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Instructions : (1) check the question paper for fairness of printing. if there is any lack of fairness, inform the hall supervisor immediately.(2) use blue or black ink to write and underline and pencil to draw diagrams.

Exam Time : 03:00:00 Hrs

Total Marks : 90

20 x 1 = 20

PART – I

ANSWER ALL THE QUESTIONS.

- If $|\text{adj}(\text{adj } A)| = |A|^9$, then the order of the square matrix A is
(a) 3 (b) 4 (c) 2 (d) 5
- If A is a 3×3 non-singular matrix such that $AA^T = A^T A$ and $B = A^{-1}A^T$, then $BB^T =$
(a) A (b) B (c) I (d) B^T
- $i^n + i^{n+1} + i^{n+2} + i^{n+3}$ is
(a) 0 (b) 1 (c) -1 (d) i
- The value of $\sum_{i=1}^{13} (i^n + i^{n-1})$ is
(a) $1+i$ (b) i (c) 1 (d) 0
- A zero of $x^3 + 64$ is
(a) 0 (b) 4 (c) 4i (d) -4
- If f and g are polynomials of degrees m and n respectively, and if $h(x) = (f \circ g)(x)$, then the degree of h is
(a) mn (b) m+n (c) m^n (d) n^m
- The value of $\sin^{-1}(\cos x), 0 \leq x \leq \pi$ is
(a) $\pi - x$ (b) $x - \frac{\pi}{2}$ (c) $\frac{\pi}{2} - x$ (d) $\pi - x$
- If $\sin^{-1} x + \sin^{-1} y = \frac{2\pi}{3}$; then $\cos^{-1} x + \cos^{-1} y$ is equal to
(a) $\frac{2\pi}{3}$ (b) $\frac{\pi}{3}$ (c) $\frac{\pi}{6}$ (d) π
- The equation of the circle passing through (1,5) and (4,1) and touching y-axis is $x^2 + y^2 - 5x - 6y + 9 + \lambda(4x + 3y - 19) = 0$ where λ is equal to
(a) $0, -\frac{40}{9}$ (b) 0 (c) $\frac{40}{9}$ (d) $-\frac{40}{9}$
- The eccentricity of the hyperbola whose latus rectum is 8 and conjugate axis is equal to half the distance between the foci is
(a) $\frac{4}{3}$ (b) $\frac{4}{\sqrt{3}}$ (c) $\frac{2}{\sqrt{3}}$ (d) $\frac{3}{2}$
- If \vec{a} and \vec{b} are parallel vectors, then $[\vec{a}, \vec{c}, \vec{b}]$ is equal to
(a) 2 (b) -1 (c) 1 (d) 0
- If a vector \vec{a} lies in the plane of $\vec{\beta}$ and $\vec{\gamma}$, then
(a) $[\vec{a}, \vec{\beta}, \vec{\gamma}] = 1$ (b) $[\vec{a}, \vec{\beta}, \vec{\gamma}] = -1$ (c) $[\vec{a}, \vec{\beta}, \vec{\gamma}] = 0$ (d) $[\vec{a}, \vec{\beta}, \vec{\gamma}] = 2$
- The volume of a sphere is increasing in volume at the rate of $3\pi \text{ cm}^3 \text{ sec}$. The rate of change of its radius when radius is $\frac{1}{2}$ cm
(a) 3 cm/s (b) 2 cm/s (c) 1 cm/s (d) $\frac{1}{2} \text{ cm/s}$

14)

A balloon rises straight up at 10 m/s. An observer is 40 m away from the spot where the balloon left the ground. Find the rate of change of the balloon's angle of elevation in radian per second when the balloon is 30 metres above the ground.

- (a) $\frac{3}{25}$ radians/sec (b) $\frac{4}{25}$ radians/sec (c) $\frac{1}{5}$ radians/sec (d) $\frac{1}{3}$ radians/sec

15) A circular template has a radius of 10 cm. The measurement of radius has an approximate error of 0.02 cm. Then the percentage error in calculating area of this template is

- (a) 0.2% (b) 0.4% (c) 0.04% (d) 0.08%

16) The percentage error of fifth root of 31 is approximately how many times the percentage error in 31?

