

# RAVI MATHS TUITION CENTER, CHENNAI – 82. PH - 8056206308

## 12TH MATHS MODEL PAPER 10

Date : 16-Nov-19

12th Standard

Maths

Reg.No. : 

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Exam Time : 03:00:00 Hrs

Total Marks : 90

20 X 1 = 20

### PART – I

### ANSWER ALL THE QUESTIONS.

- 1) In a square matrix the minor  $M_{ij}$  and the co-factor  $A_{ij}$  of an element  $a_{ij}$  are related by \_\_\_\_\_.  
 (a)  $A_{ij} = -M_{ij}$  (b)  $A_{ij} = M_{ij}$  (c)  $A_{ij} = (-1)^{i+j} M_{ij}$  (d)  $A_{ij} = (-1)^{i-j} M_{ij}$
- 2) If  $x = \cos\theta + i \sin\theta$ , then  $x^n + \frac{1}{x^n}$  is \_\_\_\_\_.  
 (a)  $2 \cos n\theta$  (b)  $2 i \sin n\theta$  (c)  $2^n \cos\theta$  (d)  $2^n i \sin\theta$
- 3)  $\sin \left\{ 2\cos^{-1} \left( \frac{-3}{5} \right) \right\} =$   
 (a)  $\frac{6}{15}$  (b)  $\frac{24}{25}$  (c)  $\frac{4}{5}$  (d)  $\frac{-24}{25}$
- 4) Equation of tangent at  $(-4, -4)$  on  $x^2 = -4y$  is  
 (a)  $2x - y + 4 = 0$  (b)  $2x + y - 4 = 0$  (c)  $2x - y - 12 = 0$  (d)  $2x + y + 4 = 0$
- 5) If  $e_1, e_2$  are eccentricities of the ellipse  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$  and the hyperbola  $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$  then  
 (a)  $e_1^2 - e_2^2 = 1$  (b)  $e_1^2 + e_2^2 = 1$  (c)  $e_1^2 - e_2^2 = 2$  (d)  $e_1^2 - e_2^2 = 2$
- 6) The value of  $|\vec{a} + \vec{i}|^2 + |\vec{a} + \vec{j}|^2 + |\vec{a} + \vec{k}|^2$  if  $|\vec{a}| = 1$  is \_\_\_\_\_.  
 (a) 0 (b) 1 (c) 2 (d) 3
- 7) The curve  $y = e^x$  is \_\_\_\_\_.  
 (a) convex (b) concave (c) convex upwards (d) concave upwards
- 8) If  $x = r \cos \theta$ ,  $y = r \sin \theta$ , then  $\frac{\partial r}{\partial x} =$  .....  
 (a)  $\sec \theta$  (b)  $\sin \theta$  (c)  $\cos \theta$  (d)  $\operatorname{cosec} \theta$
- 9) The area enclosed by the curve  $y^2 = 4x$ , the x-axis and its latus rectum is ..... sq.units.  
 (a)  $\frac{2}{3}$  (b)  $\frac{4}{3}$  (c)  $\frac{8}{3}$  (d)  $\frac{16}{3}$
- 10) The general solution of the differential equation  $\frac{dy}{dx} = \frac{y}{x}$  is  
 (a)  $xy = k$  (b)  $y = k \log x$  (c)  $y = kx$  (d)  $\log y = kx$
- 11) The general solution of  $x \frac{dy}{dx} = y$  is \_\_\_\_\_.  
 (a)  $y = cx$  (b)  $x^2 + y^2 = c$  (c)  $x^2 - y^2 = c$  (d)  $y = c^x$
- 12) A random variable  $X$  has binomial distribution with  $n = 25$  and  $p = 0.8$  then standard deviation of  $X$  is  
 (a) 6 (b) 4 (c) 3 (d) 2
- 13) Which one of the following is a binary operation on  $\mathbb{N}$ ?  
 (a) Subtraction (b) Multiplication (c) Division (d) All the above
- 14) If  $A$  is a orthogonal matrix, then  
 (1)  $AA^T = A^T A = I$   
 (2)  $A$  is non-singular  
 (3)  $|A| = 0$   
 (4)  $A^{-1} = A^T$
- 15) When  $z = x + iy$ , then  $iz$  is  
 (1)  $x - iy$   
 (2)  $i(x + iy)$

