RAVI MATHS TUITION CENTER , WHATSAPP - 8056206308

ANNUAL MINIMUM MATERIALS 5 MARKS

12th Standard

Maths

ANSWERS AVAILABLE MY YOUTUBE CHANNEL NAME - RAVI MATHS TUITION CENTER

DO DAILY 10 QUESTIONS. IN 24 DAYS YOU CAN FINISH 5 MARKS ONE TIME REVISION. 234 x 5 = 1170

1) Verify $(AB)^{-1} = B^{-1}A^{-1}$ with $A = \begin{bmatrix} 0 & -3 \\ 1 & 4 \end{bmatrix}$, $B = \begin{bmatrix} -2 & -3 \\ 0 & -1 \end{bmatrix}$.

2) If $A = \begin{bmatrix} 4 & 3 \\ 2 & 5 \end{bmatrix}$, find x and y such that $A^2 + xA + yI_2 = O_2$. Hence, find A^{-1}.

3) If $A = \frac{1}{7}\begin{bmatrix} 6 & -3 & a \\ b & -2 & 6 \\ 2 & c & 3 \end{bmatrix}$ is orthogonal, find a, b and c , and hence A^{-1}.

4) Given $A = \begin{bmatrix} 1 & -1 \\ 2 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 3 & -2 \\ 1 & 1 \end{bmatrix}$ and $C = \begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix}$, find a matrix X such that AXB = C.

5) If $A = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$, show that $A^{-1} = \frac{1}{2}(A^2 - 3I)$.

6) (a) If $A = \begin{bmatrix} -5 & 1 & 3 \\ 7 & 1 & -5 \\ 1 & -1 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 1 & 2 \\ 3 & 2 & 1 \\ 2 & 1 & 3 \end{bmatrix}$, find the products AB and BA and hence solve the system of equations x + y + 2z = 1, 3x + 2y + z = 7, 2x + y + 3z = 2.

7) A man is appointed in a job with a monthly salary of certain amount and a fixed amount of annual increment. If his salary was Rs. 19,800 per month at the end of the first month after 3 years of service and Rs. 23,400 per month at the end of the first month after 9 years of service, find his starting salary and his annual increment. (Use matrix inversion method to solve the problem.)

8) Four men and 4 women can finish a piece of work jointly in 3 days while 2 men and 5 women can finish the same work jointly in 4 days. Find the time taken by one man

alone and that of one woman alone to finish the same work by using matrix inversion method.

9) The prices of three commodities A, B and C are Rs. x, y and z per units respectively. A person P purchases 4 units of B and sells two units of A and 5 units of C . Person Q purchases 2 units of C and sells 3 units of A and one unit of B . Person R purchases one unit of A and sells 3 unit of B and one unit of C . In the process, P, Q and R earn Rs. 15,000, Rs. 1,000 and Rs. 4,000 respectively. Find the prices per unit of A, B and C . (Use matrix inversion method to solve the problem.)

10) In a T20 match, a team needed just 6 runs to win with 1 ball left to go in the last over. The last ball was bowled and the batsman at the crease hit it high up. The ball traversed along a path in a vertical plane and the equation of the path is $y = ax^2 + bx + c$ with respect to a xy-coordinate system in the vertical plane and the ball traversed through the points (10, 8), (20, 16) (40, 22) can you conclude that the team won the match?
Justify your answer. (All distances are measured in metres and the meeting point of the plane of the path with the farthest boundary line is (70, 0).)

11) In a competitive examination, one mark is awarded for every correct answer while $\frac{1}{4}$ mark is deducted for every wrong answer. A student answered 100 questions and got 80 marks. How many questions did he answer correctly ? (Use Cramer's rule to solve the problem).

12) A chemist has one solution which is 50% acid and another solution which is 25% acid. How much each should be mixed to make 10 litres of a 40% acid solution? (Use Cramer's rule to solve the problem).

13) A fish tank can be filled in 10 minutes using both pumps A and B simultaneously. However, pump B can pump water in or out at the same rate. If pump B is inadvertently run in reverse, then the tank will be filled in 30 minutes. How long would it take each pump to fill the tank by itself ? (Use Cramer's rule to solve the problem).

14) A family of 3 people went out for dinner in a restaurant. The cost of two dosai, three idlies and two vadais is Rs. 150. The cost of the two dosai, two idlies and four vadais is Rs. 200. The cost of five dosai, four idlies and two vadais is Rs. 250. The family has Rs. 350 in hand and they ate 3 dosai and six idlies and six vadais. Will they be able to manage to pay the bill within the amount they had ?

15) Solve the following system of linear equations, by Gaussian elimination method : $4x + 3y + 6z = 25$, $x + 5y + 7z = 13$, $2x + 9y + z = 1$.

16) The upward speed v(t)of a rocket at time t is approximated by $v(t) = at^2 + bt + c$, $0 \le t \le 100$ where a, b and c are constants. It has been found that the speed at times t = 3, t = 6, and t = 9 seconds are respectively, 64, 133, and 208 miles per second respectively. Find the speed at time t = 15 seconds. (Use Gaussian elimination method.)

17) If $ax^2 + bx + c$ is divided by $x + 3$, $x - 5$, and $x - 1$, the remainders are 21, 61 and 9 respectively. Find a, b and c. (Use Gaussian elimination method.)

18) An amount of Rs. 65,000 is invested in three bonds at the rates of 6%, 8% and 9% per annum respectively. The total annual income is Rs. 4,800. The income from the third bond is Rs. 600 more than that from the second bond. Determine the price of each bond. (Use Gaussian elimination method.)

19) A boy is walking along the path $y = ax^2 + bx + c$ through the points (−6, 8),(−2, −12) , and (3, 8). He wants to meet his friend at P(7, 60). Will he meet his friend? (Use Gaussian elimination method.)

20) Test for consistency of the following system of linear equations and if possible solve:
$x + 2y - z = 3$, $3x - y + 2z = 1$, $x - 2y + 3z = 3$, $x - y + z + 1 = 0$

21) By using Gaussian elimination method, balance the chemical reaction equation :
$C_5H_8 + O_2 \longrightarrow CO_2 + H_2O$.

