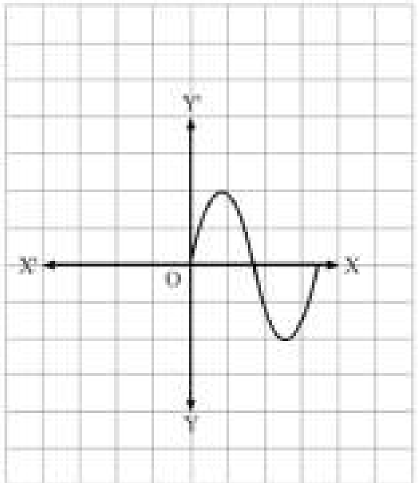
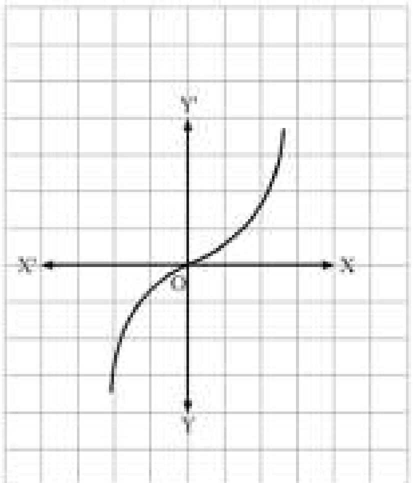
Instructions

Q1. Which of the following graphs represents a one-one f

1.



2.



Q2. Let $f(x)=\begin{cases} 1+x, & 0 \le x \le 2 \\ 3-x, & 2 < x \le 3 \end{cases}$. Find fof.

Q3. If f : R → R is given by $f(x) = x^3$, write $f^{-1}(1)$.

Q4. If f(x) = x + 7 and g(x) = x - 7, x ∈ R, write fog (7).

Q5. If f : A → B is an injection, such that range of f = {a}, determine the number of elements in A.

Q6. If f : C → C is defined by $f(x) = x^4$, write $f^{-1}(1)$.

Q7. If f : A → A, g : A → A are two bijections, then prove that:
fog is an injection.

Q8. If f : R → R, g : R → R are given by $f(x) = (x + 1)^2$ and $g(x) = x^2 + 1$, then write the value of fog(-3). **2 Marks**

Q9. Let A = {1, 2, 3}, B = {4, 5, 6, 7} and let f = {(1, 4), (2, 5), (3, 6)} be a function from A to B. State whether f is one-one or not. **2 Marks**

Q10. Which one the following relations on A = {1, 2, 3} is a function? **2 Marks**
f = {(1, 3), (2, 3), (3, 2)}, g = {(1, 2), (1, 3), (3, 1)}

Q11. If f(x) = 2x + 5 and $g(x) = x^2 + 1$ be two real functions, then describe the following functions: **2 Marks**
f^2

Also, show that fof ≠ f^2

Q12. Which of the following functions from A to B are one-one and onto? **2 Marks**
f_2 = {(2, a), (3, b), (4, c)}; A = {2, 3, 4}, B = {a, b, c}

Q13. Write the domain of the real function f defined by $f(x) = \sqrt{25 - x^2}$. **2 Marks**

Q14. If f : R → R be defined by f(x) = x^4, write $f^{-1}(1)$. **2 Marks**

Q15. Let C denote the set of all complex numbers. A function f : C → C is defined by f(x) = x^3. Write $f^{-1}(1)$. **2 Marks**

Q16. Let A and B be two sets, each with a finite number of elements. Assume that there is an injective map from A to B and that there is an injective map from B to A. Prove that there is a bijection from A to B.

Q17. If f : R → R be defined by $f(x) = (3 - x^3)^{\frac{1}{3}}$, then find fof(x).

Q18. Let A = {1, 2, 3, 4} and B = {a, b} be two sets. Write the total number of onto functions from A to B.

Q19. If f : R → R is defined by f(x) = x^2, write $f^{-1}(25)$.

Q20. Let f : R → R, g : R → R be two functions defined by f(x) = $x^2 + x + 1$ and g(x) = $1 - x^2$. Write fog (-2).

Q21. Which of the following functions from A to B are one-one and onto?
f_1 = {(1, 3), (2, 5), (3, 7)}; A = {1, 2, 3}, B = {3, 5, 7}

Q22. Write the total number of one-one functions from set A = {1, 2, 3, 4} to set B = {a, b, c}.

Q23. If f : R → R is defined by f(x) = 10x - 7, then write $f^{-1}(x)$.

Q24. If f(x) = 2x + 5 and g(x) = $x^2 + 1$ be two real functions, then describe the following functions:
gof
Also, show that fof ≠ f^2

Q25. Let f : R → R and g : R → R be defined by f(x) = x^2 and g(x) = x + 1. Show that fog ≠ gof.

Q26. Write whether f : R → R, given by $f(x) = x + \sqrt{x^2}$, is one-one, many-one, onto or into.

Q27. If f, g : R → R be two functions defined as f(x) = |x| + x and g(x) = |x| – x, $\forall\, x \in \mathbb{R}$. Then find fog and gof. Hence find fog(-3), fog(5) and gof(-2).

Q28. If f : R → R is defined by f(x) = x^2, find $f^{-1}(-25)$.

