RAVI MATHS TUITION CENTER , WHATSAPP - 8056206308

ANNUAL MINIMUM MATERIALS 3 MARKS

12th Standard

Maths

ANSWERS AVAILABLE MY YOUTUBE CHANNEL NAME - RAVI MATHS TUITION CENTER

DO DAILY 10 QUESTIONS. IN 14 DAYS YOU CAN FINISH 3 MARKS ONE TIME REVISION. 140 x 3 = 420

1) If A = $\begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$, verify thatA(adj A)=(adj A)A = $|A|\ I_3$.

2) Find a matrix A if adj(A) = $\begin{bmatrix} 7 & 7 & -7 \\ -1 & 11 & 7 \\ 11 & 5 & 7 \end{bmatrix}$.

3) If A = $\begin{bmatrix} 5 & 3 \\ -1 & -2 \end{bmatrix}$, show that $A^2 - 3A - 7I_2 = O_2$. Hence find A^{-1}.

4) If A = $\begin{bmatrix} 8 & -4 \\ -5 & 3 \end{bmatrix}$, verify that A(adj A) = (adj A)A = $|A|I_2$.

5) If A = $\begin{bmatrix} 3 & 2 \\ 7 & 5 \end{bmatrix}$ and B = $\begin{bmatrix} -1 & -3 \\ 5 & 2 \end{bmatrix}$, verify that $(AB)^{-1} = B^{-1}A^{-1}$

6) Decrypt the received encoded message $\begin{bmatrix} 2 & -3 \end{bmatrix}\begin{bmatrix} 20 & 4 \end{bmatrix}$ with the encryption matrix $\begin{bmatrix} -1 & -1 \\ 2 & 1 \end{bmatrix}$ and the decryption matrix as its inverse, where the system of codes are described by the numbers 1 - 26 to the letters A - Z respectively, and the number 0 to a blank space.

7) Find the rank of the matrix $\begin{bmatrix} 2 & -2 & 4 & 3 \\ -3 & 4 & -2 & -1 \\ 6 & 2 & -1 & 7 \end{bmatrix}$ by reducing it to an echelon form.

8) Find the inverse of A = $\begin{bmatrix} 2 & 1 & 1 \\ 3 & 2 & 1 \\ 2 & 1 & 2 \end{bmatrix}$ by Gauss-Jordan method.

9) Solve the following systems of linear equations by Gaussian elimination method:
$2x - 2y + 3z = 2$, $x + 2y - z = 3$, $3x - y + 2z = 1$

10) Investigate for what values of λ and μ the system of linear equations $x + 2y + z = 7$, $x + y + \lambda z = \mu$, $x + 3y - 5z = 5$ has
(i) no solution (ii) a unique solution (iii) an infinite number of solutions

11) Find the value of k for which the equations
$kx - 2y + z = 1$, $x - 2ky + z = -2$, $x - 2y + kz = 1$ have

(i) no solution

(ii) unique solution

(iii) infinitely many solution

12) Determine the values of λ for which the following system of equations $(3\lambda - 8)x + 3y + 3z = 0$, $3x + (3\lambda - 8)y + 3z = 0$, $3x + 3y + (3\lambda - 8)z = 0$. has a non-trivial solution.

13) Solve the following system of homogenous equations.
$2x + 3y - z = 0$, $x - y - 2z = 0$, $3x + y + 3z = 0$

14) Find the value of the real numbers x and y, if the complex number $(2+i)x+(1-i)y+2i-3$ and $x+(-1+2i)y+1+i$ are equal

15) Simplify $\left(\frac{1+i}{1-i}\right)^3 - \left(\frac{1-i}{1+i}\right)^3$ into rectangular form

16) If $\frac{z+3}{z-5i} = \frac{1+4i}{2}$, find the complex number z in the rectangular form

17) The complex numbers u, v, and w are related by $\frac{1}{u} = \frac{1}{v} + \frac{1}{w}$ If $v = 3-4i$ and $w = 4+3i$, find u in rectangular form.

