

## Applications of Integration full test

12th Standard

Maths

Reg.No. : 

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Exam Time : 03:00:00 Hrs

Total Marks : 90

ANSWER ALL

20 x 1 = 20

- 1) The value of  $\int_{-4}^4 \left[ \tan^{-1} \left( \frac{x^2}{x^4+1} \right) + \tan^{-1} \left( \frac{x^4+1}{x^2} \right) \right] dx$  is  
 (a)  $\pi$  (b)  $2\pi$  (c)  $3\pi$  (d)  $4\pi$
- 2) The value of  $\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \left( \frac{2x^7-3x^5+7x^3-x+1}{\cos^2 x} \right) dx$  is  
 (a) 4 (b) 3 (c) 2 (d) 0
- 3) If  $f(x) = \int_0^x t \cos t \, dt$ , then  $\frac{dx}{dx}$   
 (a)  $\cos x - x \sin x$  (b)  $\sin x + x \cos x$  (c)  $x \cos x$  (d)  $x \sin x$
- 4) The area between  $y^2 x = 4$  and its latus rectum is  
 (a)  $\frac{2}{3}$  (b)  $\frac{4}{3}$  (c)  $\frac{8}{3}$  (d)  $\frac{5}{3}$
- 5) The value of  $\int_0^1 x(1-x)^{99} dx$  is  
 (a)  $\frac{1}{11000}$  (b)  $\frac{1}{10100}$  (c)  $\frac{1}{10010}$  (d)  $\frac{1}{10001}$
- 6) The value of  $\int_0^\pi \frac{dx}{1+5^{\cos x}}$  is  
 (a)  $\frac{\pi}{2}$  (b)  $\pi$  (c)  $\frac{3\pi}{2}$  (d)  $2\pi$
- 7) The value of  $\frac{(n+2)}{(n)} = 90$  then n is  
 (a) 10 (b) 5 (c) 8 (d) 9
- 8) The value of  $\int_0^{\frac{\pi}{6}} \cos^3 3x dx$   
 (a)  $\frac{2}{3}$  (b)  $\frac{2}{9}$  (c)  $\frac{1}{9}$  (d)  $\frac{1}{3}$
- 9) The value of  $\int_0^\pi \sin^4 x dx$  is  
 (a)  $\frac{3\pi}{10}$  (b)  $\frac{3\pi}{8}$  (c)  $\frac{3\pi}{4}$  (d)  $\frac{3\pi}{2}$
- 10) The value of  $\int_0^\infty e^{-3x} x^2 dx$  is  
 (a)  $\frac{7}{27}$  (b)  $\frac{5}{27}$  (c)  $\frac{4}{27}$  (d)  $\frac{2}{27}$
- 11) If  $\int_a^a \frac{1}{4+x^2} dx = \frac{\pi}{8}$  then a is  
 (a) 4 (b) 1 (c) 3 (d) 2
- 12) The volume of solid of revolution of the region bounded by  $y^2 = x(a-x)$  about x-axis is  
 (a)  $\pi a^2$  (b)  $\frac{\pi a^2}{4}$  (c)  $\frac{\pi a^2}{5}$  (d)  $\frac{\pi a^2}{6}$
- 13) If  $f(x)f(x) = \int_1^x \frac{e^{\sin u}}{u} du, x > 1$  and  $\int_1^3 \frac{e^{\sin x^2}}{x} dx = \frac{1}{2}[f(a) - f(1)]$ , then one of the possible value of a is  
 (a) 3 (b) 6 (c) 9 (d) 5
- 14) The value of  $\int_0^1 (\sin^{-1} x)^2 dx$   
 (a)  $\frac{\pi^2}{4} - 1$  (b)  $\frac{\pi^2}{4} + 2$  (c)  $\frac{\pi^2}{4} + 1$  (d)  $\frac{\pi^2}{4} - 2$
- 15) The value of  $\int_0^a (\sqrt{a^2 - x^2})^2 dx$   
 (a)  $\frac{\pi a^2}{16}$  (b)  $\frac{3\pi a^4}{16}$  (c)  $\frac{3\pi a^2}{8}$  (d)  $\frac{3\pi a^4}{8}$
- 16) If  $\int_0^x f(t) dt = x + \int_x^1 t f(t) dt$  then the value of f(1) is

- (a) (b) 2 (c) 1 (d)  $\frac{3}{4}$
- 17) The value of  $\int_0^{\frac{2}{3}} \frac{dx}{\sqrt{4-9x^2}}$  is  $\frac{3}{4}$
- (a)  $\frac{\pi}{6}$  (b)  $\frac{\pi}{2}$  (c)  $\frac{\pi}{4}$  (d)  $\pi$
- 18) The value of  $\int_{-1}^2 |x|dx$  is  $\frac{3}{2}$
- (a)  $\frac{1}{2}$  (b)  $\frac{3}{2}$  (c)  $\frac{5}{2}$  (d)  $\frac{7}{2}$
- 19) For any value of  $n \in \mathbb{Z}$ ,  $\int_0^\pi \cos^{2n} x \cos^3[(2n+1)x]$  is 0
- (a)  $\frac{\pi}{2}$  (b)  $\pi$  (c) 0 (d) 2
- 20) The value of  $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin^2 x \cos x dx$  is  $\frac{2}{3}$
- (a)  $\frac{3}{2}$  (b)  $\frac{1}{2}$  (c) 0 (d)  $\frac{2}{3}$

