TEST 1- DEC 1 2025

12TH CBSE MATHS TEST PAPERS

Time Allowed : 3 Hours *Maximum Marks : 80*

General Instructions:

(i) This question paper contains **38** questions. **All** questions are compulsory.

(ii) Question paper is divided into **Five** Sections - Sections **A, B, C, D** and **E**.

(iii) In Section **A -** Question Number **1** to **18** are Multiple Choise Questions (MCQ) type and Question Number **19** & **20** are Assertion-Reason based questions of **1** mark each.

(iv) In Section **B -** Question Number **21** to **25** are Very Short Answer (VSA) type questions of **2** marks each.

(v) In Section **C -** Question Number **26** to **31** are Short Answer (SA) type questions, carrying **3** marks each.

(vi) In Section **D -** Question Number **32** to **35** are Long Answer (LA) type questions carrying **5** marks each.

(vii) In Section **E -** Question Number **36** to **38** are case study based questions carrying **4** marks each where **2 VSA** type questions are of **1** mark each and **1 SA** type question is of **2** marks. Internal choice is provided in **2** marks question in each case study.

(viii) There is no overall choice. However, an internal choice has been provided in **2** questions in Section **- B**, **3** questions in Section **- C**, **2** questions in Section **- D** and **2** questions in Section **- E**.

(ix) Use of calculators is **NOT** allowed.

SECTION - A

Select the correct option out of the four given options:

1. If A is a 3×4 matrix and B is a matrix such that A'B and AB' are both defined, then the order of the matrix B is:

(a) 3×4 (b) 3×3 (c) 4×4 (d) 4×3

2. If the area of a triangle with vertices $(2, -6)$, $(5, 4)$ and $(k, 4)$ is 35 sq. units, then k is:

(a) 12 (b) –2 (c) –12, –2 (d) 12, –2

3. If $f(x) = 2|x| + 3|\sin x| + 6$, then the right hand derivative of f(x) at $x = 0$ is:

(a) 6 (b) 5 (c) 3 (d) 2

4. If $x\begin{bmatrix}1\\2\end{bmatrix} + y\begin{bmatrix}2\\5\end{bmatrix} = \begin{bmatrix}4\\9\end{bmatrix}$, then :

(a) $x = 1, y = 2$ (b) $x = 2, y = 1$

(c) $x = 1, y = -1$ (d) $x = 3, y = 2$

5. If a matrix $A = [1\ 2\ 3]$, then the matrix AA' (where A' is the transpose of A) is:

(a) 14 (b) $\begin{bmatrix}1 & 0 & 0\\0 & 2 & 0\\0 & 0 & 3\end{bmatrix}$

(c) $\begin{bmatrix}1 & 2 & 3\\2 & 3 & 1\\3 & 1 & 2\end{bmatrix}$ (d) [14]

6. The product $\begin{bmatrix}a & b\\-b & a\end{bmatrix}\begin{bmatrix}a & -b\\b & a\end{bmatrix}$ is equal to:

(a) $\begin{bmatrix}a^2+b^2 & 0\\0 & a^2+b^2\end{bmatrix}$ (b) $\begin{bmatrix}(a+b)^2 & 0\\(a+b)^2 & 0\end{bmatrix}$

(c) $\begin{bmatrix}a^2+b^2 & 0\\a^2+b^2 & 0\end{bmatrix}$ (d) $\begin{bmatrix}a & 0\\0 & b\end{bmatrix}$

7. Distance of the point (p, q, r) from y-axis is :

(a) q (b) $|q|$ (c) $|q| + |r|$ (d) $\sqrt{\quad + r^2}$

8. The solution set of the inequation $3x + 5y < 7$ is:

(a) whole xy-plane except the points lying on the line $3x + 5y = 7$.

(b) whole xy-plane along with the points lying on the line $3x + 5y = 7$.

(c) open half plane containing the origin except the points of line $3x + 5y = 7$.

(d) open half plane not containing the origin.

9. If $\int_0^a 3x^2 dx = 8$, then the value of 'a' is:

(a) 2 (b) 4 (c) 8 (d) 10

10. The sine of the angle between the vectors $\vec{a} = 3\hat{i} + \hat{j} + 2\hat{k}$ and $\vec{b} = \hat{i} + \hat{j} + 2\hat{k}$ is:

(a) $\sqrt{\frac{5}{21}}$ (b) $\frac{5}{\sqrt{21}}$ (c) $\sqrt{\frac{3}{21}}$ (d) $\frac{4}{\sqrt{21}}$

11. The order and degree (if defined) of the differential equation, $\left(\frac{d^2y}{dx^2}\right)^2 + \left(\frac{dy}{dx}\right)^3 = x \sin\left(\frac{dy}{dx}\right)$ respectively are:

(a) 2, 2 (b) 1, 3

(c) 2, 3 (d) 2, degree not defined

12. $\int e^{5 \log x} dx$ is equal to:

(a) $\frac{x^5}{5} + C$ (b) $\frac{x^6}{6} + C$

(c) $5x^4 + C$ (d) $6x^5 + C$

13. A unit vector along the vetor $4\hat{i} - 3\hat{k}$ is:

(a) $\frac{1}{7}(4\hat{i} - 3\hat{k})$ (b) $\frac{1}{5}(4\hat{i} - 3\hat{k})$

(c) $\frac{1}{\sqrt{7}}(4\hat{i} - 3\hat{k})$ (d) $\frac{1}{\sqrt{5}}(4\hat{i} - 3\hat{k})$

14. Which of the following points satisfies both the inequations $2x + y \le 10$ and $x + 2y \ge 8$?

(a) (–2, 4) (b) (3, 2) (c) (–5, 6) (d) (4, 2)

15. If $y = \sin^2 (x^3)$, then $\frac{dy}{dx}$ is equal to:

(a) $2 \sin x^3 \cos x^3$ (b) $3x^3 \sin x^3 \cos x^3$

(c) $6x^2 \sin x^3 \cos x^3$ (d) $2x^2 \sin^2(x^3)$

16. The point (x, y, 0) on the xy-plane divides the line segment joining the points (1, 2, 3) and (3, 2, 1) in the ratio:

(a) 1 : 2 internally (b) 2 : 1 internally

(c) 3 : 1 internally (d) 3 : 1 externally

17. The events E and F independent. If P(E) = 0.3 and $P(E \cup F) = 0.5$, then P(E/F) – P(F/E):

(a) $\frac{1}{7}$ (b) $\frac{2}{7}$ (c) $\frac{3}{35}$ (d) $\frac{1}{70}$

18. The integrating factor for solving the differential equation $x\frac{dy}{dx} - y = 2x^2$ is:

(a) e^{-y} (b) e^{-x} (c) x (d) $\frac{1}{x}$

Questions number 19 and 20 are Assertion and Reason based questions carrying 1 mark each. Two statements are given, one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer from the codes (a), (b), (c) and (d) as given below.

(a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).

(b) Both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of Assertion (A)

(c) Assertion (A) is true and Reason (R) is false.

(d) Assertion (A) is false and Reason (R) is true.

19. **Assertion (A):** The lines $\vec{r} = \vec{a}_1 + \lambda\vec{b}_1$ and $\vec{r} = \vec{a}_2 + \mu\vec{b}_2$ are perpendicular, when $\vec{b}_1 . \vec{b}_2 = 0$.

Reason (R): The angle θ between the lines $\vec{r} = \vec{a}_1 + \lambda\vec{b}_1$ and $\vec{r} = \vec{a}_2 + \mu\vec{b}_2$ is given by $\cos\theta = \frac{\vec{b}_1 . \vec{b}_2}{|\vec{b}_1||\vec{b}_2|}$.

20. **Assertion (A):** All trigonometric functions have their inverses over their respective domains.

Reason (R): The inverse of $\tan^{-1}x$ exists for some $x \in R$.

SECTION - B

This section comprises of Very Short Answer (VSA) type questions of 2 marks each.

21. If $xy = e^{x-y}$, then show that $\frac{dy}{dx} = \frac{y(x-1)}{x(y+1)}$

22. (a) Find the domain of $y = \sin^{-1}(x^2 - 4)$.

OR

(b) Evaluate:

$$\cos^{-1}\left[\cos\left(-\frac{7\pi}{3}\right)\right]$$

23. If the projection of the vector $\hat{i} + \hat{j} + \hat{k}$ on the vector $p\hat{i} + \hat{j} - 2\hat{k}$ is $\frac{1}{3}$, then find the value(s) of p.

24. Find the point on the curve $y^2 = 8x$ for which the abscissa and ordinate change at the same rate.

25. (a) Find the vector equation of the line passing through the point (2, 1, 3) and perpendicular to both the lines.

$$\frac{x-1}{1} = \frac{y-2}{2} = \frac{z-3}{3}; \frac{x}{-3} = \frac{y}{2} = \frac{z}{5}.$$

OR

(b) The equations of a line are 5x -3 = 15y + 7 = 3 - 10z. Write the direction cosines of the line and find the coordinates of a point through which it passes.

SECTION - C

The section comprises of Short Answer (SA) type questions of 3 marks each.

26. Find $\int \frac{2}{(1-x)(1+x^2)}dx$.

27. (a) Evaluate $\int_{1/3}^{1} \frac{(x-x^3)^{1/3}}{x^4}dx$.

OR

(b) Evaluate: $\int_{1}^{3} \{|(x-1)| + |(x-2)|\}dx$

28. Solve the following linear programming problem graphically:

Maximise $z = 5x + 3y$

subject to the constraints

$3x + 5y \leq 15$,

$5x + 2y \leq 10$,

$x, y \geq 0$.

29. From a lot of 30 bulbs which include 6 defective bulbs, a sample of 2 bulbs is drawn at random one by one with replacement. Find the probability distribution of the number of defective bulbs and hence find the mean number of defective bulbs.

30. (a) Find the particular solution of the differential equation

$$\frac{dy}{dx} = \frac{x+y}{x}, y(1) = 0$$

OR

(b) Find the general solution of the differferntial equation

$$e^x \tan y dx + (1-e^x) \sec^2 y\, dy = 0$$

31. (a) Evaluate: $\int_{\pi/4}^{\pi/2} e^{2x}\left(\frac{1-\sin 2x}{1-\cos 2x}\right)dx$

OR

(b) Evaluate $\int_{-2}^{2} \frac{x^2}{1+5^x}dx$

SECTION - D

This section comprises of Long Answer (LA) type questions of 5 marks each.

32. (a) Find the image of the point (2, – 1, 5) in the line

$$\frac{x-11}{10} = \frac{y+2}{-4} = \frac{z+8}{-11}$$

OR

(b) Vertices B and C of ΔABC lie on the line $\frac{x+2}{2} = \frac{y-1}{1} = \frac{z}{4}$. Find the area of ΔABC given that point A has coordinates (1, – 1, 2) and the line segment BC has length of 5 units.

33. Find the inverse of the matrix $A = \begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{bmatrix}$. Using the inverse, A^{-1}, solve the sysyem of linear equations

$x - y + 2z - 1$; $2y - 3z = 1$; $3x - 2y + 4z = 3$

34. Using integration, find the area of the region bounded by the parabola $y^2 = 4ax$ and its latus rectum.

35. (a) If N denotes the set of all natural numbers and R is the relation on N × N defined by (a,b,) R (c,d), if ad(b+c) = bc(a + d). Show that R is an equivalence relation.

OR

(b) Let $f : R - \left\{-\frac{4}{3}\right\} \rightarrow R$ be a function defined as

$f(x) = \frac{4x}{3x+4}$. Show that f is one -one function. Also, check whether f is an onto function or not.

SECTION - E

This section comprises of 3 Case Study/Passage-Bassed questions of 4 marks each with two sub-parts. First two case study questions have three sub-parts (I), (II), (III) of marks 1, 1, 2 respectively. The third case study question has two sub – parts (I) and (II) of marks 2 each.

Case Study-I

36. A building contractor undertakes a job to construct 4 flats on a plot along with parking area. Due to strike the probability of many construction workers not being present for the job is 0.65. The probability that many are not present and still the work gets completed on time is 0.35. The probability that work will be completed on time when all workers are present is 0.80.

Let: E_1: represent the events when many workers were not present for the job;

E_2 : represent the events when all workers were present; and

E : represent completing the condtruction work on time.

Based on the above information, answer the following questions:

(i) What is the probability that all the workers are present for the job ?

(ii) What is the probability that construction will be completed on time?

(iii) (a) What is the probability that many workers are not present given that the costruction work is completed on time ?

OR

(IV) (b) What is the probability that all workers were present given that the construction job was completed on time?

Case Study - II

37. Let f(x) be a real valued function. Then its

- Left Hand Derivative (L.H.D.) :

$$Lf'(a) = \lim_{h \to 0} \frac{f(a-h)-f(a)}{-h}$$

- Right Hand Derivative (R.H.D) :

$$Rf'(a) = \lim_{h \to 0} \frac{f(a+b)-f(a)}{h}$$

Also, a function f(x) is said to be differentiable at x = a if its L.H.D. and R.H.D. at x = a exist and both are equal.

For the function $f(x) = \begin{cases} |x-3|, x \geq 1 \\ \frac{x^2}{4} - \frac{3x}{2} + \frac{13}{4}, x < 1 \end{cases}$

answer the following questions:

(i) What is R.H.D. of f(x) at x = 1

(ii) What is L.H.D. of f(x) at x = 1

(iii) (a) Check if the function f(x) is differentiable at x =1

OR

(iii) (b) Find the f'(2) and f'(-1)

Case Study-III

38. Sooraj's father wants to construct a rectangular garden using a brick wall on one side of the garden and wire fencing for the other three sides as shown in the figure. He has 200 meters of fencing wire.

Based on the above information, answer the following questions;

(i) Let 'x' meters denote the length of the side of the garden perpendicular to the brick wall and 'y' metres denote the length of the side parallel to the brick wall. Determine the relation representing the total length of fencing wire and also write A(x) the area of the garden.

(ii) Determine the maximum value of A(x)

General Instructions:

Read the following instructions very carefully and strictly follow them:

(i) This Question paper contains **38** questions. **All** questions are **compulsory**.

(ii) This Question paper is divided into **five** Sections - **A, B, C, D** and **E**.

(iii) In **Section A**, Questions no. **1** to **18** are **multiple choice questions (MCQs)** and Questions no. **19** and **20** are **Assertion-Reason based** questions of **1 mark each.**

(iv) In **Section B**, Questions no. **21** to **25** are **Very Short Answer (VSA)-type** questions, carrying **2 marks each.**

(v) In **Section C**, Questions no. **26** to **31** are **Short Answer (SA)-type** questions, carrying **3 marks each.**

(vi) In **Section D**, Questions no. **32** to **35** are **Long Answer (LA)-type** questions, carrying **5 marks each.**

(vii) In **Section E**, Questions no. **36** to **38** are **Case study-based questions**, carrying **4 marks each.**

(viii) There is no overall choice. However, an internal choice has been provided in 2 questions in Section B, 3 questions in Section C, 2 questions in Section D and one subpart each in 2 questions of Section E.

(ix) Use of calculators is **not** allowed.