22) If the system of equations $px + by + cz = 0$, $ax + qy + cz = 0$, $ax + by + rz = 0$ has a non-trivial solution and $p \neq a$, $q \neq b$, $r \neq c$, prove that $\frac{p}{p-a} + \frac{q}{q-b} + \frac{r}{r-c} = 2$.

23) Determine the values of λ for which the following system of equations $x + y + 3z = 0$, $4x + 3y + \lambda z = 0$, $2x + y + 2z = 0$ has
(i) a unique solution
(ii) a non-trivial solution

24) Solve the following systems of linear equations by Cramer's rule:
$\frac{3}{x} - \frac{4}{y} - \frac{2}{z} - 1 = 0$, $\frac{1}{x} + \frac{2}{y} + \frac{1}{z} - 2 = 0$, $\frac{2}{x} - \frac{5}{y} - \frac{4}{z} + 1 = 0$

25) Using Gaussian Jordan method, find the values of λ and μ so that the system of equations $2x - 3y + 5z = 12$, $3x + y + \lambda z = \mu$, $x - 7y + 8z = 17$ has
(i) unique solution
(ii) infinite solutions and
(iii) no solution.

26) Investigate for what values of λ, μ the simultaneous equations $x + y + z = 6$, $x + 2y + 3z = 10$, $x + 2y + \lambda z = \mu$ have
(i) no solution

(ii) a unique solution and

(iii) an infinite number of solutions.

27) Find the values of the real numbers x and y, if the complex numbers (3−i)x−(2−i)y+2i +5 and 2x+(−1+2i)y+3+ 2i are equal.

28) Show that $\left(\frac{19+9i}{5-3i}\right)^{15}-\left(\frac{8+i}{1+2i}\right)^{15}$ is purely imaginary.

29) Let z_1, z_2 and z_3 be complex numbers such that $|z_1| = |z_2| = |z_3| = r > 0$ and $z_1 + z_2 + z_3 \neq 0$ prove that $\left|\frac{z_1z_2+z_2z_3+z_3z_1}{z_1+z_2+z_3}\right| = r$

30) If z_1, z_2, and z_3 are three complex numbers such that $|z_1| = 1$, $|z_2| = 2|z_3| = 3$ and $|z_1 + z_2 + z_3| = 1$, show that $|9z_1z_2 + 4z_1z_2 + z_2z_3| = 6$

31) If z = x + iy is a complex number such that $\operatorname{Im}\left(\frac{2z+1}{iz+1}\right) = 0$ show that the locus of z is $2x^2+ 2y^2+ x - 2y = 0$

32) If $\cos\alpha + \cos\beta + \cos\gamma = \sin\alpha + \sin\beta + \sin\gamma = 0$ then show that

(i) $\cos 3\alpha + \cos 3\beta + \cos 3\gamma = 3\cos(\alpha + \beta + \gamma)$

(ii) $\sin 3\alpha + \sin 3\beta + \sin 3\gamma + \sin 3\gamma = 3\sin(\alpha + \beta + \gamma)$

33) If z = x + iy and $\arg\left(\frac{z-i}{z+2}\right) = \frac{\pi}{4}$, then show that $x^2 + y^2 + 3x - 3y + 2 = 0$

34) If $\omega \neq 1$ is a cube root of unity, show that $\left(1 - \omega + \omega^2\right)^6 + \left(1 + \omega - \omega^2\right)^6 = 128$

35) Find the quotient $\frac{2\left(\cos\frac{9\pi}{4}+i\sin\frac{9\pi}{4}\right)}{4\left(\cos\left(\frac{-3\pi}{2}+\right)i\sin\left(\frac{-3\pi}{2}\right)\right)}$ in rectangular form

36) Solve the equation $z^3+ 8i = 0$, where $z \in C$

37) Find the fourth roots of unity.

38) Find the cube roots of unity.

39) Simplify: $(1+i)^{18}$

40) Show that $\left(\frac{19-7i}{9+i}\right)^{12}+\left(\frac{20-5i}{7-6i}\right)^{12}$ is real

41) If $\omega \neq 1$ is a cube root of unity, show that

$$(1+\omega)\left(1+\omega^2\right)\left(1+\omega^4\right)\left(1+\omega^8\right)\ldots\left(1+\omega^{2^{11}}\right) = 1.$$

42) Simplify: $\left(-\sqrt{3}+3i\right)^{31}$

43) If the imaginary part of $\frac{2z+1}{iz+1}$ is -2 then prove that the locus of the point representing z in the complex plane is a straight line.

44) Show that the complex number 'z' satisfying $\arg\left(\frac{z-1}{z+1}\right) = \frac{\pi}{4}$ lies on a circle.

45) Solve the equation $x^3 - 9x^2 + 14x + 24 = 0$ if it is given that two of its roots are in the ratio 3:2.

46) If α, β, and γ are the roots of the polynomial equation $ax^3 + bx^2 + cx + d = 0$, find the value of $\Sigma\frac{\alpha}{\beta\gamma}$ in terms of the coefficients.

47) If 2+i and $3-\sqrt{2}$ are roots of the equation $x^6 - 13x^5 + 62x^4 - 126x^3 + 65x^2 + 127x - 140 = 0$, find all roots.

48) Solve the equation (x-2) (x-7) (x-3) (x+2)+19 = 0

49) Solve the equation (2x-3) (6x-1) (3x-2) (x-2)-5 = 0

50) Solve the equation $9x^3 - 36x^2 + 44x - 16 = 0$ if the roots form an arithmetic progression.

51) Solve the equation $3x^3 - 26x^2 + 52x - 24 = 0$ if its roots form a geometric progression.

52) Find all zeros of the polynomial $x^6 - 3x^5 - 5x^4 + 22x^3 - 39x^2 - 39x + 135$, if it is known that 1+2i and $\sqrt{3}$ are two of its zeros.

53) Solve the following equation: $x^4 - 10x^3 + 26x^2 - 10x + 1 = 0$

54) Solve the equations:

$6x^4 - 35x^3 + 62x^2 - 35x + 6 = 0$

55) Solve the equation $6x^4 - 5x^3 - 38x^2 - 5x + 6 = 0$ if it is known that $\frac{1}{3}$ is a solution.

