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**PART - I**  
**ANSWER ALL**

- 1) If  $A = \begin{bmatrix} 3 & 5 \\ 1 & 2 \end{bmatrix}$ ,  $B = \text{adj } A$  and  $C = 3A$ , then  $\frac{|\text{adj } B|}{|C|} =$
- (a)  $\frac{1}{3}$  (b)  $\frac{1}{9}$  (c)  $\frac{1}{4}$  (d) 1
- 2) Let  $A$  be a  $3 \times 3$  matrix and  $B$  its adjoint matrix. If  $|B| = 64$ , then  $|A| =$
- (a)  $\pm 2$  (b)  $\pm 4$  (c)  $\pm 8$  (d)  $\pm 12$
- 3) The area of the triangle formed by the complex numbers  $z, iz$ , and  $z+iz$  in the Argand's diagram is
- (a)  $\frac{1}{2}|z|^2$  (b)  $|z|^2$  (c)  $\frac{3}{2}|z|^2$  (d)  $2|z|^2$
- 4) If  $i^2 = -1$ , then  $i^1 + i^2 + i^3 + \dots$  up to 1000 terms is equal to
- (a) 1 (b) -1 (c)  $i$  (d) 0
- 5) A polynomial equation in  $x$  of degree  $n$  always has
- (a)  $n$  distinct roots (b)  $n$  real roots (c)  $n$  imaginary roots (d) at most one root
- 6) If  $f(x) = 0$  has  $n$  roots, then  $f'(x) = 0$  has \_\_\_\_\_ roots
- (a)  $n$  (b)  $n-1$  (c)  $n+1$  (d)  $(n-r)$
- 7)  $\sin^{-1} \frac{3}{5} - \cos^{-1} \frac{12}{13} + \sec^{-1} \frac{5}{3} - \csc^{-1} \frac{13}{2}$  is equal to
- (a)  $2\pi$  (b)  $\pi$  (c) 0 (d)  $\tan^{-1} \frac{12}{65}$
- 8) The number of solutions of the equation  $\tan^{-1} 2x + \tan^{-1} 3x = \frac{\pi}{4}$
- (a) 2 (b) 3 (c) 1 (d) none
- 9) The circle  $x^2 + y^2 = 4x + 8y + 5$  intersects the line  $3x - 4y = m$  at two distinct points if
- (a)  $15 < m < 65$  (b)  $35 < m < 85$  (c)  $-85 < m < -35$  (d)  $-35 < m < 15$
- 10)  $y^2 - 2x - 2y + 5 = 0$  is a
- (a) circle (b) parabola (c) ellipse (d) hyperbola
- 11)  $\vec{a} \cdot \vec{b} = \vec{b} \cdot \vec{c} = \vec{c} \cdot \vec{a} = 0$ , then the value of  $[\vec{a}, \vec{b}, \vec{c}]$  is
- (a)  $|\vec{a}| |\vec{b}| |\vec{c}|$  (b)  $\frac{1}{3} |\vec{a}| |\vec{b}| |\vec{c}|$  (c) 1 (d) -1
- 12) The number of vectors of unit length perpendicular to the vectors  $\begin{pmatrix} \hat{i} \\ \hat{j} \end{pmatrix}$  and  $\begin{pmatrix} \hat{j} \\ \hat{k} \end{pmatrix}$  is
- (a) 1 (b) 2 (c) 3 (d)  $\infty$
- 13) The position of a particle moving along a horizontal line of any time  $t$  is given by  $s(t) = 3t^2 - 2t - 8$ . The time at which the particle is at rest is
- (a)  $t = 0$  (b)  $t = \frac{1}{3}$  (c)  $t = 1$  (d)  $t = 3$
- 14) If the rate of increase of  $s = x^3 - 5x^2 + 5x + 8$  is twice the rate of increase of  $x$ , then one value of  $x$  is

