12TH CBSE MATHS WORKSHEET

16) Write the principal value of $\tan^{-1}\left[\sin\left(-\frac{\pi}{2}\right)\right]$

$\tan^{-1}\left[\sin\left(-\frac{\pi}{2}\right)\right] = -\frac{\pi}{4}$

Alternative Method :

$tan^{-1}\left[sin\left(-\frac{\pi}{2}\right)\right] = tan^{-1}[-1]$

$= -tan^{-1}\left(tan\frac{\pi}{4}\right)$

$= -\frac{\pi}{4}$

$\because tan^{-1}(tan\quad\theta) = \theta\quad \forall\,\theta \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

17) Find the value of the following : $cot\left(\frac{\pi}{2} - 2cot^{-1}\sqrt{3}\right)$

$cot\left(\frac{\pi}{2} - 2cot^{-1}\sqrt{3}\right) = \sqrt{3}$

Alternative Method :

$cot\left(\frac{\pi}{2} - 2cot^{-1}\sqrt{3}\right) = cot\left[\frac{\pi}{2} - 2cot^{-1}\left(cot\frac{\pi}{6}\right)\right]$

$= cot\left[\frac{\pi}{2} - 2\left(\frac{\pi}{6}\right)\right]$

$\left[\because cot^{-1}(cot\quad\theta) = \theta\,\forall\theta\, \in\, (0,\quad\pi)\right]$

$= cot\left[\frac{\pi}{2} - \frac{\pi}{3}\right]$

$= cot\left(\frac{\pi}{6}\right)$

$= \sqrt{3}$

18) Write the value of $cos^{-1}\left(-\frac{1}{2}\right) + 2sin^{-1}\left(\frac{1}{2}\right)$

$cos^{-1}\left(-\frac{1}{2}\right) + sin^{-1}\left(\frac{1}{2}\right) = \pi$

Alternative Method :

$cos^{-1}\left(-\frac{1}{2}\right) + 2sin^{-1}\left(\frac{1}{2}\right)$

$= cos^{-1}\left[-cos\left(\frac{\pi}{3}\right)\right] + 2sin^{-1}\left(sin\left(\frac{\pi}{6}\right)\right)$

$= cos^{-1}\left[cos\left(\pi - \frac{\pi}{3}\right)\right] + 2\left(\frac{\pi}{6}\right)$

$\left[\because\quad sin^{-1}(sin\theta) = \theta\forall\theta \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)\right]$

$= \pi - \frac{\pi}{3} + \frac{\pi}{3}\quad \left[\because\quad cos^{-1}(cos\quad\theta) = \theta\forall\theta(0,\quad\pi)\right]$

$= \pi$

19) Write the principal value of $\left[cos^{-1}\left(\frac{\sqrt{3}}{2}\right)+cos^{-1}\left(-\frac{1}{2}\right)\right]$

$\left[cos^{-1}\left(\frac{\sqrt{3}}{2}\right)+cos^{-1}\left(-\frac{1}{2}\right)\right]=\frac{5\pi}{6}$

Alternative Method

$\left[cos^{-1}\left(\frac{\sqrt{3}}{2}\right)+cos^{-1}\left(-\frac{1}{2}\right)\right]$

$=cos^{-1}\left[cos\left(\frac{\pi}{6}\right)\right]+cos^{-1}\left[-cos\left(\frac{\pi}{3}\right)\right]$

$=cos^{-1}\left[cos\left(\frac{\pi}{6}\right)\right]+cos^{-1}\left[cos\left(\pi-\frac{\pi}{3}\right)\right]$

$=\frac{\pi}{6}+\frac{2\pi}{3}=\frac{\pi}{6}+\frac{4\pi}{6}\quad\left[\because\quad cos^{-1}(cos\theta)=\theta\forall\theta\quad(0,\quad\pi)\right]$

$=\frac{5\pi}{6}$

20) Write the principal value of the following : $\left[tan^{-1}\left(-\sqrt{3}\right)+tan^{-1}(1)\right]$

$\left[tan^{-1}\left(-\sqrt{3}\right)+tan^{-1}(1)\right]=-\frac{\pi}{12}$

Alternative Method :

$tan^{-1}\left(-\sqrt{3}\right)+tan^{-1}(1)$

$=tan^{-1}\left[-tan\left(\frac{\pi}{3}\right)\right]+tan^{-1}\left(tan\frac{\pi}{4}\right)$

$=-\frac{\pi}{3}+\frac{\pi}{4}=-\frac{\pi}{12}\;\left[\because tan^{-1}(tan\theta)=\theta\forall\theta\in\left(-\frac{\pi}{2},\frac{\pi}{2}\right)\right]$

21) Evaluvate : $sin^{-1}\left[sin\left(\frac{3\pi}{5}\right)\right]$

$sin^{-1}\left[sin\left(\frac{3\pi}{5}\right)\right]=\frac{2\pi}{5}$

Alternative Method :

$sin^{-1}\left[sin\left(\frac{3\pi}{5}\right)\right]=sin^{-1}\left[sin\left(\pi-\frac{3\pi}{5}\right)\right]$