56) Find the domain of $f(x) = \sin^{-1}\left(\frac{|x|-2}{3}\right) + \cos^{-1}\left(\frac{1-|x|}{4}\right)$

57) Find the value of $\tan^{-1}(-1) + \cos^{-1}\left(\frac{1}{2}\right) + sin^{-}1\left(-\frac{1}{2}\right)$

58) If $a_1, a_2, a_3, \ldots a_n$ is an arithmetic progression with common difference d, prove that

$$\tan\left[tan^{-1}\left(\frac{d}{1+a_1a_2}\right)+tan^{-1}\left(\frac{d}{1+a_2a_3}\right)+\ldots tan^{-1}\left(\frac{d}{1+a_na_{n-1}}\right)\right]=\frac{a_n-a_1}{1+a_1a_n}$$

59) Prove that $\tan^{-1} x + \tan^{-1} z = \tan^{-1}\left[\frac{x+y+z-xyz}{1-xy-yz-zx}\right]$

60) If $\tan^{-1} x + \tan^{-1} y + \tan^{-1} z = \pi$, show that $x + y + z = xyz$

61) Prove that $tan^{-1}x + tan^{-1}\frac{2x}{1-x^2} = tan^{-1}\frac{3x-x^3}{1-3x^2}$, $|x| < \frac{1}{\sqrt{3}}$

62) Simplify: $tan^{-1}\frac{x}{y} - tan^{-1}\frac{x-y}{x+y}$

63) Find the number of solution of the equation $\tan^{-1}(x-1) + \tan^{-1}x + \tan^{-1}(x + 1) = \tan^{-1}(3x)$

64) Find the value of $cot^{-1}(1) + sin^{-1}\left(-\frac{\sqrt{3}}{2}\right) - sec^{-1}(-\sqrt{2})$

65) Find the value of $tan\left[\frac{1}{2}sin^{-1}\left(\frac{2a}{1+a^2}\right)+\frac{1}{2}cos^{-1}\left(\frac{1-a^2}{1+a^2}\right)\right]$

66) Solve: $tan^{-1}\left(\frac{x-1}{x-2}\right) + tan^{-1}\left(\frac{x+1}{x+2}\right) = \frac{\pi}{4}$

67) Prove that $tan^{-1}\left(\frac{1-x}{1+x}\right) - tan^{-1}\left(\frac{1-y}{1+y}\right) = sin^{-1}\left(\frac{y-x}{\sqrt{1+x^2}.\sqrt{1+y^2}}\right)$

68) Prove that $2\sin^{-1}\frac{3}{5} - \tan^{-1}\frac{17}{31} = \frac{\pi}{4}$

69) Solve for $x: \tan^{-1}x + 2\cot^{-1}x = \frac{2\pi}{3}$

70) Find the equation of the circle passing through the points (1 , 1), (2 , -1) , and (3 , 2) .

71) Find the equation of the circle through the points (1, 0),(-1, 0) , and (0, 1)

72) Find the foci, vertices and length of major and minor axis of the conic $4x^2 + 36y^2 + 40x - 288y + 532 = 0$

73) For the ellipse $4x^2+y^2+24x-2y+21 = 0$, find the centre, vertices and the foci. Also prove that the length of latus rectum is 2

74) Find the centre, foci, and eccentricity of the hyperbola $11x^2 - 25y^2 - 44x + 50y - 256 = 0$

75) Find the equations of the two tangents that can be drawn from (5, 2) to the ellipse $2x^2+7y^2 = 14$.

76) A semielliptical archway over a one-way road has a height of 3m and a width of 12m. The truck has a width of 3m and a height of 2.7m. Will the truck clear the opening of the archway?

77) The maximum and minimum distances of the Earth from the Sun respectively are 152×10^6 km and 94.5×10^6 km. The Sun is at one focus of the elliptical orbit. Find the distance from the Sun to the other focus.

78) A search light has a parabolic reflector (has a cross-section that forms a 'bowl'). The parabolic bowl is 40 cm wide from rim to rim and 30 cm deep. The bulb is located at the focus.

(1) What is the equation of the parabola used for reflector?

(2) How far from the vertex is the bulb to be placed so that the maximum distance covered?

79) A room 34m long is constructed to be a whispering gallery. The room has an elliptical ceiling, as shown in Figure. If the maximum height of the ceiling is 8 m, determine where the foci are located.

80) Two coast guard stations are located 600 km apart at points A(0, 0) and B(0, 600). A distress signal from a ship at P is received at slightly different times by two stations. It is determined that the ship is 200 km farther from station A than it is from station B. Determine the equation of hyperbola that passes through the location of the ship.

81) A bridge has a parabolic arch that is 10 m high in the centre and 30 m wide at the bottom. Find the height of the arch 6 m from the centre, on either sides.

82) A tunnel through a mountain for a four lane highway is to have a elliptical opening. The total width of the highway (not the opening) is to be 16 m, and the height at the edge of the road must be sufficient for a truck 4 m high to clear if the highest point of the opening is to be 5 m approximately. How wide must the opening be?

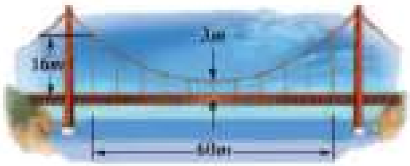
83) At a water fountain, water attains a maximum height of 4 m at horizontal distance of 0.5 m from its origin. If the path of water is a parabola, find the height of water at a horizontal distance of 0.75 m from the point of origin.

84) An engineer designs a satellite dish with a parabolic cross section. The dish is 5 m wide at the opening, and the focus is placed 1.2 m from the vertex

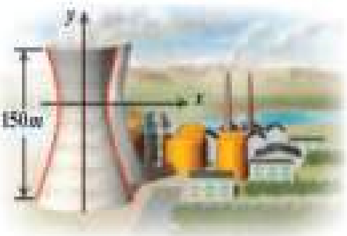
(a) Position a coordinate system with the origin at the vertex and the x -axis on the parabola's axis of symmetry and find an equation of the parabola.

(b) Find the depth of the satellite dish at the vertex.

