RAVI MATHS TUITION & TEST PAPERS , WHATSAPP 8056206308

Continuity and Differentiability PREVIOUSLY ASKED

12th Standard

Maths

Date : 15-12-24

Multiple Choice Question 22 x 1 = 22

1) If $Xe^y = 1$, then the value of $\frac{dy}{dx}$ at x = 1 is

(a) -1 (b) 1 (c) -e (d) $-\frac{1}{e}$

2) Derivative of $e^{\sin^2 x}$ with respect to cos x is

(a) $\sin x e^{\sin^2 x}$ (b) $\cos x e^{\sin^2 x}$ (c) $-2\cos x e^{\sin^2 x}$ (d) $-2\sin^2 x \cos x e^{\sin^2 x}$

3) The function f(x) = [x], where [x] denotes the greatest integer less than or equal to x, is continuous at

(a) x = 1 (b) x = 1.5 (c) x = -2 (d) x = 4

4) The value of k for which $f(x) = \begin{cases} 3x+5, & x \geq 2 \\ kx^2, & x < 2 \end{cases}$ is a continuous function, is

(a) $-\frac{11}{4}$ (b) $\frac{4}{11}$ (c) 11 (d) $\frac{11}{4}$

5) The value of k for which the function $f(x) = \begin{cases} \frac{1-\cos 4x}{8x^2}, & \text{if } x \neq 0 \\ k, & \text{if } x = 0 \end{cases}$ is continuous at x = 0 is

(a) 0 (b) -1 (c) 1 (d) 2

6) The points, at which the function f given by $f(x) = \begin{cases} \frac{x}{|x|}, & x < 0 \\ -1, & x \geq 0 \end{cases}$ is continuous, is/are

(a) $x \in R$ (b) x=0 (c) $x \in R - \{0\}$ (d) x = -1 and 1

7) The value of k (k < 0) for which the function f defined as $f(x) = \begin{cases} \frac{1-\cos kx}{x \sin x}, & x \neq 0 \\ \frac{1}{2}, & x = 0 \end{cases}$ is continuous at x = 0, is

(a) ± 1 (b) -1 (c) $\pm\frac{1}{2}$ (d) $\frac{1}{2}$

8) If $\sin(xy) = 1$, then $\frac{dy}{dx}$ is equal to

(a) $\frac{x}{y}$ (b) $-\frac{x}{y}$ (c) $\frac{y}{x}$ (d) $-\frac{y}{x}$

9) Let $f(x) = \begin{vmatrix} x^2 & \sin x \\ p & -1 \end{vmatrix}$, where p is a constant. Then, the value of p for which $f'(0) = 1$ is

(a) R (b) 1 (c) 0 (d) -1

10) The function f(x)=|x| is

(a) continuous and differentiable everywhere. (b) continuous and differentiable nowhere.

(c) continuous everywhere, but differentiable everywhere except at x=0

(d) continuous everywhere, but differentiable nowhere.

11) The value of k for which function $f(x) = \begin{cases} x^2, & x \geq 0 \\ kx, & x < 0 \end{cases}$ is differentiable at x = 0 is

(a) 1 (b) 2 (c) any real number (d) 0

12) If $f(x) = 2|x| + 3|\sin x| + 6$, then the right hand derivative of f(x) at x = 0 is

(a) 6 (b) 5 (c) 3 (d) 2

13) If $y = \frac{\cos x - \sin x}{\cos x + \sin x}$, then $\frac{dy}{dx}$ is

(a) $-\sec^2\left(\frac{\pi}{4} - x\right)$ (b) $\sec^2\left(\frac{\pi}{4} - x\right)$ (c) $\log\left|\sec\left(\frac{\pi}{4} - x\right)\right|$ (d) $-\log\left|\sec\left(\frac{\pi}{4} - x\right)\right|$

14) If $y = \log(\sin e^x)$, then $\frac{dy}{dx}$ is

(a) $\cot e^x$ (b) $\operatorname{cosec} e^x$ (c) $e^x \cot e^x$ (d) $e^x \operatorname{cosec} e^x$

15) If $y = \sin^{-1} x$, then $\left(1 - x^2\right) y_2$ is equal to

(a) xy_1 (b) xy (c) xy_2 (d) x^2

16) The set of all points, where the function f(x) = x + |x| is differentiable, is

(a) $(0, \infty)$ (b) $(-\infty, 0)$ (c) $(-\infty, 0) \cup (0, \infty)$ (d) $(-\infty, \infty)$

17) If $x = a \sec\theta$ and $y = b \tan\theta$, then $\frac{d^2y}{dx^2}$ at $\theta = \frac{\pi}{6}$ is

(a) $\frac{-3\sqrt{3}b}{a^2}$ (b) $\frac{-2\sqrt{3}b}{a}$ (c) $\frac{-3\sqrt{3}b}{a}$ (d) $\frac{-b}{3\sqrt{3}a^2}$

18) The derivative of $\sin^{-1}\left(2x\sqrt{1-x^2}\right)$ with respect to $\sin^{-1} x$, $\frac{1}{\sqrt{2}} < x < 1$ is

