MODEL PAPER 5 (REDUCED 2021)

12th Standard

Maths

Exam Time : 03:00:00 Hrs | Total Marks : 90

ANSWER ALL | 20 x 1 = 20

1) The rank of the matrix $\begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 4 & 6 & 8 \\ -1 & -2 & -3 & -4 \end{bmatrix}$ is

(a) 1 (b) 2 (c) 4 (d) 3

2) If P (A) = P ([A| B]) , then the system AX = B of linear equations is

(a) consistent and has a unique solution (b) consistent

(c) consistent and has infinitely many solution (d) inconsistent

3) In the system of liner equations with 3 unknowns If ρ (A) = ρ ([A|B]) =1, the system has _________

(a) unique solution (b) inconsistent

(c) consistent with 2 parameter -family of solution

(d) consistent with one parameter family of solution.

4) The conjugate of a complex number is $\frac{1}{i-2}$.Then the complex number is

(a) $\frac{1}{i+2}$ (b) $\frac{-1}{i+2}$ (c) $\frac{-1}{i-2}$ (d) $\frac{1}{i-2}$

5) If α and β are the roots of $x^2+x+1=0$, then $\alpha^{2020}+\beta^{2020}$ is

(a) -2 (b) -1 (c) 1 (d) 2

6) If α,β and γ are the roots of $x^3 + px^2 + qx + r$, then $\Sigma\frac{1}{\alpha}$ is

(a) $-\frac{q}{r}$ (b) $\frac{p}{r}$ (c) $\frac{q}{r}$ (d) $-\frac{q}{p}$

7) The polynomial $x^3 + 2x + 3$ has

(a) one negative and two imaginary zeros

(b) one positive and two imaginary zeros (c) three real zeros (d) no zeros

8) $\sin^{-1}(\cos x) = \frac{\pi}{2} - x$ is valid for

(a) $-\pi \le x \le 0$ (b) $0 \le x \le 0$ (c) $-\frac{\pi}{2} \le x \le \frac{\pi}{2}$ (d) $-\frac{\pi}{4} \le x \le \frac{3\pi}{4}$

9) If $\sin^{-1} x + \cot^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{2}$, then x is equal to

(a) $\frac{1}{2}$ (b) $\frac{1}{\sqrt{5}}$ (c) $\frac{2}{\sqrt{5}}$ (d) $\frac{\sqrt{3}}{2}$

10) The equation of the normal to the circle $x^2 + y^2 - 2x - 2y + 1 = 0$ which is parallel to the line $2x + 4y = 3$ is

(a) $x + 2y = 3$ (b) $x + 2y + 3 = 0$ (c) $2x + 4y + 3 = 0$ (d) $x - 2y + 3 = 0$

11) An ellipse hasOB as semi minor axes, F and F′ its foci and the angle FBF′ is a right angle. Then the eccentricity of the ellipse is

(a) $\frac{1}{\sqrt{2}}$ (b) $\frac{1}{2}$ (c) $\frac{1}{4}$ (d) $\frac{1}{\sqrt{3}}$

12) The volume of the parallelepiped with its edges represented by the vectors $\hat{i} + \hat{j}, \hat{i} + 2\hat{j}, \hat{i} + \hat{j} + \pi\hat{k}$ is

(a) $\frac{\pi}{2}$ (b) $\frac{\pi}{3}$ (c) π (d) $\frac{\pi}{4}$

13) The coordinates of the point where the line $\vec{r} = (6\hat{i} - \hat{j} - 3\hat{k}) + t(\hat{i} + 4\hat{j})$ meets the plane $\vec{r} = (\hat{i} + \hat{j} - \hat{k})$ = 3 are

(a) (2,1,0) (b) (7,1,7) (c) (1,2,6) (d) (5,1,1)

14) Find the point on the curve $6y = .x^3 + 2$ at which y-coordinate changes 8 times as fast as x-coordinate is

(a) (4,11) (b) (4,-11) (c) (-4,11) (d) (-4,-11)

15) If $g(x, y) = 3x^2 - 5y + 2y^2, x(t) = e^t$ and y(t) = cos t, then $\frac{dg}{dt}$ is equal to

(a) $6e^{2t} + 5 \sin t - 4 \cos t \sin t$ (b) $6e^{2t} - 5 \sin t + 4 \cos t \sin t$

(c) $3e^{2t} + 5 \sin t + 4 \cos t \sin t$ (d) $3e^{2t} - 5 \sin t + 4 \cos t \sin t$

16) The value of $\int_0^{\pi} sin^4 x dx$ is

(a) $\frac{3\pi}{10}$ (b) $\frac{3\pi}{8}$ (c) $\frac{3\pi}{4}$ (d) $\frac{3\pi}{2}$

17) The general solution of the differential equation $\frac{dy}{dx} = \frac{y}{x}$ is

(a) xy = k (b) y = k log x (c) y = kx (d) log y = kx

18) If the function $f(x) = \frac{1}{12}$ for $a < x < b$, represents a probability density function of a continuous random variable X, then which of the following cannot be the value of a and b?

(a) 0 and 12 (b) 5 and 17 (c) 7 and 19 (d) 16 and 24

19) The operation * defined by $a * b = \frac{ab}{7}$ is not a binary operation on

(a) Q^+ (b) Z (c) R (d) C

20) Which one of the following is not true?

(a) Negation of a negation of a statement is the statement itself

(b) If the last column of the truth table contains only T then it is a tautology.

(c) If the last column of its truth table contains only F then it is a contradiction

(d) If p and q are any two statements then $p \leftrightarrow q$ is a tautology.

ANSWER 7. Q.NO 30 COMPULSORY 7 x 2 = 14

21) Find the following $\left|\overline{(1+i)}(2+3i)(4i-3\right|$

22) Find the value of

$sin\left[\frac{\pi}{3} - sin^2\left(-\frac{1}{2}\right)\right]$

23) Find the length of the tangent from (2, -3) to the circle $x^2 + y^2 - 8x - 9y + 12 = 0$.

24) Show that the lines $\frac{x-1}{4} = \frac{2-y}{6} = \frac{z-4}{12}$ and $\frac{x-3}{-2} = \frac{y-3}{3} = \frac{5-z}{6}$ are parallel.

25) Find the angle of intersection of the curve y = sin x with the positive x -axis.