$\left(\because\quad\frac{3\pi}{5}\notin\left[-\frac{\pi}{2},\frac{\pi}{2}\right]\right)$

$sin^{-1}\left(sin\frac{2\pi}{5}\right)=\frac{2\pi}{5}$

22) Write the principal value of $tan^{-1}(1)+cos^{-1}\left(-\frac{1}{2}\right)$

$tan^{-1}(1)+cos^{-1}\left(-\frac{1}{2}\right)=\frac{11\pi}{12}$

Alternative Method :

$tan^{-1}(1)+cos^{-1}\left(-\frac{1}{2}\right)$

$=tan^{-1}\left(tan\frac{\pi}{4}\right)+cos^{-1}\left(-cos\frac{\pi}{3}\right)$

$=tan^{-1}\left(tan\frac{\pi}{4}\right)+cos^{-1}\left[cos\left(\pi-\frac{\pi}{3}\right)\right]$

$\left[\because tan^{-1}(tan\theta)=\theta\forall\theta\in\left(-\frac{\pi}{2},\frac{\pi}{2}\right)\&cos^{-1}(cos\quad\theta)=\theta\forall\theta\in(0,\pi)\right]$

$=\frac{\pi}{4}+\frac{2\pi}{3}=\frac{11\pi}{12}$

23) Write the value of $tan\left[2tan^{-1}\left(\frac{1}{5}\right)\right]$

$tan\left[2tan^{-1}\left(\frac{1}{5}\right)\right] = \frac{5}{12}$

Alternative Method :

$tan\left[2tan^{-1}\left(\frac{1}{5}\right)\right] = tan\left[tan^{-1}\left(\frac{2\left(\frac{1}{5}\right)}{1-\frac{1}{25}}\right)\right]$

$= tan\left[tan^{-1}\left(\frac{2}{5} \times \frac{25}{24}\right)\right]$

$\left[\because tan^{-1}(tan\theta) = \theta \forall \theta \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)\right]$

$= \left(\frac{5}{12}\right)$

24) Write the value of $tan^{-1}\left[2sin\left(2cos^{-1}\left(\frac{\sqrt{3}}{2}\right)\right)\right]$

$tan^{-1}\left[2sin\left(2cos^{-1}\left(\frac{\sqrt{3}}{2}\right)\right)\right] = \frac{\pi}{3}$

Alternative Method :

$tan^{-1}\left[2sin\left(2cos^{-1}\left(\frac{\sqrt{3}}{2}\right)\right)\right]$

$= tan^{-1}\left[2sin\left(2cos^{-1}\left(cos\frac{\pi}{6}\right)\right)\right]$

$= tan^{-1}\left[2sin\left(2\left(\frac{\pi}{6}\right)\right)\right] = tan^{-1}\left[2 \times \frac{\sqrt{3}}{2}\right]$

$\Rightarrow tan^{-1}\left(\sqrt{3}\right) = \frac{\pi}{3}$

25) Write the principal value of $tan^{-1}\left(tan\frac{9\pi}{8}\right)$

$tan^{-1}\left(tan\frac{9\pi}{8}\right) = \frac{\pi}{8}$

Alternative Method :

$tan^{-1}\left(tan\frac{9\pi}{8}\right) = tan^{-1}\left[tan\left(\pi + \frac{\pi}{8}\right)\right]$

$= tan^{-1}\left[tan\frac{\pi}{8}\right]$

$\left[\because tan^{-1}(tan\theta) = \theta \forall \theta \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)\right]$

$= \frac{\pi}{8}$

26) Write the principal value of $tan^{-1}\left(tan\frac{7\pi}{6}\right)$

$tan^{-1}\left(tan\frac{7\pi}{6}\right)=\frac{\pi}{6}$

Alternative Method :

$tan^{-1}\left(tan\frac{7\pi}{6}\right)=tan^{-1}\left[tan\left(\pi+\frac{\pi}{6}\right)\right]$

$=tan^{-1}\left[tan\frac{\pi}{6}\right]=\frac{\pi}{6}$

$\left[\because tan^{-1}(tan\theta)=\theta\forall\theta\in\left(-\frac{\pi}{2},\frac{\pi}{2}\right)\right]$

27) Find the principal value of $tan^{-1}\sqrt{3}-sec^{-1}(-2)$

$tan^{-1}\sqrt{3}-sec^{-1}(-2)=-\frac{\pi}{3}$

Alternative Method :

$tan^{-1}\sqrt{3}-sec^{-1}(-2)$

$=tan^{-1}\left[tan\left(\frac{\pi}{3}\right)\right]-sec^{-1}\left[-sec\left(\frac{\pi}{3}\right)\right]$

$=\frac{\pi}{3}-\left(\pi-\frac{\pi}{3}\right)$

$\left[\because \quad tan^{-1}(tan \quad \theta)=\theta\forall\theta\in\left(-\frac{\pi}{2},\frac{\pi}{2}\right) \quad and \quad sec^{-1}(sec \quad \theta)=\theta\forall\theta\in(0, \quad \pi)-\left(\frac{\pi}{2}\right)\right]$