SECTION –A

[This section comprises of Multiple Choice Questions (MCQ) 1 mark of each]

Q1	If f(x)= $\begin{cases} \frac{x^3-a^3}{x-a} & ,x\neq a \\ b & ,x = b \end{cases}$ is continuous at x =a,then b is equal to (a)a^2 (b) $2a^2$ (c) $3a^2$ (d) 4 a^2	1
Q2	If y = $Ae^{5x} + Be^{-5x}$, then d^2y/dx^2 = (a)25y (b)5y (c)-25y (d)15y	1
Q3	The lines $\frac{x-2}{1}=\frac{y-3}{1}=\frac{4-z}{k}$ and $\frac{x-1}{k}=\frac{y-4}{2}=\frac{z-5}{-2}$ are mutually perpendicular if the value of k is :	

	(a) $\frac{-2}{3}$ (b)$\frac{2}{3}$ (c) -2(d) 2	1
Q4	If A is a square matrix of order 3 and $\lvert A\rvert$=6, then the value of $\lvert Adj\ A\rvert$is : (a) 6 (b) 36 (c) 27 (d) 216	1
Q5	If A =$\begin{bmatrix}0&a&5\\-2&b&-1\\c&1&0\end{bmatrix}$is a skew symmetric matrix, then a+b+c= (a)3 (b) 0 (c) -3 (d) None of these	1
Q6	The lines $\vec{r} = 0\hat{i} + 0\hat{j} + 0\hat{k} + \lambda(\hat{i} + 2\hat{j} + 3\hat{k})$ and $\vec{r} = \hat{i} + 2\hat{j} + 3\hat{k} + \mu(-2\hat{i} - 4\hat{j} - 6\hat{k})$; (where λ & μ are scalars) are: (a) intersecting (b) parallel (c) skew (d) coincident	1
Q7	The value of ‘n’, such that $x^n\frac{dy}{dx} = y^2(\log y - \log x + 1)$;(where x,y are positive real numbers) is homogeneous: (a)0 (b) 1 (c)2 (d) 3	1
Q8	If A and B are two events such that P(A/B)=2P(B/A) and P(A) + P(B) = 2/3 , then P(B)= (a)2/9 (b)7/9 (c)4/9 (d)5/9	1
Q9	If A=$\begin{bmatrix}x&1\\-1&-x\end{bmatrix}$, such that A^2 =O,then x = (a) 0 (b) +_ 1 (c) 1 (d) -1	1
Q10	If $\vec{a}$is a unit vector and ($\vec{x} - \vec{a}$)·($\vec{x} + \vec{a}$) = 15 then $\lvert\vec{x}\rvert$ is (a) ±4 (b) 4 (c) - 4 (d) ±√7	1
Q11	A linear programming Problem is as follows: Minimise Z=2x+y Subject to constraints x≥3,x≤9,y≥0 x-y≥0,x+y≤14 The feasible region has : (a)5 corner points including (0,0) and (9,5) (b)5 corner points including (7,7) and (3,3) (c)5 corner points including (14,0) and (9,0) (d)5 corner points including (3,6) and (9,5)	1

Q12	The sum of the order and degree of differential equation $\frac{d}{dx}[(\frac{dy}{dx})^3]=0$ is: (a)2 (b)3 (c)5 (d)0	1
Q13	The value of λ for which the vectors $3\hat{\imath} - 6\hat{\jmath} + \hat{k}$ and $2\hat{\imath} - 4\hat{\jmath} + \lambda\hat{k}$ are parallel is: (a)2/3 (b)3/2 (c)5/2 (d)2/5	1
Q14	If A is a given square matrix. Then A+ A’ is a: (a)scalar matrix (b) diagonal matrix (c) symmetric matrix (d) null matrix	1
Q15	If f(x) = $\int_0^x t\sin t\, dt$, then f ′(x) is: (a) cosx + sinx (b) xsinx (c) xcosx (d) sinx + xcosx	1
Q16	If $\lvert\vec{a}\, X\vec{b}\rvert = \sqrt{3}$ and $\vec{a}\,.\,\vec{b}$ = -3 , then angle between $\vec{a}$ and $\vec{b}$ is: (a)$2\pi/3$ (b) $\pi/6$ (c) $\pi/3$ (d) $5\pi/6$	1
Q17	The area of a triangle with vertices (-3,0),(3,0) and (0,k) is9 square units. The value of k is: (a) 9 (b)3 (c) -9 (d) 6	1
Q18	The graph of the inequality 2x+3y>6 is : (a)Half plain that contains the origin. (b)Half plane that neither contains the origin nor the points of the line 2x+3y=6. (c)Whole XOY- plane excluding the points on the line 2x+3y=6. (d)Entire XOY-plane.	1
ASSERTION-REASON BASED QUESTIONS Directions: In the question no. (19) and (20) , a statement of Assertion(A) is followed by a statement of Reason(R). Choose the correct answer out of the following choices: (a) Both (A) and (R) are true and (R) is the correct explanation of (A). (b) Both (A) and (R) are true and (R) is not the correct explanation of (A).		

	(c) (A) is true but (R) is false. (d) (A) is false but (R) is true.	
Q19	Assertion (A): f(x)=tanx-x always increases Reason (R): Any function y=f(x) is increasing if $\frac{dy}{dx}$>0	1
Q20	ASSERTION (A): The function $f: R \to R$ defined by $f(x) = [x]$ $is\ neither\ one - one\ nor\ onto.$ REASON (R): The function $f: R \to R$ defined f(x) =$\lvert x \rvert$ is onto.	1
	SECTION –B [This section comprises of Very Short Answer (VSA)-type questions of 2 marks each]	
Q21	The cost (in rupees) of 'x' items is given by C(x)= 0.000014x^3 + 0.005x^2 + 5x + 1100 Find the marginal cost of 200 items. OR Find the maximum and minimum values if any of the function given by f(x)=sin2x+5 .	2
Q22	The volume of a sphere is increasing at the rate of 8 cm^3/s. Find the rate at which its surface area is increasing when the radius of sphere is 12 cm.	2
Q23	Evaluate :$\int_1^2 \frac{1}{x(logx)^4}$ dx	2
Q24	Show that the function f given by f(x) = $x^3 - 3x^2 + 3x+107$, x ∈ R is increasing on R.	2
Q25	Find the principal value of $\sin^{-1}\left[\cos\left(\sin^{-1}\frac{1}{2}\right)\right]$ OR Find the domain of $cos^{-1}(2x-3)$	2
	SECTION –C [This section comprises of Short Answer (SA)-type questions of 3 marks each]	
Q26	Evaluate :$\int \frac{dx}{5-4x-2x^2} dx$	3

	OR Evaluate: $\int_0^{\frac{\pi}{4}} log\,(1 + tanx)\,dx$	
Q27	If y = $(sin^{-1}x)^2$ show that (1 – x^2)$\frac{d^2y}{dx^2}$ – x $\frac{dy}{dx}$ -2 = 0.	3
Q28	Find :$\int \frac{x^2}{(x^2+4)(x^2+9)}\,dx$	3
Q29	Two numbers are selected at random(without replacemet) from positive integers 2, 3, 4, 5, 6 and 7. Let X denote the larger of the two numbers obtained. Find the probability distribution of X.	3
Q30	Solve the differential equation: $(\text{Cos}^2x)\frac{dy}{dx} + y = \text{tan}x$;$\left(0 \leq x < \frac{\pi}{2}\right)$ OR Solve the differential equation: $(1 + e^{2x})$dy + (1+y^2) e^xdx = 0	3
Q31	Solve the following Linear Programming Problem graphically: Minimize Z = 3x + 9y subject to the constraints: x + 3y ≤ 60, x + y ≥ 10, y - x≥ 0, x ≥ 0, y ≥ 0 OR Solve the following Linear Programming Problem graphically: MaximizeZ = 400x +300y subject to the constraints: x + y ≤ 200 ,x ≤ 40, x ≥ 20, y ≥ 0	3
	SECTION – D [This section comprises of Long Answer (LA)-type questions of 5 marks each]	
Q32	Find the area of the region bounded by the line 3x-y+2=0, the x axis and the ordinates x= -1 and x =1.	5
Q33	If A ={x∈Z : 0≤x≤15}. Show that R= {(a,b): a,b∈A ,$\lvert a - b\rvert$is divisible by 5} is an equivalence relation. Find the set of all elements related to 1. OR	5

Let N denote the set of all natural numbers and R be the relation on N×N defined by (a,b)R(c,d) if ad(b+c)=bc(a+d). Show that R is an equivalence relation.

Q34 Evaluate the product AB, where

$$A = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix} \text{ and } B = \begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix},$$ 5

Hence solve the system of linear equations:

$x - y = 3$, $2x + 3y + 4z = 17$, $y + 2z = 7$

Q35 Show that the lines $\frac{x+1}{3} = \frac{y+3}{5} = \frac{z+5}{7}$ and $\frac{x-2}{1} = \frac{y-4}{3} = \frac{z-6}{5}$ intersect. Also find their point of intersection. 5

OR

Find the shortest distance between the lines.

$\frac{x-3}{1} = \frac{y-5}{-2} = \frac{z-7}{1}$ and $\frac{x+1}{7} = \frac{y+1}{-6} = \frac{z+1}{1}$

SECTION – E

[This section comprises of 3 case-study/passage based questions of 4 marks each with sub parts]

Q36 Read the following passage and answer the questions given below:

Some students of class XIItends to stay up all night and therefore are not able to wake up on time in morning. Not only this but their dependence on tuitions further leads to absenteeism in school.Of the students in class XII,it is known that 30% of the students have 100% attendance.Previous year results report that 80% of all students who have 100% attendance attain A grade and 10% irregular students attain A grade in their annual examination.At the end of the year, one student is chosen at random from the class XII.

Using above information, answer the following.

(i) Find the conditional probability that a student attainsA grade given that he is not 100% regular student. 1

(ii) Find the probability of attaining A grade by the students of class XII. 1

(iii) Find the probability that student is 100% regular given that he attains A grade. 2

OR

	(iii) Find the probability that student is irregular, given that he attains A grade.	
Q37	Read the following passage and answer the questions given below: A mam is watching an airplane which is at the coordinate point A(2,-3,3)assuming that the Man is at O(0,0,0).At the same time he saw a bird at the coordinate point B(1,0,2). Based on the above information answer the following questions (i) Find the position vector of vector $\overrightarrow{AB}$? (ii) Find the distance between airplane and bird? (iii) Find the direction cosines of $\overrightarrow{AB}$? OR (iii) what is the angles $\overrightarrow{AB}$ makes with x,y and z axes?	1 1 2
Q38	Read the following passage and answer the questions given below. In a park, an open tank is to be constructed using metal sheet with a square base and vertical sides so that it contains 500 m³ of water. **Using above information, answer the following:** **(i)**Find the Minimum surface area of the tank. **(ii)**Find the percentage increase in volume of the tank, if size of square base of tank become twice and height remains same.	2 2

'END OF PAPER'

Sample Question Paper

12th - Mathematics

Time : 3 Hours **Max. Marks : 80**

General Instructions

1. *This Question paper contains - five sections A, B, C, D and E. Each section is compulsory. However, there are internal choices in some questions.*
2. *Section A has 18 MCQ's and 02 Assertion-Reason based questions of 1 mark each.*
3. *Section B has 5 Very Short Answer (VSA)-type questions of 2 marks each.*
4. *Section C has 6 Short Answer (SA)-type questions of 3 marks each.*
5. *Section D has 4 Long Answer (LA)-type questions of 5 marks each.*
6. *Section E has 3 source based/case based/passage based/integrated units of assessment (4 marks each) with sub parts.*

SECTION-A (Multiple Choice Questions)

Each question carries 1 mark.

1. The number of equivalence relations in the set {1, 2, 3} containing (1, 2) and (2, 1) is

(a) 2 (b) 3 (c) 1 (d) 4

2. The function f(x) = tan x – 4x is strictly decreasing on

(a) $\left(-\frac{\pi}{3},\frac{\pi}{3}\right)$ (b) $\left(\frac{\pi}{3},\frac{\pi}{2}\right)$ (c) $\left(-\frac{\pi}{3},\frac{\pi}{2}\right)$ (d) $\left(\frac{\pi}{2},\pi\right)$

3. The degree and order of the differential equation $y = px + 3\sqrt{a^2p^2+b^2},\ p=\frac{dy}{dx}$, are respectively

(a) 3, 1 (b) 1, 3 (c) 1, 1 (d) 3, 3

4. The number of surjective functions from A to B where A = {1, 2, 3, 4} and B = {a, b} is

(a) 14 (b) 12 (c) 2 (d) 15

5. The matrix $\begin{bmatrix}\lambda & -1 & 4\\ -3 & 0 & 1\\ -1 & 1 & 2\end{bmatrix}$ is invertible, if

(a) $\lambda \neq -17$ (b) $\lambda \neq -18$ (c) $\lambda \neq -19$ (d) $\lambda \neq -20$

6. On which of the following intervals is the function $x^{100}+\sin x-1$ decreasing?

(a) (0, π/2) (b) (0, 1) (c) (π/2, π) (d) None of these

7. The vector equation of the symmetrical form of equation of straight line $\frac{x-5}{3}=\frac{y+4}{7}=\frac{z-6}{2}$ is

(a) $\vec{r}=\left(3\hat{i}+7\hat{j}+2\hat{k}\right)+\mu\left(5\hat{i}+4j-6\hat{k}\right)$ (b) $\vec{r}=\left(5\hat{i}+4\hat{j}-6\hat{k}\right)+\mu\left(3\hat{i}+7j+2\hat{k}\right)$

(c) $\vec{r}=\left(5\hat{i}-4\hat{j}-6\hat{k}\right)+\mu\left(3\hat{i}-7j-2\hat{k}\right)$ (d) $\vec{r}=\left(5\hat{i}-4\hat{j}+6\hat{k}\right)+\mu\left(3\hat{i}+7j+2\hat{k}\right)$

8. The inequalities $5x + 4y \geq 20,\ x \leq 6,\ y \leq 4$ form

(a) A square (b) A rhombus (c) A triangle (d) A quadrilateral

9. Evaluate: $\int \frac{x}{1-x^4}dx$

(a) $\frac{1}{4}\log\left|\frac{1+x^2}{1-x^2}\right|+c$ (b) $\frac{1}{4}\log\left|\frac{1-x^2}{1+x^2}\right|+c$ (c) $\frac{1}{2}\log\left|\frac{1+x^2}{1-x^2}\right|+c$ (d) $\frac{1}{2}\log\left|\frac{1-x^2}{1+x^2}\right|+c$

10. If P (A) = $\frac{1}{2}$, P (B) = 0, then P (A/B) is

(a) 0 (b) $\frac{1}{2}$ (c) not defined (d) 1

11. What is the principle value of $\text{cosec}^{-1}(-\sqrt{2})$?

(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{2}$ (c) $-\frac{\pi}{4}$ (d) 0

12. A line makes angles of 45° and 60° with the positive axes of *X* and *Y* respectively. The angle made by the same line with the positive axis of *Z*, is.

(a) 30° or 60° (b) 60° or 90° (c) 90° or 120° (d) 60° or 120°

13. The order of the differential equation $\sqrt{\frac{dy}{dx}} = x$

(a) 1 (b) 2 (c) 3 (d) 4

14. If y = sec x°, then $\frac{dy}{dx}$ is equal to :

(a) sec x tan x (b) sec x° tan x° (c) $\frac{\pi}{180}\sec x^\circ \tan x^\circ$ (d) None of these

15. $\int_{\pi/4}^{3\pi/4} \frac{\phi d\phi}{1+\sin\phi}$ is equal to

(a) $\sqrt{2}-1$ (b) $\frac{1}{\sqrt{2}-1}$ (c) $\frac{\pi}{\sqrt{2}+1}$ (d) $\frac{\pi}{\sqrt{2}-1}$

16. An angle between the lines whose direction cosines are given by the equations, $l + 3m + 5n = 0$ and $5lm - 2mn + 6nl = 0$, is

(a) $\cos^{-1}\left(\frac{1}{8}\right)$ (b) $\cos^{-1}\left(\frac{1}{6}\right)$ (c) $\cos^{-1}\left(\frac{1}{3}\right)$ (d) $\cos^{-1}\left(\frac{1}{4}\right)$

17. If a circular plate is heated uniformly, its area expands 3*c* times as fast as its radius, then the value of *c* when the radius is 6 units, is

(a) 4 π (b) 2 π (c) 6 π (d) 3 π

18. If $y = e^{(1+\log_e x)}$, then $\frac{dy}{dx}$ is equal to :

(a) e (b) 1 (c) 0 (d) $\log_e x . x$

(ASSERTION-REASON BASED QUESTIONS)

In the following questions, a statement of Assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices.

(a) ***Both A and R are true and R is the correct explanation of A.***

(b) ***Both A and R are true but R is not the correct explanation of A.***

(c) ***A is true but R is false.***

(d) ***A is false but R is true.***

19. **Assertion:** Order of the differential equation whose solution is $y = c_1e^{x+c_2} + c_3e^{x+c_4}$ is 4.

Reason : Order of the differential equation is equal to the number of independent arbitrary constant mentioned in the solution of differential equation.

20. **Assertion :** The projection of the vector $a = 2\hat{i}+3\hat{j}+2\hat{k}$ on the vector $\vec{b} = \hat{i}+2\hat{j}+\hat{k}$ is $\frac{5}{3}\sqrt{6}$.

Reason The projection of vector a on vector b is $\frac{1}{|a|}(a.b)$.

SECTION-B

This section comprises of very short answer type-questions (VSA) of 2 marks each.

21. Find the principal value of $\cos^{-1}\left(\cos\frac{5\pi}{3}\right)+\sin^{-1}\left(\sin\frac{5\pi}{3}\right)$.

22. If $y = x - x^2$, then find the derivative of y^2 with respect to x^2.

OR

If $\sin y = x \sin(a+y)$, prove that $\frac{dy}{dx} = \frac{\sin^2(a+y)}{\sin a}$

23. Find the maximum area of rectangle inscribed in a circle of diameter R.

24. Find the condition when the lines x = ay + b, z = cy +d and x = a′y + b′, z = c′y +d′ are perpendicular.

25. If a line makes an angle of $\pi/4$ with the positive directions of each of *x*- axis and *y*- axis, then find the angle that the line makes with the positive direction of the *z*-axis.

OR

If either $\vec{a} = \vec{0}$ or $\vec{b} = \vec{0}$, then $\hat{a} \times \hat{b} = 0$. Is the converse true?

SECTION-C

This section comprises of short answer type questions (SA) of 3 marks each.

26. Show that the Modulus Function $f : R \to R$ given by f (x) = |x|, is neither one-one nor onto, where |x| is x, if x is positive or 0 and |x| is – x, if x is negative.

OR

Show that the relation S defined on the set $N \times N$ by (a, b) S (c, d) $\Rightarrow$ a + d = b + c is an equivalence relation.

