12th Standard

Maths

Reg.No. : ☐☐☐☐☐☐

Exam Time : 03:00:00 Hrs

Total Marks : 90

Answer ALL

20 x 1 = 20

1) Let X be random variable with probability density function

$$f(x) = \begin{cases} \dfrac{2}{x^3} & 0 < x \geq l \\ 0 & 1 \leq x < 2l \end{cases}$$

Which of the following statement is correct
(a) both mean and variance exist
(b) mean exists but variance does not exist
(c) both mean and variance do not exist
(d) variance exists but Mean does uot exist

2) A rod of length 2l is broken into two pieces at random. The probability density function of the shorter of the two pieces is

$$f(x) = \begin{cases} \dfrac{2}{x^3} & 0 < x > l \\ 0 & 1 \leq x < 2l \end{cases}$$

(a) $\dfrac{l}{2}, \dfrac{l^2}{3}$
(b) $\dfrac{l}{2}, \dfrac{l^2}{6}$
(c) $1, \dfrac{l^2}{12}$
(d) $\dfrac{1}{2}, \dfrac{l^2}{12}$

3) Consider a game where the player tosses a sixsided fair die. H the face that comes up is 6, the player wins Rs.36, otherwise he loses Rs. k^2 , where k is the face that comes up k = {I, 2,3,4, 5}.
The expected amount to win at this game in Rs is
(a) $\dfrac{19}{6}$
(b) $-\dfrac{19}{6}$
(c) $\dfrac{3}{2}$
(d) $-\dfrac{3}{2}$

4) A pair of dice numbered 1, 2, 3, 4, 5, 6 of a six-sided die and 1, 2, 3, 4 of a four-sided die is roUed and the sum is determined. Let the random variable X denote this sum. Then the number of elements in the inverse image of 7 is
(a) 1
(b) 2
(c) 3
(d) 4

5) A random variable X has binomial distribution with n = 25 and p = 0.8 then standard deviation of X is
(a) 6
(b) 4
(c) 3
(d) 2

6) Let X represent the difference between the number of heads and the number of tails obtained when a coin is tossed n times. Then the possible values of X are
(a) i + 2n, i = 0,1,2... n
(b) 2i- n, i = 0,1,2... n
(c) n - i, i = 0,1,2... n
(d) 2i + 2n, i = 0, 1, 2...n

7)
If the function $f(x) = \dfrac{1}{12}$ for. a < x < b, represents a probability density function of a continuous random variable X, then which of the followingcannot be the value of a and b?
(a) 0 and 12
(b) 5 and 17
(c) 7 and 19
(d) 16 and 24

8) Four buses carrying 160 students from the same school arrive at a football stadium. The buses carry, respectively, 42, 36, 34, and 48 students. One of the students is randomly selected. Let X denote the number of students that were on the bus carrying the randomly selected student One of the 4 bus drivers is also randomly selected. Let Y denote the number of students on that bus. Then E[X] and E[Y] respectively are

(a) 50,40   (b) 40,50   (c) 40.75,40   (d) 41,41

9) Two coins are to be flipped. The first coin will land on heads with probability 0.6, the second with probability 0.5. Assume that the results of the flips are independent, and let X equal the total number of heads that result The value of E[X] is

(a) 0.11   (b) 1.1   (c) 1.1   (d) 1

10) On a multiple-choice exam with 3 possible destructives for each of the 5 questions, the probability that a student will get 4 or more correct answers just by guessing is

(a) $\dfrac{11}{243}$ (b) $\dfrac{3}{8}$ (c) $\dfrac{1}{243}$ (d) $\dfrac{5}{243}$

11) If $P\{X = 0\} = 1 - P\{X = I\}$. If $E[X] = 3\,Var(X)$, then $P\{X = 0\}$.

(a) $\dfrac{2}{3}$ (b) $\dfrac{2}{5}$ (c) $\dfrac{1}{5}$ (d) $\dfrac{1}{3}$

12) If X is a binomial randam variable with expected value 6 and variance 2.4, then P(X=5) is

(a) $\left(\dfrac{10}{5}\right)^2\left(\dfrac{3}{5}\right)^6\left(\dfrac{2}{5}\right)^4$ (b) $\left(\dfrac{10}{5}\right)\left(\dfrac{3}{5}\right)^5$ (c) $\left(\dfrac{10}{5}\right)\left(\dfrac{3}{5}\right)^4\left(\dfrac{2}{5}\right)^6$ (d) $\left(\dfrac{10}{5}\right)\left(\dfrac{3}{5}\right)^5\left(\dfrac{2}{5}\right)^5$

13) The random variable X has the probability density function $f(x) = \begin{cases} ax + b & 0 < x < 1 \\ 0 & otherwise \end{cases}$ and $E(X) = \dfrac{7}{12}$

then a and b are respectively.

