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12TH MATHS MODEL PAPER 4 Date : 29-Nov-19

12th Standard

Maths Reg.No. :

Instructions : (1) check the question paper for fairness of printing. if there is any lack of fairness, inform the hall supervisor immediately.(2) use blue or black ink to write and underline and pencil to draw diagrams.

Exam Time : 03:00:00 Hrs Total Marks : 90

PART I 20 x 1 = 20

ANSWER ALL

1) If A = $\begin{bmatrix} 3 & 1 & -1 \\ 2 & -2 & 0 \\ 1 & 2 & -1 \end{bmatrix}$ and A^{-1} = $\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$ then the value of a_{23} is

(a) 0 (b) -2 (c) -3 (d) -1

2) If A = $\begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix}$, then adj(adj A) is

(a) $\begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix}$ (b) $\begin{bmatrix} 6 & -6 & 8 \\ 4 & -6 & 8 \\ 0 & -2 & 2 \end{bmatrix}$ (c) $\begin{bmatrix} -3 & 3 & -4 \\ -2 & 3 & -4 \\ 0 & 1 & -1 \end{bmatrix}$ (d) $\begin{bmatrix} 3 & -3 & 4 \\ 0 & -1 & 1 \\ 2 & -3 & 4 \end{bmatrix}$

3) If z is a non zero complex number, such that $2iz^2=\bar{z}$ then |z| is

(a) $\frac{1}{2}$ (b) 1 (c) 2 (d) 3

4) If x+iy =$\frac{3+5i}{7-6i}$, they y =

(a) $\frac{9}{85}$ (b) -$\frac{9}{85}$ (c) $\frac{53}{85}$ (d) none of these

5) If $x^3+12x^2+10ax+1999$ definitely has a positive zero, if and only if

(a) a≥0 (b) a > 0 (c) a< 0 (d) a≤0

6) If $ax^2 + bx + c = 0$, a, b, c E R has no real zeros, and if a + b + c < 0, then ___________

(a) c > 0 (b) c< 0 (c) c=0 (d) c≥0

7) The equation $\tan^{-1} x - \cot^{-1} x = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$ has

(a) no solution (b) unique solution (c) two solutions (d) infinite number of solutions

8) ·If $\alpha = tan^{-1}\left(\frac{\sqrt{3}}{2y-x}\right), \beta = tan^{-1}\left(\frac{2x-y}{\sqrt{3y}}\right)$ then $\alpha - \beta$

(a) $\frac{\pi}{6}$ (b) $\frac{\pi}{3}$ (c) $\frac{\pi}{2}$ (d) $\frac{-\pi}{3}$

9) If x < 0, y < 0 such that xy = 1, then $\tan^{-1}(x) + \tan^{-1}(y)$ =_____

(a) $\frac{\pi}{2}$ (b) $\frac{-\pi}{2}$ (c) $-\pi$ (d) none

10) The radius of the circle$3x^2+by^2+4bx-6by+b^2=0$ is

(a) 1 (b) 3 (c) $\sqrt{10}$ (d) $\sqrt{11}$

11) If the coordinates at one end of a diameter of the circle $x^2+y^2-8x-4y+c = 0$ are (11,2) , the coordinates of the other end are

(a) (-5,2) (b) (2,-5) (c) (5,-2) (d) (-2,5)

12) The distance between the foci of a hyperbola is 16 and e = $\sqrt{2}$ Its equation is

(a) $x^2 - y^2 = 32$ (b) $y^2 - x^2 = 32$ (c) $x^2 - y^2 = 16$ (d) $y^2 - x^2 = 16$

13) If the distance of the point (1,1,1) from the origin is half of its distance from the plane x + y + z + k =0, then the values of k are