26) Use differentials to find $\sqrt{25.2}$

27) Evaluate the following

$\int_0^1 x^2(1-x)^3dx$

28) Solve the following differential equations:

$\frac{dy}{dx} = \sqrt{\frac{1-y^2}{1-x^2}}$

29) A random variable X has the following probability mass function.

x	1	2	3	4	5
f(x)	k^2	$2k^2$	$3k^2$	2k	3k

Find (i) the value of k (ii) P(2 $\leq$ X < 5) (iii) P(3 < X)

30) Fill in the following table so that the binary operation $*$ on A = {a,b,c} is commutative.

*	a	b	c
a	b		
b	c	b	a
c	a		c

ANSWER 7. Q.NO 40 COMPULSORY 7 x 3 = 21

31) Solve the following system of linear equations by matrix inversion method:

2x − y = 8, 3x + 2y = −2

32) Evaluate the following if z=5−2i and w= −1+3i $z^2 + 2zw + w^2$

33) Find the value of $\tan^{-1}(-1) + \cos^{-1}(\frac{1}{2}) + sin^{-}1(-\frac{1}{2})$

34) Find the equation of the parabola with focus $(-\sqrt{2}, 0)$ and directrix x =$\sqrt{2}$.

35) Find the value of p so that 3x + 4y - p = 0 is a tangent to the circle $x^2 + y^2 - 64 = 0$.

36) Find the local extremum of the function f (x) = $x^4 + 32x$

37) Find the area of the region bounded by the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

38) Solve: $\frac{dv}{dx} + 2y\, cot\, x = 3x^2 cosec^2 x$

39) If the probability mass function f (x) of a random variable X isx

x	1	2	3	4

f (x)	$\frac{1}{12}$	$\frac{5}{12}$	$\frac{5}{12}$	$\frac{1}{12}$

find (i) its cumulative distribution function, hence find

(ii) $P(X \leq 3)$ and,

(iii) $P(X \geq 2)$

40) Verify whether the following compound propositions are tautologies or contradictions or contingency

$((p \vee q) \wedge \sim p) \rightarrow q$

ANSWER ALL 7 x 5 = 35

41) a) The upward speed v(t)of a rocket at time t is approximated by $v(t) = at^2 + bt + c \leq t \leq 100$ where a, b and c are constants. It has been found that the speed at times t = 3, t = 6, and t = 9 seconds are respectively, 64, 133, and 208 miles per second respectively. Find the speed at time t = 15 seconds. (Use Gaussian elimination method.)

(OR)

b) Assume that water issuing from the end of a horizontal pipe, 7 5 . m above the ground, describes a parabolic path. The vertex of the parabolic path is at the end of the pipe. At a position 2 5 . m below the line of the pipe, the flow of water has curved outward 3m beyond the vertical line through the end of the pipe. How far beyond this vertical line will the water strike the ground?

42) a) Find the vertex, focus, equation of directrix and length of the latus rectum of the following: $y^2 - 4y - 8x + 12 = 0$

(OR)

b) At 10.00 A.M. a woman took a cup of hot instant coffee from her microwave oven and placed it on a nearby Kitchen counter to cool. At this instant the temperature of the coffee was 180^o F, and 10 minutes later it was 160^oF. Assume that constant temperature of the kitchen was 70^oF.

(i) What was the temperature of the coffee at 10.15A.M.? $\left[\log \frac{9}{11} = -0.6061\right]$

(ii) The woman likes to drink coffee when its temperature is between 130°F and 140°F.between what times should she have drunk the coffee? $\left[\log \frac{6}{11} = -0.2006\right]$

43) a) Solve the equation $x^3-9x^2+14x+24=0$ if it is given that two of its roots are in the ratio 3:2.

(OR)

b) A manufacturer wants to design an open box having a square base and a surface area of 108 sq. cm. Determine the dimensions of the box for the maximum volume.

44) a) A police jeep, approaching an orthogonal intersection from the northern direction, is chasing a speeding car that has turned and moving straight east. When the jeep is 0.6 km north of the intersection and the car is 0.8 km to the east. The police determine with a radar that the distance between them and the car is increasing at 20 km/hr. If the jeep is moving at 60 km/hr at the instant of measurement, what is the speed of the car?

(OR)

b) Prove that $\int_0^{\frac{\pi}{4}} \log(1+\tan x)dx=\frac{\pi}{8}\log 2$.

45) a) If $z_1,z_2,$ and z_3 are three complex numbers such that $|z_1|=1,|z_2|=2|z_3|=3$ and $|z_1+z_2+z_3|=1$,show that $|9z_1z_2+4z_1z_2+z_2z_3|=6$

(OR)

b) Using integration find the area of the region bounded by triangle ABC, whose vertices A, B, and C are (−1,1), (3, 2), and (0,5) respectively

46) a) Find the non-parametric form of vector equation, and Cartesian equation of the plane passing through the point (2,3,6) and parallel to the straight lines

$\frac{x-1}{2} = \frac{y+1}{3} = \frac{z-3}{1}$ and $\frac{x+3}{2} = \frac{y-3}{-5} = \frac{z+1}{-3}$

(OR)

b) Solve the following differential equations

$$ye^{\frac{x}{y}}dx = \left(xe^{\frac{x}{y}} + y\right)dy$$

47) a) The probability density function of random variable X is given by

$$f(x) = \begin{cases} k & 1 \leq x \leq 5 \\ 0 & otherwise \end{cases}$$ Find

(i) Distribution function

(ii) P(X < 3)

(iii) P(2 < X < 4)

(iv) P(3 ≤ X)

(OR)

b) Using the equivalence property, show that p ↔ q ≡ (p ∧ q) v (¬p ∧ ¬q)

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ANSWER ALL 20 x 1 = 20

1) (a) 1

2) (b) consistent

3) (c) consistent with 2 parameter -family of solution

4) (b) $\frac{-1}{i+2}$

5) (b) -1

6) (a) $-\frac{q}{r}$

7) (a) one negative and two imaginary zeros

8) (b) $0 \leq x \leq 0$

9) (b) $\frac{1}{\sqrt{5}}$

10) (a) x + 2y = 3

11) (a) $\frac{1}{\sqrt{2}}$

12)
(c) π
13)
(d) (5,1,1)
14)
(a) (4,11)
15)
(a) $6e^{2t}$ + 5 sin t - 4 cos t sin t
16)
(b) $\frac{3\pi}{8}$
17)
(a) xy = k
18)
(d) 16 and 24
19)
(b) Z
20)
(d) If p and q are any two statements then p↔q is a tautology.

