RAVI MATHS TUITION CENTER, CHENNAI – 82. PH - 8056206308

12TH MATHS MODEL PAPER 5 Date : 29-Nov-19

12th Standard

Maths

Reg.No. : | | | | | | |

Exam Time : 03:00:00 Hrs Total Marks : 90

PART I 20 x 1 = 20

ANSWER ALL

1) If A, B and C are invertible matrices of some order, then which one of the following is not true?

(a) adj A = $|A|A^{-1}$ (b) adj(AB) = (adj A)(adj B) (c) det $A^{-1} = (\det A)^{-1}$ (d) $(ABC)^{-1} = C^{-1}B^{-1}A^{-1}$

2) If $(AB)^{-1} = \begin{bmatrix} 12 & -17 \\ -19 & 27 \end{bmatrix}$ and $A^{-1} = \begin{bmatrix} 1 & -1 \\ -2 & 3 \end{bmatrix}$, then B^{-1} =

(a) $\begin{bmatrix} 2 & -5 \\ -3 & 8 \end{bmatrix}$ (b) $\begin{bmatrix} 8 & 5 \\ 3 & 2 \end{bmatrix}$ (c) $\begin{bmatrix} 3 & 1 \\ 2 & 1 \end{bmatrix}$ (d) $\begin{bmatrix} 8 & -5 \\ -3 & 2 \end{bmatrix}$

3) If $|z_1|=1, |z_2|=2 |z_3|=3$ and $|9z_1z_2+4z_1z_3+z_2z_3|=12$, then the value of $|z_1+z_2+z_3|$ is

(a) 1 (b) 2 (c) 3 (d) 4

4) If z is a complex number such that $z \varepsilon C/R$ and $z + \frac{1}{z} \epsilon R$ then|z| is

(a) 0 (b) 1 (c) 2 (d) 3

5) If $x^3+12x^2+10ax+1999$ definitely has a positive zero, if and only if

(a) $a \geq 0$ (b) $a > 0$ (c) $a < 0$ (d) $a \leq 0$

6) If $x=\frac{1}{5}$, the valur of $\cos(\cos^{-1}x+2\sin^{-1}x)$ is

(a) $-\sqrt{\frac{24}{25}}$ (b) $\sqrt{\frac{24}{25}}$ (c) $\frac{1}{5}$ (d) $-\frac{1}{5}$

7) $tan^{-1}\left(\frac{1}{4}\right) + tan^{-1}\left(\frac{2}{3}\right)$ is equal to

(a) $\frac{1}{2}cos^{-1}\left(\frac{3}{5}\right)$ (b) $\frac{1}{2}sin^{-1}\left(\frac{3}{5}\right)$ (c) $\frac{1}{2}tan^{-1}\left(\frac{3}{5}\right)$ (d) $tan^{-1}\left(\frac{1}{2}\right)$

8) The radius of the circle passing through the point(6,2) two of whose diameter arex+y=6 and x+2y=4 is

(a) 10 (b) $2\sqrt{5}$ (c) 6 (d) 4

9) The area of quadrilateral formed with foci of the hyperbolas $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ and $\frac{x^2}{a^2} - \frac{y^2}{b^2} = -1$

(a) $4(a^2+b^2)$ (b) $2(a^2+b^2)$ (c) a^2+b^2 (d) $\frac{1}{2}(a^2+b^2)$

10) If $\vec{a} = \hat{i} + \hat{j} + \hat{k}$, $\vec{b} = \hat{i} + \hat{j}$, $\vec{c} = \hat{i}$ and $(\vec{a} \times \vec{b}) \times \vec{c} = \lambda\vec{a} + \mu\vec{b}$ then the value of $\lambda + \mu$ is

(a) 0 (b) 1 (c) 6 (d) 3

11) If $\vec{a}, \vec{b}, \vec{c}$ are non-coplanar, non-zero vectors such that $[\vec{a}, \vec{b}, \vec{c}] = 3$, then $\{[\vec{a} \times \vec{b}, \vec{b} \times \vec{c}, \vec{c} \times \vec{a}]\}^2$ is equal to

(a) 81 (b) 9 (c) 27 (d) 18

12) The abscissa of the point on the curve $f(x) = \sqrt{8 - 2x}$ at which the slope of the tangent is -0.25 ?