85) Parabolic cable of a 60m portion of the roadbed of a suspension bridge are positioned as shown below. Vertical Cables are to be spaced every 6m along this portion of the roadbed. Calculate the lengths of first two of these vertical cables from the vertex.



86) Cross section of a Nuclear cooling tower is in the shape of a hyperbola with equation $\frac{x^2}{30^2} - \frac{y^2}{44^2} = 1$. The tower is 150 m tall and the distance from the top of the tower to the centre of the hyperbola is half the distance from the base of the tower to the centre of the hyperbola. Find the diameter of the top and base of the tower.



87) A rod of length 1.2 m moves with its ends always touching the coordinate axes. The locus of a point P on the rod, which is 0.3 m from the end in contact with x -axis is an ellipse. Find the eccentricity.

88) Assume that water issuing from the end of a horizontal pipe, 7. 5 m above the ground, describes a parabolic path. The vertex of the parabolic path is at the end of the pipe. At a position 2. 5 m below the line of the pipe, the flow of water has curved outward 3m beyond the vertical line through the end of the pipe. How far beyond this vertical line will the water strike the ground?

89) On lighting a rocket cracker it gets projected in a parabolic path and reaches a maximum height of 4 m when it is 6 m away from the point of projection. Finally it reaches the ground 12 m away from the starting point. Find the angle of projection.

90) Find the vertex, focus, equation of directrix and length of the latus rectum of the following:
$x^2-2x+8y+17=0$

91) Find the vertex, focus, equation of directrix and length of the latus rectum of the following: $y^2-4y-8x+12=0$

92) Identify the type of conic and find centre, foci, vertices, and directrices of each of the following :

$$\frac{(x+1)^2}{100}+\frac{(y-2)^2}{64}=1$$

93) Identify the type of conic and find centre, foci, vertices, and directrices of each of the following :

$$\frac{(y-2)^2}{25}\ \frac{(x+1)^2}{16}=1$$

94) Identify the type of conic and find centre, foci, vertices, and directrices of each of the following :

$18x^2+12y^2-144x+48y+120 = 0$

95) Identify the type of conic and find centre, foci, vertices, and directrices of each of the following :

$9x^2-y^2-36x-6y+18 = 0$

96) A cable of a suspension bridge hangs in the form of a parabola when the load is uniformly distributed horizontally. The distance between two towers is 1500 ft, the points of support of
the cable on the towers are 200 ft above the road way and the lowest point on the cable is 70 ft above the roadway. Find the vertical distance to the cable (parallel to the roadway) from a pole whose height is 122 ft.

97) The ceiling in a hallway 20 ft wide is in the shape of a semi ellipse and 18 ft high at the center. Find the height of the ceiling 4 feet from either wall if the height of the side walls is 12 ft.

98) By vector method, prove that $\cos(\alpha + \beta) = \cos\alpha \cos\beta - \sin\alpha \sin\beta$

99) Prove by vector method that $\sin(\alpha - \beta) = \sin\alpha \cos\beta - \cos\alpha \sin\beta$

100) Prove by vector method that the perpendiculars (attitudes) from the vertices to the opposite sides of a triangle are concurrent.

101) Using vector method, prove that $\cos(\alpha - \beta) = \cos\alpha \cos\beta + \sin\alpha \sin\beta$

102) Prove by vector method that $\sin(\alpha + \beta) = \sin\alpha \cos\beta + \cos\alpha \sin\beta$

103) A particle acted on by constant forces $8\hat{i}+2\hat{j}-6\hat{k}$ and $6\hat{i}+2\hat{j}-2\hat{k}$ is displaced from the point (1, 2, 3) to the point (5, 4, 1). Find the total work done by the forces.

104) Forces of magnit $5\sqrt{2}$ and $5\sqrt{2}$ units acting in the directions $3\hat{i}+4\hat{j}+5\hat{k}$ and $10\hat{i}+6\hat{j}-8\hat{k}$ respectively, act on a particle which is displaced from the point with

position vector $4\hat{i} + 3\hat{j} - 2\hat{k}$ to the point with position vector $6\hat{i} + \hat{j} - 3\hat{k}$. Find the work done by the forces.

105) Find the torque of the resultant of the three forces represented by $-3\hat{i} + 6\hat{j} + 3\hat{k}$, $4\hat{i} - 10\hat{j} + 12\hat{k}$ and $4\hat{i} + 7\hat{j}$ acting at the point with position vector $8\hat{i} - 6\hat{j} - 4\hat{k}$, about the point with position vector $18\hat{i} + 3\hat{j} - 9\hat{k}$

106) Determine whether the three vectors $2\hat{i} + 3\hat{j} + \hat{k}$, $\hat{i} - 2\hat{j} + 2\hat{k}$ and $3\hat{i} + \hat{j} + 2\hat{k}$ are coplanar.

107) If $\vec{a} = 2\hat{i} + 3\hat{j} - \hat{k}, \vec{b} = 3\hat{i} + 5\hat{j} + 2\hat{k}, \vec{c} = -\hat{i} - 2\hat{j} + 3\hat{k}$, verify that

(i) $(\vec{a} \times \vec{b}) \times \vec{c} = (\vec{a}.\vec{c}) \times \vec{b} - (\vec{b}.\vec{c})\vec{a}$

(ii) $\vec{a} \times (\vec{b} \times \vec{c}) = (\vec{a}.\vec{c}) \times \vec{b} - (\vec{a}.\vec{b})\vec{c}$

108) A straight line passes through the point (1, 2, −3) and parallel to $4\hat{i} + 5\hat{j} - 7\hat{k}$. Find

(i) vector equation in parametric form

(ii) vector equation in non-parametric form

(iii) Cartesian equations of the straight line.

109) Find the equation of a straight line passing through the point of intersection of the straight lines $\vec{r} = (\hat{i} + 3\hat{j} - \hat{k}) + t(2\hat{i} + 3\hat{j} + 2\hat{k})$ and $\frac{x-2}{1} = \frac{y-4}{2} = \frac{z+3}{4}$ and perpendicular to both straight lines.