ANSWER 7. Q.NO 30 COMPULSORY 7 x 2 = 14

21)
$\left|\left(\overline{1+i}\right)(2+3i)(4i-3)\right| = \left|\left(\overline{1+i}\right)\right| |2+3i|\,|4i-3|$ ($\because$ $|z_1 z_2 z_3|=|z_1||z_2||z_3|$)

= |1+i||2+3i||-3+4i| $(\because |z| = |\overline{z}|)$

= $\left(\sqrt{1^2+1^2}\right)\left(\sqrt{2^2+3^2}\right)\left(\sqrt{(3)^2+4^2}\right)$.

22)
$sin\left[\frac{\pi}{3} - sin^{-1}\left(-\frac{1}{2}\right)\right] = sin\left[\frac{\pi}{3} - \left(-\frac{\pi}{6}\right)\right] = sin\left(\frac{\pi}{2}\right) = 1$

23)

Given circle is $x^2 + y^2 - 8x - 9y + 12 = 0$

Length of the tangent = $\sqrt{2^2+(-3)^2-8(2)-9(-3)+12}$

= $\sqrt{4+9-16+27+12}$

= $\sqrt{36}$ = 6 unit

24)

We observe that the straight line $\frac{x-1}{4}=\frac{2-y}{6}=\frac{z-4}{12}$ is parallel to the vector $4\hat{i}-6\hat{j}+12\hat{k}$ and the straight line $\frac{x-3}{-2}=\frac{y-3}{3}=\frac{5-z}{6}$ is parallel to the vector $2\hat{i}+3\hat{j}-6\hat{k}$

Since $4\hat{i}-6\hat{j}+12\hat{k}=-2(-2\hat{i}+3\hat{j}-6\hat{k})$, two vectors are parallel, and hence the two straight lines are parallel.

25)

The curve y = sin x intersects the positive x -axis. When y = 0 which gives, x = $x=n\pi, n=1,2,3,...$

Now, $\frac{dy}{dx}=cosx$. The slpoe $x=n\pi$ are $cos(n\pi)=(-1)^n$. x -Hence, the required angle of intersection is $m_2 = 0$

$\tan^{-1}(-1)^n = \begin{cases} \frac{\pi}{4}, when\ n\ is\ even \\ \frac{3\pi}{4}, when\ is\ n\ odd \end{cases}$

26)

Let y = f(x) = $\sqrt{x}$

Let x_o = 25, dx = 25.2 - 25 = 0.2

y = $\sqrt{x}$

dy = $\frac{1}{2\sqrt{x}}$ dx

dy = $\frac{1}{2\sqrt{x}}$ (0.2) = 0.02

$\therefore\sqrt{25.2}$ = $f(x_0) + f'(x_0)$ dx

= $\sqrt{25}$ + 0.02

= 5 + 0.02 = 5.02

27)

$I = \int_0^1 x^2(1-x)^3 dx$

We know $\int_0^1 x^m(1-x)^n dx = \frac{m!\times n!}{(m+n+1)!}$

$\therefore I = \frac{2!\times 3!}{(2+3+1)!} = \frac{2\times3\times2\times1}{6\times5\times4\times3\times2\times1}$

$I = \frac{1}{60}$

28)

Separating the variables we get,

$\frac{dy}{\sqrt{1-y^2}} \frac{dx}{\sqrt{1-x^2}}$

$\sin^{-1}y = \sin^{-1}x + c$

29)

Given probability mass function is

x	1	2	3	4	5
f(x)	k^2	$2k^2$	$3k^2$	2k	3k

(i) Since f(x) is a probability mass function.

$\sum_{i=1}^{5} f(x_i) = 1$

$\Rightarrow k^2+2k^2+3k^2+2k+3k=1$

$\Rightarrow 6k^2+5k=1$

$\Rightarrow 6k^2+5k-1=0$

$\Rightarrow (k+1)(6k-1)=0$

$\Rightarrow k=-1$ or $\Rightarrow k = \frac{1}{6}$ $\Rightarrow k = \frac{1}{6}$

(ii) $p(2 \leq x < 5)$

$= p(x = 2) + p(x = 3) + p(x = 4)$

$= 2k^2+3k^2+2k=5k^2+2k$

$= 5\left(\frac{1}{36}\right) + 2\left(\frac{1}{6}\right)$

$= \frac{5}{36} + \frac{1}{3} = \frac{5+12}{36}$

$= \frac{17}{36}$

(iii) $p(3 < x) = p(x > 3)$

$= p(x = 4) + p(x = 5)$

$= 2k + 3k = 5k$

$= 5\left(\frac{1}{6}\right)$

$= \frac{5}{6}$

30)

Given * on A is commutative
Given b * a = c ⇒ a * b = c;
Given c * a = a ⇒ a * c = a
Given b * c = a ⇒ c * b = a
Hence

*	a	b	c
a	b	**c**	**a**
b	c	b	a
c	a	**a**	c

ANSWER 7. Q.NO 40 COMPULSORY 7 x 3 = 21

31)

2x-y=8, 3x+2y+2=-2

The matrix form of the system is

$$\begin{bmatrix} 2 & -1 \\ 3 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 8 \\ -2 \end{bmatrix}$$

⇒ AX=B where A= $\begin{bmatrix} 2 & -1 \\ 3 & 2 \end{bmatrix}$

B= $\begin{bmatrix} 8 \\ -2 \end{bmatrix}$

⇒ X=A^{-1}N

Now, |A|= $\begin{bmatrix} 2 & -1 \\ 3 & 2 \end{bmatrix}$ =4+3=7

∴ A^{-1}= $\frac{1}{|A|}$ adj A

= $\frac{1}{7}\begin{bmatrix} 2 & 1 \\ -3 & 2 \end{bmatrix}$

∴ X=A_{-1}B= $\frac{1}{7}\begin{bmatrix} 2 & 1 \\ -3 & 2 \end{bmatrix} \begin{bmatrix} 8 \\ -2 \end{bmatrix}$

= $\frac{1}{7}\begin{bmatrix} 16-2 \\ -24-4 \end{bmatrix}$

= $\frac{1}{7}\begin{bmatrix} 14 \\ -28 \end{bmatrix} = \begin{bmatrix} \frac{14}{7} \\ \frac{-28}{7} \end{bmatrix} = \begin{bmatrix} 2 \\ -4 \end{bmatrix}$

∴ x=2, y=-4

Hence, the solution set is {2,-4}

32)