(a) -8 (b) -4 (c) -2 (d) 0

13) The slope of the line normal to the curve f(x) = 2cos 4x at $x = \frac{\pi}{12}$

(a) $-4\sqrt{3}$ (b) -4 (c) $\frac{\sqrt{3}}{12}$ (d) $4\sqrt{3}$

14) The approximate change in the volume V of a cube of side x metres caused by increasing the side by 1% is

(a) 0.3xdx m^3 (b) 0.03 xm^3 (c) $0.03.x^2$ m^3 (d) $0.03x^3m^3$

15) If $g(x, y) = 3x^2 - 5y + 2y$, $x(t) = e^t$ and $y(t) = \cos t$, then $\frac{dg}{dt}$ is equal to

(a) $6e^{2t}+5 \sin t - 4 \cos t \sin t$ (b) $6e^{2t}- 5 \sin t + 4 \cos t \sin t$ (c) $3e^{2t}+ 5 \sin t + 4 \cos t \sin t$ (d) $3e^{2t} - 5 \sin t + 4 \cos t \sin t$

16) The value of $\int_0^{\infty} e^{-3x}x^2 dx$ is

(a) $\frac{7}{27}$ (b) $\frac{5}{27}$ (c) $\frac{4}{27}$ (d) $\frac{2}{27}$

17) If $\int_a^a \frac{1}{4+x^2}dx = \frac{\pi}{8}$ then a is

(a) 4 (b) 1 (c) 3 (d) 2

18) The integrating factor of the differential equation $\frac{dy}{dx} + y = \frac{1+y}{\lambda}$ is

(a) $\frac{x}{e^{\lambda}}$ (b) $\frac{e^{\lambda}}{x}$ (c) λe^x (d) e^x

19) The integrating factor of the differential equation $\frac{dy}{dx}$ +P(x)y=Q(x)is x, then P(x)

(a) x (b) $\frac{x^2}{2}$ (c) $\frac{1}{x}$ (d) $\frac{1}{x^2}$

20) If the function $f(x) = \frac{1}{12}$ for. a < x < b, represents a probability density function of a continuous random variable X, then which of the followingcannot be the value of a and b?

(a) 0 and 12 (b) 5 and 17 (c) 7 and 19 (d) 16 and 24

PART II

7 x 2 = 14

ANSWER ANY 7 QUESTIONS IN WHICH QUESTION NO. 30 IS COMPULSORY

21) Find the rank of the matrix $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 4 \\ 3 & 0 & 5 \end{bmatrix}$ by reducing it to a row-echelon form.

22) Find the square roots of 4+3i

23) Construct a cubic equation with roots 2,−2, and 4.

24) Simplify

$$\cos^{-1}\left(\cos\left(\frac{13\pi}{3}\right)\right)$$

25) Find the vertices, foci for the hyperbola $9x^2-16y^2=144$.

26) Find the angle between the straight line $\vec{r} = (2\hat{i} + \hat{j} + \hat{k}) + t(\hat{i} - \hat{j} + \hat{k})$ and the plane 2x-y+z=5

27) Find the tangent and normal to the following curves at the given points on the curve

y = x sin x at $\left(\frac{\pi}{2}, \frac{\pi}{2}\right)$

28) A sphere is made of ice having radius 10 cm. Its radius decreases from 10 cm to 9-8 cm. Find approximations for the following: change in the volume

29) Evaluate the following definite integrals:

$$\int_0^{\frac{\pi}{2}} e^x\left(\frac{1+\sin x}{1+\cos x}\right)dx$$

30)

$\frac{dy}{dx} + \frac{y}{x} = sinx$

PART III 7 x 3 = 21

ANSWER ANY 7 QUESTIONS IN WHICH QUESTION NO. 40 IS COMPULSORY

31) Find adj(adj A) if adj A = $\begin{bmatrix} 1 & 0 & 1 \\ 0 & 2 & 0 \\ -1 & 0 & 1 \end{bmatrix}$.

32) Show that the points 1, $\frac{-1}{2} + i\frac{\sqrt{3}}{2}$, and $\frac{-1}{2} - i\frac{\sqrt{3}}{2}$ are the vertices of an equilateral triangle.

33) Solve the equation $x^4-9x^2+20=0$.