110) Determine whether the pair of straight lines $\vec{r}(2\hat{i} + 3\hat{j} - \hat{k}) + t(2\hat{i} + 3\hat{j} + 2\hat{k})$, $\vec{r} = (2\hat{j} - 3\hat{k}) + s(\hat{i} + 2\hat{j} + 3\hat{k})$are parallel. Find the shortest distance between them.

111) Show that the lines $\frac{x-3}{3} = \frac{y-3}{-1} = z - 1$ and $\frac{x-6}{2} = \frac{z-1}{3}, y - 2 = 0$ intersect. Also find the point of intersection.

112) Find the vector parametric, vector non-parametric and Cartesian form of the equation of the plane passing through the points (-1, 2, 0), (2, 2, -1)and parallel to the straight line $\frac{x-1}{1} = \frac{2y+1}{2} = \frac{z+1}{-1}$

113) Find the non-parametric form of vector equation, and Cartesian equation of the plane passing through the point (2, 3, 6) and parallel to the straight lines $\frac{x-1}{2} = \frac{y+1}{3} = \frac{z-3}{1}$ and $\frac{x+3}{2} = \frac{y-3}{-5} = \frac{z+1}{-3}$

114) Find the non-parametric form of vector equation, and Cartesian equations of the plane passing through the points (2, 2, 1), (9, 3, 6) and perpendicular to the plane 2x +

$6y + 6z = 9$

115) Find parametric form of vector equation and Cartesian equations of the plane passing through the points (2, 2, 1), (1, −2, 3) and parallel to the straight line passing through the points (2, 1, −3) and (−1, 5, −8)

116) Find the non-parametric form of vector equation of the plane passing through the point (1, −2, 4) and perpendicular to the plane $x + 2y - 3z = 11$ and parallel to the line $\frac{x+7}{3} = \frac{y+3}{-1} = \frac{z}{1}$

117) Find the parametric form of vector equation and Cartesian equations of the plane containing the line $\vec{r} = (\hat{i} - \hat{j} + 3\hat{k}) + t(2\hat{i} - \hat{j} + 4\hat{k})$ and perpendicular to plane $\vec{r}.(\hat{i} + 2\hat{j} + \hat{k}) = 8$

118) Find the parametric vector, non-parametric vector and Cartesian form of the equations of the plane passing through the points (3, 6, −2), (−1,−2, 6) , and (6, −4, −2).

119) Show that the lines $\frac{x-2}{1} = \frac{y-3}{1} = \frac{z-4}{3}$ and $\frac{x-1}{-3} = \frac{y-4}{2} = \frac{z-5}{1}$ coplanar. Also, find the plane containing these lines.

120) Find the coordinates of the foot of the perpendicular and length of the perpendicular from the point (4, 3, 2) to the plane $x + 2y + 3z = 2$.

121) If $\vec{p} = -3\hat{i} + 4\hat{j} - 7\hat{k}$ and $\vec{q} = 6\hat{i} + 2\hat{j} - 3\hat{k}$ then find $\vec{p} \times \vec{q}$. Verify that $\vec{p}$ and $\vec{p} \times \vec{q}$ are perpendicular to each other and also verify that $\vec{q}$ and $\vec{p} \times \vec{q}$ are perpendicular to each other,

122) A particle moves along a horizontal line such that its position at any time $t \geq 0$ is given by $s(t) = t^3 - 6t^2 + 9t + 1$, where s is measured in metres and t in seconds?

(1) At what time the particle is at rest?

(2) At what time the particle changes direction?

(3) Find the total distance travelled by the particle in the first 2 seconds.

123) A road running north to south crosses a road going east to west at the point P. Car A is driving north along the first road, and car B is driving east along the second road. At a particular time car A 10 kilometres to the north of P and traveling at 80 km/hr, while car B is 15 kilometres to the east of P and traveling at 100 km/hr. How fast is the distance between the two cars changing?

124) A particle moves along a line according to the law $s(t) = 2t^3 - 9t^2 + 12t - 4$, where $t \geq 0$.

(i) At what times the particle changes direction?

(ii) Find the total distance travelled by the particle in the first 4 seconds.

(iii) Find the particle's acceleration each time the velocity is zero.

125) A conical water tank with vertex down of 12 metres height has a radius of 5 metres at the top. If water flows into the tank at a rate 10 cubic m/min, how fast is the depth of the water increases when the water is 8 metres deep?

126) A ladder 17 metre long is leaning against the wall. The base of the ladder is pulled away from the wall at a rate of 5 m/s. When the base of the ladder is 8 metres from the wall.

(i) How fast is the top of the ladder moving down the wall?

(ii) At what rate, the area of the triangle formed by the ladder, wall and the floor is changing?

127) A ladder 17 metre long is leaning against the wall. The base of the ladder is pulled away from the wall at a rate of 5 m/s. When the base of the ladder is 8 metres from the wall.

128) A police jeep, approaching an orthogonal intersection from the northern direction, is chasing a speeding car that has turned and moving straight east. When the jeep is 0.6 km north of the intersection and the car is 0.8 km to the east. The police determine with a radar that the distance between them and the car is increasing at 20 km/hr. If the jeep is moving at 60 km/hr at the instant of measurement, what is the speed of the car?

129) Find the equation of the tangent and normal to the Lissajous curve given by $x = 2\cos 3t$ and $y = 3\sin 2t$, $t \in R$

130) Find the angle between $y = x^2$ and $y = (x - 3)^2$.

131) Find the angle between the curves $y = x^2$ and $x = y^2$ at their points of intersection (0,0) and (1,1).

132) If the curves $ax^2 + by^2 = 1$ and $cx^2 + dy^2 = 1$ intersect each other orthogonally then, $\frac{1}{a} - \frac{1}{b} = \frac{1}{c} - \frac{1}{d}$

133) Find the equations of the tangents to the curve $y = \frac{x+1}{x-1}$ which are parallel to the line $x + 2y = 6$.

134) Show that the two curves $x^2 - y^2 = r^2$ and $xy = c^2$ where c, r are constants, cut orthogonally

135) Find the intervals of mono tonicities and hence find the local extremum for the following function:

$f(x) = 2x^3 + 3x^2 - 12x$

136) Using the Lagrange's mean value theorem determine the values of x at which the tangent is parallel to the secant line at the end points of the given interval:
$f(x) = x^3 - 3x + 2, x \in [-2, 2]$

137) Expand log(1+ x) as a Maclaurin's series upto 4 non-zero terms for $-1 < x \leq 1$.

