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Differentials and Partial Derivatives FULL TEST

12th Standard

Maths

Reg.No. : | | | | | | |

Exam Time : 03:00:00 Hrs Total Marks : 90

ANSWER ALL 20 x 1 = 20

1) A circular template has a radius of 10 cm. The measurement of radius has an approximate error of 0.02 cm. Then the percentage error in calculating area of this template is
(a) 0.2% (b) 0.4% (c) 0.04% (d) 0.08%

2) The percentage error of fifth root of 31 is approximately how many times the percentage error in 31?
(a) $\frac{1}{31}$ (b) $\frac{1}{5}$ (c) 5 (d) 31

3) If u (x, y) = e^{x2+y2}, then $\frac{\partial u}{\partial x}$ is equal to
(a) e^{x2+y2} (b) 2xu (c) x^2u (d) y^2u

4) If v (x, y) = log (ex + ev), then $\frac{\partial v}{\partial x} + \frac{\partial v}{\partial y}$ is equal to
(a) $e^x + e^y$ (b) $\frac{1}{e^x+e^y}$ (c) 2 (d) 1

5) If w (x, y) = xy, x > 0, then $\frac{\partial w}{\partial x}$ is equal to
(a) $x^y \log x$ (b) y log x (c) yx^{y-1} (d) x log y

6) If f (x, y) = e^{xy} then $\frac{\partial^2 f}{\partial x \partial y}$ is equal to
(a) xye^{xy} (b) $(1 +xy)e^{xy}$ (c) $(1 +y)e^{xy}$ (d) $(1 + x)e^{xy}$

7) If we measure the side of a cube to be 4 cm with an error of 0.1 cm, then the error in our calculation of the volume is
(a) 0.4 cu.cm (b) 0.45 cu.cm (c) 2 cu.cm (d) 4.8 cu.cm

8) The change in the surface area S = $6x^2$ of a cube when the edge length varies from x_o to x_o+ dx is
(a) 12 x_o+dx (b) 12x_o dx (c) 6x_o dx (d) 6x_o+ dx

9) The approximate change in the volume V of a cube of side x metres caused by increasing the side by 1% is
(a) 0.3xdx m^3 (b) 0.03 xm^3 (c) 0.03.x^2 m^3 (d) 0.03x^3m^3

10) If g(x, y) = $3x^2$ - 5y + 2y, x(t) = e^t and y(t) = cos t, then $\frac{dg}{dt}$ is equal to
(a) $6e^{2t}$+5 sin t - 4 cos t sin t (b) $6e^{2t}$- 5 sin t + 4 cos t sin t (c) $3e^{2t}$+ 5 sin t + 4 cos t sin t (d) $3e^{2t}$ - 5 sin t + 4 cos t sin t

11) If f(x) = $\frac{x}{x+1}$ then its differential is given by
(a) $\frac{-1}{(x+1)^2}dx$ (b) $\frac{1}{(x+1)^2}dx$ (c) $\frac{1}{1+x}dx$ (d) $\frac{-1}{1+x}dx$

12) If u(x, y) = x^2+ 3xy + y - 2019, then $\frac{\partial u}{\partial x}_{(4, -5)}$ is equal to
(a) -4 (b) -3 (c) -7 (d) 13

13) Linear approximation for g(x) = cos x at x=$\frac{-\pi}{2}$ is
(a) x + $\frac{-\pi}{2}$ (b) - x + $\frac{\pi}{2}$ (c) x - $\frac{\pi}{2}$ (d) - x + $\frac{\pi}{2}$

14) If w (x, y, z) = x^2 (v - z) + y^2 (z - x) + z^2(x - y), then $\frac{\partial w}{\partial x} + \frac{\partial w}{\partial y} + \frac{\partial w}{\partial z}$ is
(a) xy + yz + zx (b) x(y + z) (c) y(z + x) (d) 0

15) If (x,y, z) = xy +yz +zx, then f_x - f_z is equal to
(a) z - x (b) y - z (c) x - z (d) y - x

16) If u = log $\sqrt{x^2 + y^2}$, then $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}$ is

(a) (b) 0 (c) u (d) 2u

17) If $u = \sqrt{x^2+y^2}$ log (xy + y^3 + z^3 - 3xyz) then $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z}$ =

(a) $\frac{3}{x+y+z}$ (b) x+y+z (c) $\frac{-9}{(x+y+z)^2}$ (d) $\frac{-9}{(x+y+z)^2}$

18) If u = y^x then $\frac{\partial u}{\partial y}$ = ...

(a) xy^{x-1} (b) yx^{y-1} (c) 0 (d) 1

19) If u = $(\frac{y}{x})$ then x $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}$ = ...

(a) 0 (b) 1 (c) 2u (d) u

20) If u = y sin x then $\frac{\partial^2 u}{\partial x \partial y}$ = ...

