}{3}, \frac{3}{4}, 0$  (b)  $0, \frac{3}{4}, \frac{1}{4}$  (c)  $\frac{3}{2}, \frac{1}{2}, \frac{1}{4}$  (d)  $\frac{1}{4}, \frac{2}{4}, \frac{1}{4}$

60) The following table is the probability distribution of random variable  $X$ .

|        |     |    |   |     |    |     |
|--------|-----|----|---|-----|----|-----|
| $X$    | 1   | 2  | 3 | 4   | 5  | 6   |
| $P(X)$ | 0.1 | 2k | k | 0.2 | 3k | 0.1 |

The value of  $k$  is \_\_\_\_\_.

- (a) 0.2 (b) 0.1 (c) 0.3 (d) 0.4

61) If  $X$  is a discrete random variable. then  $P(X \geq a) =$  \_\_\_\_\_.

- (a)  $P(X < a)$  (b)  $1 - P(X \leq a)$  (c)  $1 - P(X < a)$  (d) 0

62) If  $X$  is a continuous random variable. then  $P(X \geq a) =$  \_\_\_\_\_.

- (a)  $P(X < a)$  (b)  $1 - P(X > a)$  (c)  $P(X > a)$  (d)  $1 - P(X \leq a-1)$

63) If  $X$  is a discrete random variable., then which of the following is correct?

- (a)  $0 \leq F(x) \leq 1$  (b)  $F(-\infty) = 0, F(\infty) \leq 1$  (c)  $P(X = X_n) = F(X_n) - F(X_{n-1})$  (d)  $F(x)$  is a constant function

64) Which of the following are correct?

- (i)  $E(aX+b) = a E(X) + b$   
 (ii)  $\mu_2 = \mu_2^1 - (\mu_1^1)^2$   
 (iii)  $\mu^2 = \text{variance}$   
 (iv)  $V(aX+b) = a^2 V(x)$   
 (a) all (b) i, ii and iii (c) ii and iii (d) i and iv

65) A discrete random variable.  $X$  has the probability mass function  $p(x)$ , then \_\_\_\_\_ is true.

- (a)  $0 \leq P(X) \leq 1$  (b)  $P(X) \geq 0$  (c)  $P(X) \leq 1$  (d)  $0 < P(X) < 1$

39 x 2 = 78

66) The number of cars in a household is given below.

|                  |    |     |     |     |    |
|------------------|----|-----|-----|-----|----|
| No. of cars      | 0  | 1   | 2   | 3   | 4  |
| No. of Household | 30 | 120 | 380 | 190 | 80 |

67) Suppose, the life in hours of a radio tube has the following p.d.f

$$f(x) = \frac{100}{x^2}, \text{ when } x \geq 100$$

$$0, \text{ when } x < 100$$

Find the distribution function.

68) Construct cumulative distribution function for the given probability distribution.

|        |     |     |     |     |
|--------|-----|-----|-----|-----|
| X      | 0   | 1   | 2   | 3   |
| P(X=x) | 0.3 | 0.2 | 0.4 | 0.1 |

69) The discrete random variable X has the probability function

|        |   |    |    |    |
|--------|---|----|----|----|
| X      | 1 | 2  | 3  | 4  |
| P(X=x) | k | 2k | 3k | 4k |

Show that  $k = 0.1$ .

70) Two coins are tossed simultaneously. Getting a head is termed as success. Find the probability distribution of the number of successes.

71) Define random variable.

72) Explain what are the types of random variable?

73) Define discrete random variable.

74) What do you understand by continuous random variable?

75) Describe what is meant by a random variable.

76) Distinguish between discrete and continuous random variable.

77) Explain the distribution function of a random variable.

