RAVI MATHS TUITION CENTER, GKM COLONY, CH-82. PH- 8056206308

MATRICES AND DETERMINANTS FULL TEST

12th Standard

Maths

Reg.No. : | | | | | | |

Exam Time : 03:00:00 Hrs — Total Marks : 90

ANSWER ALL — 20 X 1 = 20

1) If $|adj(adj\ A)| = |A|^9$, then the order of the square matrix A is

(a) 3 (b) 4 (c) 2 (d) 5

2) If $A = \begin{bmatrix} 2 & 0 \\ 1 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 4 \\ 2 & 0 \end{bmatrix}$ then |adj (AB)| =

(a) -40 (b) -80 (c) -60 (d) -20

3) If $(AB)^{-1} = \begin{bmatrix} 12 & -17 \\ -19 & 27 \end{bmatrix}$ and $A^{-1} = \begin{bmatrix} 1 & -1 \\ -2 & 3 \end{bmatrix}$, then B^{-1} =

(a) $\begin{bmatrix} 2 & -5 \\ -3 & 8 \end{bmatrix}$ (b) $\begin{bmatrix} 8 & 5 \\ 3 & 2 \end{bmatrix}$ (c) $\begin{bmatrix} 3 & 1 \\ 2 & 1 \end{bmatrix}$ (d) $\begin{bmatrix} 8 & -5 \\ -3 & 2 \end{bmatrix}$

4) If $A = \begin{bmatrix} 1 & \tan\frac{\theta}{2} \\ -\tan\frac{\theta}{2} & 1 \end{bmatrix}$ and AB = I , then B =

(a) $(\cos^2\frac{\theta}{2})A$ (b) $(\cos^2\frac{\theta}{2})A^T$ (c) $(\cos^2\theta)I$ (d) $(\text{Sin}^2\frac{\theta}{2})A$

5) If $x^ay^b = e^m$, $x^cy^d = e^n$, $\Delta_1 = \begin{vmatrix} m & b \\ n & d \end{vmatrix}$, $\Delta_2 = \begin{vmatrix} a & m \\ c & n \end{vmatrix}$, $\Delta_3 = \begin{vmatrix} a & b \\ c & d \end{vmatrix}$, then the values of x and y are respectively,

(a) $e^{(\Delta_2/\Delta_1)}, e^{(\Delta_3/\Delta_1)}$ (b) $\log(\Delta_1/\Delta_3), \log(\Delta_2/\Delta_3)$ (c) $\log(\Delta_2/\Delta_1), \log(\Delta_3/\Delta_1)$ (d) $e^{(\Delta_1/\Delta_3)}, e^{(\Delta_2/\Delta_3)}$

6) If A^TA^{-1} is symmetric, then A^2 =

(a) A^{-1} (b) $(A^T)^2$ (c) A^T (d) $(A^{-1})^2$

7) If A is a non-singular matrix such that $A^{-1} = \begin{bmatrix} 5 & 3 \\ -2 & -1 \end{bmatrix}$, then $(A^T)^{-1}$ =

(a) $\begin{bmatrix} -5 & 3 \\ 2 & 1 \end{bmatrix}$ (b) $\begin{bmatrix} 5 & 3 \\ -2 & -1 \end{bmatrix}$ (c) $\begin{bmatrix} -1 & -3 \\ 2 & 5 \end{bmatrix}$ (d) $\begin{bmatrix} 5 & -2 \\ 3 & -1 \end{bmatrix}$

8) If $A = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$ and A(adj A) = $\begin{bmatrix} k & 0 \\ 0 & k \end{bmatrix}$ then adj (AB) is

(a) 0 (b) sin θ (c) cos θ (d) 1

9) If $A = \begin{bmatrix} 2 & 3 \\ 5 & -2 \end{bmatrix}$ be such that $\lambda A^{-1} = A$, then λ is

(a) 17 (b) 14 (c) 19 (d) 21

10) The rank of the matrix $\begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 4 & 6 & 8 \\ -1 & -2 & -3 & -4 \end{bmatrix}$ is

(a) 1 (b) 2 (c) 4 (d) 3

11) If ρ(A) = ρ([A | B]), then the system AX = B of linear equations is

(a) consistent and has a unique solution (b) consistent (c) consistent and has infinitely many solution (d) inconsistent

12) If 0 ≤ θ ≤ π and the system of equations x + (sinθ)y - (cosθ)z = 0, (cosθ)x - y +z = 0, (sinθ)x + y - z = 0 has a non-trivial solution then θ is

(a) $\frac{2\pi}{}$ (b) $\frac{3\pi}{}$ (c) $\frac{5\pi}{}$ (d) $\frac{\pi}{}$

13) If A is a 3 × 3 non-singular matrix such that $AA^T = A^TA$ and $B = A^{-1}A^T$, then $BB^T =$

