RAVI MATHS TUITION CENTER ,GKM COLONY, CHENNAI- 82. PH: 8056206308

Ordinary Differential Equations FULL TEST

12th Standard

Maths

Reg.No. : | | | | | | |

Exam Time : 03:00:00 Hrs — Total Marks : 90

ANSWER ALL — 20 x 1 = 20

1) The order and degree of the differential equation $\frac{d^2y}{dx^2}+\left(\frac{dy}{dx}\right)^{1/3}+x^{1/4}=0$ are respectively

(a) 2, 3 (b) 3, 3 (c) 2, 6 (d) 2, 4

2) The differential equation representing the family of curves y = Acos(x + B), where A and B are parameters, is

(a) $\frac{d^2y}{dx^2}-y=0$ (b) $\frac{d^2y}{dx^2}+y=0$ (c) $\frac{d^2y}{dx^2}=0$ (d) $\frac{d^2x}{dy^2}=0$

3) The order and degree of the differential equation $\sqrt{sin\quad x}(dx+dy)=\sqrt{cos\quad x}(dx-dy)$

(a) 1,2 (b) 2,2 (c) 1,1 (d) 2,1

4) The order of the differential equation of all circles with centre at (h, k) and radius ‘a’ is

(a) 2 (b) 3 (c) 4 (d) 1

5) The general solution of the differential equation $\frac{dy}{dx}=\frac{y}{x}$ is

(a) xy = k (b) y = k log x (c) y = kx (d) log y = kx

6) The solution of the differential equation 2x$\frac{dy}{dx}-y=3$ represents

(a) straight lines (b) circles (c) parabola (d) ellipse

7) The integrating factor of the differential equation $\frac{dy}{dx}+y=\frac{1+y}{\lambda}$ is

(a) $\frac{x}{e^\lambda}$ (b) $\frac{e^\lambda}{x}$ (c) λe^x (d) e^x

8) The integrating factor of the differential equation $\frac{dy}{dx}$ +P(x)y=Q(x)is x, then P(x)

(a) x (b) $\frac{x^2}{2}$ (c) $\frac{1}{x}$ (d) $\frac{1}{x^2}$

9) The degree of the differential equation y $y(x)=1+\frac{dy}{dx}+\frac{1}{1.2}\left(\frac{dy}{dx}\right)^2+\frac{1}{1.2.3}\left(\frac{dy}{dx}\right)^3+\dots$ is

(a) 2 (b) 3 (c) 1 (d) 4

10) If p and q are the order and degree of the differential equation $y=\frac{dy}{dx}+x^3\left(\frac{d^2y}{dx^2}\right)+xy=cosx$, When

(a) p < q (b) p = q (c) p > q (d) p exists and q does not exist

11) The solution of the differential equation $\frac{dy}{dx}+\frac{1}{\sqrt{1-x^2}}=0$

(a) $y + \sin^{-1} x = c$ (b) $x + \sin^{-1} y = 0$ (c) $y^2 + 2\sin^{-1} x = c$ (d) $x^2 + 2\sin^{-1} y = c$

12) The solution of the differential equation $\frac{dy}{dx}=2xy$ is

(a) $y = Ce^{x^2}$ (b) $y= 2x^2 +C$ (c) $y = Ce^{-x^2} +C$ (d) $y = x^2 +C$

13) The solution of $\frac{dy}{dx}=2^{y-x}$ is

(a) $2^x+2^y=C$ (b) $2^x-2^y=C$ (c) $\frac{1}{2^x}-\frac{1}{2^y}=C$ (d) x+y=C

14) The solution of the differential equation $\frac{dy}{dx}=\frac{y}{x}+\frac{\phi\left(\frac{y}{x}\right)}{\phi'\left(\frac{y}{x}\right)}$ is

(a) $x\phi\left(\frac{y}{x}\right)=k$ (b) $\phi\left(\frac{y}{x}\right)=kx$ (c) $y\phi\left(\frac{y}{x}\right)=k$ (d) $\phi\left(\frac{y}{x}\right)=ky$

15) If sin x is the integrating factor of the linear differential equation $\frac{dy}{dx}+Pt=Q$, Then P is

(a) log sin x (b) cos x (c) tan x (d) cot x

16) The number of arbitrary constants in the general solutions of order n and n +1are respectively

(a) n-1,n (b) n,n+1 (c) n+1,n+2 (d) n+1,n

17) Integrating factor of the differential equation $\frac{dy}{dx}=\frac{x+y+1}{x+1}$ is

(a) $\frac{1}{x+1}$ (b) x+1 (c) $\frac{1}{\sqrt{x+1}}$ (d) $\sqrt{x+1}$

18) The population P in any year t is such that the rate of increase in the population is proportional to the population. Then