27. Evaluate : $\int \frac{\sqrt{x}}{x+1} dx$

OR

Evaluate : $\int_{-\pi/2}^{\pi/2} \sin|x|\, dx$

28. Solve: $(x + 3y^2)\frac{dy}{dx} = y\ (y > 0)$

29. If $\vec{A}$, $\vec{B}$ and $\vec{C}$ are vectors such that $|\vec{B}| = |\vec{C}|$. Prove that $\left[\left(\vec{A}+\vec{B}\right)\times\left(\vec{A}+\vec{C}\right)\right]\times\left(\vec{B}\times\vec{C}\right)\cdot\left(\vec{B}+\vec{C}\right)=0.$

30. In a test, an examinee either guesses or copies or knows the answer to a multiple choice question with four choices. The probability that he makes a guess is $\frac{1}{3}$. The probability that he copies is $\frac{1}{6}$ and the probability that his answer is correct given that he copied it is $\frac{1}{8}$. Find the probability that he knew the answer to the question given that he correctly answered it.

OR

Let X denote the number of colleges where you will apply after your results and P(X = x) denotes your probability of getting admission in x number of colleges. It is given that

$$P(X = x) = \begin{cases} Kx, & \text{if } x = 0 \text{ or } 1 \\ 2Kx, & \text{if } x = 2 \\ K(5-x) & \text{if } x = 3 \text{ or } 4 \end{cases}, \quad \text{where K is +ve constant}$$

(a) Find the value of K.
(b) What is the probability that you will get admission in exactly two colleges ?
(c) Find the probability of getting at least 2 admission.

31. Dot product of a vector with vectors $\hat{i}-\hat{j}+\hat{k}$, $2\hat{i}+\hat{j}-3\hat{k}$ and $\hat{i}+\hat{j}+\hat{k}$ are respectively 4, 0 and 2. Find the vector.

SECTION-D

This section comprises of long answer-type questions (LA) of 5 marks each.

32. Evaluate: $\int_{\pi/2}^{\pi} e^{x}\left(\frac{1-\sin x}{1-\cos x}\right)dx.$

OR

Find the value of $\int \frac{1}{(\sin x+4)(\sin x-1)}dx$

33. If $A=\begin{bmatrix}6 & 5\\ 7 & 6\end{bmatrix}$, show that $A^2-12A+I=0$. Hence find A^{-1}.

OR

If $A=\begin{bmatrix}1 & -1\\ 2 & -1\end{bmatrix}$, $B=\begin{bmatrix}a & 1\\ b & -1\end{bmatrix}$ and $(A+B)^2=A^2+B^2$ then find a and b.

34. Show that height of the cylinder of greatest volume which can be inscribed in a right circular cone of height h and semi vertical angle α is one-third that of the cone and the greatest volume of cylinder is $\frac{4}{27}\pi h^3 \tan^2\alpha$.

35. Find the area of the smaller region bounded by the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ and the straight line $\frac{x}{a}+\frac{y}{b}=1$.

SECTION-E

This section comprises of 3 case study/passage - based questions of 4 marks each with two sub-parts. First two case study questions have three sub-parts (i), (ii), (iii) of marks 1, 1, 2 respectively. The third case study question has two sub-parts of 2 marks each.

36. **Case - Study 1:** Read the following passage and answer the questions given below:
Two groups of class XII students discussed the diet problem to get the proper vitamins in minimum cost.
Diet problem: Students were to mix two types of foods in such a way that vitamin contents of the mixture contain atleast 8 units of vitamin A and 10 units of vitamin C. Food 'I' contains 2 units/kg of vitamin A and 1 unit/kg of vitamin C. Food 'II' contains 1 unit/kg of vitamin A and 2 units/kg of vitamin C. It costs ₹ 50 per kg to purchase Food 'I' and ₹ 70 per kg to purchase Food 'II'.

Let Z be objective function x kg be the quantity of Food-I and y be the quantity of Food-II.
(i) What is the objective function of the diet problem?
(ii) What is the constraint for vitamin A and C?
(iii) Find the quantity of foods in the mixture for the minimum cost.

OR

Find the minimum of cost of the mixture.

37. **Case - Study 2:** Read the following passage and answer the questions given below:

A school wants to award its students for the values of cleanliness, regularity and good health with a total cash to be awarded is ₹4500. Five times the award money in total for good health and cleanliness is equal to 6 times the award money for regularity. Five times the award money for good health added to two times of regularity is equal to ₹11000. If ₹x be the cash price for cleanliness, ₹y be the cash price for regularity and ₹z be the cash price for good health.

(i) Find the determinant of coefficient matrix of the system.

(ii) Find cofactors of each entry of coefficient matrix.

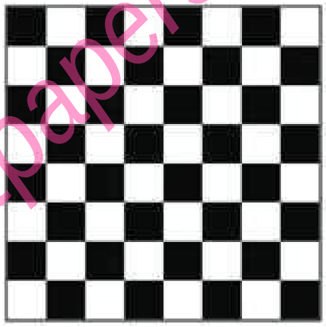
(iii) Find the inverse of coefficient matrix.

OR

Find the award money for each value using matrix method.

38. **Case - Study 3:** Read the following passage and answer the questions given below:

If the squares of a 8×8 chess board are painted either red or black at random.



(i) Find the probability that not all squares is any column are alternating in colour.

(ii) Find the probability that the chess board contains equal number of red and black squares.

SECTION-A **[1x20=20]**

(This section comprises of multiple-choice questions (MCQs) of 1 mark each)

Select the correct option (Question 1 - Question 18):

1. $\tan^{-1}\left\{sin\left(-{}^{\pi}/_{2}\right)\right\}$ is equal to

(a) -1 (b) 1 (c) ${}^{\pi}/_{2}$ (d) $-{}^{\pi}/_{4}$

2. If A is a square matrix such that A^2=A then

$(I + A)^2 - 3A$ is

(a) I (b) 2A (c) 3I (d) A

3. If A and B are matrices of order $3 \times m$ and $3 \times n$ respectively such that $m = n$, then order of 2A+7B is

(a) 3×3 (b) $m \times 3$ (c) $n \times 3$ (d) $3 \times m$

4. If $A = \begin{bmatrix} 0 & a \\ 0 & 0 \end{bmatrix}$ $then\ A^{16}\ is\ equal\ to$

(a) $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ (b) $\begin{bmatrix} 0 & a^{16} \\ 0 & 0 \end{bmatrix}$ (c) A (d) $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

5. If A is a square matrix of order 3x3 such that A. (AdjA) = 10 I then $|(Adj.A)|$ is

(a) 1 (b) 10 (c) 100 (d) 1000

6. If A and B are invertible matrices of the same order. Given $|(AB)^{-1}| = 8 \; and \; |A| = {}^{3}/_{4} \; then \; |B| \; is$

(a) 6 (b) ${}^{1}/_{6}$ (c) ${}^{4}/_{3}$ (d) $-{}^{1}/_{6}$

7. Derivative of x^x with respect to x is

$(a)\ log(1+x)$ $(b)\ x.x^{x-1}$ $(c)\ x^x(1+logx)$ $(d)\ x^x log(1+x)$

8. The total revenue in the ₹ received from the sale of x units of an article is given by $R(x)=3x^2+36x+5$. The marginal revenue when x = 15 is (in ₹)

(a) 126 (b) 116 (c) 96 (d) 90

9. The function f(x)= $cos\left(2x+{}^{\pi}/_{4}\right), x \; \epsilon \left[\frac{3\pi}{8}, \frac{5\pi}{8}\right]$

(a) Increasing (b) decreasing (c) neither increasing nor decreasing (d) none of these.

10. $\int_{-1}^{1} |1-x| \, dx$ is equal to

(a) 1 (b) 2 (c) 3 (d) -3

11. If p and q are the degree and order of the differential equation

$\left(\frac{d^2y}{dx^2}\right)^2 + 3\frac{dy}{dx} + \frac{d^3y}{dx^3} = 4$ then the value of 2p−3q is

(a) 7 (b) −7 (c) 3 (d) −3

12. The value of p for which $\vec{a} = 3\hat{\imath} + 2\hat{\jmath} + 9\hat{k} \; and \; \vec{b} = \hat{\imath} + p\hat{\jmath} + 3\hat{k}$ are parallel vectors is

(a) 3 (b) 3/2 (c) 2/3 (d) 1/3

13. A line makes angle α, β, γ, with x-axis, y-axis and z-axis respectively then

$cos2\alpha + cos2\beta + cos2\gamma$ is equal to

(a) 2 (b) 1 (c) – 2 (d) −1

14. Direction ratio of the line $\frac{4-x}{2} = \frac{y}{6} = \frac{1-z}{3}$ are

(a) 2, 6, 3 (b) − 2, 6, 3 (c) 2, − 6, 3 (d) none of these.

15. Vector equation of the line $\frac{x-5}{3} = \frac{y+4}{7} = \frac{6-z}{2}$ is

$(a)\ \vec{r} = -3\hat{\imath} + 7\hat{\jmath} - 2\hat{k} + \mu\left(5\hat{\imath} - 4\hat{\jmath} + 6\hat{k}\right)$

$(b)\ \vec{r} = 5\hat{\imath} - 4\hat{\jmath} + 6\hat{k} + \mu\left(3\hat{\imath} + 7\hat{\jmath} - 2\hat{k}\right)$

$(c)\ \vec{r} = -5\hat{\imath} + 4\hat{\jmath} - 6\hat{k} + \mu\left(-3\hat{\imath} - 7\hat{\jmath} - 2\hat{k}\right)$

$(d)\ \vec{r} = 3\hat{\imath} + 7\hat{\jmath} + 2\hat{k} + \mu\left(-5\hat{\imath} + 4\hat{\jmath} - 6\hat{k}\right)$

16. The objective function for a given linear programming problem is Z=ax +by−5. If Z attains same value at (1,2) and (3,1) then.

(a) 2a−b=0 (b) a + 2b = 0 (c) a + b = 0 (d) a = b

17. For a given LPP, corner points of a closed feasible region are A (3,5), B (4,2), C (3,0) and O (0,0), then objective function Z= px + qy attains maximum at

(a) A (b) B (c) C or O (d) It depends upon values of p and q and points A, B, C.

18. If A and B are independent events such that P(B/A) = 2/5 then P(B') is

(a) 1/5 (b) 2/5 (c) 3/5 (d) 4/5

ASSERTION-REASON BASED QUESTIONS

(Question numbers 19 and 20 are Assertion-Reason based questions carrying 1 mark each. Two statements are given, one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer from the options (A), (B), (C) and (D) as given below.)

(a) Both A and R are true and R is the correct explanation of A.

(b) Both A and R are true but R is not the correct explanation of A.

(c) A is true but R is false.

(d) A is false but R is true.

19. Assertion (A): In set A = {1, 2, 3} a relation R defined as R = {(1, 1), (2, 2)}is reflexive.

Reason (R): A relation R is reflexive in set A if (a, a)∈ R for all a∈A.

20. Assertion (A): f(x)=[x]is not differentiable at x = 2.

Reason (R): f(x) = [x] is not Continuous at x=2.

SECTION B [2x5=10]

(This section comprises of 5 very short answer (VSA) type questions of 2 marks each.)

21. Find the value of $\tan^{-1}\left[2sin\left(2cos^{-1}\frac{\sqrt{3}}{2}\right)\right]$

22. Find $\frac{dy}{dx}$ $at\ t = \frac{2\pi}{3}$ when

$x = 10\,(t - sin\,t)$ and $y = 12(1 - cos\,t)$

OR

Differentiate $\sin^{-1}x^2$, with respect to x.

23. Find $\int \frac{1+tanx}{1-tanx}$ dx

24. Find Integrating factor for the differential equation.

$x\frac{dy}{dx} + 2y = x^2$

25. Find the direction cosine of the line passing through the following points. (-2,4,5), (1,2,3)

OR

Find the cartesian equation of the line which passes through the point (-2, 4, -5) and is parallel to the line $\frac{x+3}{3} = \frac{4-y}{5} = \frac{z+8}{6}$.

SECTION C [3x6=18]

(This section comprises of 6 short answer (SA) type questions of 3 marks each.)

26. Find the intervals in which the function f given by $f(x) = x^3 - 12x^2 + 36x + 17$ is increasing or decreasing.

27. Find $\int \frac{Sinx}{(1-Cosx)(2-Cosx)}$ dx

OR

$$\int_0^{\pi} \frac{x}{a^2Cos^2x+b^2Sin^2x}\, dx$$

28. Solve the differential equation xdy$-$ydx $=\sqrt{x^2+y^2}$ dx

29. Find a vector of magnitude 6, perpendicular to each of the vector $(\vec{a}+\vec{b})$ and $(\vec{a}-\vec{b})$, where

$\vec{a}=\hat{\imath}+\hat{\jmath}+\hat{k}$ and $\vec{b}=\hat{\imath}+2\hat{\jmath}+3\hat{k}$

OR

Show that the points A, B, C with position vectors $2\hat{\imath}-\hat{\jmath}+\hat{k}, \hat{\imath}-3\hat{\jmath}-5\hat{k}$ and $3\hat{\imath}-4\hat{\jmath}-4\hat{k}$ respectively, are vertices of a right-angled triangle. Also find the area of the triangle.

30. Solve the following problem graphically: Minimise and Maximise Z = 3x+9y subject to the constraints.

$x+3y \leq 60$

$x+y \geq 10$

$x \leq y$

$x, y \geq 0$

31. Two cards are drawn simultaneously from a well shuffled pack of 52 cards. Find the mean of the number of kings.

OR

A Random variable X has the following probability distribution.

X	0	1	2	3	4	5	6	7
P(X)	0	k	2k	2k	3k	k^2	$2k^2$	$7k^2+k$

Find (i) k (ii) P(X < 3) (iii) P (0 < X < 3)

SECTION-D [5x4=20]

(This section comprises of 4 long answer (LA) type questions of 5 marks each)

32. Differentiate $(sinx)^x + sin^{-1}\sqrt{x}$ $w.r.t.x$

OR

If y= $sin(logx)$. Prove that $x^2y_2 + xy_1 + y = 0$

33. Show that lines $\vec{r} = (\hat{\imath}+\hat{\jmath}-\hat{k}) + \lambda(3\hat{\imath}-\hat{\jmath})$ and $\vec{r} = (4\hat{\imath}-\hat{k}) + \mu(2\hat{\imath}+3\hat{k})$ intersect. Also find their point of intersection.

OR

Find the length and the foot of the perpendicular drawn from the point (2, -1, 5) on the line

$$\frac{x-11}{10} = \frac{y+2}{-4} = \frac{z+8}{-11}$$

34. Sketch and find the area of the region

{(x, y): $x^2+y^2 \leq 16$, $y^2=6x$} Using integration.

35. Two schools A and B want to award their selected students on the values of Sincerity, Truthfulness and Helpfulness. The school A wants to award ₹x each, ₹y each and ₹z each for the three respective values to 3, 2 and 1 students respectively with a total award money of ₹1,600. School B wants to spend ₹2,300 to award its 4, 1 and 3 students on the respective values (by giving the same award money to the three values as before). If the total amount of award for one prize on each value is ₹900, using matrices find the award money for each value.

SECTION- E [4x3=12]

(This section comprises of 3 case-study/passage-based questions of 4 marks each with subparts. The first two case study questions have three subparts (i), (ii), (iii) of marks 1, 1, 2 respectively. The third case study question has two subparts of 2 marks each)

Case Study-1

36. Students of Grade 9, planned to plant saplings along straight lines, parallel to each other to one side of the playground ensuring that they had enough play area Let us assume that they planted one of the rows of the saplings along the line y = x−4. Let L be the set of all lines which are parallel on the ground and R be a relation on L.

Answer the following using the above information.

(i) Let relation R be defined by R= {(L_1, L_2): $L_1 || L_2$ where L_1, $L_2 \in L$} then find the set of lines related to

The liner $y = x - 4$. **[1Mark]**

(ii) Let R= {(L_1, L_2): $L_1 \perp L_2$ where L_1, $L_2 \in L$} then, show that R is symmetric but neither reflexive nor transitive. **[1Mark]**

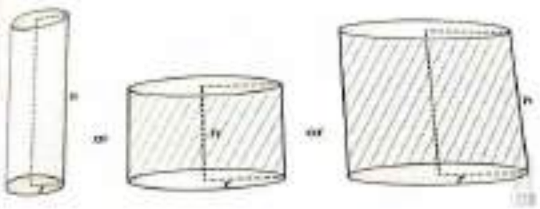
(iii) Prove that the function $f: R \rightarrow R \; defined \; by \; f(x) = x - 4$ is bijective. **[2Mark]**

OR

Let $f: R \rightarrow R$ be defined by f(x) = x - 4 Then find the range of f(x). **[2Mark]**

Case Study-2

37. Read the following passage and answer the questions given below



A company is launching a new product and decided to pack the product in the form of a closed right circular cylinder of volume 432π ml and having minimum surface area as shown. They tried different options and tried to get the solution by answering the questions given below:

(i) If r is radius of base of cylinder and h is the height of cylinder then establish the relation between r and h. **[1Mark]**

(ii) Find total surface area in terms of r only. **[1Mark]**

(iii) Find the radius r for minimum surface area. **[2Mark]**

OR

(iii) Find the minimum surface area. **[2Mark]**

Case Study-3

38. Read the following passage and answer the questions given below

In the office three employees Mehul, Janya and Charvi process incoming matter related to a particular project. Mehul processes 40% of the matter and Janya and Charvi process rest of the matter equally. It is found that 6% of matter processed by Mehul has an error whereas for Janya and Charvi error rate is 4% and 3% respectively.

