RAVI MATHS TUITION CENTER, CHENNAI – 82. PH - 8056206308

12TH MATHS MODEL PAPER 3 Date : 29-Nov-19

12th Standard

Maths

Reg.No. : | | | | | | |

Instructions : (1) check the question paper for fairness of printing. if there is any lack of fairness, inform the hall supervisor immediately.(2) use blue or black ink to write and underline and pencil to draw diagrams.

Exam Time : 03:00:00 Hrs Total Marks : 90

PART I 20 x 1 = 20

ANSWER ALL

1) If $A\begin{bmatrix}1 & -2\\ 1 & 4\end{bmatrix} = \begin{bmatrix}6 & 0\\ 0 & 6\end{bmatrix}$, then A =

(a) $\begin{bmatrix}1 & -2\\ 1 & 4\end{bmatrix}$ (b) $\begin{bmatrix}1 & 2\\ -1 & 4\end{bmatrix}$ (c) $\begin{bmatrix}4 & 2\\ -1 & 1\end{bmatrix}$ (d) $\begin{bmatrix}4 & -1\\ 2 & 1\end{bmatrix}$

2) The number of solutions of the system of equations 2x+y = 4, x - 2y = 2, 3x + 5y = 6 is

(a) 0 (b) 1 (c) 2 (d) infinitely many

3) If $z = \dfrac{\left(\sqrt{3}+i\right)^3(3i+4)^2}{(8+6i)^2}$, then |z| is equal to

(a) 0 (b) 1 (c) 2 (d) 3

4) If z=cos$\frac{\pi}{4}$ +i sin$\frac{\pi}{6}$, then

(a) $|z| =1, \arg(z) =\frac{\pi}{4}$ (b) $|z| =1, \arg(z) =\frac{\pi}{6}$ (c) $|z|=\frac{\sqrt{3}}{2}$, $\arg(z)=\frac{5\pi}{24}$ (d) $|z| =\frac{\sqrt{3}}{2}$, $\arg(z) =\tan^{-1}\left(\frac{1}{\sqrt{2}}\right)$

5) According to the rational root theorem, which number is not possible rational root of $4x^7+2x^4-10x^3-5$?

(a) -1 (b) $\frac{5}{4}$ (c) $\frac{4}{5}$ (d) 5

6) Let a > 0, b > 0, c > 0. h n both th root of th quatlon ax^2+b+C= 0 are

(a) real and negative (b) real and positive (c) rational numb rs (d) none

7) If $\alpha = tan^{-1}\left(tan\frac{5\pi}{4}\right)$ and $\beta = tan^{-1}\left(-tan\frac{2\pi}{3}\right)$ then

(a) $4\alpha = 3\beta$ (b) $3\alpha = 4\beta$ (c) $\alpha - \beta = \frac{7\pi}{12}$ (d) none

8) The centre of the circle inscribed in a square formed by the lines $x^2-8x-12=0$ and $y^2-14y+45 = 0$ is

(a) (4,7) (b) (7,4) (c) (9,4) (d) (4,9)

9) If a parabolic reflector is 20 em in diameter and 5 em deep, then its focus is

(a) (0,5) (b) (5,0) (c) (10,0) (d) (0, 10)

10) If $\vec{a}, \vec{b}, \vec{c}$ are three unit vectors such that $\vec{a}$ is perpendicular to $\vec{b}$ and is parallel to $\vec{c}$ then $\vec{a} \times (\vec{b} \times \vec{c})$ is equal to

(a) $\vec{a}$ (b) $\vec{b}$ (c) $\vec{c}$ (d) $\vec{0}$

11) The distance from the origin to the plane $\vec{r}.\left(2\hat{i} - \hat{j} + 5\hat{k}\right) = 7$ is ______________

(a) $\frac{7}{\sqrt{30}}$ (b) $\frac{\sqrt{30}}{7}$ (c) $\frac{30}{7}$ (d) $\frac{7}{30}$

12) A stone is thrown up vertically. The height it reaches at time t seconds is given by $x = 80t - 16t^2$. The stone reaches the maximum height in time t seconds is given by

(a) 2 (b) 2.5 (c) 3 (d) 3.5

13) If v (x, y) = log (ex + ev), then $\frac{\partial v}{\partial x} + \frac{\partial v}{\partial y}$ is equal to

