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APPLICATION OF VECTOR ALGEBRA FULL TEST

12th Standard

Maths

Reg.No. : [| | | | |]

Exam Time : 03:00:00 Hrs — Total Marks : 90

ANSWER ALL — 20 x 1 = 20

1) If a vector $\vec{\alpha}$ lies in the plane of $\vec{\beta}$ and $\vec{\gamma}$, then

(a) $[\vec{\alpha}, \vec{\beta}, \vec{\gamma}] = 1$ (b) $[\vec{\alpha}, \vec{\beta}, \vec{\gamma}] = -1$ (c) $[\vec{\alpha}, \vec{\beta}, \vec{\gamma}] = 0$ (d) $[\vec{\alpha}, \vec{\beta}, \vec{\gamma}] = 2$

2) $\vec{a}.\vec{b} = \vec{b}.\vec{c} = \vec{c}.\vec{a} = 0$, then the value of $[\vec{a}, \vec{b}, \vec{c}]$ is

(a) $|\vec{a}||\vec{b}||\vec{c}|$ (b) $\frac{1}{3}|\vec{a}||\vec{b}||\vec{c}|$ (c) 1 (d) -1

3) The volume of the parallelepiped with its edges represented by the vectors $\hat{i}+\hat{j}, \hat{i}+2\hat{j}, \hat{i}+\hat{j}+\pi\hat{k}$ is

(a) $\frac{\pi}{2}$ (b) $\frac{\pi}{3}$ (c) π (d) $\frac{\pi}{4}$

4) If $\vec{a}$ and $\vec{b}$ are unit vectors such that $[\vec{a}, \vec{b}, \vec{a} \times \vec{b}] = \frac{\pi}{4}$, then the angle between $\vec{a}$ and $\vec{b}$ is

(a) $\frac{\pi}{6}$ (b) $\frac{\pi}{4}$ (c) $\frac{\pi}{3}$ (d) $\frac{\pi}{2}$

5) If $\vec{a}, \vec{b}, \vec{c}$ are three non-coplanar vectors such that $\vec{a} \times (\vec{b} \times \vec{c}) = \frac{\vec{b}+\vec{c}}{\sqrt{2}}$, then the angle between

(a) $\frac{\pi}{2}$ (b) $\frac{3\pi}{6}$ (c) $\frac{\pi}{4}$ (d) π

6) If the volume of the parallelepiped with $\vec{a} \times \vec{b}, \vec{b} \times \vec{c}, \vec{c} \times \vec{a}$ as coterminous edges is 8 cubic units, then the volume of the parallelepiped with $(\vec{a} \times \vec{b}) \times (\vec{b} \times \vec{c}), (\vec{b} \times \vec{c}) \times (\vec{c} \times \vec{a})$ and $(\vec{c} \times \vec{a}) \times (\vec{a} \times \vec{b})$ as coterminous edges is,

(a) 8 cubic units (b) 512 cubic units (c) 64 cubic units (d) 24 cubic units

7) If $\vec{a} \times (\vec{b} \times \vec{c}) = (\vec{a} \times \vec{b}) \times \vec{c}$ where $\vec{a}, \vec{b}, \vec{c}$ are any three vectors such that $\vec{a}, \vec{b} \neq 0$ and $\vec{a}.\vec{b} \neq 0$ then $\vec{a}$ and $\vec{c}$ are

(a) perpendicular (b) parallel (c) inclined at an angle $\frac{\pi}{3}$ (d) inclined at an angle $\frac{\pi}{6}$

8) If $\vec{a} = 2\hat{i} + 3\hat{j} - \hat{k}, \vec{b} = \hat{i} + 2\hat{j} + 5\hat{k}, \vec{c} = 3\hat{i} + 5\hat{j} - \hat{k}$ then a vector perpendicular to $\vec{a}$ and lies in the plane containing $\vec{b}$ and $\vec{c}$ is

