

Ordinary Differential Equations FULL TEST

12th Standard

Maths

Reg.No. :

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Exam Time : 03:00:00 Hrs

Total Marks : 90

ANSWER ALL

20 x 1 = 20

- The order and degree of the differential equation $\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^{1/3} + x^{1/4} = 0$ are respectively
 (a) 2, 3 (b) 3, 3 (c) 2, 6 (d) 2, 4
- The differential equation representing the family of curves $y = A\cos(x + B)$, where A and B are parameters, is
 (a) $\frac{d^2y}{dx^2} - y = 0$ (b) $\frac{d^2y}{dx^2} + y = 0$ (c) $\frac{d^2y}{dx^2} = 0$ (d) $\frac{d^2x}{dy^2} = 0$
- The order and degree of the differential equation $\sqrt{\sin x}(dx + dy) = \sqrt{\cos x}(dx - dy)$
 (a) 1,2 (b) 2,2 (c) 1,1 (d) 2,1
- The order of the differential equation of all circles with centre at (h, k) and radius 'a' is
 (a) 2 (b) 3 (c) 4 (d) 1
- The general solution of the differential equation $\frac{dy}{dx} = \frac{y}{x}$ is
 (a) $xy = k$ (b) $y = k \log x$ (c) $y = kx$ (d) $\log y = kx$
- The solution of the differential equation $2x\frac{dy}{dx} - y = 3$ represents
 (a) straight lines (b) circles (c) parabola (d) ellipse
- The integrating factor of the differential equation $\frac{dy}{dx} + y = \frac{1+y}{\lambda}$ is
 (a) $\frac{x}{e^\lambda}$ (b) $\frac{e^\lambda}{x}$ (c) λe^x (d) e^x
- The integrating factor of the differential equation $\frac{dy}{dx} + P(x)y = Q(x)$ is x, then P(x)
 (a) x (b) $\frac{x^2}{2}$ (c) $\frac{1}{x}$ (d) $\frac{1}{x^2}$
- The degree of the differential equation $y y(x) = 1 + \frac{dy}{dx} + \frac{1}{1.2} \left(\frac{dy}{dx}\right)^2 + \frac{1}{1.2.3} \left(\frac{dy}{dx}\right)^3 + \dots$ is
 (a) 2 (b) 3 (c) 1 (d) 4
- If p and q are the order and degree of the differential equation $y = \frac{dy}{dx} + x^3 \left(\frac{d^2y}{dx^2}\right) + xy = \cos x$, When
 (a) $p < q$ (b) $p = q$ (c) $p > q$ (d) p exists and q does not exist
- The solution of the differential equation $\frac{dy}{dx} + \frac{1}{\sqrt{1-x^2}} = 0$
 (a) $y + \sin^{-1} x = c$ (b) $x + \sin^{-1} y = 0$ (c) $y^2 + 2 \sin^{-1} x = c$ (d) $x^2 + 2 \sin^{-1} y = c$
- The solution of the differential equation $\frac{dy}{dx} = 2xy$ is
 (a) $y = Ce^{x^2}$ (b) $y = 2x^2 + C$ (c) $y = Ce^{-x^2} + C$ (d) $y = x^2 + C$
- The solution of $\frac{dy}{dx} = 2^{y-x}$ is
 (a) $2^x + 2^y = C$ (b) $2^x - 2^y = C$ (c) $\frac{1}{2^x} - \frac{1}{2^y} = C$ (d) $x + y = C$
- The solution of the differential equation $\frac{dy}{dx} = \frac{y}{x} + \frac{\phi\left(\frac{y}{x}\right)}{\phi'\left(\frac{y}{x}\right)}$ is
 (a) $x\phi\left(\frac{y}{x}\right) = k$ (b) $\phi\left(\frac{y}{x}\right) = kx$ (c) $y\phi\left(\frac{y}{x}\right) = k$ (d) $\phi\left(\frac{y}{x}\right) = ky$
- If $\sin x$ is the integrating factor of the linear differential equation $\frac{dy}{dx} + Pt = Q$, Then P is
 (a) $\log \sin x$ (b) $\cos x$ (c) $\tan x$ (d) $\cot x$
- The number of arbitrary constants in the general solutions of order n and n+1 are respectively