(a) 1 and $\dfrac{1}{2}$ (b) $\dfrac{1}{2}$ and 1   (c) 2 and 1   (d) 1 and 2

14) Suppose that X takes on one of the values 0, 1, and 2. If for some constant k, $P(X = I) = k\,P(X = i\text{-}I)$ i = 1, 2 and $P(X = 0) = \dfrac{1}{7}$ then the value of k is

(a) 1   (b) 2   (c) 3   (d) 4

15) Which of the following is a discrete random variable?

I. The number of cars crossing a particular signal in a day

II. The number of customers in a queue-to buy train tickets at a moment.

III. The time taken to complete a telephone call.

(a) I and II   (b) II only   (c) III only   (d) II and III

16) If f(x) = is a probability density function of a random variable, then the value of a is

(a) 1   (b) 2   (c) 3   (d) 4

17) The probability mass function of a random variable is defined as:

x	-2	-1	0	1	2
f(x)	k	2k	3k	4k	5k

(a) $\dfrac{1}{15}$ (b) $\dfrac{1}{10}$ (c) $\dfrac{1}{3}$ (d) $\dfrac{2}{3}$

18) Let X have a Bernoulli distribution with mean 0.4, then the variance of (2X - 3) is

(a) 0.24 (b) 0.48 (c) 0.6 (d) 0.96

19) If in 6 trials, X is a binomial variate which foUows the relation $9P(X = 4) = P(X = 2)$, then the probability of success is

(a) 0.125 (b) 0.25 (c) 0.375 (d) 0.75

20) A computer salesperson knows from his past experience that he seUs computers to one in every twenty customers who enter the showroom. What is the probability that he will seU a computer to exactly two of the next three customers?

(a) $\dfrac{57}{20^3}$ (b) $\dfrac{57}{20^2}$ (c) $\dfrac{19^3}{20^3}$ (d) $\dfrac{57}{20}$

Answer any 7 questions in which question no. 30 is compulsory   7 x 2 = 14

21) An urn contains 5 mangoes and 4 apples Three fruits are taken at randaom If the number of apples taken is a random variable, then find the values of the random variable and number of points in its inverse images.

22) A six sided die is marked '2' on one face, '3' on two ofits faces, and '4' on remaining three faces. The die is thrown twice. If X denotes the total score in two throws, find the values of the random variable and number of points in its inverse images.

23) The cumulative distribution function of a discrete random variable is given by

$$F(x) = \begin{cases} 0 & -\infty < x < -10 \\ 0.15 & -1 \le x < 0 \\ 0.35 & 0 \le x < 1 \\ 0.60 & 1 \le x < 2 \\ 0.85 & 2 \le x < 3 \\ 1 & 3 \le x < \infty \end{cases}$$

Find
(i) the probability mass function
(ii) $P(X < 1)$ and
(iii) $P(X \sim 2$

24) If X is the random variable with probability density functionj{x) given by,

$$f(x) = \begin{cases} x + 1 & -1 \le x < 0 \\ -x + 1 & 0 \le x < 1 \\ 0 & otherwise \end{cases}$$

then find
(i) the distribution function F(x)
(ii) $P(-0.5 \le X \le 0.5)$

25)

For the random variable X with the given probability mass function as below, find the mean and variance.

$$f(x) = \begin{cases} \dfrac{1}{2}e^{-\frac{x}{2}} & for \quad x > 0 \\ \\ 0 & otherwise \end{cases}$$

26) If μ and σ^2 are the mean and variance of the discrete random variable X, and $E(X+3)=10$ and $E(X+3)^2 = 116$, find μ and a^2•

27) If X- B(n, p) such that $4P(X = 4) = P(X = 2)$ and $n = 6$ • Find the distribution, mean and standard deviation of X.

28) For the random variable X with the given probability mass function as below, find the mean and variance

$$f(x) = \begin{cases} 2(x-1) & 1 < x < 2 \\ 0 & otherwise \end{cases}$$

29) Four fair coins are tossed once. Find the probability mass function, mean and variance for number of heads occurred.

30) Compute $P(X = k)$ for the binomial distribution, B(n,p) where

$n=9, p = \dfrac{1}{2}$,k=7

Answer any 7 questions in which question no. 40 is compulsory   7 x 3 = 21

31) An urn contains 2 white balls and 3 red balls. A sample of 3 balls are chosen at random from the urn. If X denotes the number of red balls chosen, find the values taken by the random variable X and its number of inverse images

32) Two balls are chosen randomly from an urn containing 6 white and 4 black balls. Suppose that we win Rs.30 for each black ball selected and we lose Rs.20 for each white ball selected. If X denotes the winning amount, then find the values of X and number of points in its inverse images.
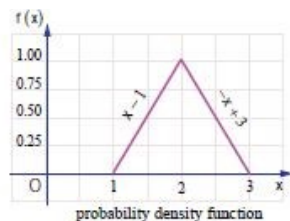
33)
Find the constant C such that the function $f(x) = \begin{cases} Cx^2 & 1 < x < 4 \\ 0 & Otherwise \end{cases}$ is a density function, and compute

(i) P(1.5 < X < 3.5)
(ii) P(X ≤2)
(iii) P(3 < X) .