(i) What is the probability of an error in processing the matter?

(ii) The processed matter is checked and the selected matter has an error, what is the probability that it was processed by Mehul?

SECTION A

20 × 1 = 20 *This section comprises of 20 Multiple Choice Questions (MCQs) of 1 mark each.*

1. The I.F. of $\frac{dy}{dx} + y \tan x = \sec x$ is:

   (a) $\tan x$ (b) $\sec x$

   (c) $\tan^2 x$ (d) $\sec x . \tan x$ **1**

2. If $|\vec{a} + \vec{b}| = |\vec{a} - \vec{b}|$, then vectors $\vec{a}$ and $\vec{b}$ are:

   (a) parallel (b) perpendicular

   (c) coplanar (d) collinear **1**

3. If $A = \begin{bmatrix} 2 & \lambda & -3 \\ 0 & 2 & 4 \\ 1 & 1 & 3 \end{bmatrix}$, then A^{-1} exists if:

   (a) $\lambda = 2$ (b) $\lambda \neq 2.5$

   (c) $\lambda \neq -2.5$ (d) $\lambda \neq 2$ **1**

4. The function $f(x) = 2x^3 - x^2 + 5$ is a polynomial of third degree then which of the following statement is not true about $f(x)$?

(a) It is continuous and decreasing in $\left(0, \frac{1}{3}\right)$.

(b) It is differentiable and increasing in $\left(\frac{1}{3}, \infty\right)$.

(c) It is continuous and increasing in $(-\infty, 0)$.

(d) It is differentiable and increasing in $(0, \infty)$. **1**

5. The direction ratios of the line $6x - 2 = 3y + 1 = 2z - 2$ are:

(a) 6, 3, 2 (b) 1, 1, 2

(c) 1, 2, 3 (d) 1, 3, 2 **1**

6. If for a square matrix B, $B = \begin{bmatrix} 432\cos\alpha & -5\sin\alpha \\ 432\sin\alpha & 5\cos\alpha \end{bmatrix}$ then $|\text{Adj } B|$ is equal to:

(a) 2160 (b) $(2160)^2$

(c) $(2160)^3$ (d) $(2160)^4$ **1**

7. The interval in which $y = x^2 (x - 3)^2$ decreases is:

(a) $(-\infty, 0) \cup [3, \infty)$ (b) $(-\infty, 0)$

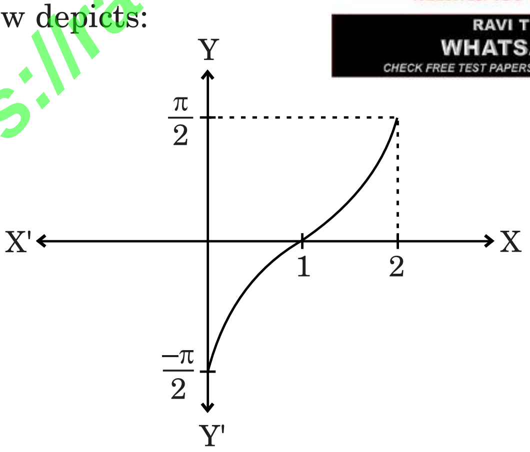
(c) $(-\infty, 0) \cup \left(\frac{3}{2}, 3\right)$ (d) $\left(0, \frac{3}{2}\right) \cup (3, \infty)$ **1**

8. The value of λ such that the vectors $\vec{a} = 2\hat{i} + \lambda\hat{j} + \hat{k}$ and $\vec{b} = \hat{i} + 2\hat{j} + 3\hat{k}$ are orthogonal is:

(a) $\frac{3}{2}$ (b) $\frac{-5}{2}$

(c) $\frac{-1}{2}$ (d) $\frac{1}{2}$

9. The graph shown below depicts:



(a) $y = \sin^{-1} x$ (b) $y = \tan^{-1} x$

(c) $y = \sin^{-1} (x - 1)$ (d) $y = \tan^{-1} (x - 1)$ **1**

10. If $x = \sin t$ and $y = 2t$, then $\dfrac{d^2y}{dx^2}$ is:

(a) $2\sin t \cos^2 t$ (b) $2\cos 2t$

(c) $2\sec^2 t \tan t$ (d) $2\sin 2t$ **1**

11. For $A = \begin{bmatrix} 1 & 4 \\ -1 & 3 \\ 0 & 5 \end{bmatrix}$, $(A^T)^T$ is equal to:

(a) A^T (b) $(A^{-1})^T$

(c) A (d) $\dfrac{1}{A}$ **1**

12. $\int \frac{1}{x} \log(\log x) dx$ is:

(a) $\log(\log x) + C$ (b) $\log x(\log (\log x)) - \log x + C$

(c) $(\log x - 1) + C$ (d) $\log(\log (\log x)) + C$ **1**

13. A florist sells a mixture of three types of flower seeds A, B and C in proportions 4 : 5 : 2. A customer randomly picks two seeds with replacement. The probability that the first seed picked is of type A and the second of type C is:

(a) $\dfrac{16}{121}$ (b) $\dfrac{8}{121}$

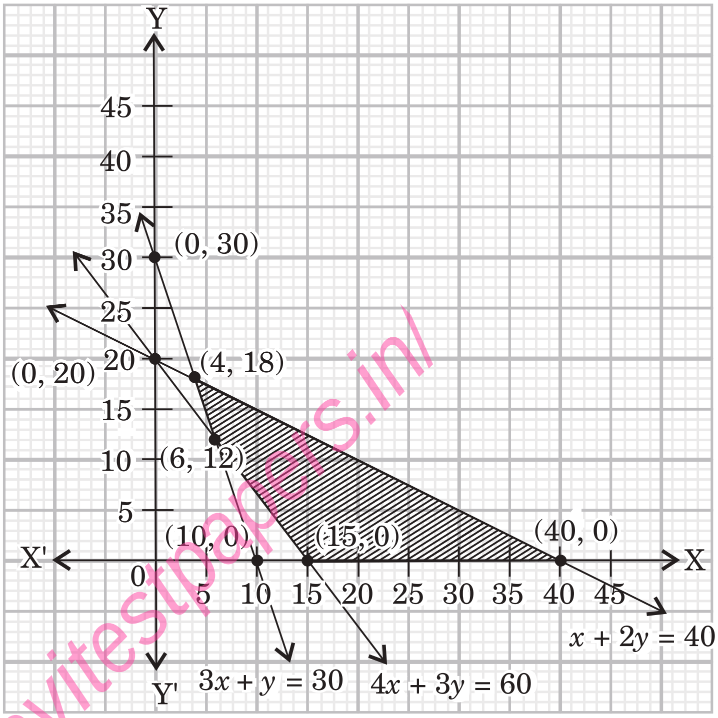
(c) $\dfrac{16}{55}$ (d) $\dfrac{8}{55}$ **1**

14. If $A = \begin{bmatrix} 1 & -2 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{bmatrix}$, then A (adj A) is:

(a) $6I_3$ (b) $8I_3$

(c) I_3 (d) $2I_3$ **1**

15. The feasible region corresponding to the linear constraints of a linear programming problem is shown:



Which of the following is not a constraint to the given Linear Programming Problem?

(a) $x + 2y < 40$

(b) $3x + y < 30$

(c) $4x + 3y > 60$

(d) $3x + y > 30$ **1**

16. The difference of the order and the degree of the differential equation $y = 2px + \sqrt{b^2p^2 + a^2}$, where, $p = \frac{dy}{dx}$ is:

(a) 0

(b) 2

(c) 1

(d) 3

17. If A = $\begin{bmatrix} \sin\theta & \cos\theta \\ \cos\theta & \sin\theta \end{bmatrix}$, then det (A. adj (A)) is:

(a) $\frac{1 + \cos 4\theta}{2}$

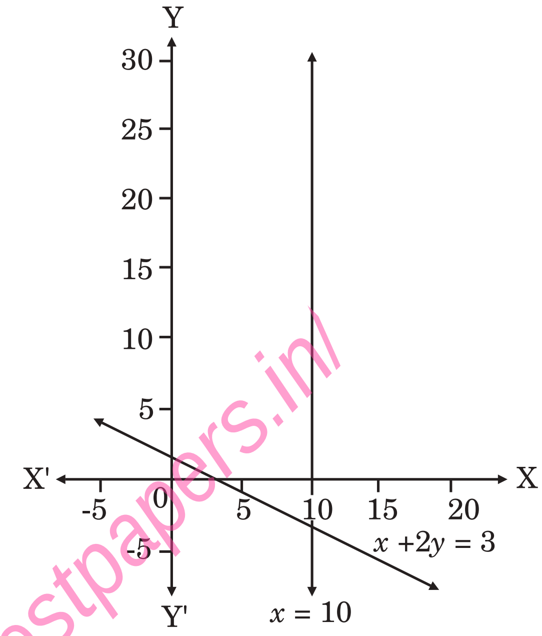
(b) $\frac{1 + \cos 2\theta}{2}$

(c) $1 + \cos 4\theta$

(d) $1 + \cos 2\theta$ **1**

18. The constraints of a linear programming problem along with their graphs is shown below:

$$x + 2y \geq 3$$
$$x \geq 10$$
$$y \geq 0$$



Which of the following objective functions has an optimal solution with respect to the above set of constraints?

(a) Minimise Z = $x + y$ (b) Minimise Z = $0.5x + y$

(c) Maximise Z = $x + y$ (d) Maximise Z = $2x + y$ **1**

Assertion-Reason Based Questions

Question numbers 19 and 20 are Assertion-Reason based questions carrying 1 mark each. Two statements are given, one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer from the options (a), (b), (c) and (d) as given below.

(a) Both (A) and (R) are true and (R) is the correct explanation of (A).
(b) Both (A) and (R) are true but (R) is not the correct explanation of (A).
(c) (A) is true but (R) is false.
(d) (A) is false but (R) is true.

19. Assertion (A): The function $f : R^* \rightarrow R^*$ defined by $f(x) = \frac{1}{x}$ is one-one and onto, where R^* is the set of all non-zero real numbers.

Reason (R): The function $g : N \rightarrow R^*$ defined by $f(x) = \frac{1}{x}$ is one-one and onto. **1**

20. Assertion (A): If $|\vec{a}+\vec{b}| = |\vec{a}-\vec{b}|$, then angle between $|\vec{a}|$ and $|\vec{b}|$ is 90°.

Reason (R): $\vec{a}+\vec{b} = \vec{b}+\vec{a}$ **1**

SECTION B

5 × 2 = 10

This section comprises of 5 Very Short Answer (VSA) type questions of 2 marks each.

21. If $\cot^{-1}\left(-\frac{1}{5}\right) = x$, then find the value of $\sin x$. **2**

OR

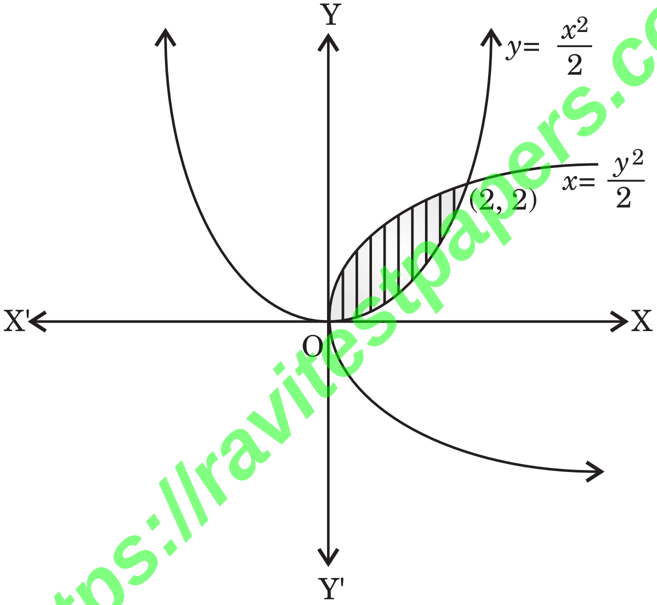
Find the value of $\{\tan^{-1}(1) - \cot^{-1}(-1)\}$. **2**

22. Prove that $\int_{0}^{\pi/4} 2\tan^3 x \, dx = 1 - \log 2$.

23. Find: $\int \frac{x^4+1}{x^2+1} dx$

OR

Find the area of shaded region as shown in figure:



2

24. Solve: $x\frac{dy}{dx} = y\left(\log\frac{y}{x} + 1\right)$. **2**

25. A drone is flying along the path represented by the vector $\vec{a} = \hat{i} + \hat{j} + \hat{k}$. Find the value of cosine of the angle that the drone's path makes with y-axis. **2**

SECTION C

6 × 3 = 18

This section comprises of 6 Short Answer (SA) type questions of 3 marks each.

26. Find the general solution of the following differential equation:

$$x\frac{dy}{dx} = y - x\sin\left(\frac{y}{x}\right)$$ **3**

OR

Find the particular solution of the differential equation $e^{x}\sqrt{1-y^{2}}\,dx + \frac{y}{x}\,dy = 0$, given that $y = 1$, when $x = 0$. **3**

27. Consider the following Linear Programming Problem:

Maximise: $Z = 3x + 2y$

Subject to: $x - 2y \leq 2$,

$3x + 2y \leq 12$,

$x, y \geq 0$

Show graphically that the maximum value of Z occurs at more than two points. **3**

28. Three satellites are floating in space, forming a triangular communication network and positions in space are given by the coordinates (3, 5, –4), (–1, 1, 2) and (–5, –5, –2). The control centre wants to calculate the direction cosines of each side of the triangle formed by the satellites in order to ensure accurate alignment of communication signals. Find the direction cosines of the sides. **3**

29. Sketch the region bounded by the lines $y = 3x + 1$, $y = x + 1$ and $x = 5$. Using integration, find the area. **3**

30. Sand is pouring from the pipe at the rate of 12 cm^3/s. The falling sand forms a cone on a ground in such a way that the height of cone is always one-sixth of the radius of the base. How fast is the height of sand cone increasing when the height is 4 cm. **3**

OR

Prove that $y = a\cos(\log x) + b\sin(\log x)$ is the solution of

$$x^{2}\left(\frac{d^{2}y}{dx^{2}}\right) + x\left(\frac{dy}{dx}\right) + y = 0.$$ **3**

31. There are two bags, one of which contains 3 black and 4 white balls, while the other contains 4 black and 3 white balls. A fair die is tossed, if the face 1 or 3 turns up, a ball is taken from the first bag, and if any other face turns up, a ball is chosen from the second bag. Find the probability of choosing a black ball. **5**

OR

In a school, 200 students study mathematics. Out of them 120 are from Arjuna classes and rest are from Lakshya classes. 108 of Arjuna classes students passed and 56 of Lakshya classes students passed in final exam.

A student is selected at random. What is the probability that the student was from Lakshya class given that he passed? **5**

SECTION D

4 × 5 = 20

This section comprises of 4 Long Answer (LA) type questions of 5 marks each.

32. If A = $\begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}$, then prove that $A^3 - 6A^2 + 7A + 2I = 0$. **5**

OR

Using matrix method, show that the following system of equations is inconsistent:

$$3x - y - 2z = 2;$$
$$2y - z = -1;$$
$$3x - 5y = 3$$

5

33. Evaluate : $\displaystyle\int_0^{\frac{\pi}{4}} e^{2x}\sin\left(\frac{\pi}{4}+x\right)dx$. **5**

34. A rectangular field is to be enclosed using 900 metres of fencing. Find the dimensions of the rectangle that will maximise the area of the field. What is the maximum area? **5**

OR

Find the maximum and minimum values, if any, of the function given by:

$$f(x) = x^3 - 6x^2 + 9x + 15$$

5

35. In an oil refinery, two pipelines lie along the lines

$$\vec{r} = (8\hat{i} - 19\hat{j} + 10\hat{k}) + \lambda(3\hat{i} - 16\hat{j} + 7\hat{k})$$

and $\vec{r} = (15\hat{i} + 29\hat{j} + 5\hat{k}) + \mu(3\hat{i} + 8\hat{j} - 5\hat{k})$

These lines are supported by a rod that is fixed perpendicular to both pipelines and passes through the point (1, 2, –4). Find the equation of the line along the support rod. **5**

SECTION E

4 × 3 = 12

This section comprises 3 case study based questions of 4 marks each.

36. Case Study – 1

A confectionery shop is a place where sweets and chocolates are sold. The table below gives information on four varieties of chocolates sold there.