34) Find the value of $\tan^{-1}(-1)+\cos^{-1}(\frac{1}{2}) + sin^{-}1(-\frac{1}{2})$

35) If the equation$3x^2+(3-p)xy+qy^2-2px = 8pq$ represents a circle, find p and q . Also determine the centre and radius of the circle

36) Prove by vector method that the parallelograms on the same base and between the same parallels are equal in area.

37) Find the equation of the plane which passes through the point (3, 4, -1) and is parallel to the plane 2x - 3y + 5z = 0. Also, find the distance between the two planes.

38) If $u=\sin^{-1}\left(\frac{x+y}{\sqrt{x}+\sqrt{y}}\right)$, Show that x $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \frac{1}{2}tanu$

39) Find, by integration, the volume of the solid generated by revolving about y-axis the region bounded by the curves y = log x, y = 0, x = 0 and y = 2.

40) Let $A = \begin{pmatrix} 1 & 0\,1 & 0 \\ 0 & 1\,0 & 1 \\ 1 & 0\,0 & 1 \end{pmatrix}, B = \begin{pmatrix} 0 & 1\,0 & 1 \\ 1 & 0\,1 & 0 \\ 1 & 0\,0 & 1 \end{pmatrix}, C = \begin{pmatrix} 1 & 1\,0 & 1 \\ 0 & 1\,1 & 0 \\ 1 & 1\,1 & 1 \end{pmatrix}$ be any three boolean matrices of the same type.

Find (A∧B)∨C

PART IV 7 x 5 = 35

ANSWER ALL

41) a) Find the area of the region bounded by y =cos x, y =sin x, the lines $x=\frac{\pi}{4}$ and $x=\frac{5\pi}{4}$.

(OR)

b) Construct the truth table for $(p \,\overline{\vee}\, q) \wedge (p \,\overline{\vee}\, \neg q)$

42) a) Let g(x,y)= x^3 - yx + sin(x+y), x(t) = e^{3t}, y(t) = t^2, t ∈ R. Find $\frac{dg}{dt}$

(OR)

b) Find the mean and variance of a random variable X , whose probability density function is $f(x) = \begin{cases} \lambda e^{-2x} & for \geq 0 \\ 0 & otherwise \end{cases}$

43) a) Find the intervals of monotonicity and local extrema of the function f (x) = x log x + 3x.

(OR)

b) Find the particular solution of $(1+ x^3)dy - x^2\,ydx$ = satisfying the condition y(1) = 2.

44) a) Find the vector parametric, vector non-parametric and Cartesian form of the equation of the plane passing through the points (-1, 2, 0), (2, 2, -1)and parallel to the straight line $\frac{x-1}{1} = \frac{2y+1}{2} = \frac{z+1}{-1}$

(OR)

b) Find $\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial^2 f}{\partial x^2}, \frac{\partial^2 f}{\partial y^2}$ at x = 2, y = 3 if f(x,y) = $2x^2 + 3y^2 - 2xy$

45) a) Find the vertex, focus, equation of directrix and length of the latus rectum of the following:
$x^2-2x+8y+17=0$

(OR)

b) Find the shortest distance between the following pairs of lines $\frac{x-3}{3}=\frac{y-8}{-1}=\frac{z-3}{1}$ and $\frac{x+3}{-3}=\frac{y+7}{2}=\frac{z-6}{4}$

46) a) Find all zeros of the polynomial $x^6-3x^5-5x^4+22x^3-39x^2-39x+135$, if it is known that 1+2i and $\sqrt{3}$ are two of its zeros.

(OR)

b) A concrete bridge is designed as a parabolic arch. The road over bridge is 40m long and the maximum height of the arch is 15m. Write the equation of the parabolic arch.

47) a) The prices of three commodities A, B and C are Rs.x, y and z per units respectively. A person P purchases 4 units of B and sells two units of A and 5 units of C . Person Q purchases 2 units of C and sells 3 units of A and one unit of B . Person R purchases one unit of A and sells 3 unit of B and one unit of C . In the process, P, Q and R earn Rs.15,000, Rs.1,000 and Rs.4,000 respectively. Find the prices per unit of A, B and C . (Use matrix inversion method to solve the problem.)

(OR)

b) Solve the equation $z^3+8i=0$,where $z \in \mathbb{C}$

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