- (3)  $-y+ix$   
 (4) Rotation of  $z$  by  $90^\circ$  in the counter clockwise direction
- 16) If  $ax + by = 1$ ,  $Cx^2 + dy^2 = 1$  have only one solution, then  
 (1)  $\frac{a^2}{c} + \frac{b^2}{d} = 1$   
 (2)  $x = \frac{a}{c}$   
 (3)  $x = \frac{c}{a}$   
 (4)  $x = \frac{b}{d}$
- 17) If  $u = \log \left( \frac{x^2+y^2}{xy} \right)$  then  
 (1)  $u$  is a homogeneous function  
 (2)  $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = 0$   
 (3)  $\frac{x^2+y^2}{xy}$  is a homogeneous function  
 (4)  $\frac{x^2+y^2}{xy}$  is a homogeneous function of degree 0.
- 18) If  $\frac{dy}{dx} = \frac{x-y}{x+y}$  then  
 1)  $x dy + y dx = x dx + y dy$   
 2)  $\int d(xy) = \int x dx + \int y dy$   
 3)  $x^2 - y^2 + 2xy = c$   
 4)  $x^2 - y^2 - 2xy = c$
- 19) Identify the false statement.  
 (1) All the stationary numbers are critical numbers.  
 (2) At the stationary point, the first derivative is zero.  
 (3) At critical numbers, the first derivative does not exist.  
 (4) All the critical numbers are stationary numbers.
- 20) If  $p$  is true and  $q$  is false, then which of the following is not true?  
 (1)  $p \rightarrow q$  is F  
 (2)  $p \vee q$  is T  
 (3)  $p \wedge q$  is F  
 (4)  $p \Leftrightarrow q$  is F

## PART - II

7 x 2 = 14

ANSWER ANY 7 QUESTIONS IN WHICH QUESTION NO. 30 IS COMPULSORY.

- 21) For any  $2 \times 2$  matrix, if  $A (\text{adj } A) = \begin{bmatrix} 10 & 0 \\ 0 & 10 \end{bmatrix}$  then find  $|A|$ .
- 22) If  $1, \omega, \omega^2$  are the cube roots of unity show that  $(1+\omega^2)^3 - (1+\omega)^3 = 0$
- 23) Prove that  $2 \tan^{-1} \left( \frac{2}{3} \right) = \tan^{-1} \left( \frac{12}{5} \right)$
- 24) Find the equation of tangent to the circle  $x^2 + y^2 + 2x - 3y - 8 = 0$  at  $(2, 3)$ .
- 25) If  $\vec{a} = \hat{i} + 2\hat{j} + 3\hat{k}$ ,  $\vec{b} = -\hat{i} + 2\hat{j} + \hat{k}$  and  $\vec{c} = 3\hat{i} + \hat{j}$  find  $\frac{\lambda}{c}$  such that  $\vec{a} + \lambda\vec{b}$  is perpendicular to  $\vec{c}$
- 26) Evaluate  $\int_0^1 \frac{e^x}{1+e^{2x}} dx$
- 27)  $\frac{dy}{dx} + \frac{y}{x \log x} = \frac{\sin 2x}{\log x}$
- 28) The time to failure in thousands of hours of an electronic equipment used in a manufactured computer has the density function  $f(x) = \begin{cases} 3e^{-3x} & x > 0 \\ 0 & \text{elsewhere} \end{cases}$   
 Find the expected life of this electronic equipment.

29) Construct the truth table for the following statements.

$$(\neg p \rightarrow r) \wedge (p \leftrightarrow q)$$

30) Let  $G = \{1, w, w^2\}$  where  $w$  is a complex cube root of unity. Then find the universe of  $w^2$ . Under usual multiplication.

### PART - III

7 x 3 = 21

ANSWER ANY 7 QUESTIONS IN WHICH QUESTION NO. 40 IS COMPULSORY

31) If the equations  $x^2 + px + q = 0$  and  $x^2 + p'x + q' = 0$  have a common root, show that it must be equal to  $\frac{pq' - p'q}{q - q'}$  or  $\frac{q - q'}{p' - p}$ .

32) Find the value of the expression in terms of  $x$ , with the help of a reference triangle.

$$\cos(\tan^{-1}(3x-1))$$

33) The equation  $y = \frac{1}{32} x^2$  models cross sections of parabolic mirrors that are used for solar energy. There is a heating tube located at the focus of each parabola; how high is this tube located above the vertex of the parabola?