- (a)  $\frac{3}{5}$  (b)  $\frac{10}{3}$  (c)  $\frac{3}{10}$  (d)  $\frac{1}{3}$
- 15) If  $u(x, y) = e^{x^2+y^2}$ , then  $\frac{\partial u}{\partial x}$  is equal to  
 (a)  $e^{x^2+y^2}$  (b)  $2xu$  (c)  $x^2u$  (d)  $y^2u$
- 16) If  $\log_e 4 = 1.3868$ , then  $\log_e 4.01 =$   
 (a) 1.3968 (b) 1.3898 (c) 1.3893 (d) none
- 17) If  $f(x) = \int_0^x t \cos t \, dt$ , then  $\frac{dx}{dx}$   
 (a)  $\cos x - x \sin x$  (b)  $\sin x + x \cos x$  (c)  $x \cos x$  (d)  $x \sin x$
- 18) The order and degree of the differential equation  $\sqrt{\sin x}(dx + dy) = \sqrt{\cos x}(dx - dy)$   
 (a) 1,2 (b) 2,2 (c) 1,1 (d) 2,1
- 19) Consider a game where the player tosses a sixsided fair die. If the face that comes up is 6, the player wins Rs.36, otherwise he loses Rs.  $k^2$ , where  $k$  is the face that comes up  $k = \{1, 2, 3, 4, 5\}$ .  
 The expected amount to win at this game in Rs is  
 (a)  $\frac{19}{6}$  (b)  $\frac{19}{6}$  (c)  $\frac{3}{2}$  (d)  $\frac{3}{2}$
- 20) Which one of the following is a binary operation on  $\mathbb{N}$ ?  
 (a) Subtraction (b) Multiplication (c) Division (d) All the above

## PART II

7 x 2 = 14

ANSWER ANY 7 QUESTIONS IN WHICH QUESTION NO. 30 IS COMPULSORY

- 21) If  $A$  is symmetric, prove that then  $\text{adj } A$  is also symmetric.
- 22) Write  $\frac{3+4i}{5-12i}$  in the  $x+iy$  form, hence find its real and imaginary parts.
- 23) Formulate into a mathematical problem to find a number such that when its cube root is added to it, the result is 6.
- 24) Find all the values of  $x$  such that  $-10\pi \leq x \leq 10\pi$  and  $\sin x = 0$
- 25) Examine the position of the point (2,3) with respect to the circle  $x^2+y^2-6x-8y+12=0$ .
- 26) If  $2\hat{i} - \hat{j} + 3\hat{k}, 3\hat{i} + 2\hat{j} + \hat{k}, \hat{i} + m\hat{j} + 4\hat{k}$  are coplanar, find the value of  $m$ .
- 27) Use differentials to find  $\sqrt{25.2}$
- 28) Prove that  $\int_{\frac{\pi}{2}}^{\pi} \log(\tan x) dx$
- 29) Two balls are chosen randomly from an urn containing 6 red and 8 black balls. Suppose that we win Rs. 15 for each red ball selected and we lose Rs. 10 for each black ball selected.  $X$  denotes the winning amount, then find the values of  $X$  and number of points in its inverse images.
- 30) Let  $A = \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix}, B = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$  be any two boolean matrices of the same type. Find  $A \vee B$  and  $A \wedge B$ .

## PART III

7 x 3 = 21

ANSWER ANY 7 QUESTIONS IN WHICH QUESTION NO. 40 IS COMPULSORY

- 31) Verify  $(AB)^{-1} = B^{-1}A^{-1}$  with  $A = \begin{bmatrix} 0 & -3 \\ 1 & 4 \end{bmatrix}, B = \begin{bmatrix} -2 & -3 \\ 0 & -1 \end{bmatrix}$ .
- 32) Find the values of the real numbers  $x$  and  $y$ , if the complex numbers  $(3-i)x - (2-i)y + 5$  and  $2x + (-1+2i)y + 3 + 2i$  are equal.
- 33) If  $\alpha, \beta$  and  $\gamma$  are the roots of the cubic equation  $x^3 + 2x^2 + 3x + 4 = 0$ , form a cubic equation whose roots are,  $2\alpha, 2\beta, 2\gamma$
- 34)

Find the domain of  $\cos^{-1}\left(\frac{2+\sin x}{3}\right)$

35) Find the centre and radius of the circle  $3x^2+(a+1)y^2+6x-9y+a+4=0$ .

36) In triangle, ABC the points, D, E, F are the midpoints of the sides, BC, CA and AB respectively. Using vector method, show that the area of  $\triangle DEF$  is equal to  $\frac{1}{4}$  (area of  $\triangle ABC$ )

37) A particle moves so that the distance moved is according to the law  $s(t) = \frac{t^3}{3} - t^2 + 3$ . At what time the velocity and acceleration are zero respectively?