78) Six men and five women apply for an executive position in a small company. Two of the applicants are selected for an interview. Let X denote the number of women in the interview pool. We have found the probability mass function of X.

|      |                |                |                |
|------|----------------|----------------|----------------|
| X=x  | 0              | 1              | 2              |
| P(x) | $\frac{2}{11}$ | $\frac{5}{11}$ | $\frac{4}{11}$ |

How many women do you expect in the interview pool?

79) The following information is the probability distribution of successes.

|                  |                |                |                |
|------------------|----------------|----------------|----------------|
| No. of Successes | 0              | 1              | 2              |
| Probability      | $\frac{6}{11}$ | $\frac{9}{22}$ | $\frac{1}{22}$ |

Determine the expected number of success.

80) Find the expected value for the random variable of an unbiased die

81) Let X be a random variable defining number of students getting A grade. Find the expected value of X from the given table

|        |     |     |     |     |
|--------|-----|-----|-----|-----|
| X=x    | 0   | 1   | 2   | 3   |
| P(X=x) | 0.2 | 0.1 | 0.4 | 0.3 |

82) Let X be a continuous random variable with probability density function  $f_x(x) \{ 2x, 0 \leq x \leq 1$

$0, \text{ otherwise}$

Find the expected value of X.

83) In an investment, a man can make a profit of Rs.5,000 with a probability of 0.62 or a loss of Rs. 8,000 with a probability of 0.38. Find the expected gain.

84) What are the properties of Mathematical expectation?

85) What do you understand by Mathematical expectation?

86) How do you define variance in terms of Mathematical expectation?

87) Define Mathematical expectation in terms of discrete random variable.

88) State the definition of Mathematical expectation using continuous random variable.

89) Let X be a random variable and  $Y = 2X + 1$ . What is the variance of Y if variance of X is 5 ?

90) Prove that if  $E(X)=0$ , then  $V(X)=E(X^2)$

91) What is the expected value of a game that works as follows: I flip a coin and, if tails pay you Rs. 2; if heads pay you Rs.1. In either case I also pay you Rs. 50.

92) Prove that,

$$V(aX)=a^2V(X)$$

93) The time to failure in thousands of hours of an important piece of electronic equipment used in a manufactured DVD player has the density function

$$f(x)=\begin{cases} 2e^{-2x}, & x > 0 \\ 0, & \text{otherwise} \end{cases}$$

Find the expected life of this piece of equipment.

94) Determine whether the following is a probability distribution of a random variable X.

|      |     |     |     |
|------|-----|-----|-----|
| X    | 0   | 1   | 2   |
| P(X) | 0.6 | 0.1 | 0.2 |

95) An unbiased die is rolled. If the random variable X is defined as

$$X(w)=\begin{cases} 1, & \text{the outcome } w \text{ is an even number} \\ 0, & \text{if the outcome } w \text{ is an odd number} \end{cases}$$

Find the probability distribution of X.

96) Two eggs are drawn at random without replacement from a bag containing two bad eggs and eight good eggs. Find the probability of getting two bad eggs?

97) A random variable X has the probability mass function

|        |               |               |                |
|--------|---------------|---------------|----------------|
| X      | -2            | 3             | 1              |
| P(X=x) | $\frac{k}{6}$ | $\frac{k}{4}$ | $\frac{k}{12}$ |

then find k

98) A discrete random variable. X has the following probability distribution

|      |   |    |    |    |    |     |     |     |     |
|------|---|----|----|----|----|-----|-----|-----|-----|
| X    | 0 | 1  | 2  | 3  | 4  | 5   | 6   | 7   | 8   |
| P(X) | a | 3a | 5a | 7a | 9a | 11a | 13a | 15a | 17a |

Find the value of a and  $P(X < 3)$

99) Verify whether  $f(x) = \begin{cases} \frac{2x}{9}, & 0 \leq x \leq \\ 0, & \text{elsewhere} \end{cases}$  is a probability density function

100) A continuous random variable. X has the p.d.f. defined by

$$f(x) = \begin{cases} Ce^{-ax}, & 0 < x < \infty \\ 0, & \text{elsewhere} \end{cases} \quad \text{Find the value of C if } a > 0$$

101) In an entrance examination a student has to answer all the 120 questions.