$=\frac{2\pi}{3}-\pi=-\frac{\pi}{3}$

28) Write the principal value of $\left[cos^{-1}\left(\frac{1}{2}\right)-2sin^{-1}\left(-\frac{1}{2}\right)\right]$

$\left[cos^{-1}\left(\frac{1}{2}\right)-2sin^{-1}\left(-\frac{1}{2}\right)\right]=\frac{2\pi}{3}$

Alternative Method :

$\left[cos^{-1}\left(\frac{1}{2}\right)-2sin^{-1}\left(-\frac{1}{2}\right)\right]$

$=\left[cos^{-1}\left(cos\frac{\pi}{3}\right)-2sin^{-1}\left(-sin\frac{\pi}{6}\right)\right]$

$=\left[\frac{\pi}{3}-2\times\left(-\frac{\pi}{6}\right)\right]$

$\left[\because cos^{-1}(cos \quad \theta)=\theta\forall\theta\in[0, \quad \pi] \quad and \quad sin^{-1}(sin \quad \theta)=\theta\forall\in\left[-\frac{\pi}{2},\frac{\pi}{2}\right]\right]$

$=\left[\frac{\pi}{3}+\frac{\pi}{3}\right]=\frac{2\pi}{3}$

31) If A is a 3×3 matrix, whose elements are given by $a_{ij}=\frac{1}{3}|-3i+j|$, then write the value a_{23}.

Given $a_{ij}=\frac{1}{3}|-3i+j|$

$\therefore \ a_{23}=\frac{1}{3}|-6+3|=1$

32) If A is a square matrix and $|A|=2$, then write the value of $|AA'|$, where A' is the transpose of matrix A.

Given $|A|=2\Rightarrow|A'|=2$

Also $|AA'|=|A||A'|=2\times2=4.$

33) If A is a square matrix of order 3 such that $|adjA|=64$, find $|A|$.

$|AdjA|=64|A|^{3-1}\Rightarrow|A|^2=64\Rightarrow|A|=\pm8$

34) If $2\begin{bmatrix}1 & 3\\0 & x\end{bmatrix}+\begin{bmatrix}y & 0\\1 & 2\end{bmatrix}=\begin{bmatrix}5 & 6\\1 & 8\end{bmatrix}$, then write the value of (x+y).

$$\begin{bmatrix}2 & 6\\0 & 2x\end{bmatrix}+\begin{bmatrix}y & 0\\1 & 2\end{bmatrix}=\begin{bmatrix}5 & 6\\1 & 8\end{bmatrix}$$
$$\Rightarrow \begin{bmatrix}2+y & 6\\1 & 2x+2\end{bmatrix}=\begin{bmatrix}5 & 6\\1 & 8\end{bmatrix}$$
$$\Rightarrow 2+y=5, 2x+2=8 \Rightarrow x=3, y=3$$
$$\therefore\ x+y=3+3=6$$

35) If $\begin{bmatrix}xy & 4\\z+6 & x+y\end{bmatrix}=\begin{bmatrix}8 & w\\0 & 6\end{bmatrix}$, write the value of x + y + z.

x + y + z = 0

Alternate method :

$$\begin{bmatrix}xy & 4\\z+6 & x+y\end{bmatrix}=\begin{bmatrix}8 & w\\0 & 6\end{bmatrix}$$

By equating z + 6 = 0 and x + y = 6

$\Rightarrow$ **z = - 6, x + y = 6**

x + y + z = 6 - 6

$\Rightarrow$ **x + y + z = 0**

36) If $\begin{pmatrix}2x & 4\end{pmatrix}\begin{pmatrix}x\\-8\end{pmatrix}=0$, find the positive of x.

$x=\pm 4$

Alternate Method :

$$\begin{pmatrix}2x & 4\end{pmatrix}\begin{pmatrix}x\\-8\end{pmatrix}=0$$
$$\Rightarrow (2x^2-32)=0$$

By equating,

$2x^2$ - 32 = 0

$\Rightarrow$ **$2x^2$ = 32**

$\Rightarrow$ **x^2 = 16**

$\Rightarrow$ $x=\pm 4 \Rightarrow$ **x = 4**

37) If $\begin{bmatrix} a+4 & 3b \\ 8 & -6 \end{bmatrix} = \begin{bmatrix} 2a+2 & b+2 \\ 8 & a-8b \end{bmatrix}$, write the value of a - 2b.

a - 2b = 0

Alternate Method :

$$\begin{bmatrix} a+4 & 3b \\ 8 & -6 \end{bmatrix} = \begin{bmatrix} 2a+2 & b+2 \\ 8 & a-8b \end{bmatrix}$$

By equating, $a + 4 = 2a + 2 \Rightarrow a = 2$

$3b = b + 2 \Rightarrow b = 1$

$\therefore\ a - 2b = 2 - 2(1)$

$= 2 - 2$

$= 0$