38) Evaluate  $\int_0^{\frac{\pi}{2}} \frac{\sec x \tan x}{1+\sec^2 x} dx$

39) Two balls are chosen randomly from an urn containing 6 white and 4 black balls. Suppose that we win Rs.30 for each black ball selected and we lose Rs.20 for each white ball selected. If X denotes the winning amount, then find the values of X and number of points in its inverse images.

40) Determine whether \* is a binary operation on the sets given below.

$a*b=b+a|b|$  on  $\mathbb{R}$

PART - IV  
ANSWER ALL

7 x 5 = 35

41) a) Using integration, find the area of the region which is bounded by x-axis, the tangent and normal to the circle  $x^2 + y^2 = 4$  drawn at  $(1, \sqrt{3})$

(OR)

b) Establish the equivalence property connecting the bi-conditional with conditional:  $p \leftrightarrow q \equiv (p \rightarrow q) \wedge (q \rightarrow p)$

42) a) Let  $f(x, y) = \sin(xy^2) + e^{x^3+5y}$  for all  $\in \mathbb{R}^2$ . Calculate  $\frac{\partial f}{\partial x}$ ,  $\frac{\partial f}{\partial y}$ ,  $\frac{\partial^2 f}{\partial y \partial x}$  and  $\frac{\partial^2 f}{\partial x \partial y}$

(OR)

b) A random variable X has the following probability mass function

x	1	2	3	4	5	6
f(x)	k	2k	6k	5k	6k	10k

Find

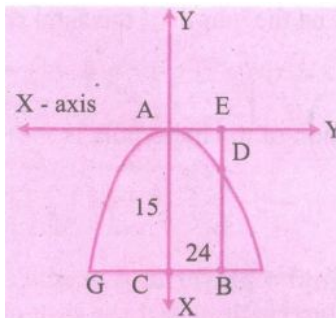
(i)  $P(2 < X < 6)$

(ii)  $P(2 \leq X < 5)$

(iii)  $P(X \leq 4)$

(iv)  $P(3 < X)$

43) a) The guides of a railway bridge is a parabola with its vertex at the highest point 15 m above the ends. If the span is 120 m, find the height of the bridge at 24 m from the middle point.



(OR)

b) Solve  $(1+x^2)\frac{dy}{dx} = 1+y^2$

44) a) Find the centre, foci, and eccentricity of the hyperbola  $11x^2 - 25y^2 - 44x + 50y - 256 = 0$

(OR)

- b) Find the acute angle between the curves  $y = x^2$  and  $x = y^2$  at their points of intersection (0,0), (1,1).

45) a) If  $2\cos\alpha = x + \frac{1}{x}$  and  $2\cos\beta = y + \frac{1}{y}$ , show that

i)  $\frac{x}{y} + \frac{y}{x} = 2\cos(\alpha - \beta)$ .

ii)  $xy - \frac{1}{xy} = 2i\sin(\alpha + \beta)$

iii)  $\frac{x^m}{y^n} - \frac{y^n}{x^m} = 2i\sin(m\alpha - n\beta)$

iv)  $x^m y^n + \frac{1}{x^m y^n} = 2\cos(m\alpha + n\beta)$

(OR)

- b) Prove by vector method that  $\sin(\alpha + \beta) = \sin\alpha \cos\beta + \cos\alpha \sin\beta$

46) a) If  $F(\alpha) = \begin{bmatrix} \cos\alpha & 0 & \sin\alpha \\ 0 & 1 & 0 \\ -\sin\alpha & 0 & \cos\alpha \end{bmatrix}$ , show that  $[F(\alpha)]^{-1} = F(-\alpha)$ .

(OR)

- b) Find a polynomial equation of minimum degree with rational coefficients, having  $\sqrt{5} - \sqrt{3}$  as a root.

47) a) Using determinants; find the quadratic defined by  $f(x) = ax^2 + bx + c$ , if  $f(1) = 0$ ,  $f(2) = -2$  and  $f(3) = -6$ .

(OR)

b) Solve  $\tan^{-1}\left(\frac{x-1}{x-2}\right) + \tan^{-1}\left(\frac{x+1}{x+2}\right) = \frac{\pi}{4}$

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