34) Find the equation of the plane passing through the line of intersection of the planes  $x + 2y + 3z = 2$  and  $x - y - z + 11 = 3$  and at a distance  $\frac{2}{\sqrt{3}}$  from the point  $(3, 1, -1)$

35) Determine the intervals of concavity of the curve  $y = 3 + \sin x$ .

36) Use linear approximation to find an approximate value of  $\sqrt{9.2}$  without using a calculator.

37) Prove that  $\int_0^{\frac{\pi}{4}} \log(1 + \tan x) dx = \frac{\pi}{8} \log 2$ .

38) The growth of a population is proportional to the number present. If the population of a colony doubles in 50 years, in how many years will the population become triple?

39) Let  $X$  be a random variable denoting the life time of an electrical equipment having probability density function

$$f(x) = \begin{cases} ke^{-2x} & \text{for } x > 0 \\ 0 & \text{for } x \leq 0 \end{cases}$$

Find

(i) the value of  $k$

(ii) Distribution function

(iii)  $P(X < 2)$

(iv) calculate the probability that  $X$  is at least for four unit of time

(v)  $P(X = 3)$

40) Determine whether  $*$  is a binary operation on the sets given below.

$$a * b = \min(a, b) \text{ on } A = \{1, 2, 3, 4, 5\}$$

### PART - IV

7 x 5 = 35

ANSWER ALL THE QUESTIONS.

41) a) Solve  $(1+x^3) \frac{dy}{dx} + 6x^2y = 1+x^2$ .

(OR)

b) A random variable  $X$  has the following probability mass function

x	1	2	3	4	5	6
f(x)	k	2k	6k	5k	6k	10k

Find

(i)  $P(2 < X < 6)$

(ii)  $P(2 \leq X < 5)$

(iii)  $P(X \leq 4)$

(iv)  $P(3 < X)$

42) a) Evaluate  $\int_0^{2a} x^2 \sqrt{2ax - x^2} dx$

(OR)

b)

Let A be  $\mathbb{Q} \setminus \{1\}$ . Define  $*$  on A by  $x*y = x + y - xy$ . Is  $*$  binary on A? If so, examine the commutative and associative properties satisfied by  $*$  on A.

- 43) a) Discuss the monotonicity and local extrema of the function  $f(x) = \log(1+x) - \frac{x}{1+x}$ ,  $x > -1$  and hence find the domain where,  $\log(1+x) > \frac{x}{1+x}$

(OR)

- b) Find the differential equation of the family of parabolas  $y^2 = ax = 4$ , where a is an arbitrary constant.

- 44) a) The maximum and minimum distances of the Earth from the Sun respectively are  $152 \times 10^6$  km and  $94.5 \times 10^6$  km. The Sun is at one focus of the elliptical orbit. Find the distance from the Sun to the other focus.

(OR)

- b) Prove by vector method that the perpendiculars (altitudes) from the vertices to the opposite sides of a triangle are concurrent.

- 45) a) Find all zeros of the polynomial  $x^6 - 3x^5 - 5x^4 + 22x^3 - 39x^2 - 39x + 135$ , if it is known that  $1+2i$  and  $\sqrt{3}$  are two of its zeros.

(OR)

- b) Let  $f(x, y) = \sin(xy^2) + e^{x^3+5y}$  for all  $(x, y) \in \mathbb{R}^2$ . Calculate  $\frac{\partial f}{\partial x}$ ,  $\frac{\partial f}{\partial y}$ ,  $\frac{\partial^2 f}{\partial y \partial x}$  and  $\frac{\partial^2 f}{\partial x \partial y}$

- 46) a) Simplify:  $(-\sqrt{3} + 3i)^{31}$

(OR)

- b) Find the vertex, focus, equation of directrix and length of the latus rectum of the following:  $y^2 - 4y - 8x + 12 = 0$

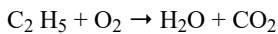
- 47) a) Determine the values of  $\lambda$  for which the following system of equations  $x + y + 3z = 0$ ,  $4x + 3y + \lambda z = 0$ ,  $2x + y + 2z = 0$  has

(i) a unique solution

(ii) a non-trivial solution

(OR)

- b) By using Gaussian elimination method, balance the chemical reaction equation:



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