18) If $|z| = 2$ show that $3 \leq |z+3+4i| \leq 7$

19) If $|z| = 1$, show that $2 \leq \left|z^2 - 3\right| \leq 4$

20) If $\omega \neq 1$ is a cube root of unity, then the show that
$$\frac{a+b\omega+c\omega^2}{b+c\omega+a\omega^2} + \frac{a+b\omega+c\omega^2}{c+a\omega+b\omega^2} = -1$$

21) Show that $|z+2-i| < 2$ represents interior points of a circle. Find its centre and radius.

22) If $z = x + iy$ and $\arg\left(\frac{z-1}{z+1}\right) = \frac{\pi}{2}$, then show that $x^2 + y^2 = 1$.

23) Simplify $\left(\frac{1+cos2\theta+isin2\theta}{1+cos2\theta-isin2\theta}\right)^{30}$

24) If $z = (cos\ \theta + isin\ \theta)$, show that $z^n + \frac{1}{z^n} = 2cos\ n\theta$ and $z^n - \frac{1}{z^n} = 2i\ sin\ n\theta$

25) Obtain the Cartesian equation for the locus of $z = x + iy$ in each of the following cases:
$|z - 4|^2 - |z - 1|^2 = 16$

26) Show that the complex numbers 3 + 2i, 5i, -3 + 2i and -i form a square.

27) Simplify: $\frac{(\cos 2\theta - i\sin 2\theta)^4 (\cos 4\theta + i\sin 4\theta)^{-5}}{(\cos 3\theta + i\sin 3\theta)^{-2} (\cos 3\theta - i\sin 3\theta)^{-9}}$

28) If $(x_1 + iy_1)(x_2 + iy_2)(x_3 + iy_3)\ldots(x_n + iy_n) = a + ib$, show that
$\sum_{r=1}^{n} tan^{-1}\left(\frac{y_r}{x_r}\right) = tan^{-1}\left(\frac{b}{a}\right) + 2k\pi, k\epsilon Z$

29) If $2\ cos\ \alpha = x + \frac{1}{x}$ and $2\ cos\ \beta = y + \frac{1}{y}$, show that $\frac{x}{y} + \frac{y}{x} = 2cos\,(\alpha - \beta)$

30) If $2cos\ \alpha = x + \frac{1}{x}$ and $2cos\ \beta = y + \frac{1}{y}$, show that $xy - \frac{1}{xy} = 2isin\,(\alpha + \beta)$

31) If $2cos\ \alpha = x + \frac{1}{x}$ and $2cos\ \beta = y + \frac{1}{y}$, show that $\frac{x^m}{y^n} - \frac{y^n}{x^m} = 2isin\,(m\alpha - n\beta)$

32) Solve the equation $3x^3 - 16x^2 + 23x - 6 = 0$ if the product of two roots is 1.

33) Find the sum of squares of roots of the equation $2x^4 - 8x^3 + 6x^2 - 3 = 0$.

34) Solve the equation $x^4 - 9x^2 + 20 = 0$.

35) Solve the equation $x^3 - 3x^2 - 33x + 35 = 0$.

36) Solve the cubic equation : $2x^3 - x^2 - 18x + 9 = 0$ if sum of two of its roots vanishes.

37) Solve the equation $7x^3 - 43x^2 = 43x - 7$

38) Solve the following equations,
$\sin^2 x - 5 \sin x + 4 = 0$

39) Solve: $8x^{\frac{3}{2x}} - 8x^{\frac{-3}{2x}} = 63$

40) Show that the polynomial $9x^9 + 2x^5 - x^4 - 7x^2 + 2$ has at least six imaginary roots.

41) Solve: $2\sqrt{\frac{x}{a}} + 3\sqrt{\frac{a}{x}} = \frac{b}{a} + \frac{6a}{b}$

42) Find all real numbers satisfying $4^x - 3(2^{x+2}) + 2^5 = 0$

43) Discuss the maximum possible number of positive and negative roots of the polynomial equation $9x^9 - 4x^8 + 4x^7 - 3x^6 + 2x^5 + x^3 + 7x^2 + 7x + 2 = 0$

44) Find the exact number of real zeros and imaginary of the polynomial $x^9 + 9x^7 + 7x^5 + 5x^3 + 3x$.