- (a) $\frac{1}{31}$ (b) $\frac{1}{5}$ (c) 5 (d) 31

17)

The value of $\int_{-4}^4 \left[\tan^{-1} \left(\frac{x^2}{x^4+1} \right) + \tan^{-1} \left(\frac{x^4+1}{x^2} \right) \right] dx$ is

- (a) π (b) 2π (c) 3π (d) 4π

18)

The order and degree of the differential equation $\frac{d^2y}{dx^2} + \left(\frac{dy}{dx} \right)^{1/3} + x^{1/4} = 0$ are respectively

- (a) 2, 3 (b) 3, 3 (c) 2, 6 (d) 2, 4

19) Let X be random variable with probability density function

$$f(x) = \begin{cases} \frac{2}{x^3} & 0 < x \leq l \\ 0 & 1 \leq x < 2l \end{cases}$$

Which of the following statement is correct

- (a) both mean and variance exist (b) mean exists but variance does not exist (c) both mean and variance do not exist (d) variance exists but Mean does not exist

20) A binary operation on a set S is a function from

- (a) $S \rightarrow S$ (b) $(S \times S) \rightarrow S$ (c) $S \rightarrow (S \times S)$ (d) $(S \times S) \rightarrow (S \times S)$

PART II

7 x 2 = 14

Answer any 7 questions in which question no. 30 is compulsory

21)

If $\text{adj } A = \begin{bmatrix} -1 & 2 & 2 \\ 1 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$, find A^{-1} .

22) If $z_1 = 1 - 3i$, $z_2 = 4i$, and $z_3 = 5$, show that $(z_1 + z_2) + z_3 = z_1 + (z_2 + z_3)$

23) If α , β , γ and δ are the roots of the polynomial equation $2x^4 + 5x^3 - 7x^2 + 8 = 0$, find a quadratic equation with integer coefficients whose roots are $\alpha + \beta + \gamma + \delta$ and $\alpha\beta\gamma\delta$.

24) Find the principal value of

$$\sin^{-1} \left(\frac{1}{\sqrt{2}} \right)$$

25) Find the general equation of the circle whose diameter is the line segment joining the points $(-4, -2)$ and $(1, 1)$.

26)

Show that the vectors $\hat{i} + 2\hat{j} - 3\hat{k}$, $\hat{i} + 2\hat{j} - 3\hat{k}$ and $3\hat{i} + \hat{j} - \hat{k}$

27) A camera is accidentally knocked off an edge of a cliff 400 ft high. The camera falls a distance of $s = 16t^2$ in t seconds.

How long does the camera fall before it hits the ground?

28)

Use the linear approximation to find approximate values of

$$\sqrt[4]{15}$$

- 29) An urn contains 5 mangoes and 4 apples. Three fruits are taken at random. If the number of apples taken is a random variable, then find the values of the random variable and number of points in its inverse images.
- 30) Examine the binary operation (closure property) of the following operations on the respective sets (if it is not, make it binary)