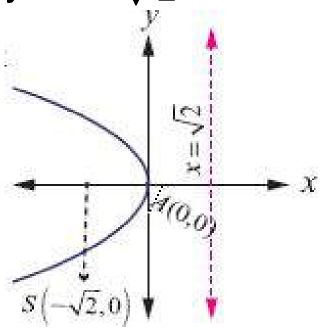
$z^2+2zw+w^2$
$(5-2i)^2+2(5-2i)(-1+3i)+(-1+3i)^2$
$=25+4i^2-20i+2\ [-5+15i+2i-6i^2]+1+9i2-6i$
$=25-4-20i+2(-5+17i+6)\ +1-9-6i$
$[\therefore\ i^2=-1]$
$=21-20i+2(1+17i)-8-6i$
$=21-20i+2+34i-8-6i$
$=15+8i$

33)

Let $\tan^{-1}(-1)=y$. Then, tan y =-1=-tan $\frac{\pi}{4} = tan(-\frac{\pi}{4})$
As -$\frac{\pi}{4} \in (-\frac{\pi}{2}, \frac{\pi}{2}), tan^-1(-1) = -\frac{\pi}{3}$
Now, $\cos^{-1}(\frac{1}{2})$=y implies cos y =$\frac{1}{2}$=cos $\frac{\pi}{3}$
As $\frac{\pi}{3} \in[0,\pi]$, $\cos^{-1}(\frac{1}{2}) = \frac{\pi}{3}$
Now,$\sin^{-1}(-\frac{1}{2})$=y implies sin y =-$\frac{1}{2}$=sin(-$\frac{\pi}{3}$).
As -$\frac{\pi}{6} \in [-\frac{\pi}{2}, \frac{\pi}{2}], sin^-1(-\frac{1}{2}) = -\frac{\pi}{6}$
Therefore, $\tan^{-1}(-1)+\cos^{-1}(\frac{1}{2}) + sin^-1(-\frac{1}{2}) = -\frac{\pi}{4} + \frac{\pi}{3} - \frac{\pi}{6} = -\frac{\pi}{12}$

34)

Parabola is open left and axis of symmetry as x -axis and vertex (0,0) .
Then the equation of the required parabola is
$(y-0)^2=-4\sqrt{2}\ (x-0)$
$y^2 = -4\sqrt{2}\ x$



35)

Equation of circle is $x^2 + y^2 = 64$

∴ a2 = 64 ⇒ a = 8

Given line is 3x+ 4y = P

4y = -3x + p

y = $y = \frac{-3}{4}x + \frac{p}{4}$

m = $\frac{-3}{4}$ and c = $\frac{p}{4}$

The condition for y = mx + c to be a tangent to the circle in $c^2 = a^2(1 + m^2)$.

∴ $\left(\frac{p}{4}\right)^2 = 64\left(1 + \frac{9}{16}\right)$

$\Rightarrow \frac{p^2}{\cancel{16}} = 64\left(\frac{16+9}{\cancel{16}}\right) \Rightarrow p^2 = 64(25)$

$p = \pm\sqrt{64(25)} = \pm 8(5)$

∴ p = ±40

36)

We have,

$f'(x) = 4x^3+32=0$ gives $x^3=-8$

⇒ x = −2

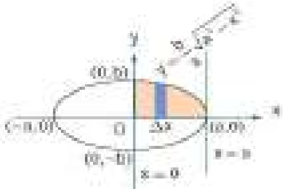
and $f''(x) = 12\,x^2$.

As f''(−2)>0 , the function has local minimum at x =−2. The local minimum value is

f (−2)=−48 . Therefore, the extreme point is (−2,−48) .

37)

The ellipse is symmetric about both major and minor axes. It is sketched. So, viewing in the positive direction of y -axis, the required area A is four times the area of the region bounded by the portion of the ellipse in the first quadrant \(\left(y=\frac { b }{ a } \sqrt { { a }^{ 2 }-{ x }^{ 2 } } ,0



x -axis, x = 0 and x = a .

Hence, by taking vertical strips, we get $A=4\int_0^a y dx=4\int_0^a \frac{b}{a}\sqrt{a^2-x^2}dx$

$= \frac{4b}{a}\left[\frac{x\sqrt{a^2-y^2}}{2}+\frac{a^2}{2}sin^{-1}\left(\frac{x}{a}\right)\right]_0^a=\frac{4b}{a}\times\frac{\pi a^2}{4}=\pi ab$

38)

Given that the equation is $\frac{dv}{dx}+2y\ cot\ x=3x^2cosec^2x$

This is a linear differential equation. Here,P=2cot x ;Q= $3x^2cosec^2x$.

$\int Pdx=\int 2cot\quad xdx=2log|sin\quad x|=log|sin\quad x|^2=logsin^2x$

Thus, $I.F=e^{\int Pdx}=e^{logsin^2}x=sin^2x$

Hence, the solution is $.ye^{\int Pdx}=\int Qe^{\int Pdx}dx+C$

That is, $ysin^2x=\int 3x^2cosec^2x.sin^2xdx+C=\int 3x^2dx+C=x^3+C$

Hence, $ysin^2x=x^3+C$ is the required solution

39)

By definition the cumulative distribution function for discrete random variable is

$F(x)P\left(X \leq x\right) = \sum_{x_1 \leq x} P(X = x_1)$

$P(X < 1) = 0$ for -∞

$F(1) = P\left(X \leq 1\right) = \sum_{x_1 \leq x} P(X = x_i) = \sum_{-\infty}^{1} P(X = x) = P\left(X < 1\right) + P\left(X = 1\right) = 0 + \frac{1}{12} = \frac{1}{12}$

$F(2) = P\left(X \leq 2\right) = \sum_{-\infty}^{2} P\left(X = x\right) = P\left(X \leq 1\right) + P\left(X = 1\right) + P\left(X = 2\right)$

= $0 + \frac{1}{12} + \frac{5}{12} = \frac{1}{2}$

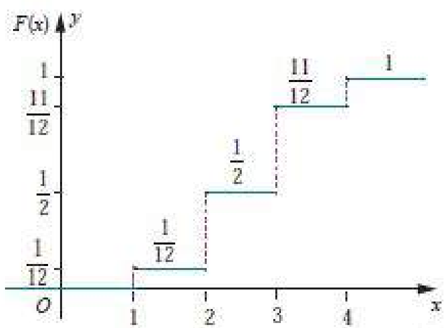
$F(3) = P\left(X \leq 3\right) = \sum_{-\infty}^{3} P(X = x) = P\left(X < 1\right) + P\left(X = 1\right) + P\left(X = 2\right) + P\left(X = 3\right)$

