Test / Exam Name: Revision Test Chapters 567 **Standard: 12th Science** **Subject: Mathematics**

Q1. Which of the following statements is true for the function $f(x) = \begin{cases} x+3, & x \neq 0 \\ 1 \quad , & x = 0 \end{cases}$? **1 Mark**

A f(x) is continuous and differentiable $\forall$ x $\in \mathbb{R}$
B f(x) is continuous $\forall$ x $\in \mathbb{R}$
C f(x) is continuous and differentiable $\forall$ x $\in \mathbb{R} - (0)$
D f(x) is discontinuous at infinitely many points

Q2. $\int x^2 e^{x^3} dx$ equals: **1 Mark**

A $\frac{1}{3}e^{x^3} + C$
B $\frac{1}{3}e^{x^4} + C$
C $\frac{1}{2}e^{x^3} + C$
D $\frac{1}{2}e^{x^2} + C$

Q3. The value of k so that f defined by $f(x) = \begin{cases} x^2 \sin\left(\frac{1}{x}\right) & \text{if} \quad x \neq 0 \\ k & \text{if} \quad x = 0 \end{cases}$ is continuous at x = 0 is **1 Mark**

A 0 **B** $\frac{1}{2}$ **C** 1 **D** 2

Q4. $\int \frac{1+\tan x}{1-\tan x} dx$ is equal to: **1 Mark**

A $\sec^2\left(\frac{\pi}{4} + x\right) + C$
B $\sec^2\left(\frac{\pi}{4} - x\right) + C$
C $\log\left|\sec\left(\frac{\pi}{4} + x\right)\right| + C$
D $\log\left|\sec\left(\frac{\pi}{4} - x\right)\right| + C$

Q5. If $x = 2\,at, y = at^2$, where a is a constant, then $\frac{d^2y}{dx^2}$ at $x = \frac{1}{2}$ is: **1 Mark**

A $\frac{1}{2}a$ **B** 1 **C** 2a **D** None of these

Q6. The least and greatest value of f(x) = $x^3 - 6x^2 + 9x$ in [0, 6], are. **1 Mark**

A 3 ,4 **B** 0, 4 **C** 0, 3 **D** 3, 6

Q7. If $\sin(x + y) = \log(x + y)$, then $\frac{dy}{dx} =$ **1 Mark**

A 2 **B** -2 **C** 1 **D** -1

Q8. $f(x) = \sin + \sqrt{3}\cos x$ is maximum when x = **1 Mark**

A $\frac{\pi}{3}$ **B** $\frac{\pi}{4}$ **C** $\frac{\pi}{6}$ **D** 0

Q9. If $y = a\sin mx + b\cos mx$, then $\frac{d^2y}{dx^2}$ is equal to: **1 Mark**

A $-m^2y$ **B** m^2y **C** -my **D** my

Q10. If $y = \sqrt{\sin x + y}$, then $\frac{dy}{dx}$ equals: **1 Mark**

A $\frac{\cos x}{2y-1}$
B $\frac{\cos x}{1-2y}$
C $\frac{\sin x}{1-2y}$
D $\frac{\sin x}{2y-1}$

Q11. If the product of two positive numbers is 9, find the numbers so that the sum of their squares is minimum. **2 Marks**

Q12. The volume of a cube is increasing at the rate of 9 cm^3/s. How fast is its surface area increasing when the length of an edge is 10 cm? **2 Marks**

Q13. Find: $\int \sin x . \log\cos x dx.$ **2 Marks**

Q14. Evaluate: **3 Marks**

$\int \frac{2x.\tan\text{-}1(x^2)}{1 + x^4} dx.$

Q15. If $y = (\tan x)^x$, then find $\frac{dy}{dx}$. **3 Marks**

Q16. If $x^{16}y^9 = (x + y)^{17}$, prove that $x\frac{dy}{dx} = 2y$ **5 Marks**

Q17. Find the intervals on which the function $f(x) = (x - 1)^3 (x - 2)^2$ is: **6 Marks**