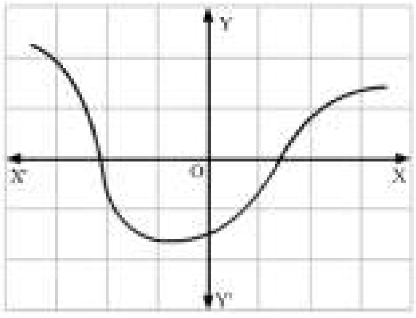
Q29. Let A = {x ∈ R : -4 ≤ x ≤ 4 and x ≠ 0} and f : A → R be defined by $f(x) = \frac{|x|}{x}$. Write the range of f.

Q30. Find fog (2) and gof (1) when : f : R → R; f(x) = $x^2 + 8$ and g : R → R; g(x) = $3x^3 + 1$.

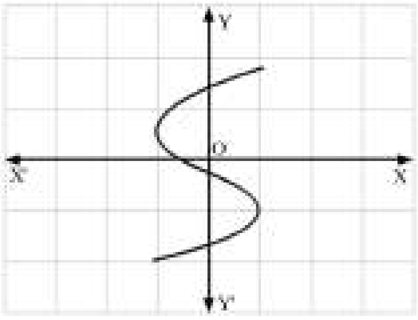
Q31. If A = {1, 2, 3, 4} and B = {a, b, c, d} define any four bijections from A to B. Also give their inverse functions. **2 Marks**

Q32. If A = {a, b, c} and B = {-2, -1, 0, 1, 2}, write the total number of one-one functions from A to B. **2 Marks**

Q33. Which one of the following graphs represents a function? **2 Marks**
1.



2.



Q34. Let $f : \mathrm{R} - \left\{-\frac{3}{5}\right\} \to \mathrm{R}$ be a function defined as $f(x) = \frac{2x}{5x+3}$. Write f^{-1}: Range of $f \to \mathrm{R} - \left\{-\frac{3}{5}\right\}$.

Q35. Let f, g : R → R be defined by f(x) = 2x + 1 and g(x) = x^2 - 2 for all x ∈ R, respectively. Then, find gof.

Q36. Let f be a real function given by $f(x) = \sqrt{x-2}$. Find the following:
f^2
Also, show that fof ≠ f^2.

Q37. If f : A → A, g : A → A are two bijections, then prove that:
fog is a surjection.

Q38. If f : R → R is defined by f(x) = 3x + 2, find f(f(x)).

Q39. Find gof and fog when f : R → R and g : R → R are defined by:
f(x) = 2x + x^2 and g(x) = x^3

Q40. What is the range of the function $f(x) = \frac{|x-1|}{x-1}$?

Q41. Let f : R → R^+ be defined by f(x) = ax, a > 0 and $a \neq 1$. Write $f^{-1}(x)$.

Q42. Let f : R - {-1} → R - {1} be given by $f(x) = \frac{x}{x+1}$. Write $f^{-1}(x)$.

Q43. Let f be a function from C (set of all complex numbers) to itself given by f(x) = x^3. Write $f^{-1}(-1)$.

Q44. If f(x) = 2x + 5 and g(x) = x^2 + 1 be two real functions, then describe the following functions:
fof
Also, show that fof ≠ f^2

Q45. Classify the following functions as injection, surjection or bijection:
f : N → N given by f(x) = x^2

Q46. Write the domain of the real function $f(x) = \sqrt{[x] - x}$.

Q47. If f(x) = 4 - $(x - 7)^3$, then write $f^{-1}(x)$. **2 Marks**

Q48. If f : R → R defined by f(x) = 3x - 4 is invertible, then write $f^{-1}(x)$. **2 Marks**

Q49. Let A = {a, b, c, d} and f : A → A be given by f = {(a, b), (b, d), (c, a), (d, c)}. Write f^{-1}. **2 Marks**

Q50. Write the domain of the real function $f(x) = \frac{1}{\sqrt{|x| - x}}$. **2 Marks**

Q51. Find the number of all onto functions from the set A = {1, 2, 3, ..., n} to itself. **2 Marks**

Q52. If A = {1, 2, 3} and B = {a, b}, write the total number of functions from A to B. **2 Marks**

Q53. If f(x) = 2x + 5 and g(x) = x^2 + 1 be two real functions, then describe the following functions: **2 Marks**
fog
Also, show that fof ≠ f^2

Q54. If f : C → C is defined by f(x) = x^2, write $f^{-1}(-4)$. Here, C denotes the set of all complex numbers. **2 Marks**

Q55. If f : C → C is defined by f(x) = $(x - 2)^3$, write $f^{-1}(-1)$. **2 Marks**

Q56. Let $f : \left(-\frac{\pi}{2}, \frac{\pi}{2}\right) \rightarrow \mathrm{R}$ be a function defined by f(x) = cos[x]. write range (f).

Q57. Show that the function f : R → {3} → R - {2} given by $f(x) = \frac{x-2}{x-3}$ is a bijection.

Q58. Let $f : \left[-\frac{\pi}{2}, \frac{\pi}{2}\right] \rightarrow \mathrm{A}$ be defined by f(x)= sinx. If f is a bijection, write set A.

Q59. Which of the following functions from A to B are one-one and onto?
f_3 = {(a, x), (b, x), (c, z), (d, z)}; A = {a, b, c, d,}, B = {x, y, z}

Q60. Are the following set of ordered pairs functions? If so, examine whether the mapping is injective or surjective:
{(a, b): a is a person, b is an ancestor of a}

Q61. Write the domain of the real function $f(x) = \sqrt{x - [x]}$.