(a) 2 (b) $\frac{\pi}{2} - 2$ (c) $\frac{\pi}{2}$ (d) -2

19) If $e^x + e^y = e^{x+y}$, then $\frac{dy}{dx}$ is

(a) e^{y-x} (b) e^{x+y} (c) $-e^{y-x}$ (d) $2e^{x-y}$

20) If $y = 5\cos x - 3\sin x$, then $\frac{d^2y}{dx^2}$ is equal to

(a) -y (b) y (c) 25 y (d) 9 y

21) If $y = \log(\cos e^x)$, then $\frac{dy}{dx}$ is

(a) $\cos e^{x-1}$ (b) $e^{-x}\cos e^x$ (c) $e^x \sin e^x$ (d) $-e^x \tan e^x$

22) If $y = Ae^{5x} + Be^{-5x}$, then $\frac{d^2y}{dx^2}$ is equal to

(a) 25 y (b) 5 y (c) -25 y (d) 15 y

2 Marks 19 x 2 = 38

23) Write the derivative of sin x with respect to cos x

24) Determine the value of 'k' for which the following function is continuous at x = 3.

$$f(x) = \begin{cases} \frac{(x+3)^2 - 36}{x-3}, & x \neq 3 \\ k, & x = 3 \end{cases}$$

25) Find the value of p for which the function $f(x) = \begin{cases} \frac{1-\cos 4x}{x^2}, x \neq 0 \\ p, x = 0 \end{cases}$ is continuous at x = 0

26) Check for differentiability of the function f defined by f(x) = |x - 5| at the point x = 5.

27) Simplify $\cos^{-1} x + \cos^{-1}\left[\frac{x}{2} + \frac{\sqrt{3-3x^2}}{2}\right]; \frac{1}{2} \leq x \leq 1$

28) Find the value(s) of λ, if the function $f(x) = \begin{cases} \frac{\sin^2 \lambda x}{x^2}, & \text{if } x \neq 0 \\ 1, & \text{if } x = 0 \end{cases}$ is continuous at x = 0.

29) If the function f defined as $f(x) = \begin{cases} \frac{x^2-9}{x-3}, & x \neq 3 \\ k, & x = 3 \end{cases}$ continuous at x = 3, find the value of k.

30) Find the number of points of discontinuity of f defined by f(x) = |x| - |x+1|.

31) If f(x) = x|x|, then find $f'(x)$.

32) Differentiate $e^{\sqrt{3x}}$, with respect to x

33) If $y = \cos(\sqrt{3x})$, then find $\frac{dy}{dx}$

34) If y = x|x|, find $\frac{dy}{dx}$ for x < 0.

35) If $f(x) = \begin{cases} x^2, \text{if } x \geq 1 \\ x, \text{if } x < 1 \end{cases}$ then show that f is not differentiable at x = 1.

36) If $\left(x^2 + y^2\right)^2 = xy$, then find $\frac{dy}{dx}$.

37) If $y = \left(x + \sqrt{x^2 - 1}\right)^2$, then show that $\left(x^2 - 1\right)\left(\frac{dy}{dx}\right)^2 = 4y^2$.

38) If $y = x^{\frac{1}{x}}$, then find $\frac{dy}{dx}$ at x = 1

39) If x = a cos t and y = b sin t, then find $\frac{d^2y}{dx^2}$.

40) If $y = \sqrt{ax + b}$, prove that $y\left(\frac{d^2y}{dx^2}\right) + \left(\frac{dy}{dx}\right)^2 = 0$

41) If $y = e^{x \sin^2 x} + (\sin x)^x$, find $\frac{dy}{dx}$.

3 Marks 89 x 3 = 267

42) Differentiate w.r.t. x, the following function:
$(i)\sqrt{3x + 2} + \frac{1}{\sqrt{2x^2+4}}$
$(ii)\log_7(\log x)$

43) Find the values of k so that the function f is continuous

$$f(x) = \begin{cases} \frac{k\cos x}{\pi - 2x}, & \text{if} \quad x \neq \frac{\pi}{2} \\ 3, & \text{if} \quad x = \frac{\pi}{2} \end{cases}$$

44) Find the value of 'k', or which:

$$f(x) = \begin{cases} \frac{\sqrt{1+kx} - \sqrt{1-kx}}{x}, & i \quad -1 \leq x < 0 \\ \frac{2x+1}{x-1}, & if \quad 0 \leq x < 1 \end{cases}$$

is continuous at x = 0.

45) Find the value of 'a' for which the function 'f' defined as:

$$f(x) = \begin{cases} a \quad sin\frac{\pi}{2}(x+), & x \leq 0 \\ \frac{tan \quad x - sin \quad x}{x^3}, & x > 0 \end{cases}$$

is continuous at x = 0.

46) If $x^{16}y^9 = \left(x^2 + y\right)^{17}$, prove that $\frac{dy}{dx} = \frac{2y}{x}$

47) Prove that $\frac{d}{dx}\left[\frac{x}{2}\sqrt{a^2 - x^2} + \frac{a^2}{2}sin^{-1}\frac{x}{a}\right] = \sqrt{a^2 - x^2}$

48) Differentiate $tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right)$ w.r.t.x.