138) Find the dimensions of the rectangle with maximum area that can be inscribed in a circle of radius 10 cm.

139) Prove that among all the rectangles of the given perimeter, the square has the maximum area.

140) Find the dimensions of the largest rectangle that can be inscribed in a semi circle of radius r cm.

141) A manufacturer wants to design an open box having a square base and a surface area of 108 sq. cm. Determine the dimensions of the box for the maximum volume.

142) The volume of a cylinder is given by the formula $V = \pi r^2 h$. Find the greatest and least values of V if r + h = 6.

143) A hollow cone with base radius a cm and , height b em is placed on a table. Show that the volume of the largest cylinder that can be hidden underneath is $\frac{4}{9}$times volume of the cone.

144) Expand sin x in ascending powers $x - \frac{\pi}{4}$ upto three non-zero terms.

145) Evaluate the following limit, if necessary use l'Hôpital Rule

$\lim_{x \to 0^+} x^x$

146) For the function $f(x)=4x^3+3x^2-6x+1$ find the intervals of monotonicity, local extrema, intervals of concavity and points of inflection.

147) Determine the intervals of concavity of the curve $f(x) = (x-1)^3 . (x-5)$, $x \in R$ and, points of inflection if any.

148) Find the local extrema of the function $f(x) = 4x^6 - 6x^4$

149) Find the intervals of concavity and points of inflexion for $f(x)=x^3-15x^2+75x-50$.

150) Find the local maximum and local minimum values for $f(x)=12x^2-2x^2-x^4$.

151) A right circular cylinder has radius r =10 cm. and height h = 20 cm. Suppose that the radius of the cylinder is increased from 10 cm to 10. 1 cm and the height does not

change. Estimate the change in the volume of the cylinder. Also, calculate the relative error and percentage error.

152) The radius of a circular plate is measured as 12.65 cm instead of the actual length 12.5 cm.find the following in calculating the area of the circular plate:
(i) Absolute error
(ii) Relative error
(iii) Percentage error

153) The time T, taken for a complete oscillation of a single pendulum with length l, is given by the equation T = $2\pi\sqrt{\frac{l}{g}}$, where g is a constant. Find the approximate percentage error in the calculated value of T corresponding to an error of 2 percent in the value of l

154) Let f(x, y) = sin(xy^2) + e^{x3+5y} for all ∈ R^2. Calculate $\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial^2 f}{\partial y \partial x}$ and $\frac{\partial^2 f}{\partial x \partial y}$

155) Let w(x, y, z) = $\frac{1}{\sqrt{x^2+y^2+z^2}}, (x, y, z) \neq (0, 0, 0)$. Show that $\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2} = 0$

156) If V(x,y) = e^x(x cos y - y siny), then prove that $\frac{\partial^2 V}{\partial x^2} = \frac{\partial^2 V}{\partial y^2} = 0$

157) If w(x, y) = xy + sin (xy), then prove that $\frac{\partial^2 w}{\partial y \partial x} = \frac{\partial^2 w}{\partial x \partial y}$

158) W(x, y, z) = xy + yz + zx, x =u -v, y = uv, z = u + v, u ∈ R. Find $\frac{\partial W}{\partial u}, \frac{\partial W}{\partial v}$, and evaluate them at $\left(\frac{1}{2}, 1\right)$

159) If u = tan $^{-1}\left(\frac{x^3+y^3}{x-y}\right)$ Prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}$ sin 2u.

160) Find the approximate value of $\sqrt[3]{1.02} + \sqrt{1.02}$

161) Using Euler's theorem, prove that x$\frac{\hat{\partial} u}{\hat{\partial} x}$ + y$\frac{\hat{\partial} u}{\partial y} = \frac{1}{2}\tan u$ if $u = \sin^{-1}\left(\frac{x-y}{\sqrt{x}+\sqrt{y}}\right)$

162) If V = ze^{ax+by} and z is a homogeneous function of degree n in x and y prove that $x\frac{\partial V}{\partial x} + \mathbf{y}\frac{\partial V}{\partial y} = (ax + by + n)V$

163) If u = $\sec^{-1}\left(\frac{x^3-y^3}{x+y}\right)$ show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial x} = 2\cot u$

164) Show that $\int_0^{\frac{\pi}{2}} \frac{dx}{4+5sinx} = \frac{1}{3}\log_e 2$.

165) Prove that $\int_0^{\frac{\pi}{4}} \log(1+\tan x)dx = \frac{\pi}{8} \log 2$.

166) Show that $\int_0^1 (tan^{-1}x + tan^{-1}(1-x))\ dx = \frac{\pi}{2} - \log_e 2$

167) Evaluate $\int_2^3 \frac{\sqrt{x}}{\sqrt{5-x}+\sqrt{x}}dx$.

168) Evaluate the following integrals using properties of integration:
$\int_0^{\pi} \frac{xsinx}{1+sinx}dx$

169) Evaluate the following integrals using properties of integration:
$\int_{\frac{\pi}{8}}^{\frac{3\pi}{8}} \frac{1}{1+\sqrt{tanx}}dx$

170) Find the area of the region bounded by the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

171) Find the area of the region bounded between the parabola $x^2 = y$ and the curve $y = |x|$.

172) The region enclosed by the circle $x^2 + y^2 = a^2$ is divided into two segments by the line $x = h$. Find the area of the smaller segment.

173) Find the area of the region in the first quadrant bounded by the parabola $y^2 = 4x$, the line $x + y = 3$ and y-axis.

174) Find, by integration, the area of the region bounded by the lines $5x - 2y = 15$, $x + y + 4 = 0$ and the x-axis

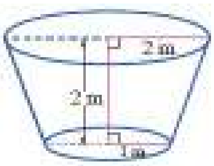
175) Find the area of the region bounded by the line $y = 2x + 5$ and the parabola $y = x^2 - 2x$.

