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12TH MATHS MODEL PAPER 6 Date : 29-Nov-19

12th Standard

Maths

Reg.No. : | | | | | | |

Instructions : (1) check the question paper for fairness of printing. if there is any lack of fairness, inform the hall supervisor immediately.(2) use blue or black ink to write and underline and pencil to draw diagrams.

Exam Time : 03:00:00 Hrs Total Marks : 90

PART – I 20 x 1 = 20

ANSWER ALL THE QUESTIONS.

1) If $(AB)^{-1} = \begin{bmatrix} 12 & -17 \\ -19 & 27 \end{bmatrix}$ and $A^{-1} = \begin{bmatrix} 1 & -1 \\ -2 & 3 \end{bmatrix}$, then B^{-1} =

(a) $\begin{bmatrix} 2 & -5 \\ -3 & 8 \end{bmatrix}$ (b) $\begin{bmatrix} 8 & 5 \\ 3 & 2 \end{bmatrix}$ (c) $\begin{bmatrix} 3 & 1 \\ 2 & 1 \end{bmatrix}$ (d) $\begin{bmatrix} 8 & -5 \\ -3 & 2 \end{bmatrix}$

2) The augmented matrix of a system of linear equations is $\begin{bmatrix} 1 & 2 & 7 & 3 \\ 0 & 1 & 4 & 6 \\ 0 & 0 & \lambda - 7 & \mu + 5 \end{bmatrix}$. The system has infinitely many solutions if

(a) $\lambda = 7, \mu \neq -5$ (b) $\lambda = 7, \mu = 5$ (c) $\lambda \neq 7, \mu \neq -5$ (d) $\lambda = 7, \mu = -5$

3) If A^T is the transpose of a square matrix A, then

(a) $|A| \neq |A^T|$ (b) $|A| = |A^T|$ (c) $|A| + |A^T| = 0$ (d) $|A| = |A^T|$ only

4) If A = [2 0 1] then the rank of AA^T is ______

(a) 1 (b) 2 (c) 3 (d) 0

5) z_1, z_2 and z_3 are complex number such that $z_1+z_2+z_3=0$ and $|z_1|=|z_2|=|z_3|=1$ then $z_1^2+z_2^2+z_3^3$ is

(a) 3 (b) 2 (c) 1 (d) 0

6) If $z^n = cos\frac{n\pi}{3} + isin\frac{n\pi}{3}$, then $z_1, z_2 ..... z_6$ is

(a) 1 (b) -1 (c) i (d) -i

7) The number of real numbers in $[0,2\pi]$ satisfying $\sin^4x-2\sin^2x+1$ is

(a) 2 (b) 4 (c) 1 (d) ∞

8) Let a > 0, b > 0, c > 0. h n both th root of th quatlon $ax^2+b+C= 0$ are

(a) real and negative (b) real and positive (c) rational numb rs (d) none

9) $sin^{-1}\left(tan\frac{\pi}{4}\right) - sin^{-1}\left(\sqrt{\frac{3}{x}}\right) = \frac{\pi}{6}$ .Then x is a root of the equation

(a) $x^2-x-6=0$ (b) $x^2-x-12=0$ (c) $x^2+x-12=0$ (d) $x^2+x-6=0$

10) The number of real solutions of the equation $\sqrt{1 + cos2x} = 2sin^{-1}(sinx),\ -\pi < x < \pi$ is

(a) 0 (b) 1 (c) 2 (d) infinte

11) An ellipse hasOB as semi minor axes, F and F′ its foci and the angle FBF′ is a right angle. Then the eccentricity of the ellipse is

(a) $\frac{1}{\sqrt{2}}$ (b) $\frac{1}{2}$ (c) $\frac{1}{4}$ (d) $\frac{1}{\sqrt{3}}$

12) In an ellipse, the distance between its foci is 6 and its minor axis is 8, then e is

(a) $\frac{4}{5}$ (b) $\frac{1}{\sqrt{52}}$ (c) $\frac{3}{5}$ (d) $\frac{1}{2}$

13) The angle between the line $\vec{r} = (\hat{i} + 2\hat{j} - 3\hat{k}) + t(2\hat{i} + \hat{j} - 2\hat{k})$ and the plane $\vec{r}.(\hat{i} + \hat{j}) + 4 = 0$ is

(a) 0° (b) 30° (c) 45° (d) 90°

14) The volume of the parallelepiped whose sides are given by $\vec{OA} = 2\hat{i} - 3\hat{j}$, $\vec{OB} = \hat{i} + \hat{j} - \hat{k}$ and $\vec{OC} = 3\hat{i} - \hat{k}$ is

(a) $\frac{4}{13}$ (b) 4 (c) $\frac{2}{7}$ (d) $\frac{4}{9}$

15) The abscissa of the point on the curve $f(x) = \sqrt{8-2x}$ at which the slope of the tangent is -0.25 ?