Q62. Find gof and fog when f : R → R and g : R → R are defined by:
f(x) = x^2 + 8 and g(x) = $3x^3$ + 1

Q63. Find fog and gof if:
f(x) = x^2, g(x) = cosx

Q64. State with reasons whether the following functions have inverse:
g : {5, 6, 7, 8} → {1, 2, 3, 4} with g = {(5, 4), (6, 3), (7, 4), (8, 2)}

Q65. If f : A → B and g : B → C are one-one functions, show that gof is a one-one function.

Q66. Find fog and gof if:
f(x) = x^2 + 2, $g(x) = 1 - \frac{1}{1-x}$

Q67. Let A = [-1, 1]. Then, discuss whether the following functions from A to itself are one-one, onto or bijective:
$f(x) = \frac{x}{2}$

Q68. Classify the following functions as injection, surjection or bijection:
f : R → R, defined by f(x) = sinx

Q69. Show that the logarithmic function $f : \mathrm{R}0^{+} \rightarrow \mathrm{R}$ given by f(x) = $\log_a x$, a > 0 is a bijection. **3 Marks**

Q70. Classify the following functions as injection, surjection or bijection: **3 Marks**
f : R → R, defined by f(x) = |x|

Q71. Find gof and fog when f : R → R and g : R → R are defined by: **3 Marks**
f(x) = x^2 + 2x - 3 and g(x) = 3x - 4

Q72. **3 Marks**

Let A = [-1, 1]. Then, discuss whether the following functions from A to itself are one-one, onto or bijective:
g(x) = |x|

Q73. Find fog and gof if: **3 Marks**
$f(x) = e^x$, $g(x) = \log_e x$

Q74. Let f be a function from R to R, such that f(x) = cos(x + 2). Is f invertible? Justify your answer. **3 Marks**

Q75. If f : A → B and g : B → C are onto functions, show that gof is a onto function. **3 Marks**

Q76. Classify the following functions as injection, surjection or bijection:
f : Z → Z given by $f(x) = x^2$

Q77. State with reasons whether the following functions have inverse:
h : {2, 3, 4, 5} → {7, 9, 11, 13} with h = {(2, 7), (3, 9), (4, 11), (5, 13)}

Q78. Find fog and gof if:
f(x) = x + 1, g(x) = 2x + 3

Q79. Classify the following functions as injection, surjection or bijection:
f : R → R, defined by $f(x) = \sin^2 x + \cos^2 x$

Q80. Find fog and gof if:
f(x) = x + 1, g(x) = sinx

Q81. Show that f : R → R, given by f(x) = x - [x], is neither one-one nor onto.

Q82. Let A = R - {3} and B = R - {1}. Consider the function f : A → B defined by $f(x) = \frac{x-2}{x-3}$. Show that f is one-one and onto and hence find f^{-1}.

Q83. Find fog and gof if:
f(x) = |x|, g(x) = sinx

Q84. Are the following set of ordered pairs functions? If so, examine whether the mapping is injective or surjective:
{(x, y): x is a person, y is the mother of x}

Q85. If a function g = {(1, 1), (2, 3), (3, 5), (4, 7)} is described by $g(x) = \alpha x + \beta \alpha x + \beta$, then find the values of α and β.

Q86. If f(x) = |x|, prove that fof = f.

Q87. Let f : R → R be defined as $f(x) = \frac{2x-3}{4}$. Write $fof^{-1}(1)$.

Q88. Classify the following functions as injection, surjection or bijection:
f : R → R, defined by $f(x) = 1 + x^2$

Q89. Let f, g, h be real functions given by f(x) = sinx, g(x) = 2x and h(x) = cosx. Prove that fog = go(fh). **3 Marks**

Q90. Let $f(x) = x^2 + x + 1$ and g(x) = sinx. Show that fog ≠ gof. **3 Marks**

Q91. Consider f : N → N, g : N → N and h : N → R defined as f(x) = 2x, g(y) = 3y + 4 and $h(z) = \sin z$ for all $x, y, z \in N$. Show that ho(gof) = (hog)of. **3 Marks**

Q92. If f(x) = sin x and g(x) = 2x be two real functions, then describe gof and fog. Are these equal functions? **3 Marks**

Q93. Let f : R → R and g : R → R be defined by f(x) = x + 1 and g(x) = x - 1. Show that fog = gof = I_R. **3 Marks**

Q94. State with reasons whether the following functions have inverse: **3 Marks**
f : {1, 2, 3, 4} → {10} with f = {(1, 10), (2, 10), (3, 10), (4, 10)}

Q95. If the mapping f : {1, 3, 4} → {1, 2, 5} and g : {1, 2, 5} → {1, 3}, given by f = {(1, 2), (3, 5), (4, 1)} and g= {(2, 3), (5, 1), (1, 3)}, then write fog. **3 Marks**

Q96. Let A = [-1, 1]. Then, discuss whether the following functions from A to itself are one-one, onto or bijective: **3 Marks**
h(x) = x^2

Q97. Find f^{-1} if it exists: f : A → B, where, A = {1, 3, 5, 7, 9}; B = {0, 1, 9, 25, 49, 81} and f(x) = x^2.

Q98. If A = {1, 2, 3}, show that a onto function f : A → A must be one-one.

Q99. Classify the following functions as injection, surjection or bijection:
f : R → R, defined by f(x) = x^3 + 1

Q100. Find fog and gof if:
$f(x) = c, c \in R, g(x) = \sin x^2$

Q101. Let f be any real function and let g be a function given by g(x) = 2x. Prove that gof = f + f.

Q102. Let A = {1, 2, 3}. Write all one-one from A to itself.

