

**RAVI MATHS TUITION CENTER, CHENNAI-82. WHATSAPP -  
8056206308**

**Application Of Derivatives**

12th Standard

Maths

8 x 2 = 16

- 1) Show that the function  $f$  given by  $f(x) = x^3 - 3x^2 + 4x, x \in R$  is strictly increasing on  $R$ .
- 2) Show that the function  $f$  given by  $f(x) = \tan^{-1}(\sin x + \cos x), x > 0$ , is always an strictly increasing function in  $(0, \frac{\pi}{4})$ .
- 3) Show that, the function  $f(x) = \log(\cos x)$  is decreasing, in  $[0, \frac{\pi}{2}]$ .
- 4) Find the least value of  $a$  such that the function  $f(x) = x + ax + 1$  is strictly increasing on  $(1, 2)$ .
- 5) Show that  $y = e^x$  has no maxima or no minima
- 6) It is given that  $x = 1$ , the function  $f(x) = x^2 - 6x^2 + ax + 9$  attains its maximum value in the interval  $[0, 2]$ . Find the value of  $a$ .
- 7) Show that  $f(x) = \frac{x}{\sin x}$  is increasing on  $(0, \frac{\pi}{2})$
- 8) Find the interval in which the function  $f(x) = 2x^3 - 15x^2 + 36x + 17$  is strictly increasing or strictly decreasing.

8 x 3 = 24

- 9) Show that  $y = \log(1 + x) - \frac{2x}{2+x}, x > -1$  is an increasing function of  $x$ , throughout its domain.
- 10) Show that the right circular cone of least curved surface and volume has an altitude equal to  $\sqrt{2}$  times the radius of the base.
- 11) Show that the height of the cylinder of maximum volume that can be inscribed in a sphere of radius  $R$  is  $\frac{2R}{\sqrt{3}}$ . Also find the maximum volume.
- 12) Find the absolute maximum value and the absolute minimum value of the following functions in the given intervals:  
 $f(x) = 4x - \frac{1}{2}x^2, x \in [-2, \frac{9}{2}]$
- 13) Find two positive numbers  $x$  and  $y$  such that their sum is 35 and the product  $x^2 y^5$  is a maximum.
- 14) Find the intervals in which the following function is strictly increasing or strictly decreasing.  
 $f(x) = -2x^3 - 9x^2 - 12x + 1$
- 15) A square piece of tin of side 18 cm is to be made into a box without top, by cutting a square from each corner and folding up the flaps to form the box. What should be the side of the square to be cut off so that the volume of the box is the maximum possible
- 16) Find the points of local maxima and local minima of the function  
 $f(x) = (x - 1)^3(x + 1)^2$ .

8 x 5 = 40

- 17) Show that the altitude of the right circular cone of maximum volume that can be inscribed in a sphere of radius  $r$  is  $\frac{4r}{3}$ . Also show that the maximum volume of the cone is  $\frac{8}{27}$  of the volume of the sphere.
- 18) Prove that the radius of the right circular cylinder of greatest curved surface area which can be inscribed in a given cone is half of that of the cone.

- 19) Find the intervals in which the function  $f$  given by  $f(x) = \sin x + \cos x, 0 \leq x \leq 2\pi$  is strictly increasing or strictly decreasing.
- 20) Prove that the function  $f$  defined by  $f(x) = x^2 - x + 1$  is neither increasing nor decreasing in  $(-1, 1)$ . Hence find the intervals in which  $f(x)$  is:
- strictly increasing
  - strictly decreasing.
- 21) Prove that the height of the cylinder of maximum volume, that can be inscribed in a sphere of radius  $R$  is  $\frac{2R}{\sqrt{3}}$ . Also find the maximum volume.
- 22) If the sum of the lengths of the hypotenuse and a side of a right-angled triangle is given, show that the area of triangle is maximum when the angle between them is  $60^\circ$  (i. e.,  $\frac{\pi}{3}$ )
- 23) Find the intervals in which the following function is strictly increasing or strictly decreasing.  
 $f(x) = \sin^4 x + \cos^4 x, 0 \leq x \leq \frac{\pi}{2}$
- 24) A rectangle is inscribed in a semicircle of radius  $r$  with one of its sides on the diameter of the semicircle. Find the dimensions of the rectangle, so that its area is maximum. Also find maximum area

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