= $0 + \frac{1}{2} + \frac{5}{12} + \frac{5}{12} = \frac{11}{12}$

$F(4) = P\left(X \leq 4\right) = \sum_{-\infty}^{4} P\left(X = x\right) = P\left(X = 1\right) + P\left(X = 2\right) + P\left(X = 3\right) + P(X = 4)$

= $0 + \frac{1}{12} + \frac{5}{12} + \frac{5}{12} + \frac{1}{12} = 1$

\(F(x)= \begin{cases}0, & -\infty



Cumulative distribution function F(x)

(ii) $P(X \leq 3) = F(3)\frac{11}{12}$

$P(X \geq 2) = 1 - P\left(X < 2\right) = 1 - P(X \leq 1) = 1 - F(1) = 1 - \frac{1}{12} = \frac{11}{12}$

40)

((p V q)∧ ~p)) → q

p	q	p V q	~p	(p V q) ∧ ~q	(p V q) ∧ ~q
T	T	T	F	F	T
T	F	T	F	F	T
F	T	T	T	T	T
F	F	F	T	F	T

The statement ((p V q)∧ ~p) → q is a tautology.

ANSWER ALL 7 x 5 = 35

41) a)

Since v(3) = 64, v(6) = 133,and v(9) = 208 , we get the following system of linear equations

9a + 3b + c = 64 ,

36a + 6b + c = 133 ,

81a + 9b + c = 208 .

We solve the above system of linear equations by Gaussian elimination method.

Reducing the augmented matrix to an equivalent row-echelon form by using elementary row operations, we get

[A | B] =

$$\left[\begin{array}{ccc|c} 9 & 3 & 1 & 64 \\ 36 & 6 & 1 & 133 \\ 81 & 9 & 1 & 208 \end{array}\right] \xrightarrow{R_2 \longrightarrow R_2 - 4R_1, R_3 \longrightarrow R_3 - 9R_1} \left[\begin{array}{ccc|c} 9 & 3 & 1 & 64 \\ 0 & -6 & -3 & -123 \\ 0 & -18 & -8 & -368 \end{array}\right] \xrightarrow{R_2 \longrightarrow R_2 \div (-3), R_3 \div (-2)} \left[\begin{array}{ccc|c} 9 & 3 & 1 & 64 \\ 0 & 2 & 1 & 41 \\ 0 & 9 & 4 & 184 \end{array}\right]$$

$$\xrightarrow{R_3 \longrightarrow 2R_3} \left[\begin{array}{ccc|c} 9 & 3 & 1 & 64 \\ 0 & 2 & 1 & 41 \\ 0 & 18 & 8 & 368 \end{array}\right] \xrightarrow{R_3 \longrightarrow R_3 - 9R_2} \left[\begin{array}{ccc|c} 9 & 3 & 1 & 64 \\ 0 & 2 & 1 & 41 \\ 0 & 0 & -1 & -1 \end{array}\right] \xrightarrow{R_3 \longrightarrow (-1)R_3} \left[\begin{array}{ccc|c} 9 & 3 & 1 & 64 \\ 0 & 2 & 1 & 41 \\ 0 & 0 & 1 & 1 \end{array}\right].$$

Writing the equivalent equations from the row-echelon matrix, we get

9a + 3b + c = 64, 2b + c = 41, c = 1.

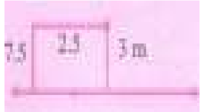
By back substitution, we get c = 1, b = $\frac{(41-c)}{2} = \frac{(41-1)}{2}$ = 20, a = $\frac{64-3b-c}{9} = \frac{64-60-1}{9} = \frac{1}{3}$.

So, we get v(t) = $\frac{1}{3}t^2$ + 20t + 1. Hence, v(15) = $\frac{1}{3}$ (225) + 20(15) + 1 = 75 + 300 + 1 = 376.

(OR)

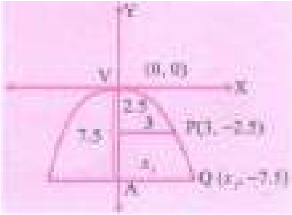
b)

As per the given information, we can take the parabola as open downward.



∴ Its equation is $x^2 = -4ay$ ...(1)

Let P be a point on the flow paths, 2.5 m below the line of the pipe and 3m beyond the vertical line through the end of the pipe.



∴ Pis (3, -2.5)

∴ (1) becomes $3^2 = -4a(-2.5)$

$\Rightarrow \frac{-9}{2.5} = 4a$

∴ (1) becomes, $x_1^2 = \frac{-9}{2.5}y$ .....(2)

Let x_1 be the distance between the bottom of the verticallineon the ground from the pipeendand the point on which this water touches the ground.

But the height of the pipe from the ground is 7.5m.

∴ (x_1 -7.5) lies on ....(2)

$\Rightarrow x_1^2 = \frac{-9}{2.5}(-7.5)$

$\Rightarrow x_1 = \sqrt{9 \times 3} = 3\sqrt{3}$ m

The water strikes the ground $3\sqrt{3}$ m beyond the vertical line.

42) a)

y^2 - 4y - 8x + 12 = 0
y^2-4y=8x-12
Adding 4 both sides, we get,
y - 4y + 4 = 8x - 12 + 4 = 8x - 8
⇒ $(y - 2)^2 = 8(x - 1)$
This is a right open parabola and latus rectum is 4a = 8 ⇒ a = 2.
(a) Vertex is (1, 2) ⇒ h = 1, k= 2
(b) focus is (h + a, 0 + k)
⇒ (1 + 2, 0 + 2)
⇒ (3, 2)
(c) Equation of directrix is x = h - a
⇒ x= 1-2
⇒ x= -1
(d) Length of latus rectum is 4a = 8 units.