Chocolate	**Cost per piece (in ₹)**
Dairy Milk (D)	5
5-Star (S)	10
Crunch (C)	20
Kit-Kat (K)	50

Let A = {D, S, C, K} be the set containing the chocolates and B = {5, 10, 20, 50} be the set containing their costs.

A relation R is defined on set A as R = {(x, y): cost of $x \leq$ cost of y}.

Based on the given information, answer the following questions:

(A) Express the relation R in roster form. **1**

(B) Is R a reflexive relation? Justify your answer. **1**

(C) Is R a symmetric relation? Justify your answer. **2**

OR

Is R a transitive relation? Justify your answer. **2**

37. *Let d_1, d_2, and d_3 be three mutually exclusive diseases.*

Let $S = \{S_1, S_2, S_3, S_4, S_5, S_6\}$ be the set of observable symptoms of these diseases. For example, S_1 is the shortness of breath, S_2 is the loss of weight, S_3 is the fatigue, etc. Suppose a random sample of 10,000 patients contains 3200 patients with disease d_1, 3500 patients with disease d_2 and 3300 patients with disease d_3. Also, 3100 patients with disease d_1, 3300 patients with disease d_2 and 3000 patients with disease d_3 show the symptom S.

Based on the given information, answer the following questions:

(A) A person is chosen at random from the sample of 10,000. What is the probability that the person chosen does not suffer with disease d_3? **1**

(B) Find the conditional probability that the patient shows the symptom S given that he suffers with disease d_1 and also calculate the conditional probability that the patient shows the symptom S given that he suffers with disease d_2. **2**

OR

If a person chosen at random shows the symptom S, then what is the probability that he does suffer with disease d_1? **2**

(C) Let D_i denote the event that the patient has disease $d_i (i = 1, 2, 3)$ and S be the event that the patient shows the symptom S. Then, find the value of $\sum_1^3 P\left(\frac{D_i}{S}\right)$. **1**

38. Case Study – 3

Three students Piyush, Prateek and Pankaj are given a rectangular sheet of sides 45 cm and 24 cm. They are asked to work independently and form an open box by cutting the squares of equal length from all four corners as shown and folding up the flaps, they want to check the volume of the box formed.

Based on the given information, answer the following questions:

(A) What should be the value of the x, so that the volume of the box is optimum? **2**

(B) Which of the points obtained gives maximum value? Also, calculate maximum volume. **2**

Series : WXYZ/S **SET~1**

प्रश्न-पत्र कोड
Q.P. Code **65/S/1**

रोल नं.
Roll No.

परीक्षार्थी प्रश्न-पत्र कोड को उत्तर-पुस्तिका के मुख-पृष्ठ पर अवश्य लिखें ।
Candidates must write the Q.P. Code on the title page of the answer-book.

गणित
MATHEMATICS

*निर्धारित समय : **3** घण्टे* *अधिकतम अंक : **80***
*Time allowed : **3** hours* *Maximum Marks : **80***

नोट

) कृपया जाँच कर लें कि इस प्रश्न-पत्र में मुद्रित पृष्ठ **23** हैं ।

I) प्रश्न-पत्र में दाहिने हाथ की ओर दिए गए प्रश्न-पत्र कोड को परीक्षार्थी उत्तर-पुस्तिका के मुख-पृष्ठ पर लिखें ।

I) कृपया जाँच कर लें कि इस प्रश्न-पत्र में **38** प्रश्न हैं ।

V) **कृपया प्रश्न का उत्तर लिखना शुरू करने से पहले, उत्तर-पुस्तिका में यथा स्थान पर प्रश्न का क्रमांक अवश्य लिखें ।**

) इस प्रश्न-पत्र को पढ़ने के लिए 15 मिनट का समय दिया गया है । प्रश्न-पत्र का वितरण पूर्वाह्न में 10.15 बजे किया जाएगा । 10.15 बजे से 10.30 बजे तक परीक्षार्थी केवल प्रश्न-पत्र को पढ़ेंगे और इस अवधि के दौरान वे उत्तर-पुस्तिका पर कोई उत्तर नहीं लिखेंगे ।

NOTE

(I) Please check that this question paper contains **23** printed pages.

(II) Q.P. Code given on the right hand side of the question paper should be written on the title page of the answer-book by the candidate.

(III) Please check that this question paper contains **38** questions.

(IV) **Please write down the Serial Number of the question in the answer-book at the given place before attempting it.**

(V) 15 minute time has been allotted to read this question paper. The question paper will be distributed at 10.15 a.m. From 10.15 a.m. to 10.30 a.m., the candidates will read the question paper only and will not write any answer on the answer-book during this period. #

सामान्य निर्देश :

निम्नलिखित निर्देशों को बहुत सावधानी से पढ़िए और उनका सख़्ती से पालन कीजिए :

(i) इस प्रश्न-पत्र में ***38*** प्रश्न हैं। ***सभी*** प्रश्न ***अनिवार्य*** हैं।

(ii) यह प्रश्न-पत्र ***पाँच*** खण्डों में विभाजित है – ***क, ख, ग, घ*** एवं ***ङ***।

(iii) ***खण्ड क*** में प्रश्न संख्या ***1*** से ***18*** तक बहुविकल्पीय *(MCQ)* तथा प्रश्न संख्या ***19*** एवं ***20*** अभिकथन एवं तर्क आधारित ***1*** अंक के प्रश्न हैं।

v) ***खण्ड ख*** में प्रश्न संख्या ***21*** से ***25*** तक अति लघु-उत्तरीय *(VSA)* प्रकार के ***2*** अंकों के प्रश्न हैं।

) ***खण्ड ग*** में प्रश्न संख्या ***26*** से ***31*** तक लघु-उत्तरीय *(SA)* प्रकार के ***3*** अंकों के प्रश्न हैं।

i) ***खण्ड घ*** में प्रश्न संख्या ***32*** से ***35*** तक दीर्घ-उत्तरीय *(LA)* प्रकार के ***5*** अंकों के प्रश्न हैं।

ii) ***खण्ड ङ*** में प्रश्न संख्या ***36*** से ***38*** तक प्रकरण अध्ययन आधारित ***4*** अंकों के प्रश्न हैं।

iii) *प्रश्न-पत्र में समग्र विकल्प नहीं दिया गया है। यद्यपि, खण्ड ख के 2 प्रश्नों में, खण्ड ग के 3 प्रश्नों में, खण्ड घ के 2 प्रश्नों में तथा खण्ड ङ के 2 प्रश्नों में आंतरिक विकल्प का प्रावधान दिया गया है।*

x) *कैल्कुलेटर का उपयोग* ***वर्जित*** *है।*

खण्ड क

स खण्ड में बहुविकल्पीय प्रश्न (MCQ) हैं, जिनमें प्रत्येक प्रश्न 1 अंक का है।

. यदि आव्यूह $A = [a_{ij}]_{2\times 2}$ में $a_{ij} = \begin{cases} 1, \text{ यदि } i \neq j \\ 0, \text{ यदि } i = j \end{cases}$ है, तो $A + A^2$ बराबर है :

(A) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ (B) $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

(C) $\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$ (D) $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

General Instructions :

Read the following instructions very carefully and strictly follow them :

(i) *This question paper contains* ***38*** *questions.* ***All*** *questions are* ***compulsory****.*

(ii) *This question paper is divided into* ***five*** *Sections –* ***A, B, C, D*** *and* ***E****.*

(iii) *In* ***Section A,*** *Questions no.* ***1*** *to* ***18*** *are multiple choice questions (MCQs) and questions number* ***19*** *and* ***20*** *are Assertion-Reason based questions of* ***1*** *mark each.*

v) *In* ***Section B,*** *Questions no.* ***21*** *to* ***25*** *are very short answer (VSA) type questions, carrying* ***2*** *marks each.*

) *In* ***Section C,*** *Questions no.* ***26*** *to* ***31*** *are short answer (SA) type questions, carrying* ***3*** *marks each.*

i) *In* ***Section D,*** *Questions no.* ***32*** *to* ***35*** *are long answer (LA) type questions carrying* ***5*** *marks each.*

ii) *In* ***Section E,*** *Questions no.* ***36*** *to* ***38*** *are case study based questions carrying* ***4*** *marks each.*

iii) *There is no overall choice. However, an internal choice has been provided in 2 questions in Section B, 3 questions in Section C, 2 questions in Section D and 2 questions in Section E.*

x) *Use of calculator is* ***not*** *allowed.*

SECTION A

his section comprises multiple choice questions (MCQs) of 1 mark each.

. If the matrix $A = [a_{ij}]_{2\times2}$ is such that $a_{ij} = \begin{cases} 1, & \text{if } i \neq j \\ 0, & \text{if } i = j \end{cases}$, then $A + A^2$ is equal to :

(A) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

(B) $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

(C) $\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$

(D) $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

2. $f(x) = \cos^{-1}(2x)$ का प्रान्त है :

(A) $[-1, 1]$

(B) $\left[0, \frac{1}{2}\right]$

(C) $[-2, 2]$

(D) $\left[-\frac{1}{2}, \frac{1}{2}\right]$

3. सारणिक $\begin{vmatrix} \cos 75^\circ & \sin 75^\circ \\ \sin 15^\circ & \cos 15^\circ \end{vmatrix}$ का मान है :

(A) 1

(B) शून्य

(C) $\frac{1}{2}$

(D) $\frac{\sqrt{3}}{2}$

. एक व्युत्क्रमणीय आव्यूह X के लिए, यदि $X^2 = I$ है, तो X^{-1} बराबर है :

(A) X

(B) X^2

(C) I

(D) O

. सारणिक $\Delta = \begin{vmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 3 & 1 & 2 \end{vmatrix}$ में अवयव a_{32} का सहखंड है :

(A) ± 5

(B) – 5

(C) 5

(D) 0

. यदि A कोटि n का एक तत्समक आव्यूह है, तो A (Adj A) है एक :

(A) तत्समक आव्यूह

(B) पंक्ति आव्यूह

(C) शून्य आव्यूह

(D) विषम सममित आव्यूह

2. The domain of $f(x) = \cos^{-1}(2x)$ is :

(A) $[-1, 1]$ (B) $\left[0, \frac{1}{2}\right]$

(C) $[-2, 2]$ (D) $\left[-\frac{1}{2}, \frac{1}{2}\right]$

. The value of the determinant $\begin{vmatrix} \cos 75^\circ & \sin 75^\circ \\ \sin 15^\circ & \cos 15^\circ \end{vmatrix}$ is :

(A) 1 (B) zero

(C) $\frac{1}{2}$ (D) $\frac{\sqrt{3}}{2}$

. For a non-singular matrix X, if $X^2 = I$, then X^{-1} is equal to :

(A) X (B) X^2

(C) I (D) O

. The cofactor of the element a_{32} in the determinant $\Delta = \begin{vmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 3 & 1 & 2 \end{vmatrix}$ is :

(A) ± 5 (B) – 5

(C) 5 (D) 0

. If A is an identity matrix of order n, then A (Adj A) is a/an :

(A) identity matrix

(B) row matrix

(C) zero matrix

(D) skew symmetric matrix

7. यदि $x = t^3$ तथा $y = t^2$ है, तो $t = 1$ पर $\frac{d^2y}{dx^2}$ का मान है :

(A) $\frac{3}{2}$

(B) $-\frac{2}{9}$

(C) $-\frac{3}{2}$

(D) $-\frac{2}{3}$

. परवलय $x^2 = y$ तथा रेखा $y = 1$ द्वारा परिबद्ध क्षेत्र का क्षेत्रफल है :

(A) $\frac{2}{3}$ वर्ग इकाई

(B) $\frac{1}{3}$ वर्ग इकाई

(C) $\frac{4}{3}$ वर्ग इकाई

(D) 2 वर्ग इकाई

. यदि एक गोले के आयतन के परिवर्तन की दर उसकी त्रिज्या के परिवर्तन की दर से दुगुनी है, तो गोले का पृष्ठीय क्षेत्रफल है :

(A) 1 वर्ग इकाई

(B) 2 वर्ग इकाई

(C) 3 वर्ग इकाई

(D) 4 वर्ग इकाई

0. $\int \frac{3 \cos \sqrt{x}}{\sqrt{x}} dx$ बराबर है :

(A) $-6 \sin \sqrt{x} + C$

(B) $-6 \cos \sqrt{x} + C$

(C) $6 \cos \sqrt{x} + C$

(D) $6 \sin \sqrt{x} + C$

7. If $x = t^3$ and $y = t^2$, then $\frac{d^2y}{dx^2}$ at $t = 1$ is :

(A) $\frac{3}{2}$

(B) $-\frac{2}{9}$

(C) $-\frac{3}{2}$

(D) $-\frac{2}{3}$

. The area bounded by the parabola $x^2 = y$ and the line $y = 1$ is :

(A) $\frac{2}{3}$ sq unit

(B) $\frac{1}{3}$ sq unit

(C) $\frac{4}{3}$ sq units

(D) 2 sq units

. If the rate of change of volume of a sphere is twice the rate of change of its radius, then the surface area of the sphere is :

(A) 1 sq unit

(B) 2 sq units

(C) 3 sq units

(D) 4 sq units

0. $\int \frac{3 \cos \sqrt{x}}{\sqrt{x}} dx$ is equal to :

(A) $-6 \sin \sqrt{x} + C$

(B) $-6 \cos \sqrt{x} + C$

(C) $6 \cos \sqrt{x} + C$

(D) $6 \sin \sqrt{x} + C$

11. यदि $\frac{d}{dx} f(x) = 3x^2 - \frac{3}{x^4}$ इस प्रकार है कि $f(1) = 0$ है, तो $f(x)$ है :

(A) $6x + \frac{12}{x^5}$

(B) $x^4 - \frac{1}{x^3} + 2$

(C) $x^3 + \frac{1}{x^3} - 2$

(D) $x^3 + \frac{1}{x^3} + 2$

2. एक LPP में, रैखिक निकाय व्यवरोधों द्वारा बने सुसंगत क्षेत्र के कोनीय बिंदु (1, 1), (3, 0) तथा (0, 3) हैं। यदि $Z = ax + by$, जहाँ $a, b > 0$ का न्यूनतमीकरण करना हो और Z का न्यूनतम मान (3, 0) तथा (1, 1) पर हो, तो a तथा b के बीच का संबंध होगा :

(A) $a = 2b$

(B) $a = \frac{b}{2}$

(C) $a = 3b$

(D) $a = b$

3. व्यवरोधों $x + y \leq 1$, $x, y \geq 0$ के अंतर्गत $Z = 3x + 4y$ का अधिकतम मान है :

(A) 3
(B) 4
(C) 7
(D) 0

14. अवकल समीकरण $\frac{dy}{dx} = 2x \, . \, e^{x^2 + y}$ का व्यापक हल है :

(A) $e^{x^2 + y} = C$
(B) $e^{x^2} + e^{-y} = C$
(C) $e^{x^2} = e^y + C$
(D) $e^{x^2 - y} = C$

11. If $\frac{d}{dx} f(x) = 3x^2 - \frac{3}{x^4}$ such that $f(1) = 0$, then $f(x)$ is :

(A) $6x + \frac{12}{x^5}$

(B) $x^4 - \frac{1}{x^3} + 2$

(C) $x^3 + \frac{1}{x^3} - 2$

(D) $x^3 + \frac{1}{x^3} + 2$

2. In an LPP, corner points of the feasible region determined by the system of linear constraints are (1, 1), (3, 0) and (0, 3). If $Z = ax + by$, where $a, b > 0$ is to be minimized, the condition on a and b, so that the minimum of Z occurs at (3, 0) and (1, 1), will be :

(A) $a = 2b$

(B) $a = \frac{b}{2}$

(C) $a = 3b$

(D) $a = b$

3. The maximum value of $Z = 3x + 4y$ subject to the constraints $x + y \leq 1$, $x, y \geq 0$ is :

(A) 3 (B) 4

(C) 7 (D) 0

14. The general solution of the differential equation $\frac{dy}{dx} = 2x \,.\, e^{x^2 + y}$ is :

(A) $e^{x^2 + y} = C$ (B) $e^{x^2} + e^{-y} = C$

(C) $e^{x^2} = e^y + C$ (D) $e^{x^2 - y} = C$

15. यदि 'm' तथा 'n' क्रमशः अवकल समीकरण $1 + \left(\frac{dy}{dx}\right)^3 = \frac{d^2y}{dx^2}$ की घात तथा कोटि हैं, तो (m + n) का मान है :

(A) 4

(B) 3

(C) 2

(D) 5

6. यदि $|\vec{a}| = 1$, $|\vec{b}| = 2$ तथा $\vec{a} \cdot \vec{b} = 2$ है, तो $|\vec{a} + \vec{b}|$ का मान है :

(A) 9

(B) 3

(C) – 3

(D) 2

7. दो सदिश $\vec{a}$ तथा $\vec{b}$ इस प्रकार हैं कि $|\vec{a} \times \vec{b}| = \vec{a} \cdot \vec{b}$ है। दोनों सदिशों के बीच का कोण है :

(A) 30° (B) 60°

(C) 45° (D) 90°

8. एक सिक्का 3 बार उछाला गया। कम-से-कम दो बार चित आने की प्रायिकता है :