(a) $-17\hat{i} + 21\hat{j} - 97\hat{k}$ (b) $17\hat{i} + 21\hat{j} - 123\hat{k}$ (c) $-17\hat{i} - 21\hat{j} + 197\hat{k}$ (d) $-17\hat{i} - 21\hat{j} - 197\hat{k}$

9) The coordinates of the point where the line $\vec{r} = (6\hat{i} - \hat{j} - 3\hat{k}) + t(\hat{i} + 4\hat{j})$ meets the plane $\vec{r} = (\hat{i} + \hat{j} - \hat{k}) = 3$ are

(a) (2,1,0) (b) (7,1,7) (c) (1,2,6) (d) (5,1,1)

10) Distance from the origin to the plane 3x - 6y + 2z 7 = 0 is

(a) 0 (b) 1 (c) 2 (d) 3

11) The distance between the planes x + 2y + 3z + 7 = 0 and 2x + 4y + 6z + 7 = 0

(a) $\frac{\sqrt{7}}{2\sqrt{2}}$ (b) $\frac{7}{2}$ (c) $\frac{\sqrt{7}}{2}$ (d) $\frac{7}{2\sqrt{2}}$

12) The vector equation $\vec{r} = (\hat{i} - 2\hat{j} - \hat{k}) + t(6\hat{i} - \hat{k})$ represents a straight line passing through the points

(a) (0,6,1)– and (1,2,1) (b) (0,6,-1) and (1,4,2) (c) (1,-2,-1) and (1,4,-2) (d) (1,-2,-1) and (0,-6,1)

13) If the distance of the point (1,1,1) from the origin is half of its distance from the plane x + y + z + k =0, then the values of k are

(a) ±3 (b) ±6 (c) -3, 9 (d) 3, 9

14) If the length of the perpendicular from the origin to the plane 2x + 3y + λz =1, λ > 0 is $\frac{1}{5}$ then the value of λ is

(a) $2\sqrt{3}$ (b) $3\sqrt{2}$ (c) 0 (d) 1

15) Let $\vec{a}$,$\vec{b}$ and $\vec{c}$ be three non- coplanar vectors and let $\vec{p}, \vec{q}, \vec{r}$ be the vectors defined by the relations $\vec{P} = \frac{\vec{b} \times \vec{c}}{[\vec{a}\vec{b}\vec{c}]}, \vec{q} = \frac{\vec{c} \times \vec{a}}{[\vec{a}\vec{b}\vec{c}]}, \vec{r} = \frac{\vec{a} \times \vec{b}}{[\vec{a}\vec{b}\vec{c}]}$ Then the value of $(\vec{a} + \vec{b}).\vec{p} + (\vec{b} + \vec{c}).\vec{q} + (\vec{c} + \vec{a}).\vec{r} =$

(a) 0 (b) 1 (c) 2 (d) 3

16) The number of vectors of unit length perpendicular to the vectors $(\hat{i} + \hat{j})$ and $(\hat{j} + \hat{k})$ is

(a) 1 (b) 2 (c) 3 (d) ∞

17) If $\vec{d} = \vec{a} \times (\vec{b} \times \vec{c}) + \vec{b} \times (\vec{c} \times \vec{a}) + \vec{c} \times (\vec{a} + \vec{b})$, then

(a) $|\vec{d}|$ (b) $\vec{d} = \vec{a} + \vec{b} + \vec{c}$ (c) $\vec{d} = \vec{0}$ (d) a, b, c are coplanar

18) If $\vec{a} = \hat{i} + 2\hat{j} + 3\hat{k}$, $\vec{b} = -\hat{i} + 2\hat{j} + \hat{k}$, $\vec{c} = 3\hat{i} + \hat{j}$ then $\vec{a} + (-\vec{b})$ will be perpendiculur to $\vec{c}$ only when t =