(a) A (b) B (c) I (d) B^T

14) If A = $\begin{bmatrix} 3 & 5 \\ 1 & 2 \end{bmatrix}$, B = adj A and C = 3A, then $\frac{|adjB|}{|C|}$ = 6 4

(a) $\frac{1}{3}$ (b) $\frac{1}{9}$ (c) $\frac{1}{4}$ (d) 1

15) If A$\begin{bmatrix} 1 & -2 \\ 1 & 4 \end{bmatrix} = \begin{bmatrix} 6 & 0 \\ 0 & 6 \end{bmatrix}$, then A =

(a) $\begin{bmatrix} 1 & -2 \\ 1 & 4 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & 2 \\ -1 & 4 \end{bmatrix}$ (c) $\begin{bmatrix} 4 & 2 \\ -1 & 1 \end{bmatrix}$ (d) $\begin{bmatrix} 4 & -1 \\ 2 & 1 \end{bmatrix}$

16) If A = $\begin{bmatrix} 7 & 3 \\ 4 & 2 \end{bmatrix}$, then 9I - A =

(a) A^{-1} (b) $\frac{A^{-1}}{2}$ (c) $3A^{-1}$ (d) $2A^{-1}$

17) If A = $\begin{bmatrix} 3 & 1 & -1 \\ 2 & -2 & 0 \\ 1 & 2 & -1 \end{bmatrix}$ and $A^{-1} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$ then the value of a_{23} is

(a) 0 (b) -2 (c) -3 (d) -1

18) If A, B and C are invertible matrices of some order, then which one of the following is not true?

(a) adj A = $|A|A^{-1}$ (b) adj(AB) = (adj A)(adj B) (c) det $A^{-1} = (\det A)^{-1}$ (d) $(ABC)^{-1} = C^{-1}B^{-1}A^{-1}$

19) If A = [2 0 1] then the rank of AA^T is ______

(a) 1 (b) 2 (c) 3 (d) 0

20) The number of solutions of the system of equations 2x+y = 4, x - 2y = 2, 3x + 5y = 6 is

(a) 0 (b) 1 (c) 2 (d) infinitely many

ANSWER ANY 7 7 X 2= 14

21) If adj(A) = $\begin{bmatrix} 0 & -2 & 0 \\ 6 & 2 & -6 \\ -3 & 0 & 6 \end{bmatrix}$, find A^{-1}.

22) A = $\begin{bmatrix} 1 & \tan x \\ -\tan x & 1 \end{bmatrix}$, show that $A^TA^{-1} = \begin{bmatrix} \cos 2x & -\sin 2x \\ \sin 2x & \cos 2x \end{bmatrix}$

23) Given A = $\begin{bmatrix} 1 & -1 \\ 2 & 0 \end{bmatrix}$, B = $\begin{bmatrix} 3 & -2 \\ 1 & 1 \end{bmatrix}$ and C = $\begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix}$, find a matrix X such that AXB = C.

24) Find the inverse of each of the following by Gauss-Jordan method:

$$\begin{bmatrix} 1 & -1 & 0 \\ 1 & 0 & -1 \\ 6 & -2 & -3 \end{bmatrix}$$

25) Solve the following system of homogenous equations.

2x + 3y − z = 0, x − y − 2z = 0, 3x + y + 3z = 0

26) If A is a non-singular matrix of odd order, prove that |adj A| is positive

27) Find the rank of the following matrices by minor method:

$$\begin{bmatrix} 0 & 1 & 2 & 1 \\ 0 & 2 & 4 & 3 \\ 8 & 1 & 0 & 2 \end{bmatrix}$$

28) For any 2 x 2 matrix, if A (adj A) = $\begin{bmatrix} 10 & 0 \\ 0 & 10 \end{bmatrix}$ then find |A|.

29)

Show that the system of equations is inconsistent. $2x + 5y = 7$, $6x + 15y = 13$.

30) Find k if the equations $x + 2y + 2z = 0$, $x - 3y - 3z = 0$, $2x + y + kz = 0$ have only the trivial solution.

ANSWER ANY 7 $\quad 7 \times 3 = 21$

31) Solve the following system of equations, using matrix inversion method:
$2x_1 + 3x_2 + 3x_3 = 5$, $x_1 - 2x_2 + x_3 = -4$, $3x_1 - x_2 - 2x_3 = 3$.

32) If $A = \begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix}$, find the productsAB and BAand hence solve the system of equations $x - y + z = 4$, $x - 2y - 2z = 9$, $2x + y + 3z = 1$.

