RAVI MATHS TUITION CENTER, CHENNAI-82. WHATSAPP - 8056206308

Continuity And Differentiability

12th Standard

Maths

10 x 2 = 20

1) Find the relationship between a and b so that the function f defined by f $f(x) = \begin{cases} ax+1, & \text{if } x \le 3 \\ bx+3, & \text{if } x > 3 \end{cases}$ is continuous at x = 3.

2) Differentiate $\sin^2 x$ w.r.t. $e^{\cos x}$

3) If y = A sin x + B cos x, then prove that $\frac{d^2y}{dx^2} + y = 0$

4) Differentiate sin {log (x^3-1)}, with respect to x.

5) Differentiate the following w.r.t. x, or find $\frac{dy}{dx}$.

$y = e^x + e^{x^2} + e^{x^3} + e^{x^4} + e^{x^5}$.

6) If y = $\tan^{-1}\sqrt{\frac{sinx}{1+cosx}}, find \frac{dy}{dx}$

7) If y = $a^x + x^a + x^x + a^a$, find dy/dx

8) If y = $log\left(tanx\frac{x}{2}\right)\ find\frac{dy}{dx}$

9) Evaluate the left hand and right hand limits of the following function at x = 2.

$f(x) = \begin{cases} 2x+3, & \text{if } x \le 2 \\ x+5, & \text{if } x > 2 \end{cases}$ Does lim f(x) exist?

10) If $x = a\sec^3\theta$ and $y = a\tan^3\theta$, find $\frac{d^2y}{dx^2}$ at $\theta = \frac{\pi}{4}$.

5 x 3 = 15

11) $Find\ \frac{dy}{dx},\ if: x = a\left(\theta + sin\theta\right),\ y = a(1 - cos\theta)$

12) $If \quad y = 3e^{2x} + 2e^{3x}, prove \quad that \quad \frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 6y = 0$

13) $If \quad y = sin^{-1}x, \quad show \quad that: \left(1 - x^2\right)\frac{d^2y}{dx^2} - x\frac{dy}{dx} = 0$

14) Find dy/dx of the function : $x = a(cost + log \quad tan\frac{t}{2}), \quad y = a \quad sin \quad t$

15) $If \quad y = A \quad e^{mx} + B \quad e^{nx}, \quad show \quad that: \frac{d^2y}{dx^2} - (m+n)\frac{dy}{dx} + mny = 0$

9 x 5 = 45

16) Differentiate (x^2 – 5x + 8) (x^3 + 7x + 9) in three ways mentioned below:
(i) by using product rule
(ii) by expanding the product to obtain a single polynomial.
(iii) by logarithmic differentiation.
Do they all give the same answer?

17) Determine the constants a and b, such that the function

$f(x) = \begin{cases} ax^2 + b, & if \quad x > 2 \\ 2 \quad , & if \quad x = 2 \\ 2ax - b, & if \quad x < 2 \end{cases}$ is continuous.

18) Is the function f (x)=$\begin{cases} \frac{[x]-1}{x-1}, & if \quad x \neq 1 \\ -1 \quad , & if \quad x = 1 \end{cases}$ continuous at x=1?

19) If y=sin (log x), prove that $x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$

20) If log y= $\tan^{-1}$ x, show that (1+x^2) y_2 + (2x-1) y_1=0

21) If $y = log\left(x + \sqrt{x^2+1}\right)$, then prove that $\left(x^2+1\right)\frac{d^2y}{dx^2} + x\frac{dy}{dx} = 0$.

22) If $y^x = e^{y-x}$, prove that $\frac{dy}{dx} = \frac{(1+log \quad y)^2}{log \quad y}$.

23) If $y = (\tan^{-1}x)^2$ show that $(x^2+1)^2\frac{d^2y}{dx^2} + 2x(x^2+1)\frac{dy}{dx} = 2$

24) Let y = $(logx)^x + x^{xcosx}$ then find $\frac{dy}{dx}$