1. Strictly increasing.
2. Strictly decreasing.

Q18. Logarithmic differentiation is a powerful technique to differentiate functions of the form $f(x) = [u(x)]^{v(x)}$, where both u(x) and v(x) are differentiable functions and f and u need to be positive functions. **5 Marks**

Let function $y = f(x) = (u(x))^{v(x)}$, then $y' = y\left[\frac{v(x)}{u(x)}u'(x) + v'(x) \cdot \log[u(x)]\right]$

On the basis of above information, answer the following questions.

1. Differentiate x^x w.r.t. x.
   1. $x^x(1 + \log x)$
   2. $x^x(1 - \log x)$
   3. $-x^x(1 + \log x)$
   4. $x^x \log x$
2. Differentiate $x^x + a^x + x^a + a^a$ w.r.t. x.
   1. $(1 + \log x) + (a^x \log a + ax^{a-1})$
   2. $x^x(1 + \log x) + \log a + ax^{a-1}$
   3. $x^x(1 + \log x) + x^a \log x + ax^{a-1}$
   4. $x^x(1 + \log x) + a^x \log a + ax^{a-1}$
3. If $x = e^{\frac{x}{y}}$, then find $\frac{dy}{dx}$.
   1. $-\frac{(x+y)}{x \log x}$
   2. $-\frac{(x-y)}{x \log x}$
   3. $\frac{(x+y)}{x \log x}$
   4. $\frac{x-y}{x \log x}$
4. If $y = (2 - x)^3(3 + 2x)^5$, then find $\frac{dy}{dx}$.
   1. $(2 - x)^3(3 + 2x)^5\left[\frac{15}{3+2x} - \frac{8}{2-x}\right]$
   2. $(2 - x)^3(3 + 2x)^5\left[\frac{15}{3+2x} + \frac{3}{2-x}\right]$
   3. $(2 - x)^3(3 + 2x)^5\left[\frac{10}{3+2x} - \frac{3}{2-x}\right]$
   4. $(2 - x)^3(3 + 2x)^5 \cdot \left[\frac{10}{3+2x} + \frac{3}{2-x}\right]$
5. If $y = x^x \cdot e^{(2x+5)}$, then find $\frac{dy}{dx}$.
   1. $x^x e^{2x+5}$
   2. $x^x e^{2x+5}(3 - \log x)$
   3. $x^x e^{2x+5}(1 - \log x)$
   4. $x^x e^{2x+5} \cdot (3 + \log x)$

Q19. An architecture design a auditorium for a school for its cultural activities. The floor of the auditorium is rectangular in shape and has a fixed perimeter P. **5 Marks**

Based on the above information, answer the following questions.

1. If x and y represents the length and breadth of the rectangular region, then relation between the variable is.
   1. x + y = P
   2. $x^2 + y^2 = P^2$
   3. 2(x + y) = P
   4. x + 2y = P
2. The area (A) of the rectangular region, as a function of x, can be expressed as.

1. $A = px + \frac{x}{2}$
2. $A = \frac{px + x^2}{2}$
3. $A = \frac{px - 2x^2}{2}$
4. $A = \frac{x^2}{2} + px^2$

3. School's manager is interested in maximising the area of floor 'A' for this to be happen, the value of x should be.

1. P
2. $\frac{P}{2}$
3. $\frac{P}{3}$
4. $\frac{P}{4}$

4. The value of y, for which the area of floor is maximum, is.

1. $\frac{P}{2}$
2. $\frac{P}{3}$
3. $\frac{P}{4}$
4. $\frac{P}{16}$

5. Maximum area of floor is.

1. $\frac{P^2}{16}$
2. $\frac{P^2}{64}$
3. $\frac{P^2}{4}$
4. $\frac{P^2}{28}$