49) If $y = tan^{-1}\left(\frac{\sqrt{1+x^2}+\sqrt{1-x^2}}{\sqrt{1+x^2}-\sqrt{1-x^2}}\right), x^2 \leq 1$ then find $\frac{dy}{dx}$

50) Find $\frac{dy}{dx}$ if $y = sin^{-1}\left[\frac{6x - 4\sqrt{1-4x^2}}{5}\right]$

51) If $x = a\,cos\theta + b\,sin\theta,\ y = a\,sin\theta - b\,cos\theta$, show that $y^2\frac{d^2y}{dx^2} - x\frac{dy}{dx} - y = 0$

52) If $y = X^x$ prove that $\frac{d^2y}{dx^2} - \frac{1}{y}\left(\frac{dy}{dx}\right)^2 - \frac{y}{x} = 0$

53) If $x = a\left(cos \quad t + log \quad tan\frac{t}{2}\right), \quad y = a \quad sin \quad t$, find $\frac{d^2y}{dt^2}$ and $\frac{d^2y}{dx^2}$

54) If $y = (\cos x)^x + \sin^{-1}\sqrt{3x}$, find $\frac{dy}{dx}$

55) If $x = a\left[\cos t + \log\left(\tan\frac{t}{2}\right)\right]$ and y = a sin t, then find $\frac{dy}{dx}$

56) If $x = ae^t(\sin t + \cos t)$ and $y = ae^t(\sin t - \cos t)$ then prove that $\frac{dy}{dx} = \frac{x+y}{x-y}$

57) If $y = e^x \sin 5x$, then find $\frac{d^2y}{dx^2}$

58) A curve has the equation $y = \frac{x+6}{2x-3}$ when $x \neq \frac{3}{2}$.

59) If $x = a(\theta + \sin\theta)$ and $y = a(1 - \cos\theta)$ then find $\frac{d^2y}{dx^2}$ at $\theta = \frac{\pi}{2}$.

60) If $y = \sin^{-1} x$, then show that $\left(1 - x^2\right)\frac{d^2y}{dx^2} - x\frac{dy}{dx} = 0$

61) If $y = Pe^{ax} + Qe^{bx}$ then show that $\frac{d^2y}{dx^2} - (a + b)\frac{dy}{dx} + aby = 0$

62) If $x = a\cos\theta + b\sin\theta$ and $y = a\sin\theta - b\cos\theta$ then show that $y^2\frac{d^2y}{dx^2} - x\frac{dy}{dx} + y = 0$

63) If $\sqrt{1-x^2}+\sqrt{1-y^2}=a(x-y)$, prove that $\frac{dy}{dx}=\sqrt{\frac{1-y^2}{1-x^2}}$

64) If x = a $\sin^3\theta$, y = b$\cos^3\theta$, then find $\frac{d^2y}{dx^2}$ at $\theta=\frac{\pi}{4}$.

65) Evaluate $\int_0^{\pi}\frac{e^{\cos x}}{e^{\cos x}+e^{-\cos x}}dx$

66) Find $\int\frac{2x+1}{(x+1)^2(x-1)}dx$

67) Find the values of a and b so that the following function is differentiable for all values of x

$$f(x)=\begin{cases} ax+b, & x>-1 \\ bx^2-3, & x\le -1 \end{cases}$$

68) If y = $(\tan^{-1}x)^2$, show that $\left(x^2+1\right)^2\frac{d^2y}{dx^2}+2x\left(x^2+1\right)\frac{dy}{dx}=2$

69) Determine the values of a and b such that the following function is continuous at x = 0.

$$f(x)=\begin{cases} \frac{x+\sin x}{\sin(a+1)x}, & \text{if } -\pi<x<0 \\ 2, & \text{if } x=0 \\ \frac{2\left(e^{\sin bx}-1\right)}{bx}, & \text{if } x>0 \end{cases}$$

70) Find the differential of $\sin^2 x$ with respect to $e^{\cos x}$.

71) Differentiate $\tan^{-1}\left(\frac{\cos x-\sin x}{\cos x+\sin x}\right)$ with respect to x.

72) Find $\frac{dy}{dx}$ at $x=1, y=\frac{\pi}{4}$, if $\sin^2 y+\cos xy=K$.

73) If $y=\sin^{-1}\left(6x\sqrt{1-9x^2}\right), -\frac{1}{3\sqrt{2}}<x<\frac{1}{3\sqrt{2}}$, then find $\frac{dy}{dx}$.

74) If $e^y(x+1)=1$, then show that $\frac{dy}{dx}=-e^y$

75) If $(\cos x)^y=(\cos y)^x$, then find $\frac{dy}{dx}$

76) Differentiate $\sec^{-1}\left(\frac{1}{\sqrt{1-x^2}}\right)$ with respect to $\sin^{-1}\left[2x\sqrt{1-x^2}\right]$

77) If $y=\tan x+\sec x$, then prove that $\frac{d^2y}{dx^2}=\frac{\cos x}{(1-\sin x)^2}$

78) If $(a+bx)e^{\frac{y}{x}}=x$, then prove that $x\frac{d^2y}{dx^2}=\left(\frac{a}{a+bx}\right)^2$

79) If $x=a\cos\theta$ and $y=b\sin\theta$, then find $\frac{d^2y}{dx^2}$

80) If $x\sqrt{1+y}+y\sqrt{1+x}=0, (x\neq y)$, then prove that $\frac{dy}{dx}=-\frac{1}{(1+x)^2}$.

81) If $y=\left(\sin^{-1}x\right)^2$, prove that $\left(1-x^2\right)\frac{d^2y}{dx^2}-x\frac{dy}{dx}-2=0$

82) Differentiate $x^{\sin x}+(\sin x)^{\cos x}$ with respect to x.