(a) -8 (b) -4 (c) -2 (d) 0

16) If f(x) = $\frac{x}{x+1}$ then its differential is given by

(a) $\frac{-1}{(x+1)^2}dx$ (b) $\frac{1}{(x+1)^2}dx$ (c) $\frac{1}{1+x}dx$ (d) $\frac{-1}{1+x}dx$

17) If f(x)$f(x) = \int_1^x \frac{e^{\sin u}}{u}du, x > 1$ *and* $\int_1^3 \frac{e^{\sin x^2}}{x}dx = \frac{1}{2}[f(a) - f(1)]$, then one of the possible value of a is

(a) 3 (b) 6 (c) 9 (d) 5

18) The solution of the differential equation $\frac{dy}{dx} = \frac{y}{x} + \frac{\phi\left(\frac{y}{x}\right)}{\phi'\left(\frac{y}{x}\right)}$ is

(a) $x\phi\left(\frac{y}{x}\right) = k$ (b) $\phi\left(\frac{y}{x}\right) = kx$ (c) $y\phi\left(\frac{y}{x}\right) = k$ (d) $\phi\left(\frac{y}{x}\right) = ky$

19) Two coins are to be flipped. The first coin will land on heads with probability 0.6, the second with probability 0.5. Assume that the results of the flips are independent, and let X equal the total number of heads that result The value of E[X] is

(a) 0.11 (b) 1.1 (c) 1.1 (d) 1

20) Which one of the following is a binary operation on N?

(a) Subtraction (b) Multiplication (c) Division (d) All the above

PART - II

7 x 2 = 14

ANSWER ANY 7 QUESTIONS IN WHICH QUESTION NO. 30 IS COMPULSORY.

21) Flod the rank of the matrix $\begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix}$.

22) If z_1 and z_2 are 1-i, -2+4i then find Im $\left(\frac{z_1 z_2}{\bar{z}_1}\right)$.

23) Prove that $2\tan^{-1}\left(\frac{2}{3}\right) = \tan^{-1}\left(\frac{12}{5}\right)$

24) If the line y = 3x + 1, touches the parabola $y^2 = 4ax$, find the length of the latus rectum?

25) Let $\vec{a}, \vec{b}, \vec{c}$ be unit vectors such $\vec{a}.\vec{b} = \vec{a}.\vec{c} = 0$ and the angle between $\vec{b}$ and $\vec{c}$ is $\frac{\pi}{6}$. Prove that $\vec{a} = \pm 2\left(\vec{b} \times \vec{c}\right)$

26) A man 2 m high walks at a uniform speed of 5 km/ hr away from a lamp post 6 m high. Find the rate at which the length of his shadow increases?

27) If f (x, y) = $2x^3 - 11x^2y + 3y^3$, prove that $x\frac{\partial f}{\partial x} + y\frac{\partial f}{\partial y} = 3f$

28) Solve the following differential equations:

$\frac{dy}{dx} = \tan^2(x+y)$

29)

A tank initially contains 50 litres of pure water. Starting at time t = 0 a brine containing with 2 grams of dissolved salt per litre flows into the tank at the rate of 3 litres per minute. The mixture is kept uniform by stirring and the well-stirred mixture simultaneously flows out of the tank at the same rate. Find the amount of salt present in the tank at any time t > 0.

30) Show that

$\neg(p \wedge q) \equiv \neg p \vee \neg q$

PART - III — 7 x 3 = 21

ANSWER ANY 7 QUESTIONS IN WHICH QUESTION NO. 40 IS COMPULSORY.

31) Solve the following systems of linear equations by Cramer's rule:

$\frac{3}{x} + 2y = 12, \frac{2}{x} + 3y = 13$

32) Find the fourth roots of unity.