Q103. Let f be an invertible real function. Write ($f^{-1}$of)(1) + ($f^{-1}$of)(2) + ... + ($f^{-1}$of)(100).

Q104. Let f : R → R be the function defined by f(x) = 4x - 3 for all x ∈ R. Then write f^{-1}.

Q105. Give examples of two one-one functions f_1 and f_2 from R to R, such that $f_1 + f_2$: R → R. defined by ($f_1 + f_2$)(x) = f_1(x) + f_2(x) is not one-one.

Q106. If A = {1, 2, 3}, show that a one-one function f : A → A must be onto.

Q107. Find f^{-1} if it exists: f : A → B, where, A = {0, -1, -3, 2}; B = {-9, -3, 0, 6} and f(x) = 3x.

Q108. If f : R → (0, 2) defined by $f(x) = \frac{e^x - e^{-x}}{e^x + e^{-x}} + 1$ is invertible, find f^{-1}.

Q109. Find fog and gof if:
f(x) = x + 1, g(x) = e^x

Q110. If f : {5, 6} → {2, 3} and g : {2, 3} → {5, 6} are given by f = {(5, 2), (6, 3)} and g = {(2, 5), (3, 6)}, then find fog.

Q111. If $f(x) = \sqrt{x+3}$ and g(x) = x^2 + 1 be two real functions, then find fog and gof.

Q112. Classify the following functions as injection, surjection or bijection: **3 Marks**
f : Z → Z, defined by f(x) = x - 5

Q113. Give examples of two functions f : N → N and g : N → N, such that gof is onto but f is not onto. **3 Marks**

Q114. Find gof and fog when f : R → R and g : R → R are defined by: **3 Marks**
f(x) = x and g(x) = |x|

Q115. Find gof and fog when f : R → R and g : R → R are defined by: **3 Marks**
f(x) = $8x^3$ and $g(x) = x^{\frac{1}{3}}$

Q116. Let R^+ be the set of all non-negative real numbers. If $f : R^+ \to R^+$ and $g : R^+ \to R^+$ are defined as $f(x) = x^2$ and $g(x) = +\sqrt{x}$, find fog and gof. Are they equal functions? **3 Marks**

Q117. Let A = {-1, 0, 1} and f = {(x, x^2): x ∈ A}. Show that f : A → A is neither one-one nor onto. **4 Marks**

Q118. Suppose f_1 and f_2 are non-zero one-one functions from R to R. Is $\frac{f_1}{f_2}$ necessarily one-one? Justify your answer. Here, $\frac{f_1}{f_2} : \mathrm{R} \to \mathrm{R}$ is given by $\left(\frac{f_1}{f_2}\right)(x) = \frac{f_1(x)}{f_2(x)}$ for all $x \in \mathrm{R}$. **4 Marks**

Q119. Classify the following functions as injection, surjection or bijection: **4 Marks**
f : Q - {3} → Q, defined by $f(x) = \frac{2x+3}{x-3}$

Q120. Prove that the function f : N → N, defined by $f(x) = x^2 + x + 1$, is one-one but not onto.

Q121. Classify the following functions as injection, surjection or bijection:
f : Z → Z, defined by $f(x) = x^2 + x$

Q122. Let A = {1, 2, 3, 4}; B = {3, 5, 7, 9}; C = {7, 23, 47, 79} and f : A → B, g : B → C be defined as f(x) = 2x + 1 and $g(x) = x^2 - 2$. Express $(gof)^{-1}$ and $f^{-1}og^{-1}$ as the sets of ordered pairs and verify that $(gof)^{-1} = f^{-1}\, og^{-1}$.

Q123. Classify the following functions as injection, surjection or bijection:
f : Q → Q, defined by $f(x) = x^3 + 1$

Q124. Find fog and gof if:
$f(x) = \sin^{-1}x$, $g(x) = x^2$

Q125. If f : R → R be the function defined by $f(x) = 4x^3 + 7$, show that f is a bijection.

Q126. Let A = {$x \in \mathrm{R}$ | −1 ≤ x ≤ 1} and let f : A → A, g : A → A be two functions defined by $f(x) = x^2$ and g(x) = $\sin\left(\frac{\pi x}{2}\right)$. Show that g^{-1} exists but f^{-1} does not exist. Also, find g^{-1}.

Q127. If $f(x) = \frac{4x+3}{6x-4}$, $x \neq \frac{2}{3}$, show that fof(x) = x for all $x \neq \frac{2}{3}$. What is the inverse of f?

Q128. Let $f : \mathrm{R} - \left\{-\frac{4}{3}\right\} \to \mathrm{R}$ be a function defined as $f(x) = \frac{4x}{3x+4}$. Show that $f : \mathrm{R} - \left\{-\frac{4}{3}\right\} \to \mathrm{Range}\ (f)$ is one-one and onto. Hence find f^{-1}.

Q129. Given A = {2, 3, 4}, B = {2, 5, 6, 7}. Construct an example of each of the following:
1. An injective map from A to B.
2. A mapping from A to B which is not injective.
3. A mapping from A to B.

Q130. Let f be a real function given by $f(x) = \sqrt{x-2}$. Find the following:
fof
Also, show that fof ≠ f^2.