83) If $\log\left(x^2+y^2\right)=2\tan^{-1}\left(\frac{y}{x}\right)$ then show that $\frac{dy}{dx}=\frac{x+y}{x-y}$.

84) If $x^y-y^x=a^b$, find $\frac{dy}{dx}$.

85) If $x=\cos t+\log\left(\tan\left(\frac{t}{2}\right)\right), y=\sin t$, then find the values of $\frac{d^2y}{dt^2}$ and $\frac{d^2y}{dx^2}$ at $t=\frac{\pi}{4}$.

86) If $y=\sin(\sin x)$, prove that $\frac{d^2y}{dx^2}+\tan x\frac{dy}{dx}+y\cos^2 x=0$

87) If $x=a(2\theta-\sin 2\theta)$ and $y=a(1-\cos 2\theta)$, find $\frac{dy}{dx}$ when $\theta=\frac{\pi}{3}$.

88) If $\sin y=x\cos(a+y)$, then show that $\frac{dy}{dx}=\frac{\cos^2(a+y)}{\cos a}$.
Also, show that $\frac{dy}{dx}=\cos a$,when x=0.

89) If $x=a\sec^3\theta$ and $y=a\tan^3\theta$, find $\frac{d^2y}{dx^2}$ at $\theta=\frac{\pi}{3}$

90) If $y=e^{\tan^{-1}x}$, prove that $\left(1+x^2\right)\frac{d^2y}{dx^2}+(2x-1)\frac{dy}{dx}=0$

91) Find a and b, if the function given by $f(x)=\begin{cases} ax^2+b, & \text{if } x<1 \\ 2x+1, & \text{if } x\ge 1 \end{cases}$ is differentiable at x = 1

92) If $y = \log\left(\sqrt{x} + \frac{1}{\sqrt{x}}\right)^2$, then prove that $x(x+1)^2 y_2 + (x+1)^2 y_1 = 2$

93) If $x^y + y^x = a^b$, then find $\frac{dy}{dx}$

94) If $e^y(x+1) = 1$, then show that $\frac{d^2y}{dx^2} = \left(\frac{dy}{dx}\right)^2$

95) Find $\frac{dy}{dx}$, if $y = e^{\sin^2 x}\left\{2\tan^{-1}\sqrt{\frac{1-x}{1+x}}\right\}$

96) Differentiate $\tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right)$ with respect to $\sin^{-1}\left(\frac{2x}{1+x^2}\right)$, when $x \neq 0$

97) If $x\cos(a+y) = \cos y$, then prove that $\frac{dy}{dx} = \frac{\cos^2(a+y)}{\sin a}$. Hence, show that $\sin a\frac{d^2y}{dx^2} + \sin 2(a+y)\frac{dy}{dx} = 0$.
Or If $\cos y = x\cos(a+y)$, where $\cos a \neq \pm 1$, prove that $\frac{dy}{dx} = \frac{\cos^2(a+y)}{\sin a}$.

98) Find the values of a and b, if the function f defined by $f(x) = \begin{cases} x^2 + 3x + a, & x \leq 1 \\ bx + 2, & x > 1 \end{cases}$ is differentiable at x = 1

99) If $x = \sin t$ and y = sin pt, then prove that $\left(1 - x^2\right)\frac{d^2y}{dx^2} - x\frac{dy}{dx} + p^2 y = 0$

100) Show that the function f(x) = |x + 1| + |x - 1|, for all $x \in R$, is not differentiable at the points x = -1 and x = 1

101) If $y = e^{m\sin^{-1}x}$, then show that $\left(1 - x^2\right)\frac{d^2y}{dx^2} - x\frac{dy}{dx} - m^2 y = 0$

102) If $y = \left(x + \sqrt{1+x^2}\right)^n$, then show that $\left(1 + x^2\right)\frac{d^2y}{dx^2} + x\frac{dy}{dx} = n^2 y$

103) If $y = (\sin x)^x + \sin^{-1}\sqrt{x}$, then find $\frac{dy}{dx}$

104) If $y = \frac{x\cos^{-1}x}{\sqrt{1-x^2}} - \log\sqrt{1-x^2}$, then prove that $\frac{dy}{dx} = \frac{\cos^{-1}x}{(1-x^2)^{3/2}}$

105) Write the derivative of sin x with respect to cos x.

106) If $y = \sin^{-1}\left[x\sqrt{1-x} - \sqrt{x}\sqrt{1-x^2}\right]$ and 0 < x < 1, then find $\frac{dy}{dx}$

107) If $e^x + e^y = e^{x+y}$, prove that $\frac{dy}{dx} + e^{y-x} = 0$

108) Find the value of $\frac{dy}{dx}$ at $\theta = \frac{\pi}{4}$, if $x = ae^\theta(\sin\theta - \cos\theta)$ and $y = ae^\theta(\sin\theta + \cos\theta)$.

109) If $x^m y^n = (x+y)^{m+n}$, prove that $\frac{dy}{dx} = \frac{y}{x}$

110) Differentiate $\tan^{-1}\left(\frac{\sqrt{1-x^2}}{x}\right)$ with respect to $\cos^{-1}\left(2x\sqrt{1-x^2}\right)$, when $x \neq 0$.

111) Differentiate $\tan^{-1}\left(\frac{x}{\sqrt{1-x^2}}\right)$ with respect to $\sin^{-1}\left(2x\sqrt{1-x^2}\right)$.