Each question has four options and only one option is correct. A student gets 1 mark for a correct answer and loses  $\frac{1}{2}$  mark for a wrong answer. What is the expectation of the mark scored by a student if he chooses the answer to each question at random?

102) In a gambling game a man wins Rs.10 if he gets all heads or all tails and loses Rs.5 if he gets 1 or 2 heads when 3 coins are tossed once. Find his expectation of gain.

103) Find the mean for the probability density function

$$f(x) = \begin{cases} \frac{1}{24}, & -12 \leq x \leq 12 \\ 0, & \text{otherwise} \end{cases}$$

104) Prove that,

$$V(X+b)=V(X)$$

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105) If  $p(x) = \begin{cases} \frac{x}{20}, & x = 0, 1, 2, 3, 4, 5 \\ 0, & \text{otherwise} \end{cases}$

Find (i)  $P(X < 3)$  and (ii)  $P(2 \leq 4)$

106) If you toss a fair coin three times, the outcome of an experiment consider as random variable which counts the number of heads on the upturned faces. Find out the probability mass function and check the properties of the probability mass function.

107) Two unbiased dice are thrown simultaneously and sum of the upturned faces considered as random variable. Construct a probability mass function.



108) A coin is tossed thrice. Let  $X$  be the number of observed heads. Find the cumulative distribution function of  $X$ .

109) A continuous random variable  $X$  has the following p.d.f

$$f(x) = ax, \quad 0 \leq x \leq 1$$

Determine the constant  $a$  and also find  $P\left[X \leq \frac{1}{2}\right]$

110) Let  $X$  be a discrete random variable with the following p.m.f

$$p(x) = \begin{cases} 0.3 & \text{for } x = 3 \\ 0.2 & \text{for } x = 5 \\ 0.3 & \text{for } x = 8 \\ 0.2 & \text{for } x = 10 \\ 0 & \text{otherwise} \end{cases}$$

Find and plot the c.d.f. of  $X$ .



111) The discrete random variable X has the following probability function  

$$P(X=x)=\begin{cases} kx & x = 2, 4, 6 \\ k(x-2) & x = 8 \\ 0 & \text{otherwise} \end{cases}$$
 where k is a constant. Show that  $k = \frac{1}{18}$

112) Explain the terms

- (i) probability mass function,
- (ii) probability density function and
- (iii) probability distribution function.

113) What are the properties of (i) discrete random variable and (ii) continuous random variable?

114) State the properties of distribution function.

115) An urn contains four balls of red, black, green and blue colours. There is an equal probability of getting any coloured ball. What is the expected value of getting a blue ball out of 30 experiments with replacement?

116) A fair die is thrown. Find out the expected value of its outcomes

117) Suppose the probability mass function of the discrete random variable is

|      |     |     |     |     |
|------|-----|-----|-----|-----|
| X=x  | 0   | 1   | 2   | 3   |
| p(x) | 0.2 | 0.1 | 0.4 | 0.3 |

What is the value of  $E(3X + 2X^2)$  ?

118) Consider a random variable X with probability density function

$$f(x) = \begin{cases} 4x^3, & \text{if } 0 < x < 1 \\ 0, & \text{otherwise} \end{cases}$$

Find  $E(X)$  and  $V(X)$

119) If  $f(x)$  is defined by  $f(x) = ke^{-2x}$ ,  $0 \leq x < \infty$  is a density function. Determine the constant k and also find mean.

120) The time to failure in thousands of hours of an important piece of electronic equipment used in a manufactured DVD player has the density function.

$$f(x) = \begin{cases} 3e^{-3x}, & x > 0 \\ 0, & \text{otherwise} \end{cases}$$

Find the expected life of the piece of equipment.