45) Find the value of $\sin^{-1}\left(sin\frac{5\pi}{9}cos\frac{\pi}{9} + cos\frac{5\pi}{9}sin\frac{\pi}{9}\right)$.

46) Find the domain of $\cos^{-1}\left(\frac{2+sinx}{3}\right)$

47) Evaluate $sin\left[sin^{-1}\left(\frac{3}{5}\right) + sec^{-1}\left(\frac{5}{4}\right)\right]$

48) If $\cos^{-1} x + \cos^{-1} y + \cos^{-1} z = \pi$ and $0 < x, y, z < 1$, show that $x^2 + y^2 + z^2 + 2xyz = 1$

49) Solve $cos\left(sin^{-1}\left(\frac{x}{\sqrt{1+x^2}}\right)\right) = sin\left\{cot^{-1}\left(\frac{3}{4}\right)\right\}$

50) Prove that
$tan^{-1}(\frac{2}{11}) + tan^{-1}(\frac{7}{24}) = tan^{-1}(\frac{1}{2})$

51) Solve $sin^{-1}\frac{5}{x} + sin^{-1}\frac{12}{x} = \frac{\pi}{2}$

52) Find the value of $cos^{-1}\left(cos\left(\frac{4\pi}{3}\right)\right) + cos^{-1}\left(cos\left(\frac{5\pi}{4}\right)\right)$

53) Find the value of
$cos\left(sin^{-1}\left(\frac{4}{5}\right) - tan^{-1}\left(\frac{3}{4}\right)\right)$

54) Find the value of
$tan\left(sin^{-1}\frac{3}{5} + cot^{-1}\frac{3}{2}\right)$

55) Solve $cot^{-1}x - cot^{-1}(x+2) = \frac{\pi}{12}, x > 0$

56) Prove that $tan^{-1}\left(\frac{m}{n}\right) - tan^{-1}\left(\frac{m-n}{m+n}\right) = \frac{\pi}{4}$

57) Prove that $2\tan^{-1}x = \cos\left(\frac{1-x^2}{1+x^2}\right), x \geq 0$

58) Solve $\tan^{-1}\left(\frac{x-1}{x-2}\right) + \tan^{-1}\left(\frac{x+1}{x+2}\right) = \frac{\pi}{4}$

59) Find the equations of the tangent and normal to the circle $x^2 + y^2 = 25$ at P(-3, 4).

60) Find the equation of the ellipse in each of the cases given below:
foci (±3, 0), e = $\frac{1}{2}$

61) Find the equation of the ellipse in each of the cases given below:
foci (0, ±4) and end points of major axis are (0, ±5).

62) Find the equation of the ellipse in each of the cases given below:
length of latus rectum 8, eccentricity = $\frac{3}{5}$, centre (0, 0) and major axis on x -axis.

63) A particle acted upon by constant forces $2\hat{j} + 5\hat{j} + 6\hat{k}$ and $-\hat{i} - 2\hat{j} - \hat{k}$ is displaced from the point (4, −3, −2) to the point (6, 1, −3) . Find the total work done by the forces.

64) Find the magnitude and the direction cosines of the torque about the point (2, 0, -1) of a force $(2\hat{i} + \hat{j} - \hat{k})$, whose line of action passes through the origin

65) Find the magnitude and direction cosines of the torque of a force represented by $3\hat{i} + 4\hat{j} - 5\hat{k}$ about the point with position vector $2\hat{i} - 3\hat{j} + 4\hat{k}$ acting through a point whose position vector is $4\hat{i} + 2\hat{j} - 3\hat{k}$.

66) The volume of the parallelepiped whose coterminus edges are $7\hat{i} + \lambda\hat{j} - 3\hat{k}, \hat{i} + 2\hat{j} - \hat{k}$, $-3\hat{i} + 7\hat{j} + 5\hat{k}$ is 90 cubic units. Find the value of λ.