(a) $P=Ce^{kt}$ (b) $P=Ce^{-kt}$ (c) P=Ckt (d) P=C

19) P is the amount of certain substance left in after time t. If the rate of evaporation of the substance is proportional to the amount remaining, then

(a) $P=Ce^{kt}$ (b) $P=ce^{-kt}$ (c) P=Ckt (d) Pt=C

20) The slope at any point of a curve y = f (x) is given by $\frac{dy}{dx} = 3x^2$ and it passes through (-1,1). Then the equation of the curve is

(a) $y=x^3+2$ (b) $y=3x^2+4$ (c) $y=3x^4+4$ (d) $y=3x^2+5$

ANSWER ANY 7 7 x 2 = 14

21) For each of the following differential equations, determine its order, degree (if exists)

$\sqrt{\frac{dy}{dx}} - 4\frac{dy}{dx} - 7x = 0$

22) For each of the following differential equations, determine its order, degree (if exists)

$\frac{d^2y}{dx^2} = xy + cos\left(\frac{dy}{dx}\right)$

23) Express each of the following physical statements in the form of differential equation.

The population P of a city increases at a rate proportional to the product of population and to the difference between 5,00,000 and the population.

24) Show that each of the following expressions is a solution of the corresponding given differential equation.

$y = 2x^2$; xy' = 2y

25) Show that y = a cos bx is a solution of the differential equation $\frac{d^2y}{dx^2} + b^2y = 0$.

26) Solve the following differential equations:

$sin\frac{dy}{dx} = a, y(0) = 1$

27) Solve the following differential equations:

x cos y dy = e^x(x log x + 1)dx

28) (x^2+y^2)dy = xy dx. It is given that y(1) = 1 and $y(x_0)$ = e. Find the value of x_0.

29) $\frac{dy}{dx} = \frac{sin^2x}{1+x^3} - \frac{3x^2}{1+x^3}y$

30) $x\frac{dy}{dx} + y = xlogx$

ANSWER ANY 7 7 x 3 = 21

31) Determine the order and degree (if exists) of the following differential equations:

$\left(\frac{d^4y}{dx^4}\right)^3 + 4\left(\frac{dy}{dx}\right)^7 + 6y = 5cos3x$

32) Determine the order and degree (if exists) of the following differential equations:

$3\left(\frac{d^2y}{dx^2}\right) = [4 + \left(\frac{dy}{dx}\right)^2]^{\frac{3}{2}}$

33) Determine the order and degree (if exists) of the following differential equations:

dy + (xy − cos x)dx = 0

34) Show that $x^2+y^2 = r^2$, where r is a constant, is a solution of the differential equation $\frac{dy}{dx}$ =-$\frac{x}{y}$.

35) Show that y=mx+$\frac{7}{m}$,m≠0 is a solution of the differential equation xy'+7$\frac{1}{y'}$ -y=0.

36) Solve $(x^2 - 3y^2)$ dx xydy= 0.

37) Solve (2x + 3y)dx + (y − x)dy = 0.

38) Solve $(1 + 2e^{x/y})dx + 2e^{x/y}\left(1 - \frac{x}{y}\right)dy = 0$

39) A radioactive isotope has an initial mass 200mg, which two years later is 50mg . Find the expression for the amount of the isotope remaining at any time. What is its half-life? (half-life means the time taken for the radioactivity of a specified isotope to fall to half its original value).

40)

In a murder investigation, a corpse was found by a detective at exactly 8 p.m. Being alert, the detective also measured the body temperature and found it to be 70°F. Two hours later, the detective measured the body temperature again and found it to be 60°F. If the room temperature is 50°F, and assuming that the body temperature of the person before death was 98.6°F, at what time did the murder occur?

[log(2.43)=0.88789; log(0.5)=-0.69315]

ANSWER ANY 7 7 x 5 = 35

41) Solve $(1+x^2)\frac{dy}{dx} = 1 + y^2$

42) Solve y ' = $\sin^2$ (x − y +)1.

43) Solve $\frac{dy}{dx} = \frac{x-y+5}{2(x-y)+7}$.

44) Solve $[y(1-x\tan x)+x^2\cos x]dx-dy=0$

45) Find the differential equation of the family of circles passing through the points (a,0) and (−a,0).

46) Solve : $\frac{dy}{dx} = \sqrt{4x+2y-1}$

47) Solve: $\frac{dy}{dx}$ $=(3x+y+4)^2$.

48) Solve $(1+x^3)\frac{dy}{dx}$ $+6x^2y=1+x^2$.

49) Solve $ye^y dx=(y^3+2xe^y)dy$

50) Solve: $(1 + e^{2x})\, dy + (1 + y^2)e^x\, dx = 0$ when $y(0) = 1$

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