176) Find the area of the region bounded by the parabola $y^2 = x$ and the line $y = x - 2$

177) Find the area of the region common to the circle $x^2 + y^2 = 16$ and the parabola $y^2 = 6x$.

178) Find the volume of the spherical cap of height h cut of from a sphere of radius r.

179) Find, by integration, the volume of the container which is in the shape of a right circular conical frustum.



180) A watermelon has an ellipsoid shape which can be obtained by revolving an ellipse with major-axis 20 cm and minor-axis 10 cm about its major-axis. Find its volume using integration.

181) Solve the following differential equations:

$$\frac{dy}{dx} = \sqrt{\frac{1-y^2}{1-x^2}}$$

182) Solve the following differential equations:

$$\frac{dy}{dx} = e^{x+y} + x^3 e^y$$

183) Solve the following differential equations:

$(e^y+1) \cos x \, dx + e^y \sin x \, dy = 0$

184) Solve the following differential equations $2xydx + (x^2 + 2y^2)dy = 0$

185) Solve the differential equation $(y^2-2xy)\,dx = (x^2-2xy)\,dy$

186) Solve the following differential equations

$$\left(1 + 3e^{\frac{y}{x}}\right)dy + 3e^{\frac{y}{x}}\left(1 - \frac{y}{x}\right)dx = 0,$$ given that y = 0- when x=1

187) Solve $(1+x^3)\frac{dy}{dx} + 6x^2y = 1+x^2$.

188) Solve the Linear differential equation:

$$\frac{dy}{dx} + \frac{y}{xlogx} = \frac{sin2x}{logx}$$

189) Solve the Linear differential equation:

$$\frac{dy}{dx} = \frac{sin^2x}{1+x^3} - \frac{3x^2}{1+x^3}y$$

190) The growth of a population is proportional to the number present. If the population of a colony doubles in 50 years, in how many years will the population become triple?

191) A radioactive isotope has an initial mass 200mg, which two years later is 50mg . Find the expression for the amount of the isotope remaining at any time. What is its half-life? (half-life means the time taken for the radioactivity of a specified isotope to fall to half its original value).

192) In a murder investigation, a corpse was found by a detective at exactly 8 p.m. Being alert, the detective also measured the body temperature and found it to be 70^oF. Two hours later, the detective measured the body temperature again and found it to be 60^oF. If the room temperature is 50^oF, and assuming that the body temperature of the

person before death was 98.6^oF, at what time did the murder occur? [log(2.43) = 0.88789; log(0.5)=-0.69315]

193) Find the population of a city at any time t, given that the rate of increase of population is proportional to the population at that instant and that in a period of 40 years the population increased from 3,00,000 to 4,00,000.

194) Suppose a person deposits 10,000 Indian rupees in a bank account at the rate of 5% per annum compounded continuously. How much money will be in his bank account 18 months later?

195) Assume that the rate at which radioactive nuclei decay is proportional to the number of such nuclei that are present in a given sample. In a certain sample 10% of the original number of radioactive nuclei have undergone disintegration in a period of 100 years. What percentage of the original radioactive nuclei will remain after 1000 years?

196) Water at temperature 100^oC cools in 10 minutes to 80^oC in a room temperature of 25^oC.
Find
(i) The temperature of water after 20 minutes
(ii) The time when the temperature is 40^oC

$$\left[log_e\frac{11}{15} = -0.3101; log_e 5 = 1.6094\right]$$

197) A pot of boiling water at 100^oC is removed from a stove at time t = 0 and left to cool in the kitchen. After 5 minutes, the water temperature has decreased to 80^oC , and another 5 minutes later it has dropped to 65^oC . Determine the temperature of the kitchen.

198) Two balls are chosen randomly from an urn containing 6 red and 8 black balls. Suppose that we win Rs. 15 for each red ball selected and we lose Rs. 10 for each black ball selected. X denotes the winning amount, then find the values of X and number of points in its inverse images.

199) A six sided die is marked '1' on one face, '3' on two of its faces, and '5' on remaining three faces. The die is thrown twice. If X denotes the total score in two throws, find
(i) the probability mass function
(ii) the cumulative distribution function
(iii) $P(4 \le X < 10)$
(iv) $P(X \ge 6)$

200) Find the probability mass function and cumulative distribution function of number of girl child in families with 4 children, assuming equal probabilities for boys and girls.

201) A random variable X has the following probability mass function.

x	1	2	3	4	5
f(x)	k^2	$2k^2$	$3k^2$	2k	3k

Find

(i) the value of k

(ii) $P(2 \leq X < 5)$

(iii) $P(3 < X)$

202) The probability density function of X is given

$$f(x) = \begin{cases} Ke^{\frac{-x}{3}} & for \quad x > 0 \\ 0 \quad for & x \leq 0 \end{cases}$$

Find

(i) the value of k

(ii) the distribution function.

(iii) $P(X < 3)$

(iv) $P(5 \leq X)$

(v) $P(X \leq 4)$

203) If X is the random variable with probability density functionj{x) given by,

$$f(x) = \begin{cases} x+1 & -1 \leq x < 0 \\ -x+1 & 0 \leq x < 1 \\ 0 \quad otherwise & \end{cases}$$

then find

(i) the distribution function F(x)

(ii) $P(-0.5 \leq X \leq 0.5)$

204) If μ and $σ^2$ are the mean and variance of the discrete random variable X, and E(X + 3) =10 and $E(X + 3)^2 = 116$, find μ and σ^2

205) Compute P(X = k) for the binomial distribution, B(n, p) where

n = 9, $p = \frac{1}{2}$, k = 7

206) The probability that Mr.Q hits a target at any trial is $\frac{1}{4}$. Suppose he tries at the target 10 times. Find the probability that he hits the target

(i) exactly 4 times

(ii) at least one time.

207) A retailer purchases a certain kind of electronic device from a manufacturer. The manufacturer, indicates that the defective rate of the device is 5%. The inspector of the retailer randomly picks 10 items from a shipment. What is the probability that there will be

(i) at least one defective item

(ii) exactly two defective items.

208) If the probability that a fluorescent light has a useful life of at least 600 hours is 0.9, find the probabilities that among 12 such lights

(i) exactly 10 will have a useful life of at least 600 hours;

(ii) at least 11 will have a useful life of at least 600 hours

(iii) at least 2 will not have a useful life of at least 600 hours.