67) Prove that $[\vec{a} \times \vec{b}, \vec{b} \times \vec{c}, \vec{c} \times \vec{a}]$ = $[\vec{a}, \vec{b}, \vec{c}]^2$

68) Find the angle between the lines $\vec{r} = (\hat{i} + 2\hat{j} + 4\hat{k}) + t(2\hat{i} + 2\hat{j} + \hat{k})$ and the straight line passing through the points (5, 1, 4) and (9, 2, 12)

69) Show that the straight line passing through the points A (6, 7, 5) and B(8, 10, 6) is perpendicular to the straight line passing through the points C(10, 2, -5) and D(8, 3, -4)

70) Find the equation of the plane passing through the intersection of the planes $\vec{r}.(\hat{i} + \hat{j} + \hat{k}) + 1 = 0$ and $\vec{r}.(2\hat{i} - 3\hat{j} + 5\hat{k}) = 2$ and the point (-1, 2, 1).

71) Find the tangent and normal to the following curves at the given points on the curve
x = cos t, y = 2sin t^2 at t = $\frac{\pi}{3}$

72) Compute the value of 'c' satisfied by the Rolle's theorem for the function f (x) = x^2 (1 - $x)^2$, x ∈ [0,1]

73) Evaluate: $\lim\limits_{x\to 0^+}\left(\frac{1}{x} - \frac{1}{e^x-1}\right)$.

74) Evaluate the following limit, if necessary use l 'Hôpital Rule

$\lim\limits_{x \to 1^+} \left(\frac{2}{x^2-1} - \frac{x}{x-1} \right)$

75) Evaluate the following limit, if necessary use l 'Hôpital Rule

$\lim\limits_{x \to \frac{\pi}{2}} (sinx)^{tanx}$

76) Evaluate the following limit, if necessary use l 'Hôpital Rule

$\lim\limits_{x \to 0^+} (cosx)^{\frac{1}{x^2}}$

77) Find the absolute maximum and absolute minimum values of the function f (x) = $2x^3 + 3x^2 - 12x$ on [−3, 2]

78) We have a 12 square unit piece of thin material and want to make an open box by cutting small squares from the corners of our material and folding the sides up. The question is, which cut produces the box of maximum volume?

79) Find the local maximum and minimum of the function $x^2 y^2$ on the line x + y = 10

80) Evaluate the following limits, if necessary use L'Hopitals rule

(i) $\lim\limits_{x \to 0^+} x^{sinx}$

(ii) $\lim\limits_{x \to 0} \frac{cotx}{cot2x}$

(iii) $\lim\limits_{x \to \frac{\pi}{2}^-} (tanx)^{cosx}$

81) Find the points of local maxima and local minima (if any) for the function $f(x) = 2x^3 - 21x^2 + 36x - 20$

82) Discuss the curve y = $x^4 - 4x^3$ with respect to local extrema.

83) Verify Rolle's theorem for the function $f(x) = x^3 - 6x^2 + 11x - 6$ on the interval [1, 3]

84) Find the linear approximation for f(x) = $\sqrt{1+x}, x \geq -1$ at x_0 = 3. Use the linear approximation to estimate f(3.2)

85) Let $f(x) = \sqrt[3]{x}$. Find the linear approximation at x = 27. Use the linear approximation to approximate $\sqrt[3]{27.2}$

86) Find Δf and df for the function f for the indicated values of x, Δx and compare

(1) f(x) = $x^3 - 2x^2$; x = 2, Δ x = dx = 0.5

(2) f(x) = $x^2 + 2x + 3$; x = -0.5, Δx = dx = 0.1

87) For each of the following functions find the f_x, f_y, and show that $f_{xy} = f_{yx}$

f(x, y) = $\frac{3x}{y+sinx}$

88) If U(x, y, z) = $\frac{x^2+y^2}{xy} + 3z^2y$, find $\frac{\partial U}{\partial x}$; $\frac{\partial U}{\partial y}$ and $\frac{\partial U}{\partial z}$

89) Let U(x, y, z) = xyz, x = e^{-t}, y = e^{-t} cos t, z = sin t, t ∈ R. Find $\frac{dU}{dt}$