(a) ±3 (b) ±6 (c) -3, 9 (d) 3, 9

14) Let $\vec{a}$, $\vec{b}$ and $\vec{c}$ be three non- coplanar vectors and let $\vec{p}, \vec{q}, \vec{r}$ be the vectors defined by the relations $\vec{P} = \frac{\vec{b}\times\vec{c}}{[\vec{a}\vec{b}\vec{c}]}, \vec{q} = \frac{\vec{c}\times\vec{a}}{[\vec{a}\vec{b}\vec{c}]}, \vec{r} = \frac{\vec{a}\times\vec{b}}{[\vec{a}\vec{b}\vec{c}]}$ Then the value of $\left(\vec{a}+\vec{b}\right).\vec{p} + \left(\vec{b}+\vec{c}\right).\vec{q} + (\vec{c}+\vec{a}).\vec{r} =$

(a) 0 (b) 1 (c) 2 (d) 3

15) If $\vec{p} \times \vec{q} = 2\hat{i} + 3\hat{j}, \vec{r} \times \vec{s} = 3\hat{i} + 2\hat{k}$ then $\vec{p}.(\vec{q}(\vec{r} \times \vec{s}))$ is

(a) 9 (b) 6 (c) 2 (d) 5

16) Find the point on the curve 6y = .x^3 + 2 at which y-coordinate changes 8 times as fast as x-coordinate is

(a) (4,11) (b) (4,-11) (c) (-4,11) (d) (-4,-11)

17) If w (x, y) = xy, x > 0, then $\frac{\partial w}{\partial x}$ is equal to

(a) x^y log x (b) y log x (c) yx^{y-1} (d) x log y

18) The value of $\int_0^1 x(1-x)^{99}dx$ is

(a) $\frac{1}{11000}$ (b) $\frac{1}{10100}$ (c) $\frac{1}{10010}$ (d) $\frac{1}{10001}$

19) The general solution of the differential equation $\frac{dy}{dx} = \frac{y}{x}$ is

(a) xy = k (b) y = k log x (c) y = kx (d) log y = kx

20) The operation * defined by a*b =$\frac{ab}{7}$ is not a binary operation on

(a) Q^+ (b) Z (c) R (d) C

PART II 7 x 2 = 14

ANSWER ANY 7 QUESTIONS IN WHICH QUESTION NO. 30 IS COMPULSORY.

21) Find the rank of each of the following matrices:

$$\begin{bmatrix} 3 & 2 & 5 \\ 1 & 1 & 1 \\ 3 & 3 & 6 \end{bmatrix}$$

22) Find the following $\left|\frac{2+i}{-1+2i}\right|$

23) If x^2+2(k+2)x+9k=0 has equal roots, find k.

24) Is $\cos^{-1}$(-x)=π -$\cos^{-1}$(x) true? Justify your answer.

25) If y=2$\sqrt{2}$ x +c is a tangent to the circlex^2+y^2=16 , find the value of c.

26) Show that the lines $\frac{x-1}{4} = \frac{2-y}{6} = \frac{z-4}{12}$ and $\frac{x-1}{4} = \frac{2-y}{6} = \frac{z-4}{12}$ are parallel.

27) If the volume of a cube of side length x is v = x^3. Find the rate of change of the volume with respect to x when x = 5 units.

28) The time T, taken for a complete oscillation of a single pendulum with length l, is given by the equation T = 2π$\sqrt{\frac{1}{g}}$, where g is a constant. Find the approximate percentage error in the calculated value of T corresponding to an error of 2 percent in the value of 1

29) Evaluate the following integrals using properties of integration:

$$\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} sin^2 x dx$$

30) Form the differential equation of all straight lines touching the circle $x^2 + y^2 = r^2$.

PART III 7 x 3 = 21

ANSWER ANY 7 QUESTIONS IN WHICH QUESTION NO. 40 IS COMPULSORY.

31) A = $\begin{bmatrix} 1 & \tan x \\ -\tan x & 1 \end{bmatrix}$, show that A^TA^{-1} = $\begin{bmatrix} \cos 2x & -\sin 2x \\ \sin 2x & \cos 2x \end{bmatrix}$

32) The complex numbers u,v, and w are related by $\frac{1}{u} = \frac{1}{v} + \frac{1}{w}$ If v=3−4i and w=4+3i, find u in rectangular form.

33) If p and q are the roots of the equation lx^2+nx+n = 0, show that $\sqrt{\frac{p}{q}} + \sqrt{\frac{q}{p}} + \sqrt{\frac{n}{l}}$ =0.

