RAVI MATHS TUITION CENTER ,GKM COLONY, CHENNAI- 82. PH: 8056206308

12th Standard

Maths

Exam Time : 01:15:00 Hrs | Total Marks : 50

10 x 1 = 10

1) The volume of a sphere is increasing in volume at the rate of 3 πcm^3 sec. The rate of change of its radius when radius is $\frac{1}{2}$ cm

(a) 3 cm/s (b) 2 cm/s (c) 1 cm/s (d) $\frac{1}{2} cm/s$

2) A balloon rises straight up at 10 m/s. An observer is 40 m away from the spot where the balloon left the ground. Find the rate of change of the balloon's angle of elevation in radian per second when the balloon is 30 metres above the ground.

(a) $\frac{3}{25} radians/sec$ (b) $\frac{4}{25} radians/sec$ (c) $\frac{1}{5} radians/sec$ (d) $\frac{1}{3} radians/sec$

3) The position of a particle moving along a horizontal line of any time t is given by set) = $3t^2$ -2t- 8. The time at which the particle is at rest is

(a) t= 0 (b) $t = \frac{1}{3}$ (c) t =1 (d) t = 3

4) A stone is thrown up vertically. The height it reaches at time t seconds is given by x = 80t -$16t^2$. The stone reaches the maximum height in time t seconds is given by

(a) 2 (b) 2.5 (c) 3 (d) 3.5

5) Find the point on the curve 6y = .x^3 + 2 at which y-coordinate changes 8 times as fast as x-coordinate is

(a) (4,11) (b) (4,-11) (c) (-4,11) (d) (-4,-11)

6) A circular template has a radius of 10 cm. The measurement of radius has an approximate error of 0.02 cm. Then the percentage error in calculating area of this template is

(a) 0.2% (b) 0.4% (c) 0.04% (d) 0.08%

7) The percentage error of fifth root of 31 is approximately how many times the percentage error in 31?

(a) $\frac{1}{31}$ (b) $\frac{1}{5}$ (c) 5 (d) 31

8) If u (x, y) = e^{x2+y2}, then $\frac{\partial u}{\partial x}$ is equal to

(a) e^{x2+y2} (b) 2xu (c) x^2u (d) y^2u

9) If v (x, y) = log (ex + ev), then $\frac{\partial v}{\partial x} + \frac{\partial v}{\partial y}$ is equal to

(a) $e^x + e^y$ (b) $\frac{1}{e^x + e^y}$ (c) 2 (d) 1

10) If w (x, y) = xy, x > 0, then $\frac{\partial w}{\partial x}$ is equal to

(a) $x^y \log x$ (b) y log x (c) yx^{y-1} (d) x log y

ANY 5 | 5 x 2 = 10

11) A point moves along a straight line in such a way that after t seconds its distance from the origin is s = $2t^2$ + 3t metres
Find the average velocity of the points between t = 3 and t = 6 seconds.

12) If the mass m(x) (in kilograms) of a thin rod of length x (in metres) is given by, m(x) = $\sqrt{3}x$ then what is the rate of change of mass with respect to the length when it is x = 3 and x = 27 metres.

13)

Find the tangent and normal to the following curves at the given points on the curve

$x = \cos t, y = 2\sin t^2$ at $t = \frac{\pi}{3}$

14) Using the Rolle's theorem, determine the values of x at which the tangent is parallel to the x -axis for the following functions:

$f(x) = \frac{x^2-2x}{x+2}, x \in [-1, 6]$

15) Let $f(x) = \sqrt[3]{x}$. Find the linear approximation at x = 27. Use the linear approximation to approximate $\sqrt[3]{27.2}$

16) The radius of a circular plate is measured as 12.65 cm instead of the actual length 12.5 cm.find the following in calculating the area of the circular plate:

Absolute error

17) Evaluate $\lim_{(x,y) \to (1,2)}$, if the limit exists, where $(x, y) = \frac{3x^2-xy}{x^2+y^2+3}$

18) Let V(x,y, z) =xy+ yz + zx, x, y, z ∈ R. Find the differential dV.

ANY 5 5 x 3 = 15

19) The temperature in celsius in a long rod of length 10 m, insulated at both ends, is a function of length x given by T = x(10 − x). Prove that the rate of change of temperature at the midpoint of the rod is zero.

20) The price of a product is related to the number of units available (supply) by the equation Px + 3P −16x = 234, where P is the price of the product per unit in Rupees(Rs) and x is the number of units. Find the rate at which the price is changing with respect to time when 90 units are available and the supply is increasing at a rate of 15 units/week.

21) $lim\theta \to 0(\frac{1-cos\, m\theta}{1-cos\, n\theta}) = 1$, then prove that, $m = \pm n$

22) Find the local maximum and minimum of the function $x^2 y^2$ on the line x + y =10

23) Sketch the graph of the function $y = \frac{3x}{x^2-1}$

24) Find the linear approximation for $f(x) = \sqrt{1+x}, x \geq -1$ at $x_0 = 3$. Use the linear approximation to estimate f(3.2)

25) Use linear approximation to find an approximate value of $\sqrt{9.2}$ without using a calculator.

26) Let us assume that the shape of a soap bubble is a sphere. Use linear approximation to approximate the increase in the surface area of a soap bubble as its radius increases from 5 cm to 5.2 cm. Also, calculate the percentage error.

3 x 5 = 15

27) Find the equation of the tangent and normal to the Lissajous curve given by x = 2cos3t and y = 3sin 2t, t ∈ R

28) Find the intervals of monotonicity and local extrema of the function f (x) = x log x + 3x.

29) A steel plant is capable of producing x tonnes per day of a low-grade steel and y tonnes per day of a high-grade steel, where $y = \frac{40-5x}{10-x}$ If the fixed market price of low-grade steel is half that of high-grade steel, then what should be optimal productions in low-grade steel and high-grade steel in order to have maximum receipts.

30) Let $F(x, y) = x^3 y + y^2 x + 7$ for all $(x, y) \in R^2$. Calculate $\frac{\partial F}{\partial x}$ (-1,3) and $\frac{\partial F}{\partial y}$ (-2,1).

31) Let $f(x, y) = \sin(xy^2) + e^{x3+5y}$ for all $\in R^2$. Calculate $\frac{\partial f}{\partial x}$, $\frac{\partial f}{\partial y}$, $\frac{\partial^2 f}{\partial y \partial x}$ and $\frac{\partial^2 f}{\partial x \partial y}$

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