90) Let z(x, y) = $x^3 - 3x^2y^3$, where x = se^t, y = se^{-t}, s, t ∈ R. Find $\frac{\partial z}{\partial s}$ and $\frac{\partial z}{\partial t}$

91) If v(x, y) = log $\left(\frac{x^2+y^2}{x+y}\right)$, prove that $x\frac{\partial v}{\partial x} + y\frac{\partial u}{\partial y}$ = 1

92) If u = $\sin^{-1}\left(\frac{x+y}{\sqrt{x}+\sqrt{y}}\right)$, Show that x $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \frac{1}{2}tanu$

93) If f = $\frac{x}{x^2+y^2}$ then show that = $x\frac{\partial f}{\partial x} + y\frac{\partial f}{\partial y}$ = -f

94) If w= $\log(x^2+y^2)$ and x=rcosθ and y=rsinθ then, find $\frac{\partial w}{\partial r} and \frac{\partial w}{\partial \theta}$

95) If u = log (tan x + tan y + tan z), prove that $\sum \sin 2x \frac{\partial u}{\partial x} = 2$

96) If U = (x - y) (y -z) (z- x) then show that $U_x + U_y + U_z = 0$

97) Using Euler's Theorem prove the following.

(i) If u = $\tan^{-1}\left(\frac{x^3+y^3}{x-y}\right)$. Prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \sin 2u$

(ii) u = $xy^2 \sin(x/y)$ Show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 3u$

(iii) If $u = \sqrt{x^2+y^2}$ show that $\mathbf{x}\frac{\partial u}{\partial x} + \mathbf{y}\frac{\partial u}{\partial y} = u$

(iv) If u = $\mathbf{u} = e^{(x/y)}\sin(x/y) + e^{(y/x)}\cos(y/x)$ Show that x $\frac{\hat{c}u}{\partial x} + \mathbf{y}\frac{\partial u}{\partial y} = 0$

98) If u $= \sin 3x \ \cos 4y$ verify $\frac{\partial^2 u}{\partial x \partial y} = \frac{\partial^2 u}{\partial y \partial x}$

99) Find an approximate value of $\int_1^{1.5} x dx$ by applying the left-end rule with the partition {1.1, 1.2, 1.3, 1.4, 1.5}.

100) Evaluate$\int_1^4 (2x^2+3)$ dx, as the limit of a sum

101) Evaluate the following integrals as the limits of sums.

$\int_1^2 (4x^2-1)dx$

102) Evaluate: $\int_0^{\frac{\pi}{2}} \frac{cos\theta}{(1+sin\theta)(2+sin\theta)} d\theta$

103) Evaluate: $\int_0^{\frac{\pi}{2}} (\sqrt{tan\ x} + \sqrt{cot\ x})dx$

104) Prove that $\int_0^{\frac{\pi}{4}} \frac{sin2xdx}{sin^4x+cos^4x}$ = $\frac{\pi}{4}$

105) Prove that $\int_0^{\frac{\pi}{4}} \frac{dx}{a^2sin^2x+b^2cos^2x}$ = $\frac{1}{ab}tan^{-1}(\frac{a}{b})$ where a, b > 0

106) Evaluate the following $\int_0^{\frac{\pi}{\sqrt{2}}} \frac{dx}{5+4sin^2x}$

107) Find the area of the region bounded by the line 6x + 5y = 30, x − axis and the lines x = −1 and x = 3.

108) Find the area of the region bounded by the y-axis and the parabola x = 5 − 4y − y^2.

109) Find the area of the region bounded between the parabolas y^2 = 4x and x^2 = 4y.

110) Using integration find the area of the region bounded by triangle ABC, whose vertices A, B, and C are (−1, 1), (3, 2), and (0, 5) respectively

111) Find the volume of a sphere of radius a.

112) Find the volume of a right-circular cone of base radius r and height h.

113) Evaluate $\int_0^{\frac{\pi}{2}} \frac{\sqrt{cot\ x}}{\sqrt{cot\ x}+\sqrt{tan\ x}} dx$

114) Evaluate $\int_3^6 \frac{\sqrt{x}}{\sqrt{9-x}+\sqrt{x}} dx$

115) Evaluate $\int_0^{\frac{\pi}{2}} \frac{cos^{3/2}x}{cos^{3/2}x+sin^{3/2}x} dx$

116) Solve : $\frac{dy}{dx} = \sqrt{4x+2y-1}$

117) Solve $(x^2 - 3y^2)\ dx + 2xydy = 0$.