Q131. Show that if f_1 and f_2 are one-one maps from R to R, then the product $f_1 \times f_2$: R → R defined by $(f_1 \times f_2)(x) = f_1(x)f_2(x)$ need not be one-one. **4 Marks**

Q132. Verify associativity for the following three mappings : f : N → Z_0 (the set of non-zero integers), g : Z_0 → Q and h : Q → R given by f(x) = 2x, $g(x) = \frac{1}{x}$ and $h(x) = e^x$. **4 Marks**

Q133. Classify the following functions as injection, surjection or bijection: **4 Marks**
f : Z → Z given by $f(x) = x^3$

Q134. Let f be a real function given by $f(x) = \sqrt{x-2}$. Find the following: **4 Marks**
fofof
Also, show that fof ≠ f^2.

Q135. Classify the following functions as injection, surjection or bijection: **4 Marks**
f : N → N given by $f(x) = x^3$

Q136. Classify the following functions as injection, surjection or bijection: **4 Marks**
f : R → R, defined by $f(x) = 5x^3 + 4$

Q137. Classify the following functions as injection, surjection or bijection: **4 Marks**
f : R → R, defined by f(x) = 3 - 4x

Q138. If f : $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right) \to R$ and g : [-1, 1] → R be defined as f(x) = tanx and $g(x) = \sqrt{1-x^2}$ respectively, describe fog and gof.

Q139. Give examples of two functions f : N → Z and g : Z → Z, such that gof is injective but g is not injective.

Q140. Find gof and fog when f : R → R and g : R → R are defined by:
f(x) = 2x + 3 and $g(x) = x^2 + 5$

Q141. If f : R → R be defined by $f(x) = x^3 - 3$, then prove that f^{-1} exists and find a formula for f^{-1}. Hence, find $f^{-1}(24)$ and $f^{-1}(5)$.

Q142. Consider $f : R \to R_+ \to [4, \infty)$ given by $f(x) = x^2 + 4$. Show that f is invertible with inverse of f given by $f^{-1}(x) = \sqrt{x-4}$, where R^+ is the set of all non-negative real numbers.

Q143. Give an example of a function:
Which is neither one-one nor onto.

Q144. Give examples of two surjective functions f_1 and f_2 from Z to Z such that $f_1 + f_2$ is not surjective.

Q145. Let f : N → N be defined by $f(n) = \begin{cases} n+1, & \text{if n is odd} \\ n-1, & \text{if n is even} \end{cases}$ Show that f is a bijection.

Q146. Show that the function f : Q → Q, defined by f(x) = 3x + 5, is invertible. Also, find f^{-1}.

Q147. Classify the following functions as injection, surjection or bijection:
f : R → R, defined by $f(x) = x^3 - x$

Q148. Let f = {(1, -1), (4, -2), (9, -3), (16, 4)} and g = {(-1, -2), (-2, -4), (-3, -6), (4, 8)}. Show that gof is defined while fog is not defined. Also, find gof.

Q149. Consider f : R → R given by f(x) = 4x + 3. Show that f is invertible. Find the inverse of f.

Q150. Let f be a real function given by $f(x) = \sqrt{x-2}$. Find the following:
(fofof)(38)
Also, show that fof ≠ f^2.

Q151. Let f = {(3, 1), (9, 3), (12, 4)} and g = {(1, 3), (3, 3) (4, 9) (5, 9)}. Show that gof and fog are both defined. Also, find fog and gof. **5 Marks**

Q152. Give an example of a function: **5 Marks**
Which is one-one but not onto.

Q153. Give an example of a function: **5 Marks**
Which is not one-one but onto.

Q154. A function f : R → R is defined as $f(x) = x^3 + 4$. Is it a bijection or not? In case it is a bijection, find $f^{-1}(3)$. **5 Marks**

Q155. If f : Q → Q, g : Q → Q are two functions defined by f(x) = 2x and g(x) = x + 2, show that f and g are bijective maps. Verify that $(gof)^{-1} = f^{-1}og^{-1}$. **6 Marks**

Q156. Let A = {a, b, c}, B = {u, v, w} and let f and g be two functions from A to B and from B to A, respectively, defined as: **6 Marks**

f = {(a, v), (b, u), (c, w)}, g = {(u, b), (v, a), (w, c)}.

Show that f and g both are bijections and find fog and gof.

Q157. Let $f : [-1, \infty) \rightarrow [-1, \infty)$ be given by $f(x) = (x + 1)^2 - 1$, $x \geq -1$. Show that f is invertible. Also, find the set S = {x : f(x) = $f^{-1}(x)$}.

Q158. Show that the exponential function f : R → R, given by $f(x) = e^x$, is one-one but not onto. What happens if the co-domain is replaced by $R0^+$ (set of all positive real numbers)?

Q159. If f : R → (-1, 1) defined by $f(x) = \frac{10^x - 10^{-x}}{10^x + 10^{-x}}$ is invertible, find f^{-1}.

Q160. Consider $f : R_+ \rightarrow [-5, \infty)$ given by $f(x) = 9x^2 + 6x - 5$. Show that f is invertible with $f^{-1}(x) = \frac{\sqrt{x+6} - 1}{3}$.

Q161. Classify the following functions as injection, surjection or bijection:

f : R → R, defined by $f(x) = \frac{x}{x^2 + 1}$

Q162. Consider f : {1, 2, 3} → {a, b, c} and g : {a, b, c} → {apple, ball, cat} defined as f(1) = a, f(2) = b, f(3) = c, g(a) = apple, g(b) = ball and g(c) = cat. Show that f, g and g o f are invertible. Find f^{-1}, g^{-1} and gof^{-1} and show that $(gof)^{-1} = f^{-1}og^{-1}$.