33) The upward speed v(t)of a rocket at time t is approximated by $v(t) = at^2 + bt + c \le t \le 100$ where a, b and c are constants. It has been found that the speed at times $t = 3$, $t = 6$, and $t = 9$ seconds are respectively, 64, 133, and 208 miles per second respectively. Find the speed at time $t = 15$ seconds. (Use Gaussian elimination method.)

34) Determine the values of λ for which the following system of equations $(3\lambda - 8)x + 3y + 3z = 0$, $3x + (3\lambda - 8)y + 3z = 0$, $3x + 3y + (3\lambda - 8)z = 0$. has a non-trivial solution.

35) By using Gaussian elimination method, balance the chemical reaction equation: $C_5H_8 + O_2 \longrightarrow CO_2 + H_2O$. (The above is the reaction that is taking place in the burning of organic compound called isoprene.)

36) Find the rank of the matrix $\begin{bmatrix} 2 & -2 & 4 & 3 \\ -3 & 4 & -2 & -1 \\ 6 & 2 & -1 & 7 \end{bmatrix}$ by reducing it to an echelon form.

37) Verify $(AB)^{-1} = B^{-1}A^{-1}$ with $A = \begin{bmatrix} 0 & -3 \\ 1 & 4 \end{bmatrix}$, $B = \begin{bmatrix} -2 & -3 \\ 0 & -1 \end{bmatrix}$.

38) 4 men and 4 women can finish a piece of work jointly in 3 days while 2 men and 5 women can finish the same work jointly in 4 days. Find the time taken by one man alone and that of one woman alone to finish the same work by using matrix inversion method.

39) If $A = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$, show that $A^{-1} = \frac{1}{2}(A^2 - 3I)$.

40) Decrypt the received encoded message $\begin{bmatrix} 2 & -3 \end{bmatrix}\begin{bmatrix} 20 & 4 \end{bmatrix}$ with the encryption matrix $\begin{bmatrix} -1 & -1 \\ 2 & 1 \end{bmatrix}$ and the decryption matrix as its inverse, where the system of codes are described by the numbers 1 - 26 to the letters A - Z respectively, and the number 0 to a blank space.

ANSWER ANY 7 $\quad 7 \times 5 = 35$

41) Reduce the matrix $\begin{bmatrix} 0 & 3 & 1 & 6 \\ -1 & 0 & 2 & 5 \\ 4 & 2 & 0 & 0 \end{bmatrix}$ to row-echelon form.

42) Solve, by Cramer's rule, the system of equations
$x_1 - x_2 = 3$, $2x_1 + 3x_2 + 4x_3 = 17$, $x_2 + 2x_3 = 7$.

43) Find the condition on a, b and c so that the following system of linear equations has one parameter family of solutions: $x + y + z = a$, $x + 2y + 3z = b$, $3x + 5y + 7z = c$.

44) Investigate for what values of λ and μ the system of linear equations
$x + 2y + z = 7$, $x + y + \lambda z = \mu$, $x + 3y - 5z = 5$ has
(i) no solution

(ii) a unique solution

(iii) an infinite number of solutions

45) If $A = \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & 4 \\ 2 & -4 & 3 \end{bmatrix}$, verify that $A(\text{adj } A) = (\text{adj } A)A = |A|\, I_3$.

46) Find the inverse of $A = \begin{bmatrix} 2 & 1 & 1 \\ 3 & 2 & 1 \\ 2 & 1 & 2 \end{bmatrix}$ by Gauss-Jordan method.

47) Find the value of k for which the equations kx - 2y + z = 1, x - 2ky + z = -2, x - 2y + kz = 1 have

(i) no solution

(ii) unique solution

(iii) infinitely many solution

48) Investigate the values of λ and m the system of linear equations 2x + 3y + 5z = 9, 7x + 3y - 5z = 8, 2x + 3y + λz = μ, have

(i) no solution

(ii) a unique solution

(iii) an infinite number of solutions.

49) In a T20 match, Chennai Super Kings needed just 6 runs to win with 1 ball left to go in the last over. The last ball was bowled and the batsman at the crease hit it high up. The ball traversed along a path in a vertical plane and the equation of the path is $y = ax^2 + bx + c$ with respect to a xy-coordinate system in the vertical plane and the ball traversed through the points (10, 8), (20, 16) (30, 18) can you conclude that Chennai Super Kings won the match?

Justify your answer. (All distances are measured in metres and the meeting point of the plane of the path with the farthest boundary line is (70, 0).)

50) If the system of equations px + by + cz = 0, ax + qy + cz = 0, ax + by + rz = 0 has a non-trivial solution and $p \neq a, q \neq b, r \neq c$, prove that $\frac{p}{p-a} + \frac{q}{q-b} + \frac{r}{r-c} = 2$.

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