(a) $e^x + e^y$ (b) $\frac{1}{e^x+e^y}$ (c) 2 (d) 1

14) If $u = (x-y)^4+(y-z)^4+(z-x)^4$ then $\sum \frac{\partial u}{\partial x}$ =

(a) 4 (b) 1 (c) 0 (d) -4

15) The area between y^2 x = 4 and its latus rectum is

(a) $\frac{2}{3}$ (b) $\frac{4}{3}$ (c) $\frac{8}{3}$ (d) $\frac{5}{3}$

16) The value of $\int_{-\pi}^{\pi} sin^3x\ cos^3x\ dx$ is

(a) 0 (b) π (c) 2π (d) 4π

17) The order of the differential equation of all circles with centre at (h, k) and radius 'a' is

(a) 2 (b) 3 (c) 4 (d) 1

18) The general solution of $4\frac{d^2y}{dx^2}$ +y=0 is

(a) $y = e^{\frac{x}{2}}\left[A \quad cos\frac{x}{2} + B \quad sin\frac{x}{2}\right]$ (b) $y = e^{\frac{x}{2}}\left[A \quad cos\frac{x}{2} - B \quad sin\frac{x}{2}\right]$ (c) $y = Acos\frac{x}{2} + Bsin\frac{x}{2}$ (d) $t = Ae^{\frac{x}{2}} + Be^{\frac{-x}{2}}$

19) A pair of dice numbered 1, 2, 3, 4, 5, 6 of a six-sided die and 1, 2, 3, 4 of a four-sided die is roUed and the sum is determined. Let the random variable X denote this sum. Then the number of elements in the inverse image of 7 is

(a) 1 (b) 2 (c) 3 (d) 4

20) In the set R of real numbers '*' is defined as follows. Which one of the following is not a binary operation on R?

(a) a*b=min (a.b) (b) a*b= max (a,b) (c) a*b= a (d) $a*b=a^b$

PART II

7 x 2 = 14

ANSWER ANY 7 QUESTIONS IN WHICH QUESTION NO. 30 IS COMPULSORY

21) Prove that $\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$ is orthogonal

22) Simplify $\left(\frac{1+i}{1-i}\right)^3 - \left(\frac{1-i}{1+i}\right)^3$

23) A 12 metre tall tree was broken into two parts. It was found that the height of the part which was left standing was the cube root of the length of the part that was cut away. Formulate this into a mathematical problem to find the height of the part which was cut away.

24) Find the Cartesian equation of a.line passing through the pointsA(2, -1, 3) and B(4, 2, 1)

25) Find a linear approximation for the following functions at the indicated points.

$f(x) = x^3 - 5x + 12, x_0 = 2$

26) If $f (x, y) = 2x^3 - 11x^2y + 3y^3$, prove that $x\frac{\partial f}{\partial x} + y\frac{\partial f}{\partial y} = 3f$

27) Evaluate the following integrals as the limits of sums.

$\int_1^2 4x^2 - 1)dx$

28) For each of the following differential equations, determine its order, degree (if exists)

$$x^2\frac{d^2y}{dx^2} + \left[1 + \left(\frac{dy}{dx}\right)^2\right]^{\frac{1}{2}} = 0$$

29) A six sided die is marked '2' on one face, '3' on two ofits faces, and '4' on remaining three faces. The die is thrown twice. If X denotes the total score in two throws, find the values of the random variable and number of points in its inverse images.

30) Fill in the following table so that the binary operation $*$ on A = {a,b,c} is commutative.

*	a	b	c
a	b		
b	c	b	a
c	a		c

PART III 7 x 3 = 21

ANSWER ANY 7 QUESTIONS IN WHICH QUESTION NO. 30 IS COMPULSORY

31) If $\frac{z+3}{z-5i} = \frac{1+4i}{2}$, find the complex number z

32) Find the condition that the roots of $x^3+ax^2+bx+c = 0$ are in the ratio p:q:r.

33) For what value of x , the inequality $\frac{\pi}{2} < cos^{-1}(3x-1) < \pi$

34) Find the equation of the circle with centre (2,3) and passing through the intersection of the lines 3x−2y−1=0and 4x+y−27=0.

35) Find the magnitude and the direction cosines of the torque about the point (2, 0, -1) of a force $(2\hat{i}+\hat{j}-\hat{k})$, whose line of action passes through the origin

36) If we blow air into a balloon of spherical shape at a rate of 1000^3 cm per second. At what rate the radius of the baloon changes when the radius is 7cm? Also compute the rate at which the surface area changes.