(a) cos x (b) cos y (c) sin x (d) 0

ANSWER ANY 7 7 x 2 = 14

21) Use the linear approximation to find approximate values of $\sqrt[3]{26}$

22) A sphere is made of ice having radius 10 cm. Its radius decreases from 10 cm to 9-8 cm. Find approximations for the following:
change in the volume

23) The time T, taken for a complete oscillation of a single pendulum with length l, is given by the equation T = $2\pi\sqrt{\frac{1}{g}}$, where g is a constant. Find the approximate percentage error in the calculated value of T corresponding to an error of 2 percent in the value of 1

24) Find df for f(x) = x^2 + x 3 and evaluate it for
x = 3 and dx = 0.02

25) An egg of a particular bird is very nearly spherical. If the radius to the inside of the shell is 5 mm and radius to the outside of the shell is 5.3 mm, find the volume of the shell approximately.

26) Assume that the cross section of the artery of human is circular. A drug is given to a patient to dilate his arteries. If the radius of an artery is increased from 2 mm to 2.1 mm, how much is cross-sectional area increased approximately?

27) Evaluate $\lim_{(x,y) \to (1,2)}$, if the limit exists, where $(x, y) = \frac{3x^2-xy}{x^2+y^2+3}$

28) Let g(x, y) = $\frac{e^y sinx}{x}$, for x ≠ 0 and g(0, 0) = 1. Show that g is continuous at (0,0).

29) Find the partial derivatives of the following functions at the indicated point
h (x, y, z) = x sin (xy) + z^2x, $\left(2, \frac{\pi}{4}, 1\right)$

30) If U(x, y, z) = log (x^3 + y^3 + z^3), find $\frac{\partial U}{\partial x} + \frac{\partial U}{\partial y} + \frac{\partial U}{\partial z}$

ANSWER ANY 7 7 x 3 = 21

31) Use linear approximation to find an approximate value of $\sqrt{9.2}$ without using a calculator.

32) Let us assume that the shape of a soap bubble is a sphere. Use linear approximation to approximate the increase in the surface area of a soap bubble as its radius increases from 5 cm to 5.2 cm. Also, calculate the percentage error.

33) Consider g(x,y) = $\frac{2x^2y}{x^2+y^2}$, if (x,y) ≠ (0,0) and g(0,0) = 0 Show that g is continuous on R^2

34) If w(x, y, z) = x^2 y + y^2z + z^2x, x, y, z∈R, find the differential dw .

35) Let U(x, y, z) = x^2 − xy + 3 sin z, x, y, z ∈ R Find the linear approximation for U at (2,−1,0).

36)

f(x,y) = $\frac{xy}{x^2+y^2}$, (x,y) ≠ (0,0) and f (0,0) = 0 Show that f is not continuous at f, -(0,0) and continuous at all other points of R^2

37) If u=$\sin^{-1}\left(\frac{x+y}{\sqrt{x}+\sqrt{y}}\right)$, Show that x $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \frac{1}{2}tanu$

38) Use differentials to find the value of $\sqrt{0.037}$

39) Find the approximate value of f (3.02) where f(x) = $3x^2$ + 5x +3.

40) If w = xy + z where x = cos t; y = sin t; z = t find $\frac{dw}{dt}$

ANSWER ANY 7 7 x 5 = 35

41) Let f , g : (a,b)→R be differentiable functions. Show that d(fg) = fdg + gdf

42) Let g(x) = x^2 + sin x. Calculate the differential dg.

43) If the radius of a sphere, with radius 10 cm, has to decrease by 0 1. cm, approximately how much will its volume decrease?

44) Let f (x, y) = 0 if xy ≠ 0 and f (x, y) =1 if xy = 0.
(i) Calculate: $\frac{\partial f}{\partial x}(0,0)$, $\frac{\partial f}{\partial y}(0,0)$.
(ii) Show that f is not continuous at (0,0)

45) Let F(x, y) = x^3 y + y^2x + 7 for all (x, y)∈ R^2. Calculate $\frac{\partial F}{\partial x}$ (-1,3) and $\frac{\partial F}{\partial y}$ (-2,1).

46) Let f(x, y) = sin(xy^2) + e^{x3+5y} for all ∈ R^2. Calculate $\frac{\partial f}{\partial x}$, $\frac{\partial f}{\partial y}$, $\frac{\partial^2 f}{\partial y \partial x}$ and $\frac{\partial^2 f}{\partial x \partial y}$

47) Let w(x, y) = xy+ $\frac{e^y}{y^2+1}$ for all (x, y) ∈ R^2. Calculate $\frac{\partial^2 w}{\partial y \partial x}$ and $\frac{\partial^2 w}{\partial x \partial y}$

48) Let (x, y) = e^{-2y} cos(2x) for all (x, y) ∈ R^2. Prove that u is a harmonic function in R^2.

49) Let g(x,y)= x^3 - yx + sin(x+y), x(t) = e^{3t}, y(t) = t^2, t ∈ R. Find $\frac{dg}{dt}$

50) If u = $\tan^{-1}\left(\frac{x^3+y^3}{x-y}\right)$
Prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}$ sin 2u.

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