(OR)

b)
Let T be the temperature of the coffee at time t and $T_{m'}$ the temperature of the kitchen.
By Newton's law of cooling

$$\frac{dT}{dt} = K(T - T_m)$$
$$\Rightarrow \frac{dT}{dt} = K(T - 70)$$
$$\Rightarrow \int \frac{dT}{T-70} = K \int dt$$
$$\Rightarrow log(T - 70) = kt + logC$$
$$\Rightarrow log(T - 70) - logC = Kt$$
$$\Rightarrow log\left(\frac{T-70}{C}\right) = Kt$$
$$\Rightarrow \frac{T-70}{C} = e^{Kt}$$
$$\Rightarrow T - 70 = Ce^{Kt} \ldots (1)$$
$$When, \quad t = 0, \quad T = 180^o F$$

$When \quad t = 0, \quad T = 180 F$

$\therefore 180^o - 70^o = Ce^0$

$\Rightarrow C = 110^0$

$\therefore (1) \Rightarrow T - 70 = 110e^{Kt} .. (2)$

$When \quad t = 0, T = 160$

$\therefore 160 - 70 = 110e^{10K}$

$90 = 110e^{10K}$

$\Rightarrow e^{10K} = \frac{9}{11}$

$\Rightarrow e^{K} = \left(\frac{9}{11}\right)^{\frac{1}{10}} \ldots (3)$

(i) when t=15, (2)becomes,

$\Rightarrow T - 70 = 110\left(\frac{9}{11}\right)^{\frac{1}{10} \times 15}$

$= 110\left(\frac{9}{11}\right)^{\frac{3}{2}}$

$= 110 \times \left(\frac{9}{11}\right) \left(\sqrt{\frac{9}{11}}\right)$

$= 110 \times \frac{9}{11} \times \frac{3}{\sqrt{11}}$

$= \frac{270}{\sqrt{11}} = \frac{270}{3.32} = 81.33$

$\Rightarrow$ T=81.33+70=151.3F

$\therefore$ T=151.3F

$\therefore$The temperature of the coffee at 10.15 am is 151.3F

(ii) when T=130F, (2) becomes

T-70=110e^{kt} ...(2)

$\Rightarrow$130-70=110e^{kt}

60=110e^{kt}

e^{kt} = $\frac{6}{11}$

$\left(\frac{9}{11}\right)^{\frac{t}{10}} = \frac{6}{11}$

$\frac{t}{10} = \frac{log\left(\frac{6}{11}\right)}{log\left(\frac{9}{11}\right)}$

10 $\quad log\left(\frac{9}{11}\right)$

$= \frac{log(0.545)}{log(0.818)} = \frac{-0.264}{-0.087}$

=3.34

t=30.34min

T=140F,(2)becomes

140-70=110e^{kt} ...(2)

$\Rightarrow 70 = 110e^{kt}$

$e^{kt} = \frac{7}{11}$

$\left(\frac{9}{11}\right)^{\frac{t}{10}} = \frac{7}{11}$

$\frac{t}{10} = \frac{log\left(\frac{7}{11}\right)}{log\left(\frac{7}{11}\right)} = \frac{-0.197}{-0.087}$

=2.26

t=22.6min

$\therefore$ Between 10.22 min to 10.30 min, the woman should have drunk the coffee.

43) a)

Let ∝,β,ɤ be the roots of the equation

Given $\frac{\alpha}{\beta} = \frac{3}{2} \Rightarrow 2\alpha = 3\beta \Rightarrow \alpha = \frac{3}{2}\beta$

$\therefore \frac{3}{2}\beta, \beta, \gamma$ are the roots of the given equation

Then by vieta's formula,

$\frac{3}{2}\beta + \beta + \gamma = \frac{-b}{a} = \frac{-(-9)}{1} = 9$

$\frac{5}{2}\beta + \gamma = 9 \Rightarrow \gamma = 9 - \frac{5}{2}\beta$

$\Rightarrow \gamma = \frac{18-5\beta}{2} \ldots (2)$

Also $\frac{3}{2}\beta(\beta) + \beta\gamma + \left(\frac{3}{2}\beta\right)\gamma = \frac{c}{a} = \frac{14}{1} = 14$

$\Rightarrow \frac{3}{2}\beta^2 + \frac{5}{2}\beta\left(\frac{18-5\beta}{2}\right) = 14 \quad [using \quad (2)]$

$\Rightarrow \frac{3}{2}\beta^2 + \frac{90\beta}{} - \frac{25\beta^2}{} = 14$

Multiplying by $4, 6\beta^2 + 90\beta - 25\beta^2 = 56$

$19\beta^2 - 90\beta + 56 = 0$

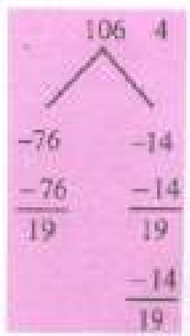
$\Rightarrow (\beta - 4)(19\beta - 14) = 0$

$\Rightarrow \beta = 4$

$\beta = \frac{14}{19}$

When β=4, the other roots are $\frac{3}{2}(4), 4, \frac{18-5}{2}(4)$

$\Rightarrow 6, 4, -1$



When $\beta = \frac{14}{19}$, the other roots are $\frac{3}{2}\beta, \beta\frac{18-5\beta}{2}[by(2)]$

$\Rightarrow \frac{3}{2}\left(\frac{14}{19}\right), \frac{14}{19}, \frac{18-5\left(\frac{14}{19}\right)}{2} \Rightarrow \frac{21}{19}, \frac{14}{19}, \frac{136}{19}$

(OR)

b)

Since the open box has square area, let the length, breadth and height of the box be l, 1 and b cm respectively.

ஃSurface area = l^2 + 41b = 108

$\Rightarrow l + 4b = \frac{108}{l}$

$\Rightarrow 4b = \frac{108}{l} - 1$

$\Rightarrow b = \frac{108}{4l} - \frac{l}{4}$

$\Rightarrow b = \frac{27}{l} - \frac{l}{4}$

Let f(1) = Volume of the box = 1 x 1x b = l^2 b

= $l^2 \left(\frac{27}{l} - \frac{l}{4}\right)$

$f(1) = 27l - \frac{l^3}{4}$

$f'(l) = 27 - \frac{3l^2}{4}$

f'(1) = 0

$\Rightarrow 27 - \frac{3l^2}{4} = 0$

$\Rightarrow \frac{3l^2}{4} = 27$

$\Rightarrow l^2 = 27 \times \frac{4}{3} = 36$

$\Rightarrow 1 = \pm 6$

$\Rightarrow 1 = 6$

ஃThe critical number is 6

$f''(l) = \frac{6l}{4} = \frac{3l}{2}$

$\therefore f''(6) = -\frac{3(6)}{2} < 0$

ஃ f"(1) is maximum when 1= 6

When l = 6, $b = \frac{27}{6} - \frac{6}{4}$

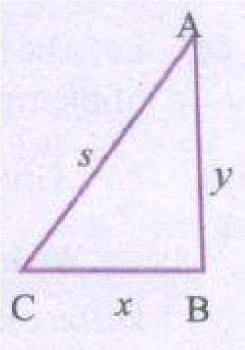
= $\frac{9}{2} - \frac{3}{2} = \frac{6}{2} = 3cm$

Hence the dimensions of the required box are 6 cm, 6 cm and 3 cm respectively.