112) If $x = \cos t\left(3 - 2\cos^2 t\right)$ and $y = \sin t\left(3 - 2\sin^2 t\right)$, then find the value of $\frac{dy}{dx}$ at $t = \frac{\pi}{4}$.

113) If $(x-y)e^{\frac{x}{x-y}} = a$, prove that $y\frac{dy}{dx} + x = 2y$

114) If $(x-y)e^{\frac{x}{x-y}} = a$, prove that $y\frac{dy}{dx} + x = 2y$

115) If $y = \tan^{-1}\left(\frac{a}{x}\right) + \log\sqrt{\frac{x-a}{x+a}}$, prove that $\frac{dy}{dx} = \frac{2a^3}{x^4 - a^4}$.

116) If $\left(\tan^{-1}x\right)^y + y^{\cot x} = 1$, then find dy/dx

117) If $x = 2\cos\theta - \cos 2\theta$ and $y = 2\sin\theta - \sin 2\theta$, then prove that $\frac{dy}{dx} = \tan\left(\frac{3\theta}{2}\right)$

118) If $y = x\log\left(\frac{x}{a+bx}\right)$, then prove that $x^3\frac{d^2y}{dx^2} = \left(x\frac{dy}{dx} - y\right)^2$

119) If $x = \cos\theta$ and $y = \sin^3\theta$, then prove that $y\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2 = 3\sin^2\theta\left(5\cos^2\theta - 1\right)$

120) Differentiate the following function with respect to x. $(\log x)^x + x^{\log x}$

121) If $y = \log\left[x + \sqrt{x^2 + a^2}\right]$, then show that $\left(x^2 + a^2\right)\frac{d^2y}{dx^2} + x\frac{dy}{dx} = 0$

122) Differentiate the following with respect to x.
$\sin^{-1}\left[\frac{2^{x+1}.3^x}{1+(36)^x}\right]$

123) If $x = a\cos^3\theta$ and $y = a\sin^3\theta$, then find the value of $\frac{d^2y}{dx^2}$ at $\theta = \frac{\pi}{6}$.

124) If $x\sin(a+y) + \sin a\cos(a+y) = 0$, then prove that $\frac{dy}{dx} = \frac{\sin^2(a+y)}{\sin a}$

125) If $x = \sqrt{a^{\sin^{-1}t}}$ and $y = \sqrt{a^{\cos^{-1}t}}$, then show that $\frac{dy}{dx} = \frac{-y}{x}$.

126) If $y = x^{\sin x - \cos x} + \frac{x^2-1}{x^2+1}$, then find $\frac{dy}{dx}$

127) If $x = a(\cos t + t\sin t)$ and $y = a(\sin t - t\cos t)$, then find $\frac{d^2x}{dt^2}, \frac{d^2y}{dt^2}$ and $\frac{d^2y}{dx^2}$

128) If $y = \left(\tan^{-1}x\right)^2$, then show that $\left(x^2+1\right)^2\frac{d^2y}{dx^2} + 2x\left(x^2+1\right)\frac{dy}{dx} = 2$

129) Find $\frac{dy}{dx}$, when $y = x^{\cot x} + \frac{2x^2-3}{x^2+x+2}$

130) If $x = a(\theta - \sin\theta), y = a(1+\cos\theta)$, then find $\frac{d^2y}{dx^2}$

5 Marks 94 x 5 = 470

131) For what value of k, is the following function continuous at x=2 ?

$$f(x) = \begin{cases} 2x+1, & if \quad x < 2 \\ k \quad , & if \quad x = 2 \\ 3x-1, & if \quad x > 2 \end{cases}$$

132) Let $f(x) = \begin{cases} \frac{1-sin^3x}{3cos^2x} & ,if \quad x < \frac{\pi}{2} \\ a & ,if \quad x = \frac{\pi}{2} \\ \frac{b(1-sin\quad x)}{(\pi-2x)^2} & ,if \quad x > \frac{\pi}{2} \end{cases}$ If f(x) be a continuous function at x = $\frac{\pi}{2}$, find a and b.

133) Find the value of a for which the function f defined by $f(x) = \begin{cases} a \quad sin\frac{\pi}{2}(x+1) & , \quad x \le 0 \\ \frac{tan\quad x\quad -\quad sin\quad x}{x^3} & , \quad x > 0 \end{cases}$ is continuous at x = 0.

134) Find the value of k such that the function f defined by

$$f(x) = \begin{cases} \frac{k\quad cos\quad x}{\pi-2x}, & if \quad x \neq \frac{\pi}{2} \\ 3 \quad , & if \quad x = \frac{\pi}{2} \end{cases} \; is \quad continuous \quad at \quad x = \frac{\pi}{2} \quad .$$

135) Show that the function f(x) = 2x-|x| is continuous but not differentiable at x = 0.

136) If f(x)=$f(x) = \begin{cases} \frac{1-cos\quad 4x}{x^2} & ,when \quad x < 0 \\ a & ,when \quad x = 0 \\ \frac{\sqrt{x}}{(\sqrt{16+\sqrt{x}})-4} & ,when \quad x > 0 \end{cases}$ and function is continuous at x=0, find the value of a.

137) Find the derivative of each of the following function w.r.t. x, or find $\frac{dy}{dx}$:
$y=(\sin x)^{\tan x} + (\cos x)^{\sec x}$.