(a) 5 (b) 4 (c) 3 (d) $\frac{7}{3}$

19) If $\vec{a} = |\vec{a}|\vec{e}$ then $\vec{e}.\vec{e}$

(a) 0 (b) e (c) 1 (d) $\vec{0}$

20) If the work done by a force $\vec{F} = \hat{i} + m\hat{j} - \hat{k}$ in moving the point of application from(1, 1, 1) to (3, 3, 3) along a straight line is 12 units, then m is

(a) 5 (b) 2 (c) 3 (d) 6

ANSWER 7 ONLY 7 x 2 = 14

21) If $\vec{a}, \vec{b}, \vec{c}$ are three vectors, prove that $[\vec{a} + \vec{c}, \vec{a} + \vec{b}, \vec{a} + \vec{b} + \vec{c}] = [\vec{a}, \vec{b}, \vec{c}]$

22) If $\vec{a} = \hat{i} - 2\hat{j} + 3\hat{k}, \vec{6} = 2\hat{i} + \hat{j} - 2\hat{k}, \vec{c} = 3\hat{i} + 2\hat{j} + \hat{k}$ find $\vec{a}.(\vec{b} \times \vec{c})$.

23) Find the volume of the parallelepiped whose coterminous edges are represented by the vectors $-6\hat{i} + 14\hat{j} + 10\hat{k}, 14\hat{i} - 10\hat{j} - 6\hat{k}$ and $2\hat{i} + 4\hat{j} - 2\hat{k}$

24) Prove that $(\vec{a}.(\vec{b} \times \vec{c}))\vec{a} = (\vec{a} \times \vec{b}) \times (\vec{a} \times \vec{c})$

25) Show that the lines $\frac{x-1}{4} = \frac{2-y}{6} = \frac{z-4}{12}$ and $\frac{x-1}{4} = \frac{2-y}{6} = \frac{z-4}{12}$ are parallel.

26) Find the angle between the following lines.

$\vec{r} = (4\hat{i} - \hat{j}) + t(\hat{i} + 2\hat{j} - 2\hat{k})$, $\hat{r} = (\hat{i} + 2\hat{j} - 2\hat{k}) + s(-\hat{i} - 2\hat{j} + 2\hat{k})$

27) Show that the points (2, 3, 4),(−1, 4, 5) and (8,1, 2) are collinear.

28) Find the angle between the planes $\vec{r}.(\hat{i} + \hat{j} - 2\hat{k}) = 3$ and 2x - 2y + z =2

29) Find the distance of a point (2,5, −3) from the plane $\vec{r}.(6\hat{i} - 3\hat{j} + 2\hat{k})$ =5

30) Find the distance between the parallel planes x+2y-2z=0 and 2x+4y-4z+5=0

ANSWER 7 ONLY 7 x 3 = 21

31) A particle acted upon by constant forces $2\hat{j} + 5\hat{j} + 6\hat{k}$ and $-\hat{i} - 2\hat{j} - \hat{k}$ is displaced from the point (4, −3, −2) to the point (6, 1, −3) . Find the total work done by the forces.

32) Prove by vector method that if a line is drawn from the centre of a circle to the midpoint of a chord, then the line is perpendicular to the chord.

33) Prove by vector method that an angle in a semi-circle is a right angle.

34) Prove by vector method that the area of the quadrilateral ABCD having diagonals AC and BD is

$\frac{1}{2}\left|\vec{AC} \times \vec{BC}\right|$.

35) Forces of magnit $5\sqrt{2}$ and $5\sqrt{2}$ units acting in the directions $3\hat{i} + 4\hat{j} + 5\hat{k}$ and $10\hat{i} + 6\hat{j} - 8\hat{k}$ respectively, act on a particle which is displaced from the point with position vector $4\hat{i} + 3\hat{j} - 2\hat{k}$ to the point with position vector $6\hat{i} + \hat{j} - 3\hat{k}$. Find the work done by the forces.

36) Find the magnitude and direction cosines of the torque of a force represented by $3\hat{i} + 4\hat{j} - 5\hat{k}$ about the point with position vector $2\hat{i} - 3\hat{j} + 4\hat{k}$ acting through a point whose position vector is $4\hat{i} + 2\hat{j} - 3\hat{k}$.