33) Solve the equation $x^4-9x^2+20=0$.

34) Find the value of the expression in terms of x, with the help of a reference triangle.

$\tan\left(sin^{-1}\left(x+\frac{1}{2}\right)\right)$

35) Prove that $tan^{-1}\left(\frac{m}{n}\right) - tan^{-1}\left(\frac{m-n}{m+n}\right) = \frac{\pi}{4}$

36) Find the equation of the circle described on the chord 3x+y+5= 0 of the circle $x^2+y^2=16$ as diameter.

37) If $\vec{a}, \vec{b}, \vec{c}$ are three non-coplanar vectors represented by concurrent edges of a parallelepiped of volume 4 cubic units, find the value of $(\vec{a}+\vec{b}).(\vec{b}\times\vec{c})+(\vec{b}+\vec{c}).(\vec{c}\times\vec{a})+(\vec{c}+\vec{a})(\vec{a}\times\vec{b})$

38) Compute the limit $lim_{x\to 1}\left(\frac{x^2-3x+2}{x^2-4x+3}\right)$.

39) Evaluate :$\int_0^9 \frac{1}{x+\sqrt{x}}dx$

40) Let *be defined on R by (a*b)=a+b+ab-7. is*binary on R? If so, find $3\left(\frac{-7}{15}\right)$.

PART – IV — 7 x 5 = 35

ANSWER ALL THE QUESTIONS.

41) a) Solve $\frac{dy}{dx} = \frac{x-y+5}{2(x-y)+7}$.

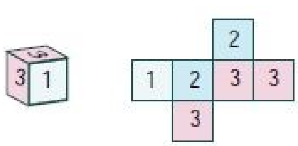
(OR)

b) A six sided die is marked '1' on one face, '2' on two of its faces, and '3' on remaining three faces. The die is rolled twice. If X denotes the total score in two throws.

(i) Find the probability mass function.

(ii) Find the cumulative distribution function.

(iii) Find $P(3 \le X < 6)$ (iv) Find $P(X \ge 4)$.



42)

a) How many rows are needed for following statement formulae?

$((p \wedge q) \vee (\neg r \vee \neg s)) \wedge (\neg t \wedge v))$

(OR)

b) Define an operation$*$ on Q as follows: $a*b=\left(\frac{a+b}{2}\right)$; a,b $\in$Q. Examine the existence of identity and the existence of inverse for the operation * on Q.

43) a) If the radius of a sphere, with radius 10 cm, has to decrease by 0 1. cm, approximately how much will its volume decrease?

(OR)

b) Find the area of the region bounded by the y -axis and the parabola $x=5-4y-y^2$.

44) a) With usual notations, in any triangle ABC, prove by vector method that $\frac{a}{sinA} = \frac{b}{sinB} = \frac{c}{sinc}$

(OR)

b) Find the coordinates of the foot of the perpendicular and length of the perpendicular from the point (4,3,2) to the plane x + 2y + 3z = 2

45) a) Solve the following equation: $x^4-10x^3+26x^2-10x+1=0$

(OR)

b) Identify the type of conic and find centre, foci, vertices, and directrices of each of the following :
$18x^2+12y^2-144x+48y+120 = 0$

46) a) Suppose z1,Z2, and z3 are the vertices of an equilateral triangle inscribed in the circle |z|= 2. If $z_1 = 1+i\sqrt{3}$ then find z_2 and z_3

(OR)

b) An engineer designs a satellite dish with a parabolic cross section. The dish is 5m wide at the opening, and the focus is placed 1 2 . m from the vertex

(a) Position a coordinate system with the origin at the vertex and the x -axis on the parabola's axis of symmetry and find an equation of the parabola.

(b) Find the depth of the satellite dish at the vertex.

47) a) The upward speed v(t)of a rocket at time t is approximated by $v(t) = at^2 + bt + c \leq t \leq 100$ where a, b and c are constants. It has been found that the speed at times t = 3, t = 6, and t = 9 seconds are respectively, 64, 133, and 208 miles per second respectively. Find the speed at time t = 15 seconds. (Use Gaussian elimination method.)

(OR)

b) Verify that arg(1+i) + arg(1-i) = arg[(1+i) (1-i)]

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