Q163. Let f : N → N be a function as $f(x) = 9x^2 + 6x - 5$. Show that f : N → S, where S is the range of f, is invertible. Find the inverse of f and hence find $f^{-1}(43)$ and $f^{-1}(163)$.

Q164. If $f(x) = \sqrt{1 - x}$ and $g(x) = \log_e x$ are two real functions, then describe, functions fog and gof.

Q165. Consider the function $f : R^+ \rightarrow [-9, \infty]$ given by f(x) = 5x2 + 6x - 9. Prove that f is invertible with $f^{-1}(y) = \frac{\sqrt{54 + 5y} - 3}{5}$.

Q166. Let f : R - {n} → R be a function defined by $f(x) = \frac{x - m}{x - n}$, where $m \neq n$. Then,

A f is one-one onto. **B** f is one-one into. **C** f is many one onto. **D** f is many one into.

Q167. The function f : A → B defined by $f(x) = -x^2 + 6x - 8$ is a bijection if,

A $A = (-\infty, 3]$ and $B = (-\infty, 1]$

B $A = [-3, \infty)$ and $B = (-\infty, 1]$

C $A = (-\infty, 3]$ and $B = [1, \infty)$

D $A = [3, \infty)$ and $B = [1, \infty)$

Q168. If f : R → R is given by f(x) = 3x - 5, then $f^{-1}(x)$

A is given by $\frac{1}{3x - 5}$

B is given by $\frac{x + 5}{3}$

C does not exist because f is not one-one.

D does not exist because f is not onto.

Q169. Let f : R → R be given by $f(x) = [x^2] + [x + 1] - 3$ where [x] denotes the greatest integer less than or equal to x. Then, f(x) is: **1 Mark**

A Many-one and onto. **B** Many-one and into. **C** One-one and into. **D** One-one and onto.

Q170. f : R → R is defined by $f(x) = \frac{e^{x^2} - e^{-x^2}}{e^{x^2} + e^{-x^2}}$ is: **1 Mark**

A One-one but not onto.

B Many-one but onto.

C One-one and onto.

D Neither one-one nor onto.

Q171. **1 Mark**

If the function f : R → A given by $f(x) = \frac{x^2}{x^2+1}$ is a surjection, then A =

A R **B** [0, 1] **C** [0, 1) **D** [0, 1)

Q172. Let f : R → R be a function defined by $f(x) = \frac{x^2-8}{x^2+2}$. Then, f is: **1 Mark**

A One-one but not onto. **B** One-one and onto.
C Onto but not one-one. **D** Neither one-one nor onto.

Q173. Let f : R → R be given by f(x) = x^2 - 3. Then, f^{-1} is given by: **1 Mark**

A $\sqrt{x+3}$ **B** $\sqrt{x}+3$
C $x+\sqrt{3}$ **D** None of these

Q174. Which of the following functions form Z to itself are bijections?

A f(x) = x^3 **B** f(x) = x + 2 **C** f(x) = 2x + 1 **D** f(x) = x^2 + x

Q175. A function f from the set of natural numbers to integers defined by $f(n) = \begin{cases} \frac{n-1}{2}, & \text{when n is odd} \\ -\frac{n}{2}, & \text{when n is even} \end{cases}$

A Neither one-one nor onto. **B** One-one but not onto.
C Onto but not one-one. **D** One-one and onto both.

Q176. If $g(f(x)) = |\sin x|$ and $f(g(x)) = (\sin\sqrt{x})^2$, then

A $f(x) = \sin^2 x,\ g(x) = \sqrt{x}$ **B** $f(x) = \sin x,\ g(x) = |x|$
C $f(x) = x^2,\ g(x) = \sin\sqrt{x}$ **D** f and g cannot be determined.

Q177. Let $f : R - \left\{\frac{3}{5}\right\} \to R$ be defined by $f(x) = \frac{3x+2}{5x-3}$. Then,

A $f^{-1}(x) = f(x)$ **B** $f^{-1}(x) = -f(x)$
C fof(x) = -x **D** $f^{-1}(x) = \frac{1}{19}f(x)$

Q178. Let f : Z → Z be given by $f(x) = \begin{cases} \frac{x}{2}, & \text{if x is even} \\ 0, & \text{if x is odd} \end{cases}$. Then, f is:

A Onto but not one-one. **B** One-one but not onto.
C One-one and onto. **D** Neither one-one nor onto.

Q179. The function f : R → R defined by f(x) = (x - 1)(x - 2)(x - 3) is:

A One-one but not onto. **B** Onto but not one-one.
C Both one and onto. **D** Neither one-one nor onto.

Q180. Let f be an injective map with domain {x, y, z} and range {1, 2, 3}, such that exactly one of the following statements is correct and the remaining are false.
$f(x) = 1,\ f(y) \neq 1,\ f(z) \neq 2.$
The value of $f^{-1}(1)$is:

A x **B** y **C** z **D** None of these.

Q181. The function f : R → R defined by f(x) = $2^x + 2^{|x|}$ is:

A One-one and onto. **B** Many-one and onto. **C** One-one and into. **D** Many-one and into.

Q182. Let A = {1, 2, ..., n} and B = {a, b}. Then the number of subjections from A into B is: **1 Mark**

A ${}^nP_2$ **B** $2^n - 2$
C $2^n - 1$ **D** ${}^nC_2$

Q183. If the function f : R → R be such that f(x) = x - [x], where [x] denotes the greatest integer less than or equal to x, then $f^{-1}(x)$ is: **1 Mark**

A $\frac{1}{x-[x]}$ **B** [x] - x
C Not defined. **D** None of these.