138) Find the derivative of each of the following function w.r.t. x, or find $\frac{dy}{dx}$:
$y=\tan^{-1}\left[\frac{\sqrt{1+x}-\sqrt{1-x}}{\sqrt{1+x}+\sqrt{1-x}}\right]$

139) Find the derivative of each of the following function w.r.t. x, or find $\frac{dy}{dx}$:
$y = x^x + (\sin x)^x$

140) Find the derivative of each of the following function w.r.t. x, or find $\frac{dy}{dx}$:
$(x^2+y^2)^2 = xy$.

141) Find the derivative of each of the following function w.r.t. x, or find $\frac{dy}{dx}$:
$(\cos x)^y=(\sin y)^x$.

142) Find the derivative of each of the following function w.r.t. x, or find $\frac{dy}{dx}$:
$y = sin^{-1}\left[x\sqrt{1-x} - \sqrt{x}\sqrt{1-x^2}\right]$

143) Find the derivative of each of the following function w.r.t. x, or find $\frac{dy}{dx}$:
$y = (cos\quad x)^x + (sin\quad x)^{\frac{1}{x}}$

144) Find the derivative of each of the following function w.r.t. x, or find $\frac{dy}{dx}$:
$y = cos^{-1}\left(\frac{2^{x+1}}{1+4^x}\right)$

145) Find the derivative of each of the following function w.r.t. x, or find $\frac{dy}{dx}$:
$f(x) = tan^{-1}\left(\frac{1-x}{1+x}\right) - tan^{-1}\left(\frac{x+2}{1-2x}\right)$.

146) Find the derivative of each of the following function w.r.t. x, or find $\frac{dy}{dx}$:
$y = x^{x\ cos\ x} + \frac{x^2+1}{x^2-1}$

147) Find the derivative of each of the following function w.r.t. x, or find $\frac{dy}{dx}$:
$(\cos x)^y=(\cos y)^x$

148) Find the derivative of each of the following function w.r.t. x, or find $\frac{dy}{dx}$
$y=(\log x)^x + x^{\log x}$

149) If y=sin (log x), prove that $x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$

150) If $y=Ae^{mx}+Be^{nx}$, prove that $\frac{d^2y}{dx^2} - (m+n)\frac{dy}{dx} + mny = 0$

151) If $x\sqrt{1+y} + y\sqrt{1+x} = 0$, for -1< x < 1, show that $\frac{dy}{dx} = \frac{-1}{(1+x)^2}$.

152) If $y=[\log (x + \sqrt{x^2+1}\ )]^2$, show that $(1+x^2)\ \frac{d^2y}{dx^2} + x\frac{dy}{dx} - 2 = 0.$

153) If y=3 cos (log x) + 4 sin (log x), show that $x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$

154) If $\log (x^2+y^2)=2\ \tan^{-1}\left(\frac{y}{x}\right)$, then show that $\frac{dy}{dx} = \frac{x+y}{x-y}$.

155) If y = log tan $\left(\frac{\pi}{4} + \frac{x}{2}\right)$, show that $\frac{dy}{dx}$ = sec x. Also find the value of $\frac{d^2y}{dx^2}$ at $x = \frac{\pi}{4}$.

156) Prove that $\frac{d}{dx}\left[\frac{x}{2}\sqrt{a^2-x^2} + \frac{a^2}{2}sin^{-1}\frac{x}{a}\right] = \sqrt{a^2-x^2}$

157) If $y = log\left(x + \sqrt{x^2+1}\right)$, then prove that $\left(x^2+1\right)\frac{d^2y}{dx^2} + x\frac{dy}{dx} = 0$.

158) If $y= (\tan^{-1} x)^2$, prove that $(x^2+1)^2\ \frac{d^2y}{dx^2} + 2x\left(x^2+1\right)\frac{dy}{dx} = 2$.

159) If $y= \sin^{-1} x$, show that $\left(1-x^2\right)^2\frac{d^2y}{dx^2} - x\frac{dy}{dx} = 0$

160) If $x = \sqrt{a^{sin^{-1}t}}, \quad y = \sqrt{a^{cos^{-1}tan}}, \quad show \quad that \quad \frac{dy}{dx} = \frac{-y}{x}.$

161) If x=a (cos t + sin t) and y=a(sin t - t cos t), 0< t < $\frac{\pi}{2}, find\ \frac{d^2x}{dt^2}, \frac{d^2y}{dt^2}, and\ \frac{d^2y}{dx^2}.$

162) If $y = e^{a\ \ cos^{-1}x}, -1 \le x \le 1,$ show that $(1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} - a^2y = 0.$

163) If $x^{16}\ y^9=(x^2+y)^{17}$, prove that $\frac{dy}{dx} = \frac{2y}{x}$

164) If x=a sin t and y=a (cos t + log tan $\frac{t}{2}$), find $\frac{d^2y}{dx^2}$.

165) If $y^x= e^{y-x}$, prove that $\frac{dy}{dx} = \frac{(1+log\ \ y)^2}{log\ \ y}$.

166) If $x^py^q = (x + y)^{p+q}$, prove that $(i)\frac{dy}{dx} = \frac{y}{x} and\quad (ii)\frac{d^2y}{dx^2} = 0.$

167) Differentiate, $\tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right)$ with respect to $\tan^{-1}$ x, when $x \neq 0$.