(A) $\frac{1}{2}$

(B) $\frac{3}{8}$

(C) $\frac{1}{8}$

(D) $\frac{1}{4}$

15. If 'm' and 'n' are the degree and order respectively of the differential equation $1 + \left(\frac{dy}{dx}\right)^3 = \frac{d^2y}{dx^2}$, then the value of (m + n) is :

(A) 4

(B) 3

(C) 2

(D) 5

6. If $|\vec{a}| = 1$, $|\vec{b}| = 2$ and $\vec{a} \cdot \vec{b} = 2$, then the value of $|\vec{a} + \vec{b}|$ is :

(A) 9

(B) 3

(C) – 3

(D) 2

7. Two vectors $\vec{a}$ and $\vec{b}$ are such that $|\vec{a} \times \vec{b}| = \vec{a} \cdot \vec{b}$. The angle between the two vectors is :

(A) 30° (B) 60°

(C) 45° (D) 90°

8. A coin is tossed three times. The probability of getting at least two heads is :

(A) $\frac{1}{2}$

(B) $\frac{3}{8}$

(C) $\frac{1}{8}$

(D) $\frac{1}{4}$

प्रश्न संख्या ***19*** *और* ***20*** *अभिकथन एवं तर्क आधारित प्रश्न हैं। दो कथन दिए गए हैं जिनमें एक को अभिकथन (A) तथा दूसरे को तर्क (R) द्वारा अंकित किया गया है। इन प्रश्नों के सही उत्तर नीचे दिए गए कोड (A), (B), (C) और (D) में से चुनकर दीजिए।*

(A) अभिकथन (A) और तर्क (R) दोनों सही हैं और तर्क (R), अभिकथन (A) की सही व्याख्या करता है।

(B) अभिकथन (A) और तर्क (R) दोनों सही हैं, परन्तु तर्क (R), अभिकथन (A) की सही व्याख्या ***नहीं*** करता है।

(C) अभिकथन (A) सही है, परन्तु तर्क (R) ग़लत है।

(D) अभिकथन (A) ग़लत है, परन्तु तर्क (R) सही है।

9. फलन $f : R \longrightarrow R$ पर विचार कीजिए, जिसे $f(x) = x^3$ के रूप में परिभाषित किया गया है।

अभिकथन (A) : f(x) एक एकैकी फलन है।

तर्क (R) : यदि फलन का सहप्रान्त इसके परिसर के समान हो, तो f(x) एकैकी फलन होता है।

0. *अभिकथन (A) :* महत्तम पूर्णांक फलन $f(x) = [x], x \in \mathbb{R}$ में, $x = 2$ पर अवकलनीय नहीं है।

तर्क (R) : महत्तम पूर्णांक फलन किसी भी पूर्णांकीय मान पर संतत नहीं होता।

खण्ड ख

न खण्ड में अति लघु-उत्तरीय (VSA) प्रकार के 5 प्रश्न हैं, जिनमें प्रत्येक के 2 अंक हैं।

1. (क) $\cos^{-1}\left(-\frac{1}{2}\right) + 2\sin^{-1}\left(\frac{1}{2}\right)$ का मुख्य मान ज्ञात कीजिए।

अथवा

(ख) सिद्ध कीजिए कि :

$$\tan^{-1}\sqrt{x} = \frac{1}{2}\cos^{-1}\left(\frac{1-x}{1+x}\right), x \in [0, 1]$$

*Questions number **19** and **20** are Assertion and Reason based questions. Two statements are given, one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer from the codes (A), (B), (C) and (D) as given below.*

(A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).

(B) Both Assertion (A) and Reason (R) are true, but Reason (R) is ***not*** the correct explanation of the Assertion (A).

(C) Assertion (A) is true, but Reason (R) is false.

(D) Assertion (A) is false, but Reason (R) is true.

9. Consider the function $f : R \longrightarrow R$, defined as $f(x) = x^3$.

Assertion (A) : f(x) is a one-one function.

Reason (R) : f(x) is a one-one function, if co-domain = range.

0. *Assertion (A) :* $f(x) = [x]$, $x \in \mathbb{R}$, the greatest integer function is not differentiable at $x = 2$.

Reason (R) : The greatest integer function is not continuous at any integral value.

SECTION B

his section comprises 5 very short answer (VSA) type questions of 2 marks each.

1. (a) Find the principal value of $\cos^{-1}\left(-\frac{1}{2}\right) + 2\sin^{-1}\left(\frac{1}{2}\right)$.

OR

(b) Prove that :

$$\tan^{-1}\sqrt{x} = \frac{1}{2}\cos^{-1}\left(\frac{1-x}{1+x}\right),\ x \in [0, 1]$$

22. यदि $e^y (x + 1) = 1$ है, तो सिद्ध कीजिए कि $\frac{dy}{dx} = -e^y$.

23. 13 m लंबी एक सीढ़ी दीवार के सहारे झुकी है। सीढ़ी का नीचे का सिरा, भूमि के अनुदिश दीवार से दूर 2 m/s की दर से खींचा जाता है। दीवार पर इसकी ऊँचाई किस दर से घट रही है, जबकि सीढ़ी के नीचे का सिरा दीवार से 12 m दूर है ?

4. (क) यदि बिंदु $(-1, -1, 2)$, $(2, 8, \lambda)$ तथा $(3, 11, 6)$ संरेख हैं, तो λ का मान ज्ञात कीजिए।

अथवा

(ख) $\vec{a}$ तथा $\vec{b}$ दो सह-प्रारंभिक सदिश (co-initial vectors) हैं जो एक समांतर चतुर्भुज की संलग्न भुजाएँ बनाते हैं और $|\vec{a}| = 10$, $|\vec{b}| = 2$ तथा $\vec{a} \cdot \vec{b} = 12$ है। समांतर चतुर्भुज का क्षेत्रफल ज्ञात कीजिए।

5. रेखाओं $\vec{r} = (3 + 2\lambda)\hat{i} - (2 - 2\lambda)\hat{j} + (6 + 2\lambda)\hat{k}$ तथा

$\vec{r} = (2\hat{j} - 5\hat{k}) + \mu(6\hat{i} + 3\hat{j} + 2\hat{k})$ के बीच का कोण ज्ञात कीजिए।

खण्ड ग

स खण्ड में लघु-उत्तरीय (SA) प्रकार के 6 प्रश्न हैं, जिनमें प्रत्येक के 3 अंक हैं।

6. वक्र $y = -x^3 + 3x^2 + 9x - 30$ की प्रवणता (ढाल) का अधिकतम मान ज्ञात कीजिए।

7. (क) ज्ञात कीजिए :

$$\int \sqrt{4x^2 - 4x + 10}\, dx$$

अथवा

(ख) मान ज्ञात कीजिए :

$$\int_0^{\pi} \frac{x \sin x}{1 + \cos^2 x}\, dx$$

22. If $e^y (x + 1) = 1$, prove that $\frac{dy}{dx} = - e^y$.

23. A ladder 13 m long is leaning against the wall. The bottom of the ladder is pulled along the ground away from the wall at the rate of 2 m/s. How fast is the height on the wall decreasing when the foot of the ladder is 12 m away from the wall ?

4. (a) Find the value of λ, if the points $(-1, -1, 2)$, $(2, 8, \lambda)$ and $(3, 11, 6)$ are collinear.

OR

(b) $\vec{a}$ and $\vec{b}$ are two co-initial vectors forming the adjacent sides of a parallelogram such that $|\vec{a}| = 10$, $|\vec{b}| = 2$ and $\vec{a} \cdot \vec{b} = 12$. Find the area of the parallelogram.

5. Find the angle between the lines

$$\vec{r} = (3 + 2\lambda)\hat{i} - (2 - 2\lambda)\hat{j} + (6 + 2\lambda)\hat{k} \quad \text{and}$$

$$\vec{r} = (2\hat{j} - 5\hat{k}) + \mu(6\hat{i} + 3\hat{j} + 2\hat{k}).$$

SECTION C

his section comprises 6 short answer (SA) type questions of 3 marks each.

6. Find the maximum slope of the curve $y = -x^3 + 3x^2 + 9x - 30$.

7. (a) Find :

$$\int \sqrt{4x^2 - 4x + 10}\, dx$$

OR

(b) Evaluate :

$$\int_0^{\pi} \frac{x \sin x}{1 + \cos^2 x}\, dx$$

28. निम्नलिखित रैखिक प्रोग्रामन समस्या को ग्राफ द्वारा हल कीजिए :

व्यवरोधों $x + 4y \leq 8$

$2x + 3y \leq 12$

$3x + y \leq 9$

$x \geq 0, y \geq 0$

के अंतर्गत $Z = 2x + 3y$ का अधिकतमीकरण कीजिए।

9. (क) अवकल समीकरण $(2x^2 + y)\, dx = x\, dy$ का व्यापक हल ज्ञात कीजिए।

अथवा

(ख) अवकल समीकरण $\frac{dy}{dx} - \frac{y}{x} + \text{cosec}\left(\frac{y}{x}\right) = 0$ का विशिष्ट हल ज्ञात कीजिए, यदि $x = 1$ के लिए $y = 0$ दिया गया है।

0. यदि $\hat{a}$, $\hat{b}$ तथा $\hat{c}$ मात्रक सदिश हैं तथा $\hat{a} . \hat{b} = \hat{a} . \hat{c} = 0$ है, $\hat{b}$ तथा $\hat{c}$ के बीच का कोण $\frac{\pi}{6}$ है, तो सिद्ध कीजिए कि $\hat{a} = \pm 2\, (\hat{b} \times \hat{c})$.

1. (क) कक्षा XII के चार विद्यार्थियों को एक समस्या स्वतंत्र रूप से हल करने के लिए दी गई है। उनकी समस्या को हल कर पाने की संभावनाएँ क्रमश: $\frac{1}{2}, \frac{1}{3}, \frac{2}{3}$ तथा $\frac{1}{5}$ हैं। उनमें से अधिकतम एक द्वारा समस्या हल कर पाने की प्रायिकता ज्ञात कीजिए।

अथवा

(ख) एक यादृच्छिक चर X का प्रायिकता बंटन नीचे दिया गया है :

X	1	2	4	2k	3k	5k
P(X)	$\frac{1}{2}$	$\frac{1}{5}$	$\frac{3}{25}$	$\frac{1}{10}$	$\frac{1}{25}$	$\frac{1}{25}$

यदि $E(X) = 2{\cdot}94$ है, तो k ज्ञात कीजिए और $P(X \leq 4)$ भी ज्ञात कीजिए।

28. Solve the following LPP graphically :

Maximize Z = 2x + 3y

subject to the constraints $x + 4y \leq 8$

$2x + 3y \leq 12$

$3x + y \leq 9$

$x \geq 0, y \geq 0.$

9. (a) Find the general solution of the differential equation

$(2x^2 + y)\, dx = x\, dy.$

OR

(b) For the differential equation $\frac{dy}{dx} - \frac{y}{x} + \text{cosec}\left(\frac{y}{x}\right) = 0$, find the particular solution, given that y = 0 when x = 1.

0. If $\hat{a}$, $\hat{b}$ and $\hat{c}$ are unit vectors such that $\hat{a} . \hat{b} = \hat{a} . \hat{c} = 0$ and the angle between $\hat{b}$ and $\hat{c}$ is $\frac{\pi}{6}$, then prove that $\hat{a} = \pm 2\, (\hat{b} \times \hat{c})$.

1. (a) Four students of class XII are given a problem to solve independently. Their chances of solving the problem respectively are $\frac{1}{2}, \frac{1}{3}, \frac{2}{3}$ and $\frac{1}{5}$. Find the probability that at most one of them will solve the problem.

OR

(b) The probability distribution of a random variable X is given below :

X	1	2	4	2k	3k	5k
P(X)	$\frac{1}{2}$	$\frac{1}{5}$	$\frac{3}{25}$	$\frac{1}{10}$	$\frac{1}{25}$	$\frac{1}{25}$

Find k, if E(X) = 2·94 and also find $P(X \leq 4)$.

खण्ड घ

इस खण्ड में 4 दीर्घ-उत्तरीय (LA) प्रकार के प्रश्न हैं, जिनमें प्रत्येक के 5 अंक हैं।

32. यदि $A = \begin{bmatrix} 3 & 2 & 1 \\ 4 & -1 & 2 \\ 7 & 3 & -3 \end{bmatrix}$ है, तो A^{-1} ज्ञात कीजिए। A^{-1} के प्रयोग से दिए गए समीकरणों के निकाय $3x + 4y + 7z = 14;\hat{}\ 2x - y + 3z = 4;\ \hat{}x + 2y - 3z = 0$ को हल कीजिए।

3. (क) यदि $y = \cos x^2 + \cos^2 x + \cos^2 (x^2) + \cos (x^x)$ है, तो $\frac{dy}{dx}$ ज्ञात कीजिए।

अथवा

(ख) उन अंतरालों को ज्ञात कीजिए जिनमें दिया गया फलन :

$$f(x) = \frac{3}{10}x^4 - \frac{4}{5}x^3 - 3x^2 + \frac{36}{5}x + 11$$

(i) निरंतर वर्धमान है।

(ii) निरंतर ह्रासमान है।

4. समाकलन के प्रयोग से क्षेत्र $\{(x, y) : 0 \le y \le x^2,\ 0 \le y \le x,\ 0 \le x \le 3\}$ का क्षेत्रफल ज्ञात कीजिए।

5. (क) नीचे दी गई रेखाओं l_1 तथा l_2 में न्यूनतम दूरी ज्ञात कीजिए :

$$l_1 : \vec{r} = \hat{i} + 2\hat{j} - 4\hat{k} + \lambda(4\hat{i} + 6\hat{j} + 12\hat{k})$$

$$\text{और } l_2 : \vec{r} = 3\hat{i} + 3\hat{j} - 5\hat{k} + \mu(6\hat{i} + 9\hat{j} + 18\hat{k})$$

अथवा

(ख) दर्शाइए कि रेखाएँ $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ तथा $\frac{x-4}{5} = \frac{y-1}{2} = \frac{z}{1}$ प्रतिच्छेदी रेखाएँ हैं। रेखाओं का प्रतिच्छेदन बिंदु भी ज्ञात कीजिए।

SECTION D

This section comprises 4 long answer (LA) type questions of 5 marks each.

32. If $A = \begin{bmatrix} 3 & 2 & 1 \\ 4 & -1 & 2 \\ 7 & 3 & -3 \end{bmatrix}$, find A^{-1}. Using A^{-1}, solve the given system of equations $3x + 4y + 7z = 14$; $2x - y + 3z = 4$; $x + 2y - 3z = 0$.

3. (a) If $y = \cos x^2 + \cos^2 x + \cos^2 (x^2) + \cos (x^x)$, find $\frac{dy}{dx}$.

OR

(b) Find the intervals in which the function given by

$$f(x) = \frac{3}{10}x^4 - \frac{4}{5}x^3 - 3x^2 + \frac{36}{5}x + 11 \text{ is :}$$

(i) strictly increasing.

(ii) strictly decreasing.

4. Using integration, find the area of the region

$$\left\{(x, y) : 0 \leq y \leq x^2,\ \ 0 \leq y \leq x,\ \ 0 \leq x \leq 3\right\}.$$

5. (a) Find the shortest distance between the lines l_1 and l_2 given by :

$$l_1 : \vec{r} = \hat{i} + 2\hat{j} - 4\hat{k} + \lambda(4\hat{i} + 6\hat{j} + 12\hat{k})$$

and $l_2 : \vec{r} = 3\hat{i} + 3\hat{j} - 5\hat{k} + \mu(6\hat{i} + 9\hat{j} + 18\hat{k})$

OR

(b) Show that the lines $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and $\frac{x-4}{5} = \frac{y-1}{2} = \frac{z}{1}$ intersect. Also, find their point of intersection.