Q184. Let $A = \{x \in R : -1 \leq x \leq 1\} = B$ and $C = \{x \in R : x \geq 0\}$ and let **1 Mark**
$S = \{(x, y) \in A \times B : x^2 + y^2 = 1\}$ and $S_0 = \{(x, y) \in A \times C : x^2 + y^2 = 1\}$. Then,

A S defines a function from A to B.
B S_0 defines a function from A to C.
C S_0 defines a function from A to B.
D S defines a function from A to C.

Q185. The distinct linear functions that map [-1, 1] onto [0, 2] are: **1 Mark**

A f(x) = x + 1, g(x) = -x + 1
B f(x) = x - 1, g(x) = x + 1
C f(x) = -x - 1, g(x) = x - 1
D None of these.

Q186. Let the function f : R - {-b} → R - {1} be defined by $f(x) = \frac{x+a}{x+b}$, $a \neq b$. Then, **1 Mark**

A f is one-one but not onto.
B f is onto but not one-one.
C f is both one-one and onto.
D None of these.

Q187. If $F : [1, \infty) \to [2, \infty)$ is given by $f(x) = x + \frac{1}{x}$, then $f^{-1}(x)$ equals:

A $\frac{x+\sqrt{x^2-4}}{2}$
B $\frac{x}{1+x^2}$
C $\frac{x-\sqrt{x^2-4}}{2}$
D $1 + \sqrt{x^2 - 4}$

Q188. Let $A = \{x \in R : x \leq 1\}$ and f : A → A be defined as f(x) = x(2 - x). Then $f^{-1}(x)$ is:

A $1 + \sqrt{1 - x}$
B $1 - \sqrt{1 - x}$
C $\sqrt{1 - x}$
D $1 \pm \sqrt{1 - x}$

Q189. The range of the function $f(x) = {}^{7-x}P_{x-3}$ is:

A {1, 2, 3, 4, 5}
B {1, 2, 3, 4, 5, 6}
C {1, 2, 3, 4}
D {1, 2, 3}

Q190. Let $f(x) = x^3$ be a function with domain {0, 1, 2, 3}. Then domain of f^{-1} is:

A {3, 2, 1, 0}
B {0, -1, -2, -3}
C {0, 1, 8, 27}
D {0, -1, -8, -27}

Q191. Which of the following functions from $A = \{x \in R : -1 \leq x \leq 1\}$ to itself are bijections?

A $f(x) = |x|$
B $f(x) = \sin \frac{\pi x}{2}$
C $f(x) = \sin \frac{\pi x}{4}$
D None of these.

Q192. If the set A contains 5 elements and the set B contains 6 elements, then the number of one-one and onto mappings from A to B is:

A 720
B 120
C 0
D None of these.

Q193. Which of the following functions from $A = \{x : -1 \leq x \leq 1\}$ to itself are bijections?

A $f(x) = \frac{x}{2}$
B $g(x) = \sin\left(\frac{\pi x}{2}\right)$
C $h(x) = |x|$
D $k(x) = x^2$

Q194. If f : R → R is given by $f(x) = x^3 + 3$, then $f^{-1}(x)$ is equal to:

A $x^{\frac{1}{3}} - 3$
B $x^{\frac{1}{3}} + 3$
C $(x - 3)^{\frac{1}{3}}$
D $x + 3^{\frac{1}{3}}$

Q195. Let $f(x) = x^2$ and $g(x) = 2^x$. Then, the solution set of the equation fog(x) = gof(x) is:

A R
B {0}
C {0, 2}
D None of these.

Q196. Let f : R → R be given by $f(x) = \tan x$. Then, $f^{-1}(1)$ is:

A $\frac{\pi}{4}$
B $\{n\pi + \frac{\pi}{4} : n \in Z\}$
C Does not exist.
D None of these.

Q197. A function f from the set of natural numbers to the set of integers defined by **1 Mark**

$$f(n) \begin{cases} \frac{n-1}{2}, & \text{when n is odd} \\ -\frac{n}{2}, & \text{when n is even} \end{cases}$$ is:

A Neither one-one nor onto.
B One-one but not onto.
C Onto but not one-one.
D One-one and onto.

Q198. If f : R → (-1, 1) is defined by $f(x) = \frac{-x|x|}{1+x^2}$, then $f^{-1}(x)$ equals, **1 Mark**

A $\sqrt{\frac{|x|}{1-|x|}}$

B $-\mathrm{Sgn}\,(x)\sqrt{\frac{|x|}{1-|x|}}$

C $-\sqrt{\frac{x}{1-x}}$

D None of these

Q199. Let f : R → R be a function defined by $f(x)=\frac{e^{|x|}-e^{-x}}{e^{x}+e^{-x}}$. Then, **1 Mark**

A f is a bijection.

B f is an injection only.

C f is surjection on only.

D f is neither an injection nor a surjection.

Q200. The function $f:[0,\infty)\to R$ given by $f(x)=\frac{x}{x+1}$ is: **1 Mark**

A One-one and onto. **B** One-one but not onto. **C** Onto but not one-one. **D** Onto but not one-one.