168) If x=2 cos θ- cos 2 θ and y =2 sin θ -sin 2 θ, then prove that $\frac{dy}{dx} = tan\left(\frac{3\theta}{2}\right)$

169) If y = x log $\left(\frac{x}{a+bx}\right)$, , then prove that $x^3\frac{d^2y}{dx^2} = \left(x\frac{dy}{dx} - y\right)^2$.

170) The function f(x) is defined as
$$f(x) = \begin{cases} x^2 + ax + b, 0 \le x \le 2 \\ 3x + 2, 2 \le x \le 4 \\ 2ax + 5b, 4 \le x \le 8 \end{cases}$$
if f(x) is continuous on [0,8], find the values of a and b.

171) Find the value of the constant k, so that the function f(x) defined below is continuous at function f(x) defined below is continuous at

x=0, where f(x) = $\begin{cases} (\frac{1-cos4x}{8x^2}) if x \neq 0 \\ k, if x = 0 \end{cases}$

172) Find the value of k, so that the function f is continuous at x = $\pi/2$.

$f(x) = \begin{cases} \frac{kcosx}{\pi-2x}, if x \neq \frac{\pi}{2} \\ 5, if x = \frac{\pi}{2} \end{cases}$

173) Differentiate the following with respect to x: $\sin^{-1}[\frac{2^{x+1}.3^x}{1+(36)^x}]$.

174) If $y = x^{\sin x - \cos x} + \frac{x^2-1}{x^2+1}$ find $\frac{dy}{dx}$

175) Find $\frac{dy}{dx}$, when $y = x^{\cot x} + \frac{2x^2-3}{x^2+x+2}$

176) If $x = \cos t + \log \tan \frac{t}{2}, y = \sin t,$ then find the value of $\frac{d^2y}{dt^2}$ and $\frac{d^2y}{dx^2}$ at $t = \frac{\pi}{4}$

177) Find $\frac{dy}{dx}$, if $(\text{Cos x})^y = (\text{cos y})^x$.

178) If $y = (\tan^{-1} x)^2$ show that $(x^2+1)^2 \frac{d^2y}{dx^2} + 2x(x^2+1)\frac{dy}{dx} = 2$

179) If $y = 3\cos(\log x) + 4\sin(\log x)$ show that: $x^2 \frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$

180) If $y = \sin^{-1} x$ show that: $(1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} = 0$

181) If $x = \sqrt{a^{\sin^{-1}t}}, y = \sqrt{a^{\cos^{-1}t}}$ show that $\frac{dy}{dx} = -\frac{y}{x}$

182) Differentiate $\tan^{-1}[\frac{\sqrt{1+x^2}-1}{x}]$ with respect to x.

183) If $x = a[\cos t + t \sin t], y = a[\cos t - t \sin t],\ 0$ Find $\frac{d^2x}{dt^2}, \frac{d^2y}{dt^2}$ and $\frac{d^2y}{dx^2}$

184) Differentiate $\tan^{-1}[\frac{\sqrt{1+x^2}-\sqrt{1-x^2}}{\sqrt{1+x^2}+\sqrt{1-x^2}}]$ with respect to $\cos^{-1} x^2$

185) If $y = \frac{\sin^{-1} x}{\sqrt{1-x^2}}$ show that $(1-x^2)\frac{d^2y}{dx^2} - 3x\frac{dy}{dx} - y = 0$

186) If $y = ae^{2x} + be^{-x}$ show that $\frac{d^2y}{dx^2} - \frac{dy}{dx} - 2y = 0$

187) If $x = a(t + \frac{1}{t})$ and $y = a(1 - \frac{1}{t^2})$ Prove that $\frac{dy}{dx} = \frac{x}{y}$

188) If $y = e^{a \cos^{-1}} 3x$ prove that $(1-9x^2)\frac{d^2y}{dx^2} - 9x\frac{dy}{dx} - 9a^2y = 0$

189) If $\sqrt{1-x^4} + \sqrt{1-y^4} = a(x^2-y^2)$ prove that $\frac{dy}{dx} = \frac{x}{y}\sqrt{\frac{1-y^4}{1-x^4}}$

190) If $x = a \sin pt, y = b \cos pt$ then show that $(a^2 - x^2)y = \frac{d^2y}{dx^2} + b^2 = 0$

191) If x = a sin 2t (1 + cos 2t) and y = b cos 2t (1 - cos 2t), find $\frac{dy}{dx}$ at $t = \frac{\pi}{4}$

192) If x = a sin pi, y = b cos pt, then find $\frac{dy}{dx}$ at t = 0 = -bp sin pt

193) If x = $asec^3\theta$, $y = atan^3\theta$ find $\frac{dy}{dx}\theta = \frac{\pi}{4}$

194) If $x = ac^{-1}(sint + cost)$ and y = $ac^{-1}(sint + cost)$ then prove that $\frac{dy}{dx} = \frac{x+y}{x-y}$

195) If x = a(cost t + t sin t) and y = a(sin t - t cas t) then find the value of $\frac{d^2y}{dx^2}t = \frac{\pi}{4}$

196) If x = a $\left(cost + log \quad tan\frac{t}{2}\right)$ and y = sin t, then find $\frac{d^2y}{dx^2}t = \frac{\pi}{3}$