44) a)

Let x represent the distance covered by the car, y represent the distance covered by the police jeep, and s represent the distance between the car and jeep.

∴ Given = x = 0.8 km, y = 0.6 km,



$\frac{dy}{dt}$ = -60km/hr,

$\frac{ds}{dt}$ = 20 km/hr,

In ΔABC, $S^2 = x^2 + y^2$ ......(1)

⇒ $S^2 = (0.8)^2 + (0.6)^2$

= 0.64 + 0.36

⇒ $S^2 = 1$

⇒ s = 1 ....(2)

Differentiating (1) with respect to 't' we get,

$2s\frac{ds}{dt} = 2x\frac{dx}{dt} + 2y\frac{dy}{dt}$

⇒ $s\frac{ds}{dt} = x\frac{dx}{dt} + y\frac{dy}{dt}$ [Divided by 2]

⇒$1\left(\frac{ds}{dt}\right) = (0.8)\left(\frac{dx}{dt}\right) + (0.6)(-60)$

⇒ 1(20) = (0.8) $\left(\frac{dx}{dt}\right)$ + (0.6)(-60)

⇒ 20 = (0.8) $\left(\frac{dx}{dt}\right)$ - 36

⇒ 20 + 36 = (0.8) $\frac{dx}{dt}$

⇒ $\frac{dx}{dt} = \frac{56}{0.8}$ = 70km/hr.

⇒Speed of the car is 70 km/hr.

(OR)

b)

Let us put I = $\int_0^{\frac{\pi}{4}}$ log(1+tan x) dx

Applying the property $\int_0^a$ f(x) dx=$\int_0^a$f(a-x) dx in equation (1), we get

I=$\int_0^{\frac{\pi}{4}}$ log $[1 + tan(\frac{\pi}{4} - x)]$dx=$\int_0^{\frac{\pi}{4}}$ log$[1 + \frac{tan\frac{\pi}{4} - tanx}{1 + tan\frac{\pi}{4}tanx}]$ dx

=$\int_0^{\frac{\pi}{4}}$ log $[1 + \frac{1-tanx}{1+tanx}]$dx=$\int_0^{\frac{\pi}{4}}$ log$[\frac{1+tanx+1-tanx}{1+tanx}]$ dx

=$\int_0^{\frac{\pi}{4}}$ log $[\frac{2}{1+tanx}]$ dx=$\int_0^{\frac{\pi}{4}}$ [log 2 - log (1+tan x)] dx

= log 2 $\int_0^{\frac{\pi}{4}}$ dx - $\int_0^{\frac{\pi}{4}}$log (1+tan x)] dx

= $\frac{\pi}{4}$log 2 - I

So, we get 2I = $\frac{\pi}{4}$log2.Hence,we get I = $\frac{\pi}{8}$ log2.

45) a)

Given $|z_1|=1$, $|z_2|=2$, $|z_3|=3$, $|z_1+z_2+z_3|=1$

$|z_1|^2=1^2 \Rightarrow z_1\,\overline{z_1}=1 \Rightarrow z_1=\frac{1}{\overline{z_1}}$

$|z_2|^2=4 \Rightarrow z_2\,\overline{z_2}=1 \Rightarrow z_2=\frac{4}{\overline{z_2}}$

$|z_3|^2=9 \Rightarrow z_3\,\overline{z_3}=1 \Rightarrow z_3=\frac{9}{\overline{z_3}}$

$\therefore \left|9, \frac{1}{\overline{z_1}}\cdot\frac{4}{\overline{z_2}} + 4.\frac{1}{\overline{z_1}}\cdot\frac{9}{\overline{z_3}} + \frac{4}{\overline{z_2}}\cdot\frac{9}{\overline{z_3}}\right|$

$\left|\frac{36}{\overline{z_1}\,\overline{z_2}} + \frac{36}{\overline{z_1}\,\overline{z_3}} + \frac{36}{\overline{z_2}\,\overline{z_3}}\right| = \left|36\left(\frac{\overline{z_3}+\overline{z_2}+\overline{z_1}}{\overline{z_1}\,\overline{z_2}\,\overline{z_3}}\right)\right|$

$\left[\because \left|\overline{z_1}+\overline{z_2}+\overline{z_3}\right| = \left|\overline{z_1+z_2+z_3}\right|\right]$

$= \frac{36\left|\overline{z_1+z_2+z_3}\right|}{|\overline{z_1}||\overline{z_2}||\overline{z_3}|} = 36\frac{\overline{\left|\overline{z_1}+\overline{z_2}+\overline{z_3}\right|}}{|\overline{z_1}||\overline{z_2}||\overline{z_3}|}$

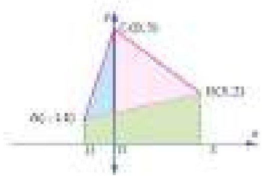
$\left[\because \left|\overline{z_1}\right| = |z_1|, \left|\overline{z_2} = |z_{21}|, \left|\overline{z_3}\right| = \left|\overline{z_3}\right|\right.\right]$

$= \frac{36(1)}{1(2)(3)} = \frac{36}{6} = 6$

$\therefore |9z_1+z_2+4z_1z_3+z_2z_3|=6$

(OR)

b)



Equation of AB is $\frac{y-1}{2-1}=\frac{x+1}{3+1} or \quad y=\frac{1}{4}(x+5)$

Equation of BC is $\frac{y-5}{2-5}=\frac{x-0}{3-0} or \quad y=-x+5$

Equation of AC is $\frac{y-1}{5-1}=\frac{x+1}{0+1} or \quad y=4x+5$

$\therefore$Area of ΔABC = Area DACO+ Area of OCBE − Area of DABE

$=\int_{-1}^{0}(4x+5)dx+\int_{0}^{3}(-x+5)dx-\frac{1}{4}\int_{-1}^{3}(x+5)dx$

$=\left[\frac{4x^2}{2}+5x\right]_{-1}^{0}+\left[-\frac{x^2}{2}+5x\right]_{0}^{3}-\frac{1}{4}\left[\frac{x^2}{2}+5x\right]_{-1}^{3}$

$=0-(+2-5)+\left(-\frac{9}{2}+15\right)-0-\frac{1}{4}\left[\frac{9}{2}+15\right]+\frac{1}{4}\left[\frac{1}{2}-5\right]=\frac{15}{2}$

46) a)

The plane passes through the point.