Q201. Let g(x) = 1 + x - [x] and $f(x)=\begin{cases}-1, & x<0\\ 0, & x=0\\ 1, & x>0\end{cases}$ where [x] denotes the greatest integer less than or equal to x. Then for all x, f(g(x)) is equal to:

A x **B** 1 **C** f(x) **D** g(x)

Q202. If f : A → B given by $3^{f(x)}+2^{-x}=4$ is a bijection, then

A $A=\{x\in R:-1<x<\infty\}$, $B=\{x\in R:2<x<4\}$

B $A=\{x\in R:-3<x<\infty\}$, $B=\{x\in R:2<x<4\}$

C $A=\{x\in R:-2<x<\infty\}$, $B=\{x\in R:2<x<4\}$

D None of these.

Q203. Let $A=\{x:-1\le x\le 1\}$ and f : A → A such that $f(x)=x|x|$, then f is:

A A bijection.

B Injective but not surjective.

C Surjective but not injective.

D Neither injective nor surjective.

Q204. Let $A=\{x\in R:x\ge 1\}$. The inverse of the function f : A → A given by $f(x)=2^{x(x-1)}$, is:

A $\left(\frac{1}{2}\right)^{x(x-1)}$

B $\frac{1}{2}\{1+\sqrt{1+4\log_2 x}\}$

C $\frac{1}{2}\{1-\sqrt{1+4\log_2 x}\}$

D Not defined.

Q205. If a function $f:[2,\infty)\to B$ defined by $f(x)=x^2-4x+5$ is a bijection, then B =

A R **B** $[1,\infty)$ **C** $[4,\infty)$ **D** $[5,\infty)$

Q206. Let [x] denote the greatest integer less than or equal to x. If $f(x)=\sin^{-1}x$, $g(x)=[x^2]$ and $h(x)=2x,\ \frac{1}{2}\le x\le\frac{1}{\sqrt{2}}$, then

A $fogoh(x)=\frac{\pi}{2}$

B $fogoh(x)=\pi$

C $hofog=hogof$

D $hofog\ne hogof$

Q207. The function f : R → R, $f(x)=x^2$ is:

A Injective but not surjective.

B Surjective but not injective.

C Injective as well as surjective.

D Neither injective nor surjective.

Q208. f : R → R given by $f(x)=x+\sqrt{x^2}$ is:

A Injective. **B** Surjective. **C** Bijective. **D** None of these.

Q209. The function $f:\left[\frac{-1}{2},\frac{1}{2},\frac{1}{2}\right]\to\left[\frac{-\pi}{2},\frac{\pi}{2}\right]$, defined by $f(x)=\sin^{-1}(3x-4x^3)$, is: **1 Mark**

A Bijection.

B Injection but not a surjection.

C Surjection but not an injection.

D Neither an injection nor a surjection.

Q210. If $g(x)=x^2+x-2$ and $\frac{1}{2}gof(x)=2x^2-5x+2$, then f(x) is equal to: **1 Mark**

A 2x - 3 **B** 2x + 3 **C** $2x^2+3x+1$ **D** $2x^2-3x-1$

Q211. Let $f:[2,\infty)\to X$ be defined by $f(x)=4x-x^2$. Then, f is invertible if X = **1 Mark**

A $[2,\infty)$ **B** $(-\infty,2]$ **C** $(-\infty,4]$ **D** $[4,\infty)$

Q212. Let $f(x) = \frac{\alpha x}{x+1}$, $x \neq -1$. Then, for what value of α is f(f(x)) = x? **1 Mark**

A $\sqrt{2}$ **B** $-\sqrt{2}$ **C** 1 **D** -1

Q213. The inverse of the function $f: R \to \{x \in R : x < 1\}$ given by $f(x) = \frac{e^x - e^{-x}}{e^x + e^{-x}}$ is: **1 Mark**

A $\frac{1}{2}\log\frac{1+x}{1-x}$
B $\frac{1}{2}\log\frac{2+x}{2-x}$
C $\frac{1}{2}\log\frac{1-x}{1+x}$
D None of these.

Q214. If the set A contains 7 elements and the set B contains 10 elements, then the number one-one functions from A to B is: **1 Mark**

A ${}^{10}C_7$
B ${}^{10}C_7 \times 7!$
C 7^{10}
D 10^7

Q215. Let f : R → R be defined as $f(x) = \begin{cases} 2x, & \text{if } x > 3 \\ x^2, & \text{if } 1 < x \leq 3 \\ 3x, & \text{if } x \leq 1 \end{cases}$. Then, find f(-1) + f(2) + f(4):

A 9 **B** 14 **C** 5 **D** None of these.

Q216. Let $A = \{x \in R : -1 \leq x \leq 1\} = B$. Then, the mapping f : A → B given by f(x) = x|x| is:

A Injective but not surjective.
B Surjective but not injective.
C Bijective.
D None of these.

Q217. If f(x) = $\sin^2 x$ and the composite function $g(f(x)) = |\sin x|$, then g(x) is equal to:

A $\sqrt{x-1}$ **B** $\sqrt{x}$ **C** $\sqrt{x+1}$ **D** $-\sqrt{x}$

Q218. Let $f(x) = \frac{1}{1-x}$. Then, {fo(fof)}(x):

A x for all $x \in R$
B x for all $x \in R - \{1\}$
C x for all $x \in R - \{0, 1\}$
D None of these.

Q219. Let M be the set of all 2 × 2 matrices with entries from the set R of real numbers. Then, the function f : M→ R defined by f(A) = |A| for every A ∈ M, is:

A One-one and onto.
B Neither one-one nor onto.
C One-one but-not onto.
D Onto but not one-one.

Q220. The function f : R → R defined by f(x) = $6^x + 6^{|x|}$ is:

A One-one and onto. **B** Many one and onto **C** One-one and into **D** Many one and into.