197) If y= $tan^{-1}\left(\frac{a}{x}\right) + log\sqrt{\frac{x-a}{x+a}}$ prove that $\frac{dy}{dx} = \frac{2a^3}{x^4-a^4}$

198) If y = $sin^{-1} sin(x\sqrt{1-x} - \sqrt{x}\sqrt{1-x^2}$ and 0 < x < 1, then find $\frac{dy}{dx}$

199) Find the value of $\frac{dy}{dx}$ at $\theta = \frac{\pi}{4}$, if $x = ae^\theta(\sin\theta - \cos\theta)$ and $y = ae^\theta(\sin\theta + \cos\theta)$

200) Differentiate $tan^{-1}\left(\frac{\sqrt{1-x^2}}{x}\right)$ with respect $cos^{-1}(2 \times 1 - x^2)$ when $x \neq 0$

201) If $x = \cos t(3 - 2\cos^2 t)$ and $y = \sin t(3 - 2\sin^2 t)$ find $\frac{dy}{dx}$ at $t = \frac{\pi}{4}$

202) If $y = Pe^{ax} + Qe^{bx}$, show that $\frac{d^2y}{dx^2} - (a+b)\frac{dy}{dx} + aby = 0$

203) If $y = \log[x + \sqrt{x^2 + a^2}]$ then prove that: $(x^2 + a^2)\frac{d^2y}{dx^2} + x\frac{dy}{dx} = 0$

204) Differentiate $tan^{-1}\left(\frac{\sqrt{1+tan^2\theta}-1}{tan\theta}\right)$ with respect to $\sin^{-1}\left(\frac{2x}{1+x^2}\right)$ $x \neq 0$

205) Differentiate $\log(x^{sinx} + \cot^2 x)$ w.r.t x

206) Differentiate the following function with respect to x: $(\log x)^x + x^{\log x}$

207) If y = x^2, prove that $\frac{d^2y}{dx^2}\frac{1}{y}\left(\frac{dy}{dx}\right)^2 - \frac{y}{x} = 0$

208) If $x^m y^n = (x + y)^{m+n}$, prove that $\frac{dy}{dx} = \frac{y}{x}$

209) Siny = x sin (a+y) prove that: $\frac{sin^2(a+y)}{sin \quad a}$

210) Find $\frac{dy}{dx}$ if $y = \sin^{-1}\left(\frac{6x-4\sqrt{1-4x^2}}{5}\right)$

211) If x = a sin pt, y = b cos pt, then show that $\left(a^2 - x^2\right)y\frac{d^2y}{dx^2} + b^2 = 0$

212) Find $\frac{dy}{dx}$ if $y = e^{sin^2}x\left\{2tan^{-1}\sqrt{\frac{1-x}{1+x}}\right\}$

213) If x = a(cos t+t sin t) and y = a(sin t- t cos t) then find the value of $\frac{d^2y}{dx^2}$ at t = $t = \frac{\pi}{4}$

214) Find the value of k for which the function

$$f(x) = \begin{cases} \frac{\sin x - \cos x}{4x - \pi}, x \neq \frac{\pi}{4} \\ k, x = \frac{\pi}{4} \end{cases}$$ is continuous at $x = \frac{\pi}{4}$.

215) Find the value of m, such that the function

$$f(x) = \begin{cases} \lambda\left(x^2 - 2x\right), & \text{if } x \leq 0 \\ 4x + 1, & \text{if } x > 0 \end{cases}$$ continuous at x = 0? What about continuity at x = 1?

216) For what value of k, the following function is continuous at x = 0?

$$f(x) = \begin{cases} \frac{1 - \cos 4x}{8x^2}, x \neq 0 \\ k, x = 0 \end{cases}$$

217) Find the-value of a, if the function f(x) defined by $f(x) = \begin{cases} 2x - 1, & \text{if } x < 2 \\ a, & \text{if } x = 2 \\ x + 1, & \text{if } x > 2 \end{cases}$ is continuous at x = 2.

218) Find $\frac{dy}{dx}$, if $y = \sin^{-1}\left[\frac{6x-4\sqrt{1-4x^2}}{5}\right]$

219) $\sqrt{1 - x^4} + \sqrt{1 - y^4} = a\left(x^2 - y^2\right)$ then prove that $\frac{dy}{dx} = \frac{x}{y}\sqrt{\frac{1-y^4}{1-x^4}}$

220) If $x = \cos t\left(3 - 2\cos^2 t\right)$ and $y = \sin t\left(3 - 2\sin^2 t\right)$ then find the value of $\frac{dy}{dx}$ at $x = \frac{\pi}{4}$

221) If $x = a\sin 2t(1 + \cos 2t)$ and $y = b\cos 2t(1 - \cos 2t)$ find the values of $\frac{dy}{dx}$ at $t = \frac{\pi}{4}$ and $t = \frac{\pi}{3}$

222) If $x = a\sin 2t(1 + \cos 2t)$ and $y = b\cos 2t(1 - \cos 2t)$ then show that at $t = \frac{\pi}{4}$, $\frac{dy}{dx} = \frac{b}{a}$

223) If y = sin (sin x), prove that $\frac{d^2y}{dx^2} + \tan x\frac{dy}{dx} + y\cos^2 x = 0$

224) Differentiate $\tan^{-1}\left(\frac{x}{\sqrt{1-x^2}}\right)$ w.r.t. $\sin^{-1}\left(2x\sqrt{1 - x^2}\right)$

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