209) The mean and standard deviation of a binomial variate X are respectively 6 and 2. Find

(i) the probability mass function

(ii) $P(X = 3)$

(iii) $P(X \geq 2)$.

210) Suppose a pair of unbiased dice is rolled once. If X denotes the total score of two dice, write down

(i) the sample space

(ii) the values taken by the random variable X,

(iii) the inverse image of 10, and

(iv) the number of elements in inverse image of X.

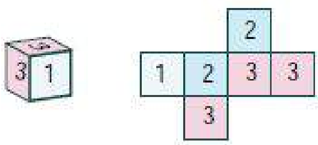
211) A six sided die is marked '1' on one face, '2' on two of its faces, and '3' on remaining three faces. The die is rolled twice. If X denotes the total score in two throws.

(i) Find the probability mass function.

(ii) Find the cumulative distribution function.

(iii) Find $P(3 \leq X < 6)$

(iv) Find $P(X \geq 4)$.



212) A random variable X has the following probability mass function

x	1	2	3	4	5	6
f(x)	k	2k	6k	5k	6k	10k

Find

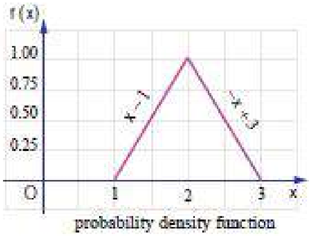
(i) $P(2 < X < 6)$

(ii) $P(2 \leq X < 5)$

(iii) $P(X \leq 4)$

(iv) $P(3 < X)$

213) If X is the random variable with probability density functionf(x)given by,



$$f(x) = \begin{cases} x-1 & 1 \leq x < 2 \\ -x+3 & 2 \leq x < 3 \\ 0 & Otherwise \end{cases}$$

find (i) the distribution function F (x)

(ii) $P(1.5 \leq X \leq 2.5)$

214) Let X be a random variable denoting the life time of an electrical equipment having probability density function

$$f(x) = \begin{cases} ke^{-2x} & for x > 0 \\ 0 & for x \leq 0 \end{cases}$$

Find

(i) the value of k

(ii) Distribution function

(iii) $P(X < 2)$

(iv) calculate the probability that X is at least for four unit of time

(v) $P(X = 3)$

215) Suppose that f (x) given below represents a probability mass function

x	1	2	3	4	5	6
f(x)	c^2	$2c^2$	$3c^2$	$4c^2$	c	2c

Find

(i) the value of c

(ii) Mean and variance.

216) On the average, 20% of the products manufactured by ABC Company are found to be defective. If we select 6 of these products at random and X denote the number of defective products find the probability that

(i) two products are defective

(ii) at most one product is defective

(iii) at least two products are defective.

217) A random variable X has the following probability distribution values of X.

x	0	1	2	3	4	5	6	7

f(x)	0	k	2k	2k	3k	k^2	$2k^2$	$7k^2 + k$

Find

(i) k

(ii) P(X< 6)

(iii) $P(X \geq 6)$

(iv) P(0 < X< 5)

218) Verify

(i) closure property

(ii) commutative property, and

(iii) associative property of the following operation on the given set. $(a*b) = a^b$;∀a, b∈N (exponentiation property)

219) Verify

(i) closure property

(ii) commutative property

(iii) associative property

(iv) existence of identity, and

(v) existence of inverse for following operation on the given set m*n = m + n - mn; m, n ∈Z

220) Using the equivalence property, show that p ↔ q ≡ (p ∧ q) v (¬p ∧ ¬q)

221) Let p: Jupiter is a planet and q: India is an island be any two simple statements. Give verbal sentence describing each of the following statements.

(i) ¬p

(ii) p ∧ ¬q

(iii) ¬p ∨ q

(iv) p→ ¬q

(v) p↔q

222) Define an operation $*$ on Q as follows: a * b = $\left(\frac{a+b}{2}\right)$; a,b ∈Q. Examine the closure, commutative, and associative properties satisfied by $*$ on Q.

223) Define an operation$*$ on Q as follows: a*b = $\left(\frac{a+b}{2}\right)$; a,b ∈Q. Examine the existence of identity and the existence of inverse for the operation * on Q.

224) Verify whether the following compound propositions are tautologies or contradictions or contingency

(p ∧ q) ∧ ¬ (p ∨ q)

225) Verify whether the following compound propositions are tautologies or contradictions or contingency

$((p \vee q) \wedge \neg p) \rightarrow q$

226) Verify whether the following compound propositions are tautologies or contradictions or contingency

$((p \longrightarrow q) \wedge (q \longrightarrow r)) \longrightarrow (p \longrightarrow r)$

227) Let M = $\left\{ \begin{pmatrix} x & x \\ x & x \end{pmatrix} : x \in R - \{0\} \right\}$ and let * be the matrix multiplication. Determine whether M is closed under ∗. If so, examine the commutative and associative properties satisfied by ∗ on M.

228) Let M = $\left\{ \begin{pmatrix} x & x \\ x & x \end{pmatrix} : x \in R - \{0\} \right\}$ and let ∗ be the matrix multiplication. Determine whether M is closed under ∗ . If so, examine the existence of identity, existence of inverse properties for the operation ∗ on M.

229) Let A be $Q\backslash\{1\}$. Define ∗ on A by x*y = x + y − xy. Is ∗ binary on A? If so, examine the commutative and associative properties satisfied by ∗ on A.

230) Let A be $Q\backslash\{1\}$. Define ∗ on A by x*y = x + y − xy . Is ∗ binary on A? If so, examine the existence of identity, existence of inverse properties for the operation ∗ on A.

231) Prove $p \longrightarrow (q \longrightarrow r) \equiv (p \wedge q) \longrightarrow r$ without using truth table.

232) Prove that $p \rightarrow (\neg q \vee r) \equiv \neg p \vee (\neg q \vee r)$ using truth table.

233) Construct the truth table for $(p \wedge q) \vee r$.

234) Verify $(p \wedge \sim p) \wedge (\sim q \wedge p)$ is a tautlogy, contradiction or contingency.

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