RAVI MATHS TUITION CENTER ,GKM COLONY, CH- 82. PH: 8056206308
COMPLEX NUMBER FULL CHAPTER TEST

12th Standard

Maths

Reg.No. : | | | | | | |

Exam Time : 03:00:00 Hrs — Total Marks : 90

ANSWER ALL — 20 x 1 = 20

1) $i^n+i^{n+1}+i^{n+2}+i^{n+3}$ is

(a) 0 (b) 1 (c) -1 (d) i

2) The conjugate of a complex number is $\frac{1}{i-2}$ /Then the complex number is

(a) $\frac{1}{i+2}$ (b) $\frac{-1}{i+2}$ (c) $\frac{-1}{i-2}$ (d) $\frac{1}{i-2}$

3) If |z-2+i|≤2, then the greatest value of |z| is

(a) $\sqrt{3}-2$ (b) $\sqrt{3}+2$ (c) $\sqrt{5}-2$ (d) $\sqrt{5}+2$

4) If |z|=1, then the value of $\frac{1+z}{1+\bar{z}}$ is

(a) z (b) $\bar{z}$ (c) $\frac{1}{z}$ (d) 1

5) If $|z_{1|}=1,|z_2|=2|z_3|=3$ and $|9z_1z_2+4z_1z_3+z_2z_3|=12$, then the value of $|z_1+z_2+z_3|$ is

(a) 1 (b) 2 (c) 3 (d) 4

6) If z is a complex number such that $z\varepsilon C/R$ and $z+\frac{1}{z}\epsilon R$ then|z| is

(a) 0 (b) 1 (c) 2 (d) 3

7) If $\frac{z-1}{z+1}$ is purely imaginary, then |z| is

(a) $\frac{1}{2}$ (b) 1 (c) 2 (d) 3

8) The principal argument of $\frac{3}{-1+i}$

(a) $\frac{-5\pi}{6}$ (b) $\frac{-2\pi}{3}$ (c) $\frac{-3\pi}{4}$ (d) $\frac{-\pi}{2}$

9) The principal argument of $(\sin 40°+i\cos 40°)^5$ is

(a) −110° (b) −70° (c) 70° (d) 110°

10)

The principal argument of the complex number $\frac{\left(1+i\sqrt{3}\right)^2}{4i\left(1-i\sqrt{3}\right)}$ is

(a) $\frac{2\pi}{3}$ (b) $\frac{\pi}{6}$ (c) $\frac{5\pi}{6}$ (d) $\frac{\pi}{2}$

11) If α and β are the roots of $x^2+x+1=0$, then $\alpha^{2020}+\beta^{2020}$ is

(a) -2 (b) -1 (c) 1 (d) 2

12) The product of all four values of $\left(cos\frac{\pi}{3}+isin\frac{\pi}{3}\right)^{\frac{3}{4}}$ is

(a) -2 (b) -1 (c) 1 (d) 2

13) If $\omega \neq 1$ is a cubic root of unity and $\begin{vmatrix} 1 & 1 & 1 \\ 1 & -\omega^2 & \omega^2 \\ 1 & \omega^2 & \omega^2 \end{vmatrix}$ =3k, then k is equal to

(a) 1 (b) -1 (c) $\sqrt{3i}$ (d) $-\sqrt{3i}$

14) The value of $\left(\frac{1+3\sqrt{i}}{1-\sqrt{3i}}\right)^{10}$

(a) $cis\frac{2\pi}{3}$ (b) $cis\frac{4\pi}{3}$ (c) $-cis\frac{2\pi}{3}$ (d) $-cis\frac{4\pi}{3}$

15) If $\omega = cis\frac{2\pi}{3}$, then the number of distinct roots of $\begin{vmatrix} z+1 & \omega & \omega^2 \\ \omega & z+\omega^2 & 1 \\ \omega^2 & 1 & z+\omega \end{vmatrix}$

(a) 1 (b) 2 (c) 3 (d) 4

16) The value of $\sum_{i=1}^{13}\left(i^n+i^{n-1}\right)$ is

(a) 1+ i (b) i (c) 1 (d) 0

17) The area of the triangle formed by the complex numbers z,iz, and z+iz in the Argand's diagram is

(a) $\frac{1}{2}|z|^2$ (b) $|z|^2$ (c) $\frac{3}{2}|z|^2$ (d) $2|z|^2$

18) If $z=\frac{\left(\sqrt{3}+i\right)^3(3i+4)^2}{(8+6i)^2}$, then |z| is equal to

(a) 0 (b) 1 (c) 2 (d) 3

19) If z is a non zero complex number, such that $2iz^2=\bar{z}$ then |z| is

(a) $\frac{1}{2}$ (b) 1 (c) 2 (d) 3

20) The solution of the equation |z|-z=1+2i is

(a) $\frac{3}{2} - 2i$ (b) $-\frac{3}{2} + 2i$ (c) $2 - \frac{3}{2}i$ (d) $2 + \frac{3}{2}i$

ANSWER ANY 7 7 x 2 = 14

21) If $z_1=2+5i$, $z_2=-3-4i$, and $z_3=1+i$, find the additive and multiplicate inverse of z_1,z_2 and z_3

22) If z=x+iy, find the following in rectangular form.