ANSWER ANY 7

7 x 2 = 14

21) Find an approximate value of  $\int_1^{1.5} x^2 dx$  by applying the right-end rule with the partition  $\{1.1, 1.2, 1.3, 1.4, 1.5\}$ .

22) Evaluate the following integrals as the limits of sums.

$$\int_1^2 4x^2 - 1) dx$$

23) Evaluate the following definite integrals:

$$\int_0^{\frac{\pi}{2}} e^x \left( \frac{1+\sin x}{1+\cos x} \right) dx$$

24) Evaluate the following integrals using properties of integration:

$$\int_0^{2\pi} x \log \left( \frac{3+\cos x}{3-\cos x} \right) dx$$

25) Evaluate the following integrals using properties of integration:

$$\int_0^1 |5x - 3| dx$$

26) Evaluate the following integrals using properties of integration:

$$\int_{\frac{\pi}{8}}^{\frac{3\pi}{8}} \frac{1}{1+\sqrt{\tan x}} dx$$

27) Evaluate the following:

$$\int_0^{\frac{1}{2}} \frac{e^a \sin^{-1} x \sin^{-1} x}{\sqrt{1-x^2}} dx$$

28) Evaluate the following

$$\int_0^{\pi/4} \sin^6 x dx$$

29) Find the area of the region bounded by  $3x - 2y + 6 = 0$ ,  $x = -3$ ,  $x = 1$  and x-axis.

30) Find the area of the region bounded by the line  $y = 2x + 5$  and the parabola  $y = x^2 - 2x$ .

ANSWER ANY 7

7 x 3 = 21

31) Evaluate:  $\int_0^{\frac{\pi}{3}} \frac{\sec x \tan x}{1+\sec^2 x} dx$

32) Evaluate:  $\int_0^{\frac{\pi}{2}} \frac{\cos \theta}{(1+\sin \theta)(2+\sin \theta)} d\theta$

33) Show that  $\int_0^{\frac{\pi}{2}} \frac{dx}{4+5\sin x} = \frac{1}{3} \log_e 2$

34) Prove that  $\int_0^{\frac{\pi}{4}} \frac{\sin 2x dx}{\sin^4 x + \cos^4 x} = \frac{\pi}{4}$

35) Evaluate  $\int_0^\pi \frac{x}{1+\sin x} dx$

36) Prove that  $\int_0^{\frac{\pi}{4}} \log(1+\tan x) dx = \frac{\pi}{8} \log 2$ .

- 37) Evaluate  $\int_0^{\frac{\pi}{2}} \frac{dx}{\sqrt{1 + 5 \sin^2 x}}$
- 38) Find the volume of a right-circular cone of base radius  $r$  and height  $h$ .
- 39) Find the volume of the solid formed by revolving the region bounded by the ellipse  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ ,  $a > b$  about the major axis.

- 40) Find, by integration, the volume of the solid generated by revolving about y-axis the region bounded between the curve  $y = \frac{3}{4}\sqrt{x^2 - 16}$ ,  $x \geq 4$  the y-axis, and the lines  $y = 1$  and  $y = 6$ .

ANSWER ANY 7

7 x 5 = 35

- 41) Evaluate  $\int_1^4 (2x^2 - 3) dx$ , as the limit of a sum
- 42) Evaluate  $\int_0^1 e^{-2x} (1 + x - 2x^3) dx$
- 43) Evaluate  $\int_0^{\frac{\pi}{2}} (\sin^2 x + \cos^4 x) dx$
- 44) Evaluate  $\int_0^{2a} x^2 \sqrt{2ax - x^2} dx$
- 45) Find the area of the region bounded by the ellipse  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$
- 46) Find the area of the region bounded by x-axis, the curve  $y = \cos x$ , the lines  $x = 0$  and  $x = \pi$ .
- 47) Find the area of the region bounded between the parabolas  $y^2 x = 4$  and  $x^2 y = 4$ .
- 48) Find the area of the region in the first quadrant bounded by the parabola  $y^2 x = 4$ , the line  $x + y = 3$  and y-axis
- 49) Find, by integration, the area of the region bounded by the lines  $5x - 2y = 15$ ,  $x + y + 4 = 0$  and the x-axis
- 50) Using integration find the area of the region bounded by triangle ABC, whose vertices A, B, and C are  $(-1, 1)$ ,  $(3, 2)$ , and  $(0, 5)$  respectively

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