37) Prove that $[\vec{a} \times \vec{b}, \vec{b} \times \vec{c}, \vec{c} \times \vec{a}] = [\vec{a}, \vec{b}, \vec{c}]^2$

38) For any vector $\vec{a}$, prove that $\hat{i} \times (\vec{a} \times \hat{i}) + \hat{j} \times (\vec{a} \times \hat{j}) + \hat{k} \times \vec{a} \times \hat{k} = 2\vec{a}$.

39) Show that the straight lines x + 1= 2y = −12z and x = y + 2 = 6z − 6 are skew and hence find the shortest distance between them.

40) Find the equation of the plane passing through the intersection of the planes $\vec{r}.(\hat{i} + \hat{j} + \hat{k}) + 1 = 0$ and $\vec{r}.(2\hat{i} - 3\hat{j} + 5\hat{k}) = 2$ and the point (-1, 2, 1)

ANSWER 7 ONLY 7 x 5 = 35

41) By vector method, prove that cos(α + β) = cos α cos β - sin α sin β

42) If $\vec{a} = \vec{i} - \vec{j}, \vec{b} = \hat{i} - \hat{j} - 4\hat{k}, \vec{c} = 3\hat{j} - \hat{k}$ and $\vec{d} = 2\hat{i} + 5\hat{j} + \hat{k}$

(i) $(\vec{a} \times \vec{b}) \times (\vec{c} \times \vec{d}) = [\vec{a}, \vec{b}, \vec{d}]\vec{c} - [\vec{a}, \vec{b}, \vec{c}]\vec{d}$

(ii) $(\vec{a} \times \vec{b}) \times (\vec{c} \times \vec{d}) = [\vec{a}, \vec{c}, \vec{d}]\vec{b} - [\vec{b}, \vec{c}, \vec{d}]\vec{a}$

43) Determine whether the pair of straight lines $\vec{r}(2\hat{i} + 3\hat{j} - \hat{k}) + t(2\hat{i} + 3\hat{j} + 2\hat{k})$, $\vec{r} = (2\hat{j} - 3\hat{k}) + s(\hat{i} + 2\hat{j} + 3\hat{k})$ are parallel. Find the shortest distance between them.

44) Show that the lines $\frac{x-3}{3} = \frac{y-3}{-1} = z - 1$ and $\frac{x-6}{2} = \frac{z-1}{3}, y - 2 = 0$ intersect. Also find the point of intersection

45) Find the vector parametric, vector non-parametric and Cartesian form of the equation of the plane passing through the points (-1, 2, 0), (2, 2, -1)and parallel to the straight line $\frac{x-1}{1} = \frac{2y+1}{2} = \frac{z+1}{-1}$

46) Find parametric form of vector equation and Cartesian equations of the plane passing through the points (2, 2,1), (1, −2,3) and parallel to the straight line passing through the points (2,1,−3) and (−1,5,−8)

47) Find the parametric vector, non-parametric vector and Cartesian form of the equations of the plane passing through the points (3,6,−2), (−1,−2,6) , and (6,−4,−2).

48) If the straight lines $\frac{x-1}{2} = \frac{y+1}{\lambda} = \frac{z}{2}$ and $\frac{x-1}{2} = \frac{y+1}{\lambda} = \frac{z}{\lambda}$ are coplanar, find λ and equations of the planes containing these two lines.

49) Show that the lines $\vec{r} = (\hat{i} - 3\hat{j} - 5\hat{k}) + s(3\hat{i} + 5\hat{j} + 7\hat{k})$ and $\vec{r} = (2\hat{i} + 4\hat{j} + 6\hat{k}) + t(\hat{i} + 4\hat{j} + 7\hat{k})$ are coplanar. Also,find the non-parametric form of vector equation of the plane containing these lines

50) Find the coordinates of the foot of the perpendicular and length of the perpendicular from the point (4,3,2) to the plane x + 2y + 3z = 2

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