$Re\left(\frac{1}{z}\right)$

23) The complex numbers u,v, and w are related by $\frac{1}{u} = \frac{1}{v} + \frac{1}{w}$ If v=3−4i and w=4+3i, find u in rectangular form.

24) If |z|= 3, show that $7 \leq |z + 6 - 8i| \leq 13$.

25) If the area of the triangle formed by the vertices z,iz and z+iz is 50 square units, find the value of |z|

26) Find the square roots of 4+3i

27) Show that the following equations represent a circle, and, find its centre and radius|

$|z - 2 - i| = 3$

28) Write in polar form of the following complex numbers

$2 + i2\sqrt{3}$

29) If $cos\alpha + cos\beta + cos\gamma = sin\alpha + sin\beta + sin\gamma = 0$ then show that

(i) $cos3\alpha + cos3\beta + cos3\gamma = 3cos(\alpha + \beta + \gamma)$

(ii) $sin3\alpha + sin3\beta + sin3\gamma + sin3\gamma = 3sin(\alpha + \beta + \gamma)$

30) If $2cosa = x + \frac{1}{x}$ and $2cos\beta = y + \frac{1}{y}$, show that

i) $\frac{x}{y} + \frac{y}{x} = 2cos(\alpha - \beta)$.

ii) $xy - \frac{1}{xy} = 2isin(\alpha + \beta)$

iii) $\frac{x^m}{y^n} - \frac{y^n}{x^m} = 2isin(m\alpha - n\beta)$

iv) $x^my^n + \frac{1}{x^my^n} = 2cos(m\alpha + n\beta)$

ANSWER ANY 7 7 x 3 = 21

31) Simplify $\left(\frac{1+i}{1-i}\right)^3 - \left(\frac{1-i}{1+i}\right)^3$

32) If $\frac{z+3}{z-5i} = \frac{1+4i}{2}$, find the complex number z

33) Solve the equation $z^3+8i=0$,where $z \in \mathbb{C}$

34) Find the cube roots of unity.

35) Simplify $\left(\frac{1+cos2\theta+isin2\theta}{1+cos2\theta-isin2\theta}\right)^{30}$

36) If $z = (cos\theta + isin\theta)$, show that $z^n + \frac{1}{z^n} = 2cosn\theta$ and $z^n - \frac{1}{z^n} = 2isinn\theta$

37) Simplify $\left(sin\frac{\pi}{6} + icos\frac{\pi}{6}\right)^{18}$

38) Simplify the following:
$i\, i^2 i^3 \ldots i^{40}$

39) Given the complex number z=3+2i, represent the complex numbers z,iz, and z+ iz in one Argand diagram. Show that these complex numbers form the vertices of an isosceles right triangle.

40) If $\omega \neq 1$ is a cube root of unity, show that $(1+\omega)\left(1+\omega^2\right)\left(1+\omega^4\right)\left(1+\omega^8\right)\ldots\left(1+\omega^{2^{11}}\right) = 11$.

ANSWER ANY 7 $7 \times 5 = 35$

41) If z_1,z_2 and z_3 ,are complex numbers such that $|z_1|=|z_2|=|z_3|=|z_1+z_2+z_3|=1$ find the value of $\left|\frac{1}{z_1}+\frac{1}{z_2}+\frac{1}{z_3}\right|$

42) Show that the points $1, \frac{-1}{2}+i\frac{\sqrt{3}}{2},$ and $\frac{-1}{2}-i\frac{\sqrt{3}}{2}$ are the vertices of an equilateral triangle.

43) Let z_1,z_2, and z_3 be complex numbers such that $|z_1| = |z_2| = |z_3| = r > 0$ and $z_1+z_2+z_3 \neq 0$ prove that $\left|\frac{z_1z_2+z_2z_3+z_3z_1}{z_1+z_2+z_3}\right|$ =r

44) Find the quotient $\frac{2\left(cos\frac{9\pi}{4}+isin\frac{9\pi}{4}\right)}{4\left(cos\left(\frac{-3\pi}{2}\right)+isin\left(\frac{-3\pi}{2}\right)\right)}$ in rectangular form

45)

If z=x+iy and $\arg\left(\frac{z-1}{z+1}\right)=\frac{\pi}{2}$,then show that $x^2+y^2=1$.

46) Find the following $\left|\overline{(1+i)}(2+3i)(4i-3\right|$

47) Find the following $\left|\frac{i(2+i)^3}{(1+i)^2}\right|$

48) If z=x+iy is a complex number such that $\operatorname{Im}\left(\frac{2z+1}{iz+1}\right)=0$ show that the locus of z is $2x^2+2y^2+x-2y=0$

49) Show that $\left(2+i\sqrt{3}\right)^{10}+\left(2-i\sqrt{3}\right)^{10}$ is real ii) $\left(\frac{19+9i}{5-3i}\right)^{15}-\left(\frac{8+i}{I+2i}\right)^{15}$ is purely imaginary.

50) Solve the equation $z^3+27=0$.

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