खण्ड ङ

इस खण्ड में 3 प्रकरण-अध्ययन आधारित प्रश्न हैं, जिनमें प्रत्येक के 4 अंक हैं।

प्रकरण अध्ययन – 1

36. एक खिड़की एक आयत के रूप में है जिसके ऊपर लंबाई पर एक समबाहु त्रिभुज अध्यारोपित है। माना आयताकार भाग की लंबाई x मीटर तथा चौड़ाई y मीटर है।

उपर्युक्त सूचना के आधार पर, निम्नलिखित प्रश्नों के उत्तर दीजिए :

(i) यदि खिड़की का परिमाप 12 m है, तो x तथा y के बीच संबंध ज्ञात कीजिए। *1*

(ii) (i) में प्राप्त व्यंजक के प्रयोग से, खिड़की के क्षेत्रफल का केवल x के फलन के रूप में व्यंजक लिखिए। *1*

(iii) (क) आयत की वह विमाएँ ज्ञात कीजिए जिनसे खिड़की से अधिक-से-अधिक प्रकाश आ सके। ((ii) में प्राप्त व्यंजक का प्रयोग कीजिए) *2*

अथवा

(iii) (ख) यदि यह दिया गया हो कि खिड़की का क्षेत्रफल 50 m^2 है, तो खिड़की के परिमाप का x के पदों में व्यंजक ज्ञात कीजिए। *2*

प्रकरण अध्ययन – 2

7. त्योहारों के मौसम में, एक सोसाइटी के आवासीय कल्याण संघ ने साथ वाले पार्क में एक मेले का आयोजन किया। मेले का मुख्य आकर्षण, पार्क के एक कोने में लगा झूला था जो झूलते समय फलन $x^2 = y$ का परवलय पथ पूरा करता था।

उपर्युक्त सूचना के आधार पर, निम्नलिखित प्रश्नों के उत्तर दीजिए :

(i) माना $f : N \rightarrow R,\ f(x) = x^2$ द्वारा परिभाषित है। इसका परिसर क्या होगा ? *1*

(ii) माना $f : N \rightarrow N,\ f(x) = x^2$ द्वारा परिभाषित है, तो जाँच कीजिए कि फलन एकैकी है या नहीं। *1*

(iii) (क) माना $f : \{1, 2, 3, 4 .....\} \longrightarrow \{1, 4, 9, 16 .....\}$ में $f(x) = x^2$ द्वारा परिभाषित है, तो सिद्ध कीजिए कि फलन एकैकी-आच्छादी है। *2*

अथवा

(iii) (ख) माना $f : \mathbb{R} \longrightarrow \mathbb{R}$ में $f(x) = x^2$ द्वारा परिभाषित है, तो दर्शाइए कि f न तो एकैकी है और न ही आच्छादी है। *2*

SECTION E

This section comprises 3 case study based questions of 4 marks each.

Case Study – 1

36. A window is in the form of a rectangle surmounted by an equilateral triangle on its length. Let the rectangular part have length and breadth x and y metres respectively.

Based on the given information, answer the following questions :

(i) If the perimeter of the window is 12 m, find the relation between x and y. *1*

(ii) Using the expression obtained in (i), write an expression for the area of the window as a function of x only. *1*

(iii) (a) Find the dimensions of the rectangle that will allow maximum light through the window. (use expression obtained in (ii)) *2*

OR

(iii) (b) If it is given that the area of the window is 50 m^2, find an expression for its perimeter in terms of x. *2*

Case Study – 2

7. During the festival season, there was a mela organized by the Resident Welfare Association at a park, near the society. The main attraction of the mela was a huge swing installed at one corner of the park. The swing is traced to follow the path of a parabola given by $x^2 = y$.

Based on the above information, answer the following questions :

(i) Let $f : N \longrightarrow R$ is defined by $f(x) = x^2$. What will be the range ? *1*

(ii) Let $f : N \longrightarrow N$ is defined by $f(x) = x^2$. Check if the function is injective or not. *1*

(iii) (a) Let $f : \{1, 2, 3, 4 .....\} \longrightarrow \{1, 4, 9, 16 .....\}$ be defined by $f(x) = x^2$. Prove that the function is bijective. *2*

OR

(iii) (b) Let $f : \mathbb{R} \longrightarrow \mathbb{R}$ is defined by $f(x) = x^2$. Show that f is neither injective nor surjective. *2*

प्रकरण अध्ययन – 3

38. एक संगठन की प्रबंध समिति के पद के लिए दो व्यक्ति प्रतिस्पर्धा कर रहे हैं। पहले तथा दूसरे व्यक्ति के चयन की प्रायिकता क्रमशः 0·5 तथा 0·6 है। इसके अतिरिक्त, यदि पहले व्यक्ति का चयन होता है, तो इसके द्वारा अपशिष्ट उपचार संयंत्र शुरू करने की प्रायिकता 0·7 है, जबकि दूसरे व्यक्ति का चयन होता है, तो उसकी संबंधित प्रायिकता 0·4 है।

उपर्युक्त सूचना के आधार पर, निम्नलिखित प्रश्नों के उत्तर दीजिए :

(i) अपशिष्ट उपचार संयंत्र के शुरू होने की प्रायिकता क्या है ? *2*

(ii) चयन के बाद, यदि अपशिष्ट उपचार संयंत्र शुरू हो गया है तो इसकी क्या प्रायिकता है कि पहले व्यक्ति ने इसे शुरू किया है ? *2*

Case Study – 3

38. Two persons are competing for a position on the Managing Committee of an organisation. The probabilities that the first and the second person will be appointed are 0·5 and 0·6 respectively. Also, if the first person gets appointed, then the probability of introducing waste treatment plant is 0·7 and the corresponding probability is 0·4, if the second person gets appointed.

Based on the above information, answer the following questions :

(i) What is the probability that the waste treatment plant is introduced ? *2*

(ii) After the selection, if the waste treatment plant is introduced, what is the probability that the first person had introduced it ? *2*

KENDRIYA VIDYALAYA SANGATHAN, BHUBANESWAR REGION

PRE- BOARD EXAMINATION- 2024

CLASS-XII

SUBJECT- MATHEMATICS (041)

TIME- 3:00 Hrs M.M. 80

General Instructions:

1. This paper contains 38 questions. All questions are compulsory.
2. This question paper is divided into five sections – A, B, C, D and E.
3. In Section A, Questions no.1 to 18 are multiple choice questions (MCQs) and Questions no. 19 and 20 are Assertion-Reason based questions of 1 mark each. .
4. In Section B, Questions no. 21 to 25 are very short (VSA)-type questions, carrying 2 marks each.
5. In Section C, Questions no. 26 to 31 are short answer (SA)- type questions carrying 3 marks each.
6. In Sections D, Questions no. 32 to 35 are Long Answer (LA)- type questions, carrying 5 marks each.
7. In Section E, Question no. 36 to 38 are Case study- based questions, carrying 4 marks each.
8. There is no overall choice, however, an internal choice has been provided in 2 questions in section B,3 questions in section C, 2 questions in Section D and one subpart each in 2 questions of section E.
9. Use of calculator is not allowed.

Q.NO.	**Section-A (MCQs)**	
1	What is the domain of $\cos^{-1}(2x-3)$? a) $[-1,1]$ b) $(1,2)$ c) $(-1,1)$ d) $[1,2]$.	1
2	If $f(\alpha)=\begin{pmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{pmatrix}$ then $f(\alpha)f(\beta)=$ a) $f(\alpha)$ b)$f(\alpha\beta)$ c) $f(\alpha+\beta)$ d)$f(\alpha-\beta)$	1
3	If $A=\begin{pmatrix} 2 & 3 \\ -1 & 2 \end{pmatrix}$ then $A^2-4A+7I$ is a) null matrix b) an identity matrix c) diagonal matrix d) none of these	1
4	If $A=\begin{pmatrix} 0 & 2 \\ 3 & -4 \end{pmatrix}$ and If $kA=\begin{pmatrix} 0 & 3a \\ 2b & 24 \end{pmatrix}$, then the values of k, a and b are a) -6, -12, -8 b) -6,-4, -9 c) -6, 4, 9 d) -6, 12, 18	1
5	If $\lvert A\rvert=\lvert kA\rvert$ and A is a 2×2 matrix then sum of all possible values of k is a) 1 b) -1 c) 2 d) 0	1
6	If A is a skew symmetric matrix of order 3×3 and$\lvert A\rvert=x$ then $(2025)^x=$ a)$\frac{1}{2025}$ b) 2025 c)$(2025)^2$ d) 1	1
7	If $y=e^{-x}$ then $\frac{d^2y}{dx^2}=$ a) $-y$ b) y c) x d)$-x$	1
8	The rate of change of area of a circle with respect to its radius at $r=3cm$ is a)3π b) 4π c) 6π d) 12π	1
9	$\int 3^{x+2}dx=$ a) $3^{x+2}+c$ b) $3^{x+2}log3+c$ c)$\frac{3^{x+2}}{log3}+c$ d) $\frac{3^{x+2}}{2log3}+c$	1
10	$\int_0^{\frac{\pi}{3}} sec^2\left(\frac{\pi}{3}-x\right)dx=$ a) $\frac{1}{\sqrt{3}}$ b) $\sqrt{3}$ c) $-\sqrt{3}$ d) 1	1
11	The area of the curve $y=sinx$ between 0 and π is a) 1 sq. unit b) 2 sq. unit c) 4 sq. unit d) 8 sq. unit	1

12	The solution of differential equation $\frac{dy}{dx}+\frac{2y}{x}=0$ is a) $y=\frac{c}{x^2}$ b) $x=\frac{c}{y^2}$ c) $xy=cx$ d) $y=c$	1
13	The integrating factor of the differential equation $x\frac{dy}{dx}+2y=x^2$ is a) $\frac{1}{x}$ b) x c) x^2 d) $\frac{1}{x^2}$	1
14.	The value of $\left(\hat{i}\times\hat{j}\right).\hat{k}+2.\left(\hat{j}\times\hat{i}\right).\hat{k}$ is a) 1 b) -1 c) 2 d) -2	1
15.	Projection of $2\hat{\imath}+\hat{\jmath}$ on the vector $\hat{\imath}-2\hat{\jmath}$ is a) 4 b) 0 c) -4 d) 2	1
16.	The maximum value of $z=3x+4y$ subject to constraints $x+y\le 1$ and $x,y\ge 0$ is a) 7 b) 3 c) 4 d) 10	1
17.	The optimal value of the objective function is attained at the points a) given by intersection of inequation with y-axis only b) given by intersection of inequation with x-axis only c) given by corner points of the feasible region d) None of these.	1
18.	Two dice are thrown. If it is known that the sum of numbers on the dice was less than 5, the probability of getting a sum 3 is a) $\frac{1}{6}$ b) $\frac{2}{3}$ c) $\frac{1}{3}$ d) $\frac{5}{6}$	1
19.	The following question contains STATEMENT-1 (Assertion) and STATEMENT-2 (Reason) and has the following choice (a), (b), (c) and (d), only one of which is the correct answer. Mark the correct choice. a) Both A and R are true and R is the correct explanation of A. b) Both A and R are true but R is not the correct explanation of A. c) A is true but R is false. d) A is false but R is true. **Assertion(A):** Principal value of $tan^{-1}(-1)=\frac{\pi}{4}$ **Reason**(R): $tan^{-1}:R\to(-\frac{\pi}{2},\frac{\pi}{2})$	1
20.	The following question contains STATEMENT-1 (Assertion) and STATEMENT-2 (Reason) and has the following choice (a), (b), (c) and (d), only one of which is the correct answer. Mark the correct choice. a) Both A and R are true and R is the correct explanation of A. b) Both A and R are true but R is not the correct explanation of A. c) A is true but R is false. d) A is false but R is true. **Assertion(A)** : |sinx| is continuous for all x ∈R **Reason**(R) : sinx and |x| are continuous in R .	1
	Section-B (VSA)	
21.	Find the value of $\sin\left\{2\cot^{-1}\left(-\frac{5}{12}\right)\right\}$	2
22.	Determine the values of the constants k so that the given function is continuous at $x=0$. $f(x)=\begin{cases}\frac{sin3x}{sin5x} & , \ if\ x<0\\ k & , \ if\ x\ge 0\end{cases}$	2
23.	Find $\frac{dy}{dx}$ where $x^6y^5=(x+y)^{11}$. OR Given $e^x+e^y=e^{x+y}$. Show that $\frac{dy}{dx}+e^{y-x}=0$	2
24	If $\vec{a}=8\hat{\jmath}+x\hat{k}$ and $\vec{b}=y\hat{\imath}-2\hat{\jmath}+\hat{k}$ are mutually perpendicular and $\|\vec{a}\|=\|\vec{b}\|$, then find the values of x and y	2

	OR If $\|\vec a\| = 3, \|\vec b\| = 5, \|\vec c\| = 7$ and $\vec a + \vec b + \vec c = \vec 0$ then find the angle between $\vec a$ and $\vec b.$	
25	Find a unit vector perpendicular to both of the vectors $\vec p + \vec q$ and $\vec p - \vec q$ where $\vec p = 2\hat i - \hat j + 2\hat k$, $\vec q = 3\hat i + 4\hat j + 5\hat k$.	2
	Section-C (SA)	
26	The area of an expanding rectangle is increasing at the rate of $48cm^2/s$. The length of the rectangle is always equal to square of breadth. At what rate, the length is increasing when breadth is 4.5cm.	3
27.	Find the interval in which the function $f(x) = tan^{-1}(sinx + cosx), x \in (0, \pi)$ is increasing or decreasing.	3
28	. Evaluate $\int(\frac{1}{logx} - \frac{1}{(logx)^2})dx$	3
29.	Find the points on the line $\frac{x+2}{3} = \frac{y+1}{2} = \frac{z-3}{2}$ at a distance of 5 units from $P(1,3,3)$. OR Find the equation of a line passing through $(1, 2, -4)$ and perpendicular to the lines $\vec r = 8\hat i + 2\hat j - 5\hat k + \lambda(3\hat i - 16\hat j + 7\hat k)$ $\vec r = 3\hat i - \hat j + 5\hat k + \mu(3\hat i + 8\hat j - 5\hat k)$.	3
30.	Solve the following LPP graphically. Maximize $Z = 10x + 15y$ Subject to constraints: $3x + y \le 12, \quad x + 2y \le 10 \ and\ x, y \ge 0$	3
31.	Assume that each born child is equally likely to be a boy or a girl. If a family has two children, what is the conditional probability that both are girls given that i)The youngest is a girl? ii)At least one is a girl? OR Two defective bulbs are mixed with 8 good ones. Find the probability distribution of number of defective bulbs if two bulbs are drawn at random. What is the average number of defective bulbs drawn?	3
	Section-D (LA)	
32.	If $A = \begin{bmatrix} 2 & 4 & 6 \\ 3 & -6 & 9 \\ 10 & 5 & -20 \end{bmatrix}$. Find A^{-1} and hence solve the equations $\frac{2}{x} + \frac{3}{y} + \frac{10}{z} = 2$, $\frac{4}{x} - \frac{6}{y} + \frac{5}{z} = 5$, $\frac{6}{x} + \frac{9}{y} - \frac{20}{z} = -4$	5
33	If $x = sint$ and $y = sinpt$ then prove that $(1 - x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} + p^2y = 0.$ OR If $x = acos\theta + bsin\theta$ and $y = asin\theta - bcos\theta$ then show that $y^2\frac{d^2y}{dx^2} - x\frac{dy}{dx} + y = 0.$	5
34.	Using integration, find the area bounded between two curves $x^2 = 4y$ and the line $x = 4y - 2$	5
35. .	Find the foot of the perpendicular from $A(1, 2, -3)$ on the line $\frac{x+1}{2} = \frac{y-3}{-2} = \frac{z-0}{-1}$ Also find the image of the point A in the line. OR Find the value of 'a' so that the lines $\frac{x-1}{2} = \frac{y-a}{3} = \frac{z-3}{4}$ and $\frac{x-4}{5} = \frac{y-1}{2} = \frac{z-0}{1}$ and are intersecting lines. Also, find the point of intersection	5

Section-E (Case-Based) (4 marks)

36 Rahul visited the amusement park along with his family. The amusement park had a huge swing, which attracted many children. He found that the swing traced the path of a parabola as given by $y = 3x^2$

Answer the following questions using the above information.

(i) If f: R→R be defined by $f(x) = 3x^2$, then check whether f is an injective function or not. 1

(ii) Let f: N→N be defined by $(x) = 3x^2$. Check whether f is a bijective function or not. 1

(iii)Let $f: \{1, 2, 3, \ldots\} \rightarrow \{3, 12, 27, \ldots\}$ be defined by $f(x) = 3x^2$. Check whether the function f is bijective or not by giving suitable reason. 2

OR

Let f: N→R be defined by $f(x) = 3x^2$. Determine the range of the function f. Also find f (3) 2

37\. The relation between the heights of the plant (y in cm) with respect to exposure to sunlight is governed by the following equation $y = 4x - \frac{1}{2}x^2$, where x is the number of days exposed to sunlight

Answer the following based on above information:

(i) Find the rate of growth of the plant with respect to sunlight. 1

(ii) What are the number of days it will take for the plant to grow to the maximum height? 1

(iii) What is the maximum height of the plant? 2

OR

What will be the height of the plant after 2 days? 2

38 In answering a question on a multiple-choice test for class XII, a student either knows the answer or guesses. Let 3/5 be the probability that he knows the answer and 2/5 be the probability that he guesses. Assume that a student who guesses at the answer will be correct with probability 1/3. Let E_1,E_2 and E be the events that the student knows the answer, guesses the answer and answers.

Based on the above information, answer the following:

(i) What is the value of $P(E_1)$? 1

(ii) Find the value of $P(E|E_1)$? 1

(iii) Find the value of $\sum_{k=1}^{2} P(E/E_k)P(E_k)$ 2

OR

What is the probability that the student knows the answer given that he answered it correctly? 2

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Series QSS4R/4

Set – 1

प्रश्न-पत्र कोड
Q.P. Code **65/4/1**

अनुक्रमांक
Roll No.

परीक्षार्थी प्रश्न-पत्र कोड को उत्तर-पुस्तिका के मुख-पृष्ठ पर अवश्य लिखें ।
Candidates must write the Q.P. Code on the title page of the answer-book.