34) Find the value of

$tan\left(cos^{-1}\left(\frac{1}{2}\right)-sin^{-1}\left(-\frac{1}{2}\right)\right)$

35) Find the vertex, focus, equation of directrix and length of the latus rectum of the following:

$y^2 = 16x$

36) Prove by vector method that an angle in a semi-circle is a right angle.

37) The price of a product is related to the number of units available (supply) by the equation Px + 3P −16x = 234, where P is the price of the product per unit in Rupees(Rs) and x is the number of units. Find the rate at which the price is changing with respect to time when 90 units are available and the supply is increasing at a rate of 15 units/week.

38) Prove that $\frac{x}{1+x}$ < < log (1+ x) for x > 0.

39) Evaluate $\int_0^{\frac{\pi}{2}} \frac{dx}{4sin^2x+5cos^2x}$

40) Let $A=\begin{pmatrix}1 & 0\,1 & 0\\ 0 & 1\,0 & 1\\ 1 & 0\,0 & 1\end{pmatrix}, B=\begin{pmatrix}0 & 1\,0 & 1\\ 1 & 0\,1 & 0\\ 1 & 0\,0 & 1\end{pmatrix}, C=\begin{pmatrix}1 & 1\,0 & 1\\ 0 & 1\,1 & 0\\ 1 & 1\,1 & 1\end{pmatrix}$ be any three boolean matrices of the same type.

Find AVB

PART IV 7 x 5 = 35

ANSWER ALL

41) a) Solve y ' = $\sin^2$ (x − y +)1.

(OR)

b) Construct the truth table for (p ∧ q) v r.

42) a) Let f (x, y) = 0 if xy ≠ 0 and f (x, y) =1 if xy = 0.

(i) Calculate: $\frac{\partial f}{\partial x}(0,0), \frac{\partial f}{\partial y}(0,0)$.

(ii) Show that f is not continuous at (0,0)

(OR)

b) Find the area of the region bounded by the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$

43) a) Provethat $tan^{-1}\left(\frac{1-x}{1+x}\right)-tan^{-1}\left(\frac{1-y}{1+y}\right)=sin^{-1}\left(\frac{y-x}{\sqrt{1+x^2}.\sqrt{1+y^2}}\right)$

(OR)

b) The probability density function of random variable X is given by $f(x)=\{\begin{matrix}k & 1\le x\le 5\\ 0 & otherwise\end{matrix}$ Find

(i) Distribution function

(ii) P(X < 3)

(iii) P(2 < X < 4)

(iv) P(3 ≤ X)

44) a) Identify the type of conic and find centre, foci, vertices, and directrices of each of the following :

$9x^2-y^2-36x-6y+18 = 0$

(OR)

b) Find the non-parametric form of vector equation, and Cartesian equation of the plane passing through the point (2,3,6) and parallel to the straight lines $\frac{x-1}{2}=\frac{y+1}{3}=\frac{z-3}{1}$ and $\frac{x+3}{2}=\frac{y-3}{-5}=\frac{z+1}{-3}$

45) a) Solve the equation (x-2)(x-7)(x-3)(x+2)+19=0

(OR)

b) Prove that the function f (x) = x^2 + 2 is strictly increasing in the interval (2,7) and strictly decreasing in the interval (−2, 0)

46) a)

If A = $\begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix}$ and B = $\begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix}$, find the productsAB and BAand hence solve the system of equations x - y + z = 4, x - 2y - 2z = 9, 2x + y + 3z = 1.

(OR)

b) Let M=$\{\begin{pmatrix} x & x \\ x & x \end{pmatrix} : x \in R - \{0\}\}$ and let * be the matrix multiplication. Determine whether M is closed under $*$. If so, examine the commutative and associative properties satisfied by $*$ on M.

47) a) Find all cube roots of $\sqrt{3} + i$

(OR)

b) At a water fountain, water attains a maximum height of 4m at horizontal distance of 0 5 . m from its origin. If the path of water is a parabola, find the height of water at a horizontal distance of 0.75m from the point of origin.

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