$$a * b = \left(\frac{a-1}{b-1} \right), \forall a, b \in Q$$

PART III

7 x 3 = 21

Answer any 7 questions in which question no. 40 is compulsory

- 31) Verify the property $(A^T)^{-1} = (A^{-1})^T$ with $A = \begin{bmatrix} 2 & 9 \\ 1 & 7 \end{bmatrix}$.
- 32) Given the complex number $z = 2 + 3i$, represent the complex numbers in Argand diagram z , iz , and $z + iz$.
- 33) If the sides of a cubic box are increased by 1, 2, 3 units respectively to form a cuboid, then the volume is increased by 52 cubic units. Find the volume of the cuboid.
- 34) Find the domain of the following
- $$\sin^{-1} \left(\frac{x^2 + 1}{2x} \right)$$
- 35) A circle of radius 3 units touches both the axes. Find the equations of all possible circles formed in the general form.
- 36) If D is the midpoint of the side BC of a triangle ABC, then show by vector method that $|\vec{AB}|^2 + |\vec{AC}|^2 = 2(|\vec{AD}|^2 + |\vec{BD}|^2)$.
- 37) Let us assume that the shape of a soap bubble is a sphere. Use linear approximation to approximate the increase in the surface area of a soap bubble as its radius increases from 5 cm to 5.2 cm. Also, calculate the percentage error.
- 38) Evaluate $\int_0^1 \frac{2x+7}{5x^2+9} dx$
- 39) An urn contains 2 white balls and 3 red balls. A sample of 3 balls are chosen at random from the urn. If X denotes the number of red balls chosen, find the values taken by the random variable X and its number of inverse images.
- 40) Verify the
- (i) closure property,
 - (ii) commutative property,
 - (iii) associative property
 - (iv) existence of identity and
 - (v) existence of inverse for the arithmetic operation + on
- Z_0 = the set of all even integers

PART – IV

7 x 5 = 35

ANSWER ALL THE QUESTIONS.

- 41) a) Evaluate $\int_1^4 (2x^2 - 3) dx$, as the limit of a sum
- (OR)
- b) Let A be $Q \setminus \{1\}$. Define * on A by $x * y = x + y - xy$. Is * binary on A? If so, examine the existence of identity, existence of inverse properties for the operation * on A.
- 42) a) Prove that among all the rectangles of the given area square has the least perimeter.

(OR)

- b) If the probability mass function $f(x)$ of a random variable X is

x	1	2	3	4
$f(x)$	$\frac{1}{12}$	$\frac{5}{12}$	$\frac{5}{12}$	$\frac{1}{12}$

find (i) its cumulative distribution function, hence find

(ii) $P(X \leq 3)$ and,

(iii) $P(X \geq 2)$

- 43) a) Find the foci, vertices and length of major and minor axis of the conic

$$4x^2 + 36y^2 + 40x - 288y + 532 = 0.$$

(OR)

- b) Let $F(x, y) = x^3 y + y^2 x + 7$ for all $(x, y) \in \mathbb{R}^2$. Calculate $\frac{\partial F}{\partial x}(-1, 3)$ and $\frac{\partial F}{\partial y}(-2, 1)$.

- 44) a) Solve the equation $3x^2 - 16x^2 + 23x - 6 = 0$ if the product of two roots is 1.

(OR)

- b) Solve: $\frac{dy}{dx} = (3x + y + 4)^2$.

- 45) a) Investigate the values of λ and m in the system of linear equations $2x + 3y + 5z = 9$, $7x + 3y - 5z = 8$, $2x + 3y + \lambda z = \mu$, have

(i) no solution

(ii) a unique solution

(iii) an infinite number of solutions.

(OR)

- b) Find (i) $\cos^{-1}\left(-\frac{1}{\sqrt{2}}\right)$

ii) $\cos^{-1}\left(\cos\left(-\frac{\pi}{3}\right)\right)$

iii) $\cos^{-1}\left(\cos\left(-\frac{7\pi}{6}\right)\right)$

- 46) a) Find the acute angle between $y = x^2$ and $y = (x - 3)^2$.

(OR)

- b) By vector method, prove that $\cos(\alpha + \beta) = \cos \alpha \cos \beta - \sin \alpha \sin \beta$

- 47) a)

If $A = \frac{1}{7} \begin{bmatrix} 6 & -3 & a \\ b & -2 & 6 \\ 2 & c & 3 \end{bmatrix}$ is orthogonal, find a , b and c , and hence A^{-1} .

(OR)

- b) If $z = x + iy$ and $\arg\left(\frac{z-i}{z+2}\right) = \frac{\pi}{4}$, then show that $x^2 + y^2 + 3x - 3y + 2 = 0$