37) Let f (x,y) = $\frac{3x-5y+8}{x^2+y^2+1}$ for all (x, y) ∈R^2 Show that f is continuous on R^2

38) Evaluate: $\int_0^{\frac{\pi}{2}} \frac{cos\theta}{(1+sin\theta)(2+sin\theta)} d\theta$

39) Show that $x^2+y^2=r^2$, where r is a constant, is a solution of the differential equation $\frac{dy}{dx} = -\frac{x}{y}$.

40) On Z, define ⊗by (m⊗n)=mn+nm: ∀m, n∈Z. Is ⊗binary on Z?

PART IV 7 x 5 = 35

ANSWER ALL

41) a)

Solve $cos\left(sin^{-1}\left(\frac{x}{\sqrt{1+x^2}}\right)\right) = sin\left\{cot^{-1}\left(\frac{3}{4}\right)\right\}$

(OR)

b) An engineer designs a satellite dish with a parabolic cross section. The dish is 5m wide at the opening, and the focus is placed 1 2 . m from the vertex

(a) Position a coordinate system with the origin at the vertex and the x -axis on the parabola's axis of symmetry and find an equation of the parabola.

(b) Find the depth of the satellite dish at the vertex.

42) a)

If z=x+iy and arg$\left(\frac{z-1}{z+1}\right) = \frac{\pi}{2}$,then show that $x^2+y^2=1$.

(OR)

b)

Define an operation*on Q as follows: a*b=$\left(\frac{a+b}{2}\right)$; a,b ∈Q. Examine the closure, commutative, and associative properties satisfied by*on Q.

43) a) Using integration, find the area of the triangle with sides y = 2x+1, y = 3x + 1 and x = 4.

(OR)

b) Solve: $(1 + e^{2x})\, dy + (1 + y^2)e^x\, dx = 0$ when y(0) = 1

44) a) Let $g(x, y) = 2y + x^2$, x = 2r -s, $y = r^2 + 2s$, r, s ∈ R. Find $\frac{\partial g}{\partial r}, \frac{\partial g}{\partial s}$

(OR)

b) The probability density function of random variable X is given by $f(x) = \begin{cases} k & 1 \le x \le 5 \\ 0 & otherwise \end{cases}$ Find

(i) Distribution function

(ii) P(X < 3)

(iii) P(2 < X < 4)

(iv) P(3 ≤ X)

45) a) If $\vec{a} = \vec{i} - \vec{j}, \vec{b} = \hat{i} - \hat{j} - 4\hat{k}, \vec{c} = 3\hat{j} - \hat{k}$ and $\vec{d} = 2\hat{i} + 5\hat{j} + \hat{k}$

(i) $(\vec{a} \times \vec{b}) \times (\vec{c} \times \vec{d}) = [\vec{a}, \vec{b}, \vec{d}]\vec{c} - [\vec{a}, \vec{b}, \vec{c}]\vec{d}$

(ii) $(\vec{a} \times \vec{b}) \times (\vec{c} \times \vec{d}) = [\vec{a}, \vec{c}, \vec{d}]\vec{b} - [\vec{b}, \vec{c}, \vec{d}]\vec{a}$

(OR)

b) A steel plant is capable of producing x tonnes per day of a low-grade steel and y tonnes per day of a high-grade steel, where $y = \frac{40 - 5x}{10 - x}$ If the fixed market price of low-grade steel is half that of high-grade steel, then what should be optimal productions in low-grade steel and high-grade steel in order to have maximum receipts.

46) a) If A = $\begin{bmatrix} 5 & 3 \\ -1 & -2 \end{bmatrix}$, show that $A^2 - 3A - 7I_2 = O_2$. Hence find A^{-1}.

(OR)

b) Solve the following systems of linear equations by Cramer's rule:

$\frac{3}{x} - \frac{4}{y} - \frac{2}{z}$ - 0, $\frac{1}{x} + \frac{2}{y} + \frac{1}{z}$ - 2 = 0, $\frac{2}{x} - \frac{5}{y} - \frac{4}{z}$ + 1 = 0

47) a) Find the area of the region bounded between the parabola $x^2=y$ and the curve y = x.

(OR)

b) Solve: $\frac{dv}{dx} + 2y \cot x = 3x^2 cosec^2 x$

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