- (a) $n-1, n$ (b) $n, n+1$ (c) $n+1, n+2$ (d) $n+1, n$
- 17) Integrating factor of the differential equation $\frac{dy}{dx} = \frac{x+y+1}{x+1}$ is
 (a) $\frac{1}{x+1}$ (b) $x+1$ (c) $\frac{1}{\sqrt{x+1}}$ (d) $\sqrt{x+1}$
- 18) The population P in any year t is such that the rate of increase in the population is proportional to the population. Then
 (a) $P = Ce^{kt}$ (b) $P = Ce^{-kt}$ (c) $P = Ckt$ (d) $P = C$
- 19) P is the amount of certain substance left in after time t . If the rate of evaporation of the substance is proportional to the amount remaining, then
 (a) $P = Ce^{kt}$ (b) $P = ce^{-kt}$ (c) $P = Ckt$ (d) $Pt = C$
- 20) The slope at any point of a curve $y = f(x)$ is given by $\frac{dy}{dx} = 3x^2$ and it passes through $(-1, 1)$. Then the equation of the curve is
 (a) $y = x^3 + 2$ (b) $y = 3x^2 + 4$ (c) $y = 3x^4 + 4$ (d) $y = 3x^2 + 5$

ANSWER ANY 7

7 x 2 = 14

- 21) For each of the following differential equations, determine its order, degree (if exists)
 $\sqrt{\frac{dy}{dx}} - 4\frac{dy}{dx} - 7x = 0$
- 22) For each of the following differential equations, determine its order, degree (if exists)
 $\frac{d^2y}{dx^2} = xy + \cos\left(\frac{dy}{dx}\right)$
- 23) Express each of the following physical statements in the form of differential equation.
 The population P of a city increases at a rate proportional to the product of population and to the difference between 5,00,000 and the population.
- 24) Show that each of the following expressions is a solution of the corresponding given differential equation.
 $y = 2x^2$; $xy' = 2y$
- 25) Show that $y = a \cos bx$ is a solution of the differential equation $\frac{d^2y}{dx^2} + b^2y = 0$.
- 26) Solve the following differential equations:
 $\sin \frac{dy}{dx} = a, y(0) = 1$
- 27) Solve the following differential equations:
 $x \cos y \, dy = e^x(x \log x + 1)dx$
- 28) $(x^2 + y^2)dy = xy \, dx$. It is given that $y(1) = 1$ and $y(x_0) = e$. Find the value of x_0 .
- 29) $\frac{dy}{dx} = \frac{\sin^2 x}{1+x^3} - \frac{3x^2}{1+x^3}y$
- 30) $x \frac{dy}{dx} + y = x \log x$

ANSWER ANY 7

7 x 3 = 21

- 31) Determine the order and degree (if exists) of the following differential equations:
 $\left(\frac{d^4y}{dx^4}\right)^3 + 4\left(\frac{dy}{dx}\right)^7 + 6y = 5\cos 3x$
- 32) Determine the order and degree (if exists) of the following differential equations:
 $3\left(\frac{d^2y}{dx^2}\right) = \left[4 + \left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}}$
- 33) Determine the order and degree (if exists) of the following differential equations:
 $dy + (xy - \cos x)dx = 0$
- 34) Show that $x^2 + y^2 = r^2$, where r is a constant, is a solution of the differential equation $\frac{dy}{dx} = -\frac{x}{y}$.

35) Show that $y=mx+\frac{7}{x}$, $m \neq 0$ is a solution of the differential equation $xy'+7\frac{1}{y}-y=0$.

36) Solve $(x^2-3y^2)dx+\frac{7}{xy}dy=0$.

37) Solve $(2x+3y)dx+(y-x)dy=0$.

38) Solve $(1+2e^{x/y})dx+2e^{x/y}\left(1-\frac{x}{y}\right)dy=0$

39) A radioactive isotope has an initial mass 200mg, which two years later is 50mg. Find the expression for the amount of the isotope remaining at any time. What is its half-life? (half-life means the time taken for the radioactivity of a specified isotope to fall to half its original value).

40) In a murder investigation, a corpse was found by a detective at exactly 8 p.m. Being alert, the detective also measured the body temperature and found it to be 70°F. Two hours later, the detective measured the body temperature again and found it to be 60°F. If the room temperature is 50°F, and assuming that the body temperature of the person before death was 98.6°F, at what time did the murder occur?

[$\log(2.43)=0.88789$; $\log(0.5)=-0.69315$]

ANSWER ANY 7

7 x 5 = 35

41) Solve $(1+x^2)\frac{dy}{dx}=1+y^2$

42) Solve $y'=\sin^2(x-y+1)$.

43) Solve $\frac{dy}{dx}=\frac{x-y+5}{2(x-y)+7}$.

44) Solve $[y(1-x \tan x)+x^2 \cos x]dx-dy=0$

45) Find the differential equation of the family of circles passing through the points (a,0) and (-a,0).

46) Solve: $\frac{dy}{dx}=\sqrt{4x+2y-1}$

47) Solve: $\frac{dy}{dx}=(3x+y+4)^2$.

48) Solve $(1+x^3)\frac{dy}{dx}+6x^2y=1+x^2$.

49) Solve $ye^y dx=(y^3+2xe^y)dy$

50) Solve: $(1+e^{2x})dy+(1+y^2)e^x dx=0$ when $y(0)=1$