34) If X is the random variable with probability density function $f(x)$ given by,

probability density function

$$f(x) = \begin{cases} x - 1 & 1 \le x < 2 \\ -x + 3 & 2 \le x < 3 \\ 0 & Otherwise \end{cases}$$

find (i) the distribution function F (x)

(ii) P(1.5 ≤ X ≤ 2.5)

35) If X is the random variable with distribution function F(x) given by,

$$F(x) = \begin{cases} 0 & x < 0 \\ x & 0 \le x < 1 \\ 1 & 1 \le x \end{cases}$$

then find (i) the probability density function f (x) (ii) P(0.2≤ X ≤0.7)

36) Suppose that f (x) given below represents a probability mass function

x	1	2	3	4	5	6
f(x)	c^2	$2c^2$	$3c^2$	$4c^2$	c	$2c$

Find

(i) the value of c

(ii) Mean and variance.

37) The mean and variance of a binomial variate X are respectively 2 and 1.5. Find

(i) P(X = 0)

(ii) P(X =1)

(iii) P(X ≥1)

38) A multiple choice examination has ten questions, each question has four distractors with exactly one correct answer. Suppose a student answers by guessing and if X denotes the number of correct answers, find (i) binomial distribution (ii) probability that the student will get seven correct answers (iii) the probability of getting at least one correct answer

39) On the average, 20% of the products manufactured by ABC Company are found to be defective. If we select 6 of these products at random and X denote the number of defective products find the probability that (i) two products are defective (ii) at most one product is defective (iii) at least two products are defective.

40) Suppose two coins are tossed once. If X denotes the number of tails,

(i) write down the sample space

(ii) find the inverse image of 1

(iii) the values of the random variable and number of elements in its inverse images

A mapping $X(.)$ from S to $\mathbf{R}$
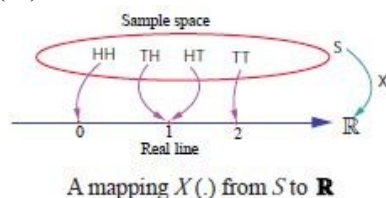
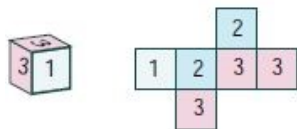ANSWER ANY 7   7 x 5 = 35

41) A six sided die is marked '1' on one face, '2' on two of its faces, and '3' on remaining three faces. The die is rolled twice. If X denotes the total score in two throws.

(i) Find the probability mass function.

(ii) Find the cumulative distribution function.

(iii) Find P(3 ≤ X< 6) (iv) Find P(X ≥ 4) .

42) A random variable X has the following probability mass function

x	1	2	3	4	5	6
f(x)	k	2k	6k	5k	6k	10k

Find

(i) $P(2 < X < 6)$

(ii) $P(2 \leq X < 5)$

(iii) $P(X \leq 4)$

(iv) $P(3 < X)$

43)

The probability density function of random variable X is given by $f(x) = \begin{cases} k & 1 \leq x \leq 5 \\ 0 & otherwise \end{cases}$ Find

(i) Distribution function

(ii) $P(X < 3)$

(iii) $P(2 < X < 4)$

(iv) $P(3 \leq X)$

44) Two balls are chosen randomly from an urn containing 8 white and 4 black balls. Suppose that we win Rs 20 for each black ball selected and we lose Rs10 for each white ball selected. Find the expected winning amount and variance

45) Find the probability mass function f (x) of the discrete random variable X whose cumulative distribution function F(x) is given by

$$F(x) = \begin{cases} 0 & -\infty < x < -2 \\ 0.25 & -2 \leq x < -1 \\ 0.60 & -1 \leq x < 0 \\ 0.90 & 0 \leq x < 1 \\ 1 & 1 \leq x < \infty \end{cases}$$

Also find (i) P(X < 0) and (ii)$P(X \geq -1$

46) Find the mean and variance of a random variable X , whose probability density function is

$$f(x) = \begin{cases} \lambda e^{-2x} & for \geq 0 \\ 0 & otherwise \end{cases}$$

47) Two fair coins are tossed simultaneously (equivalent to a fair coin is tossed twice). Find the probability mass function for number of heads occurred.

48) A pair of fair dice is rolled once. Find the probability mass function to get the number of fours.

49) If the probability mass function f (x) of a random variable X isx

x	1	2	3	4
f (x)	$\dfrac{1}{12}$	$\dfrac{5}{12}$	$\dfrac{5}{12}$	$\dfrac{1}{12}$

find (i) its cumulative distribution function, hence find
(ii) $P(X \leq 3)$ and,
(iii) $P(X \geq 2)$

****