121) A commuter train arrives punctually at a station every 25 minutes. Each morning, a commuter leaves his house and casually walks to the train station. Let X denote the amount of time, in minutes, that commuter waits for the train from the time he reaches the train station. It is known that the probability density function of X is

$$f(x) = \begin{cases} \frac{1}{25}, & \text{for } 0 < x < 25 \\ 0, & \text{otherwise} \end{cases}$$

Obtain and interpret the expected value of the random variable X.

122) The following table is describing about the probability mass function of the random variable X

|      |     |     |     |
|------|-----|-----|-----|
| x    | 3   | 4   | 5   |
| P(x) | 0.1 | 0.1 | 0.2 |

Find the standard deviation of x.

123) Let X be a continuous random variable with probability density function

$$f(x) = \begin{cases} \frac{3}{x^4}, & x \geq 1 \\ 0, & \text{otherwise} \end{cases}$$

Find the mean and variance of X.

124) In a business venture a man can make a profit of Rs. 2,000 with a probability of 0.4 or have a loss of Rs. 1,000 with a probability of 0.6. What is his expected, variance and standard deviation of profit?

125) The number of miles an automobile tire lasts before it reaches a critical point in tread wear can be represented by a p.d.f.

$$f(x) = \begin{cases} \frac{1}{30} e^{-\frac{x}{30}}, & \text{for } x > 0 \\ 0, & \text{for } x \leq 0 \end{cases}$$

Find the expected number of miles (in thousands) a tire would last until it reaches the critical tread wear point.

126) A person tosses a coin and is to receive Rs. 4 for a head and is to pay Rs. 2 for a tail. Find the expectation and variance of his gains.

127) The p.d.f. of X is defined as

$$f(x) = \begin{cases} k & \text{for } 0 < x \leq 4 \\ 0, & \text{otherwise} \end{cases}$$

Find the value of k and also find  $P(2 \leq X \leq 4)$ .

128) The probability distribution function of a discrete random variable X is

$$f(x) = \begin{cases} 2k, & x = 1 \\ 3k, & x = 3 \\ 4k, & x = 5 \\ 0, & \text{otherwise} \end{cases}$$

$$3k, x = 3$$

$$4k, x = 5$$

$$0, \text{ otherwise}$$

where k is some constant. Find (a) k and (b)  $P(X > 2)$ .

129) Consider a random variable X with p.d.f

$$f(x) = \begin{cases} 3x^2, & \text{if } 0 < x < 1 \\ 0, & \text{otherwise} \end{cases}$$

Find  $E(X)$  and  $V(3X-2)$

130) The probability distribution of a discrete random variable. X is given by

|        |               |               |               |
|--------|---------------|---------------|---------------|
| X      | -2            | 2             | 5             |
| P(X=x) | $\frac{1}{4}$ | $\frac{1}{4}$ | $\frac{1}{2}$ |

then find  $4E(X^2) - \text{Var}(2X)$

131) A random variable. X has following distribution

|        |               |               |               |               |
|--------|---------------|---------------|---------------|---------------|
| X      | -1            | 0             | 1             | 2             |
| P(X=x) | $\frac{1}{3}$ | $\frac{1}{6}$ | $\frac{1}{6}$ | $\frac{1}{3}$ |

Find  $E(2X+3)^2$

132) If a continuous random variable. X has the p.d.f.  $f(x) = 4k(x-1)^3$ ,  $1 \leq x \leq 3$  then find  $p[-2 \leq X \leq 2]$

133) A player tosses two unbiased coins. He wins Rs.5 if two heads appear, rs.2 if one head appear and Rs.1 if no head appear. Find the expected amount to win.