$\vec{a}=2\hat{i}+3\hat{j}+6\hat{k}$ and parallel to the lines $\frac{x-1}{2}$

$=\frac{y+1}{3}=\frac{z-3}{1} and \frac{x+3}{2}=\frac{y-3}{-5}=\frac{z+1}{-3}$

$\Rightarrow \vec{b}=2\hat{i}+3\hat{j}+\hat{k} and \quad \vec{c}=2\hat{i}-5\hat{j}-3\hat{k}$

$$\vec{b}\times\vec{c}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 2 & 3 & 4\\ 2 & -5 & -3\end{vmatrix}$$

$=\hat{i}(-9+5)-\hat{j}(-6-2)+\hat{k}(-10-6)$

$=-4\hat{i}+8\hat{j}-16\hat{k}$

The non-parametric vector equation of the plane is

$(\vec{r}.\vec{a}).(\vec{b}\times\vec{c})=0,$

$\Rightarrow [\vec{r}(2\hat{i}+3\hat{j}+16\hat{k}).(-4\hat{i}+8\hat{j}-16\hat{k})]=0$

$\Rightarrow [\vec{r}.(-4\hat{i}+8\hat{j}-16\hat{k})]-(-8+24-96)=0$

$\Rightarrow \vec{r}.(-4\hat{i}+8\hat{j}-16\hat{k})=-80$

$\div -4$, We get

$\vec{r}.(\hat{i}-2\hat{j}+4\hat{k})=20$

$Let \quad \vec{r}=x\hat{i}+y\hat{j}+z\hat{k}$

$\Rightarrow (x\hat{i}+y\hat{j}+z\hat{k}).(\hat{i}-2\hat{j}+4\hat{k})=20$

$\Rightarrow x=2y+4z=20$

$\Rightarrow x-2y+4z-20=0$

(OR)

b)

$$\frac{dx}{dy} = \frac{xe^{\frac{x}{y}} + y}{ye^{\frac{x}{y}}} \ldots (1)$$

This is a homogeneous differential equation

put x=vy

$\Rightarrow \frac{dx}{dy} = v + y.\frac{dv}{dy}$

$\therefore$ (1) becomes,

$v + y\frac{dv}{dy} = \frac{vye^v + y}{ye^v}$

$= y\frac{(ve^v + 1)}{ye^v} = \frac{ve^v + 1}{e^v}$

$\therefore y\frac{dv}{dy} = \frac{ve^v + 1}{e^v} - v$

$= \frac{ve^v + 1 - ve^v}{e^v} = \frac{1}{e^v}$

Separating the variables,

$e^v dv = \frac{dy}{y}$

$\int e^v dv = \int \frac{dy}{y}$

$e^v = log \quad y + log \quad c$

$e^v = log \quad yc$

$\Rightarrow e^{\frac{x}{y}} = log|cy|$

$|\therefore v = \frac{x}{y}|$

47) a)

Since f (x) is a probability density function, f (x) ≥ 0 and $\int_{-\infty}^{\infty} f(x)dx = 1$

That is $\int_{-\infty}^{1} 0dx + \int_{1}^{5} kdx + \int_{5}^{\infty} 0dx = 1$

$0 + k(x)_1^5 + 0 = 1 \Rightarrow 4k = 1 \Rightarrow k = \frac{1}{4}$

Therefore the probability density function is

$$f(x) = \begin{cases} \frac{1}{4} & 1 \leq x \leq 5 \\ 0 & Otherwise \end{cases}$$

(i) Distribution function

The distribution function

$F(x) = P(X \leq x) = \int_{-\infty}^{x} f(u)dx$

When x <1, $F(x) = \int_{-\infty}^{x} f(u)du = \int_{-\infty}^{x} oldu = 0$

When 1≤x ≤ 5

$F(x) = \int_{-\infty}^{x} f(u)du = \int_{-\infty}^{x} 0du + \int_{1}^{x} odu + \int_{1}^{x} \frac{1}{4}du = \frac{1}{4}(x-1)$

When x ≥ 5

$F(x) = \int_{-\infty}^{x} f(u)du = \int_{-\infty}^{x} odu + \int_{1}^{5} \frac{1}{4}du + \int_{1}^{5} \frac{1}{4}du + \int_{5}^{5} odu = 1$

Thus

$$F(x) = \begin{cases} 0 & x < 1 \\ \frac{x-1}{1} & 1 \leq x \leq 5 \\ 1 & x > 5 \end{cases}$$

(ii) P(X < 3) = P(X ≤ 3) = F(3) = $\frac{3-1}{2} = \frac{1}{2}$ (Since F(x) is continuous)

(iii) P(2 < X < 4) = P(2 ≤X ≤4) F(4)- F(2)= $\frac{3}{4} - \frac{1}{4} = \frac{1}{2}$

(iv) P(3 ≤X) = P(X ≥ 3) = 1− P(X < 3)=1-$1 - \frac{1}{2} = \frac{1}{2}$

(OR)

b)

It can be obtained by using examples 12.15 and 12.16 that

$p \leftrightarrow q \equiv (\neg p \vee q) \wedge (\neg q \vee p)$ ... (1)

$\equiv (\neg p \vee q) \wedge (p \vee \neg q)$ (by Commutative Law) ... (2)

$\equiv (\neg p \wedge (p \vee \neg q)) \vee (q \wedge (p \vee \neg q))$ (by Distributive Law)

$\equiv (\neg p \wedge p) \vee (\neg p \wedge \neg q) \vee (q \wedge p) \vee (q \wedge \neg q)$ (by Distributive Law)

$\equiv F \vee (\neg p \wedge \neg q) \vee (q \wedge p) \vee F$; (by Complement Law)

$\equiv (\neg p \wedge \neg q) \vee (q \wedge p)$; (by Identity Law)

$\equiv (p \wedge q) \vee (\neg p \wedge \neg q)$; (by Commutative Law)

Finally (1) becomes $p \leftrightarrow q \equiv (p \wedge q) \vee (\neg p \wedge \neg q)$