118) Solve $\frac{dy}{dx} + 2y = e^{-x}$

119) Solve the Linear differential equation:

$\cos x \frac{dy}{dx} + y \sin x = 1$

120) Form the differential equation for $y = e^{-2x}$ [A cos 3x-B sin 3x]

121) Solve: $\frac{dy}{dx} = (4x + y + 1)^2$

122) Solve :$(x^2-yx^2)dy+(y^2+x^2y^2)dx=0$

123) Find the differential equation of the family of curves $y = Ae^{-x} + Be^{x}$ where A and B are arbitrary constants.

124) Verify that y = A cos 2x - B sin 2x is the general solution of the differential equation $\frac{d^2y}{dx^2} + 4y = 0$

125) Form the differential equation of $y = e^{3x}$ (C cos 2x + D sin 2x), where C and D are atbitrary constants.

126) The probability density function of X is given by $f(x) = \begin{cases} kxe^{-2x} & for x > 0 \\ 0 \quad for & x \leq 0 \end{cases}$ Find the value of k.

127) An urn contains 2 white balls and 3 red balls. A sample of 3 balls are chosen at random from the urn. If X denotes the number of red balls chosen, find the values taken by the random variable X and its number of inverse images

128) Two balls are chosen randomly from an urn containing 6 white and 4 black balls. Suppose that we win Rs. 30 for each black ball selected and we lose Rs. 20 for each white ball selected. If X denotes the winning amount, then find the values of X and number of points in its inverse images.

129) Two fair coins are tossed simultaneously (equivalent to a fair coin is tossed twice). Find the probability mass function for number of heads occurred.

130) A random variable X has the following probability distribution

x	0	1	2	3	4	5	6	7

P(X)	0	k	2k	2k	3k	k^2	$2k^2$	$7k^2+k$

Evaluate (i) k

(ii) $P(X \geq 6)$

(iii) P(0

131) Establish the equivalence property p → q ≡ ¬ p v q

132) Establish the equivalence property connecting the bi-conditional with conditional: p ↔ q ≡ (p → q) ∧ (q⟶ p)

133) Let ∗ be defined on R by (a∗b)=a+b+ab-7. Is ∗ binary on R? If so, find $3*\left(\frac{-7}{15}\right)$.

134) Let $A = \begin{pmatrix} 1 & 0\,1 & 0 \\ 0 & 1\,0 & 1 \\ 1 & 0\,0 & 1 \end{pmatrix}, B = \begin{pmatrix} 0 & 1\,0 & 1 \\ 1 & 0\,1 & 0 \\ 1 & 0\,0 & 1 \end{pmatrix}, C = \begin{pmatrix} 1 & 1\,0 & 1 \\ 0 & 1\,1 & 0 \\ 1 & 1\,1 & 1 \end{pmatrix}$ be any three boolean matrices of the same type.

Find (A∨B)∧C

135) Let $A = \begin{pmatrix} 1 & 0\,1 & 0 \\ 0 & 1\,0 & 1 \\ 1 & 0\,0 & 1 \end{pmatrix}, B = \begin{pmatrix} 0 & 1\,0 & 1 \\ 1 & 0\,1 & 0 \\ 1 & 0\,0 & 1 \end{pmatrix}, C = \begin{pmatrix} 1 & 1\,0 & 1 \\ 0 & 1\,1 & 0 \\ 1 & 1\,1 & 1 \end{pmatrix}$ be any three boolean matrices of the same type.

Find (A∧B)∨C

136) Show that ¬(p ∧ q) ≡ ¬p V ¬q

137) Prove that q → p ≡ ¬p → ¬q

138) Show that p → q and q → p are not equivalent

139) Show that ¬(p↔q) ≡ p↔¬q

140) Construct the truth table for (-p) v (q ∧ r)

****