134) If the probability density function of a random variable. X is given by  $f(x) = \frac{2x}{9}, 0$

135) If a random variable. X has the probability distribution

|        |   |    |    |    |    |    |
|--------|---|----|----|----|----|----|
| X      | 0 | 1  | 2  | 3  | 4  | 5  |
| P(X=x) | a | 2a | 3a | 4a | 5a | 6a |

then find F(4)

136) Let X denote the number of hours you study during a randomly selected school day. The probability distribution function is

$$P(X = x) = \begin{cases} 0.1 & \text{if } x = 0 \\ kx & \text{if } x = 1 \text{ or } 2 \\ k(5 - x) & \text{if } x = 3 \text{ or } 4 \\ 0, & \text{otherwise} \end{cases}$$

Find the value of k and what is the probability that you study atleast 2 hours.

137) A random variable X can take all nonnegative integral values and the probabilities that X takes the value r is proportional to  $a^r$  ( $0 < a < 1$ ). Find  $P(X = 0)$

138) Two cards are drawn from a pack of 52 playing cards. Find the probability distribution of the number of aces.

139) An urn contains 4 white and 6 red balls. Four balls are drawn at random from the urn. Find the probability distribution of the number of white balls.

$$24 \times 5 = 120$$

140) A random variable X has the following probability function

|             |   |    |    |    |    |                |                 |                    |
|-------------|---|----|----|----|----|----------------|-----------------|--------------------|
| Values of X | 0 | 1  | 2  | 3  | 4  | 5              | 6               | 7                  |
| p(x)        |   | 0a | 2a | 2a | 3a | a <sup>2</sup> | 2a <sup>2</sup> | 7a <sup>2</sup> +a |

141) Construct the distribution function for the discrete random variable X whose probability distribution is given below. Also draw a graph of p(x) and F(x).

|       |     |      |     |     |      |      |      |
|-------|-----|------|-----|-----|------|------|------|
| X = x | 1   | 2    | 3   | 4   | 5    | 6    | 7    |
| P(x)  | 0.1 | 0.12 | 0.2 | 0.3 | 0.15 | 0.08 | 0.05 |

142) A continuous random variable X has p.d.f

$$f(x) = 5x^4, 0 \leq x \leq 1$$

Find a1 and a2 such that i)  $P[X \leq a_1] = P[X > a_1]$  ii)  $P[X > a_2] = 0.05$

143) The amount of bread (in hundreds of pounds) x that a certain bakery is able to sell in a day is found to be a numerical valued random phenomenon, with a probability function specified by the probability density function f(x) is given by  $f(x) = \{Ax, \text{ for } 0 \leq x \leq 10$

$$A(20 - x), \text{ for } 10 \leq x < 20$$

$$0, \text{ otherwise}$$

(a) Find the value of A.

(b) What is the probability that the number of pounds of bread that will be sold tomorrow is

(i) More than 10 pounds,

(ii) Less than 10 pounds, and

(iii) Between 5 and 15 pounds?

144) A continuous random variable X has the following probability function

|              |   |    |    |    |    |                |                 |                    |
|--------------|---|----|----|----|----|----------------|-----------------|--------------------|
| Value of X=x | 0 | 1  | 2  | 3  | 4  | 5              | 6               | 7                  |
| P(x)         |   | 0k | 2k | 2k | 3k | k <sup>2</sup> | 2k <sup>2</sup> | 7k <sup>2</sup> +k |

(i) Find k

(ii) Evaluate  $p(x < 6)$ ,  $p(x \geq 6)$  and  $p(0)$  (iii) If  $P(X \leq x) = \frac{1}{2}$ , then find the minimum value of x.

145) The distribution of a continuous random variable X in range  $(-3, 3)$  is given by p.d.f.

$$f(x) = \begin{cases} \frac{1}{16}(3+x)^2, & -3 \leq x \leq -1 \\ \frac{1}{16}(6-2x)^2, & -1 \leq x \leq 1 \\ \frac{1}{16}(3-x)^2, & 1 \leq x \leq 3 \end{cases}$$

Verify that the area under the curve is unity.

