Standard: 12th Science Subject: Mathematics

Q1. If $x = \frac{\sin^3 t}{\sqrt{\cos^2 t}}, y = \frac{\cos^3 t}{\sqrt{\cos 2t}}$, find $\frac{dy}{dx}$ **7 Marks**

Q2. Find the local maxima and local minima, if any, of the following function. Also find the local maximum and the local minimum values, as the case may be: **6 Marks**
$f(x) = \sin x + \frac{1}{2}\cos 2x,\ 0 \le x \le \frac{\pi}{2}$

Q3. Of all the closed right circular cylindrical cans of volume l28π cm^3, find the dimensions of the can which has minimum surface area. **6 Marks**

Q4. For what value of k is the following function continuous at $x = 2$

$$f(x) = \begin{cases} 2x+1 & ; \ x < 2 \\ k & ; \ x = 2 \\ 3x-1 & ; \ x > 2 \end{cases}$$

Q5. Find the absolute maximum and absolute minimum values of the function f given by
$f(x) = \sin^2 x - \cos x\ , x \in [0, \pi]$.

Q6. Find the intervals on which the function f(x) = $(x-1)^3 (x-2)^2$ is:
1. Strictly increasing.
2. Strictly decreasing.

Q7. If $x = a\sec^3\theta, y = a\tan^3\theta$, then find $\frac{d^2y}{dx^2}$ at $\theta = \frac{\pi}{4}$.

Q8. If $y = e^{x^2 \cos x} + (\cos x)^x$, then find $\frac{dy}{dx}$.

Q9. If y = $3e^{2x} + 2e^{3x}$, prove that
$\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 6y = 0.$

Q10. If $y = \log\left[x+\sqrt{x^2+a^2}\right]$, show that $(x^2+a^2)\frac{d^2y}{dx^2} + x\frac{dy}{dx} = 0.$

Q11. If $y = \sin(\log x)$, prove that **4 Marks**
$x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$

Q12. If $y = x^3(\cos x)^x + \sin^{-1}\sqrt{x}$, Find $\frac{dy}{dx}$. **4 Marks**

Q13. if $y = \sqrt{\frac{(x-3)(x^2+4)}{3x^2+4x+5}}$, find $\frac{dy}{dx}$ **4 Marks**

Q14. If $x^y + y^x = a^b$, then find $\frac{dy}{dx}$. **4 Marks**

Q15. If $x = a(2\theta - \sin 2\theta)$ and $y = a(1-\cos 2\theta)$, find $\frac{dy}{dx}$ when $\theta = \frac{\pi}{3}$. **4 Marks**

Q16. If $y = 2\cos(\log x) + 3\sin(\log x)$, Prove that $x^2\frac{d^2y}{dx}^2 + x\frac{dy}{dx} + y = 0.$ **4 Marks**

Q17. If $x^y = e^{x-y}$, show that $\frac{dy}{dx} = \frac{\log x}{\{\log(x\,e)\}^2}$. **4 Marks**

Q18. If $x^m y^n = (x+y)^{m+n}$, prove that $\frac{d^2y}{dx^2} = 0$. **4 Marks**