146) A continuous random variable X has the following distribution function:

$$f(x) = \begin{cases} 0 & \text{if } x \leq 1 \\ k(x-1)^4 & \text{if } 1 < x \leq 3 \\ 1, & \text{if } x > 3 \end{cases}$$

Find (i) k and (ii) the probability density function.

147) The length of time (in minutes) that a certain person speaks on the telephone is found to be random phenomenon, with a probability function specified by the probability density function  $f(x)$  as

$$f(x) = \begin{cases} Ae^{-x/5}, & \text{for } x \geq 0 \\ 0, & \text{otherwise} \end{cases}$$

(a) Find the value of A that makes  $f(x)$  a p.d.f,

(b) What is the probability that the number of minutes that person will talk over the phone is (i) more than 10 minutes, (ii) less than 5 minutes and (iii) between 5 and 10 minutes.

148) Suppose that the time in minutes that a person has to wait at a certain station for a train is found to be a random phenomenon with a probability

$$\text{function specified by the distribution function } F(x) = \begin{cases} 0, & \text{for } x < 0 \\ \frac{1}{2}, & \text{for } 0 \leq x < 1 \\ 0, & \text{for } 1 \leq x < 2 \\ \frac{1}{4}, & \text{for } 2 \leq x < 4 \\ 0, & \text{for } x \geq 4 \end{cases}$$

(a) Is the distribution function continuous? If so, give its probability density function?

(b) What is the probability that a person will have to wait

(i) more than 3 minutes,

(ii) less than 3 minutes and

(iii) between 1 and 3 minutes?

149) Determine the mean and variance of the random variable X having the following probability distribution.

|     |   |   |   |   |   |   |   |   |   |    |
|-----|---|---|---|---|---|---|---|---|---|----|
| X=x | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|-----|---|---|---|---|---|---|---|---|---|----|

|      |      |      |      |      |      |      |      |      |      |      |
|------|------|------|------|------|------|------|------|------|------|------|
| P(x) | 0.15 | 0.10 | 0.10 | 0.01 | 0.08 | 0.01 | 0.05 | 0.02 | 0.28 | 0.20 |
|------|------|------|------|------|------|------|------|------|------|------|

150) Determine the mean and variance of a discrete random variable, given its distribution as follows.

|                    |               |               |               |               |               |   |
|--------------------|---------------|---------------|---------------|---------------|---------------|---|
| X=x                | 1             | 2             | 3             | 4             | 5             | 6 |
| F <sub>x</sub> (x) | $\frac{1}{6}$ | $\frac{2}{6}$ | $\frac{3}{6}$ | $\frac{4}{6}$ | $\frac{5}{6}$ | 1 |

151) Suppose the life in hours of a radio tube has the probability density function

$$f(x) = \begin{cases} e^{-\frac{x}{100}}, & \text{when } x \geq 100 \\ 0, & \text{when } x < 100 \end{cases}$$

Find the mean of the life of a radio tube.

152) The probability density function of a random variable X is

$$f(x) = ke^{-|x|}, \quad -\infty < x < \infty$$

Find the value of k and also find mean and variance for the random variable.

153) The probability function of a random variable X is given by

$$p(x) = \begin{cases} \frac{1}{4}, & \text{for } x = -2 \\ \frac{1}{4}, & \text{for } x = 0 \\ \frac{1}{2}, & \text{for } x = 10 \\ 0, & \text{elsewhere} \end{cases}$$

Evaluate the following probabilities.

$$P(\leq 0)$$

154) Let X be a random variable with cumulative distribution function

$$F(x) = \begin{cases} 0, & \text{if } x < 0 \\ \frac{x}{8}, & \text{if } 0 \leq x < 1 \\ \frac{1}{4} + \frac{x}{8}, & \text{if } 1 \leq x < 2 \\ \frac{3}{4} + \frac{x}{12}, & \text{if } 2 \leq x < 3 \\ 1, & \text{for } 3 \leq x \end{cases}$$

(a) Compute: (i)  $P(1 \leq X \leq 2)$  and

(ii)  $P(X=3)$

(b) Is X a discrete random variable? Justify your answer.

155) The probability density function of a continuous random variable X is

$$f(x) = \begin{cases} a + bx^2, & 0 \leq x \leq 1; \\ 0, & \text{otherwise} \end{cases}$$

where a and b are some constants. Find (i) a and b if  $E(X) = \frac{3}{5}$

(ii)  $\text{Var}(X)$ .

156) The probability function of a random variable X is given by

$$p(x) = \begin{cases} \frac{1}{4}, & \text{for } x = -2 \\ \frac{1}{4}, & \text{for } x = 0 \\ \frac{1}{2}, & \text{for } x = 10 \\ 0, & \text{elsewhere} \end{cases}$$

Evaluate the following probabilities.

$$P(X < 0)$$

157) The probability function of a random variable X is given by

$$p(x) = \begin{cases} \frac{1}{4}, & \text{for } x = -2 \\ \frac{1}{4}, & \text{for } x = 0 \\ \frac{1}{2}, & \text{for } x = 10 \\ 0, & \text{elsewhere} \end{cases}$$

Evaluate the following probabilities.

$$P(|X| \leq 2)$$

158) The probability function of a random variable X is given by

$$p(x) = \begin{cases} \frac{1}{4}, & \text{for } x = -2 \\ \frac{1}{4}, & \text{for } x = 0 \\ \frac{1}{2}, & \text{for } x = 10 \\ 0, & \text{elsewhere} \end{cases}$$

Evaluate the following probabilities.

$$P(0 \leq X \leq 10)$$

159) A discrete random variable X has the following probability distribution.

|      |   |    |    |    |                |                 |                    |
|------|---|----|----|----|----------------|-----------------|--------------------|
| x    | 1 | 2  | 3  | 4  | 5              | 6               | 7                  |
| P(X) | c | 2c | 2c | 3c | c <sup>2</sup> | 2c <sup>2</sup> | 7c <sup>2</sup> +c |

Find the value of c. Also, find the mean of the distribution.

160) The probability distribution of a random variation X is given below.

|      |     |      |     |     |      |
|------|-----|------|-----|-----|------|
| X    | 0   | 1    | 2   | 3   | 4    |
| P(X) | 0.1 | 0.25 | 0.3 | 0.2 | 0.15 |

Find (i) V(X)

$$\text{ii) } V\left(\frac{X}{2}\right)$$

161) The probability distribution of the discrete random variables X and Y are given below

|      |               |               |               |               |
|------|---------------|---------------|---------------|---------------|
| X    | 0             | 1             | 2             | 3             |
| P(X) | $\frac{1}{5}$ | $\frac{2}{5}$ | $\frac{1}{5}$ | $\frac{1}{5}$ |

  

|      |               |                |               |                |
|------|---------------|----------------|---------------|----------------|
| Y    | 0             | 1              | 2             | 3              |
| P(Y) | $\frac{1}{5}$ | $\frac{3}{10}$ | $\frac{2}{5}$ | $\frac{1}{10}$ |

Prove that  $E(Y^2) = 2E(X)$ .

162) The random variable X can take only the values 0, 1, 2. Given that  $P(X = 0) = P(X = 1) = P$  and  $E(X^2) = E(X)$ , find the value of p.

163) The probability distribution of a random variable X is

|      |               |               |                |                |                |                |
|------|---------------|---------------|----------------|----------------|----------------|----------------|
| X    | 1             | 2             | 4              | 2A             | 3A             | 5A             |
| P(X) | $\frac{1}{2}$ | $\frac{1}{5}$ | $\frac{3}{25}$ | $\frac{1}{10}$ | $\frac{1}{25}$ | $\frac{1}{25}$ |

Calculate (i) A if  $E(X)=2.94$

(ii)  $V(X)$

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