- *कृपया जाँच कर लें कि इस प्रश्न-पत्र में मुद्रित पृष्ठ 23 हैं ।*
- *कृपया जाँच कर लें कि इस प्रश्न-पत्र में 38 प्रश्न हैं ।*
- *प्रश्न-पत्र में दाहिने हाथ की ओर दिए गए प्रश्न-पत्र कोड को परीक्षार्थी उत्तर-पुस्तिका के मुख-पृष्ठ पर लिखें ।*
- ***कृपया प्रश्न का उत्तर लिखना शुरू करने से पहले, उत्तर-पुस्तिका में प्रश्न का क्रमांक अवश्य लिखें ।***
- *इस प्रश्न-पत्र को पढ़ने के लिए 15 मिनट का समय दिया गया है । प्रश्न-पत्र का वितरण पूर्वाह्न में 10.15 बजे किया जाएगा । 10.15 बजे से 10.30 बजे तक परीक्षार्थी केवल प्रश्न-पत्र को पढ़ेंगे और इस अवधि के दौरान वे उत्तर-पुस्तिका पर कोई उत्तर नहीं लिखेंगे ।*
- *Please check that this question paper contains **23** printed pages.*
- *Please check that this question paper contains **38** questions.*
- *Q.P. Code given on the right hand side of the question paper should be written on the title page of the answer-book by the candidate.*
- ***Please write down the serial number of the question in the answer-book before attempting it.***
- *15 minute time has been allotted to read this question paper. The question paper will be distributed at 10.15 a.m. From 10.15 a.m. to 10.30 a.m., the candidates will read the question paper only and will not write any answer on the answer-book during this period.*

गणित
MATHEMATICS

*निर्धारित समय : **3** घण्टे* *अधिकतम अंक : **80***

*Time allowed : **3** hours* *Maximum Marks : **80***

सामान्य निर्देश :

निम्नलिखित निर्देशों को बहुत सावधानी से पढ़िए और उनका सख़्ती से पालन कीजिए :

(i) *इस प्रश्न-पत्र में 38 प्रश्न हैं । **सभी** प्रश्न **अनिवार्य** हैं ।*

(ii) *यह प्रश्न-पत्र **पाँच** खण्डों में विभाजित है – खण्ड **क, ख, ग, घ** तथा **ङ** ।*

(iii) ***खण्ड-क** में प्रश्न संख्या 1 से 18 तक बहुविकल्पीय तथा प्रश्न संख्या 19 एवं 20 अभिकथन एवं तर्क आधारित 1 अंक के प्रश्न हैं ।*

(iv) ***खण्ड-ख** में प्रश्न संख्या 21 से 25 तक अति लघु-उत्तरीय (VSA) प्रकार के 2 अंकों के प्रश्न हैं ।*

(v) ***खण्ड-ग** में प्रश्न संख्या 26 से 31 तक लघु-उत्तरीय (SA) प्रकार के 3 अंकों के प्रश्न हैं ।*

(vi) ***खण्ड-घ** में प्रश्न संख्या 32 से 35 तक दीर्घ-उत्तरीय (LA) प्रकार के 5 अंकों के प्रश्न हैं ।*

(vii) ***खण्ड-ङ** में प्रश्न संख्या 36 से 38 तक प्रकरण अध्ययन आधारित 4 अंकों के प्रश्न हैं ।*

(viii) *प्रश्न-पत्र में समग्र विकल्प नहीं दिया गया है । यद्यपि, **खण्ड-ख** के 2 प्रश्नों में, **खण्ड-ग** के 3 प्रश्नों में, **खण्ड-घ** के 3 प्रश्नों में तथा **खण्ड-ङ** के 2 प्रश्नों में आतंरिक विकल्प का प्रावधान दिया गया है ।*

(ix) *कैल्कुलेटर का उपयोग **वर्जित** है ।*

खण्ड – क

इस खण्ड में **20** बहुविकल्पी प्रश्न हैं । प्रत्येक प्रश्न का **1** अंक है । **20 × 1 = 20**

1. यदि $\begin{bmatrix} a & c & 0 \\ b & d & 0 \\ 0 & 0 & 5 \end{bmatrix}$ एक अदिश आव्यूह (scalar matrix) है, तो $a + 2b + 3c + 4d$ का मान है

(A) 0 (B) 5

(C) 10 (D) 25

General Instructions :

Read the following instructions very carefully and strictly follow them :

- *(i)* *This Question Paper contains **38** questions. **All** questions are **compulsory**.*
- *(ii)* *Question Paper is divided into **five** Sections – Section **A**, **B**, **C**, **D** and **E**.*
- *(iii)* *In **Section A** – Questions no. **1** to **18** are Multiple Choice Questions (MCQs) and Questions no. **19** & **20** are Assertion-Reason based questions of **1** mark each.*
- *(iv)* *In **Section B** – Questions no. **21** to **25** are Very Short Answer (VSA) type questions, carrying **2** marks each.*
- *(v)* *In **Section C** – Questions no. **26** to **31** are Short Answer (SA) type questions, carrying **3** marks each.*
- *(vi)* *In **Section D** – Questions no. **32** to **35** are Long Answer (LA) type questions, carrying **5** marks each.*
- *(vii)* *In **Section E** – Questions no. **36** to **38** are case study based questions, carrying **4** marks each.*
- *(viii)* *There is no overall choice. However, an internal choice has been provided in **2** questions in Section **B**, **3** questions in Section **C**, **3** questions in Section **D** and **2** questions in Section **E**.*
- *(ix)* *Use of calculators is **not** allowed.*

SECTION – A

This section consists of **20** multiple choice questions of **1** mark each. **20 × 1 = 20**

1. If $\begin{bmatrix} a & c & 0 \\ b & d & 0 \\ 0 & 0 & 5 \end{bmatrix}$ is a scalar matrix, then the value of a + 2b + 3c + 4d is :

(A) 0 (B) 5

(C) 10 (D) 25

2. दिया है कि $A^{-1} = \frac{1}{7}\begin{bmatrix} 2 & 1 \\ -3 & 2 \end{bmatrix}$ है, तो आव्यूह A है :

(A) $7\begin{bmatrix} 2 & -1 \\ 3 & 2 \end{bmatrix}$

(B) $\begin{bmatrix} 2 & -1 \\ 3 & 2 \end{bmatrix}$

(C) $\frac{1}{7}\begin{bmatrix} 2 & -1 \\ 3 & 2 \end{bmatrix}$

(D) $\frac{1}{49}\begin{bmatrix} 2 & -1 \\ 3 & 2 \end{bmatrix}$

3. यदि $A = \begin{bmatrix} 2 & 1 \\ -4 & -2 \end{bmatrix}$ है, तो $I - A + A^2 - A^3 + \dots$ है :

(A) $\begin{bmatrix} -1 & -1 \\ 4 & 3 \end{bmatrix}$

(B) $\begin{bmatrix} 3 & 1 \\ -4 & -1 \end{bmatrix}$

(C) $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

(D) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

4. यदि $A = \begin{bmatrix} -2 & 0 & 0 \\ 1 & 2 & 3 \\ 5 & 1 & -1 \end{bmatrix}$ है, तो $|A\,(\text{adj. } A)|$ का मान है :

(A) 100 I

(B) 10 I

(C) 10

(D) 1000

5. दिया है कि $\begin{bmatrix} 1 & x \end{bmatrix}\begin{bmatrix} 4 & 0 \\ -2 & 0 \end{bmatrix} = 0$, तो x का मान है :

(A) –4

(B) –2

(C) 2

(D) 4

6. e^{2x} का e^x के सापेक्ष अवकलज है :

(A) e^x

(B) $2e^x$

(C) $2e^{2x}$

(D) $2e^{3x}$

2. Given that $A^{-1} = \frac{1}{7}\begin{bmatrix} 2 & 1 \\ -3 & 2 \end{bmatrix}$, matrix A is :

(A) $7\begin{bmatrix} 2 & -1 \\ 3 & 2 \end{bmatrix}$

(B) $\begin{bmatrix} 2 & -1 \\ 3 & 2 \end{bmatrix}$

(C) $\frac{1}{7}\begin{bmatrix} 2 & -1 \\ 3 & 2 \end{bmatrix}$

(D) $\frac{1}{49}\begin{bmatrix} 2 & -1 \\ 3 & 2 \end{bmatrix}$

3. If $A = \begin{bmatrix} 2 & 1 \\ -4 & -2 \end{bmatrix}$, then the value of $I - A + A^2 - A^3 + \dots$ is :

(A) $\begin{bmatrix} -1 & -1 \\ 4 & 3 \end{bmatrix}$

(B) $\begin{bmatrix} 3 & 1 \\ -4 & -1 \end{bmatrix}$

(C) $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

(D) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

4. If $A = \begin{bmatrix} -2 & 0 & 0 \\ 1 & 2 & 3 \\ 5 & 1 & -1 \end{bmatrix}$, then the value of $|A(\text{adj. } A)|$ is :

(A) 100 I

(B) 10 I

(C) 10

(D) 1000

5. Given that $[1 \quad x]\begin{bmatrix} 4 & 0 \\ -2 & 0 \end{bmatrix} = 0$, the value of x is :

(A) –4

(B) –2

(C) 2

(D) 4

6. Derivative of e^{2x} with respect to e^x, is :

(A) e^x

(B) $2e^x$

(C) $2e^{2x}$

(D) $2e^{3x}$

7. k के किस मान के लिए, फलन $f(x) = \begin{cases} \frac{\sqrt{4+x}-2}{x}, & x \neq 0 \\ k \quad , & x = 0 \end{cases}$

$x = 0$ पर सतत है ?

(A) 0 (B) $\frac{1}{4}$

(C) 1 (D) 4

8. $\int_0^3 \frac{dx}{\sqrt{9-x^2}}$ का मान है :

(A) $\frac{\pi}{6}$ (B) $\frac{\pi}{4}$

(C) $\frac{\pi}{2}$ (D) $\frac{\pi}{18}$

9. अवकल समीकरण $x\,dy + y\,dx = 0$ का व्यापक हल है :

(A) $xy = c$ (B) $x + y = c$

(C) $x^2 + y^2 = c^2$ (D) $\log y = \log x + c$

10. अवकल समीकरण $(x + 2y^2)\frac{dy}{dx} = y \ (y > 0)$ का समाकलन गुणक है :

(A) $\frac{1}{x}$ (B) x

(C) y (D) $\frac{1}{y}$

11. यदि $\vec{a}$ तथा $\vec{b}$ दो ऐसे सदिश हैं कि $|\vec{a}| = 1$, $|\vec{b}| = 2$ तथा $\vec{a} \cdot \vec{b} = \sqrt{3}$ है, तो $2\vec{a}$ तथा $-\vec{b}$ के बीच का कोण है :

(A) $\frac{\pi}{6}$ (B) $\frac{\pi}{3}$

(C) $\frac{5\pi}{6}$ (D) $\frac{11\pi}{6}$

7. For what value of k, the function given below is continuous at $x = 0$?

$$f(x) = \begin{cases} \dfrac{\sqrt{4+x}-2}{x}, & x \neq 0 \\ k \quad , & x = 0 \end{cases}$$

(A) 0 (B) $\frac{1}{4}$

(C) 1 (D) 4

8. The value of $\int_0^3 \frac{dx}{\sqrt{9-x^2}}$ is :

(A) $\frac{\pi}{6}$ (B) $\frac{\pi}{4}$

(C) $\frac{\pi}{2}$ (D) $\frac{\pi}{18}$

9. The general solution of the differential equation $x\,dy + y\,dx = 0$ is :

(A) $xy = c$ (B) $x + y = c$

(C) $x^2 + y^2 = c^2$ (D) $\log y = \log x + c$

10. The integrating factor of the differential equation $(x + 2y^2)\frac{dy}{dx} = y \ (y > 0)$ is :

(A) $\frac{1}{x}$ (B) x

(C) y (D) $\frac{1}{y}$

11. If $\vec{a}$ and $\vec{b}$ are two vectors such that $|\vec{a}| = 1$, $|\vec{b}| = 2$ and $\vec{a} \cdot \vec{b} = \sqrt{3}$, then the angle between $2\vec{a}$ and $-\vec{b}$ is :

(A) $\frac{\pi}{6}$ (B) $\frac{\pi}{3}$

(C) $\frac{5\pi}{6}$ (D) $\frac{11\pi}{6}$

12. सदिश $\vec{a} = 2\hat{i} - \hat{j} + \hat{k}$, $\vec{b} = \hat{i} - 3\hat{j} - 5\hat{k}$ तथा $\vec{c} = -3\hat{i} + 4\hat{j} + 4\hat{k}$ जिस त्रिभुज की भुजाओं को निरूपित करते हैं, वह है :

(A) एक समबाहु त्रिभुज
(B) एक अधिक-कोण त्रिभुज
(C) एक समद्विबाहु त्रिभुज
(D) एक समकोण त्रिभुज

13. माना $\vec{a}$ एक ऐसा सदिश है जिसके लिए $|\vec{a}| = a$ है, तो

$$|\vec{a} \times \hat{i}|^2 + |\vec{a} \times \hat{j}|^2 + |\vec{a} \times \hat{k}|^2$$ का मान है :

(A) a^2
(B) $2a^2$
(C) $3a^2$
(D) 0

14. बिंदु (1, –1, 0) से होकर जाने वाली तथा Y-अक्ष के समांतर रेखा का सदिश समीकरण है :

(A) $\vec{r} = \hat{i} - \hat{j} + \lambda(\hat{i} - \hat{j})$
(B) $\vec{r} = \hat{i} - \hat{j} + \lambda\hat{j}$
(C) $\vec{r} = \hat{i} - \hat{j} + \lambda\hat{k}$
(D) $\vec{r} = \lambda\hat{j}$

15. रेखाएँ $\frac{1-x}{2} = \frac{y-1}{3} = \frac{z}{1}$ तथा $\frac{2x-3}{2p} = \frac{y}{-1} = \frac{z-4}{7}$, p के जिस मान के लिए परस्पर लंबवत हैं, वह है :

(A) $-\frac{1}{2}$
(B) $\frac{1}{2}$
(C) 2
(D) 3

12. The vectors $\vec{a} = 2\hat{i} - \hat{j} + \hat{k}$, $\vec{b} = \hat{i} - 3\hat{j} - 5\hat{k}$ and $\vec{c} = -3\hat{i} + 4\hat{j} + 4\hat{k}$ represents the sides of

(A) an equilateral triangle

(B) an obtuse-angled triangle

(C) an isosceles triangle

(D) a right-angled triangle

13. Let $\vec{a}$ be any vector such that $|\vec{a}| = a$. The value of

$$|\vec{a} \times \hat{i}|^2 + |\vec{a} \times \hat{j}|^2 + |\vec{a} \times \hat{k}|^2 \text{ is :}$$

(A) a^2 (B) $2a^2$

(C) $3a^2$ (D) 0

14. The vector equation of a line passing through the point (1, –1, 0) and parallel to Y-axis is :

(A) $\vec{r} = \hat{i} - \hat{j} + \lambda(\hat{i} - \hat{j})$ (B) $\vec{r} = \hat{i} - \hat{j} + \lambda\hat{j}$

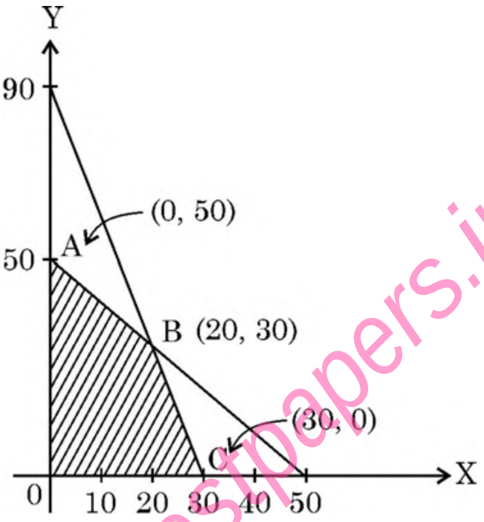
(C) $\vec{r} = \hat{i} - \hat{j} + \lambda\hat{k}$ (D) $\vec{r} = \lambda\hat{j}$

15. The lines $\frac{1-x}{2} = \frac{y-1}{3} = \frac{z}{1}$ and $\frac{2x-3}{2p} = \frac{y}{-1} = \frac{z-4}{7}$ are perpendicular to each other for p equal to :

(A) $-\frac{1}{2}$ (B) $\frac{1}{2}$

(C) 2 (D) 3

16. रैखिक प्रोग्रामन समस्या (LPP) जिसका सुसंगत क्षेत्र दर्शाया गया है, के उद्देश्य फलन $Z = 4x + y$ का अधिकतम मान है :



(A) 50
(B) 110
(C) 120
(D) 170

17. यदि एक यादृच्छिक चर X का प्रायिकता बंटन, निम्न है :

X	0	1	2	3	4
P(X)	0.1	k	2k	k	0.1

जहाँ k एक अज्ञात अचर है ।

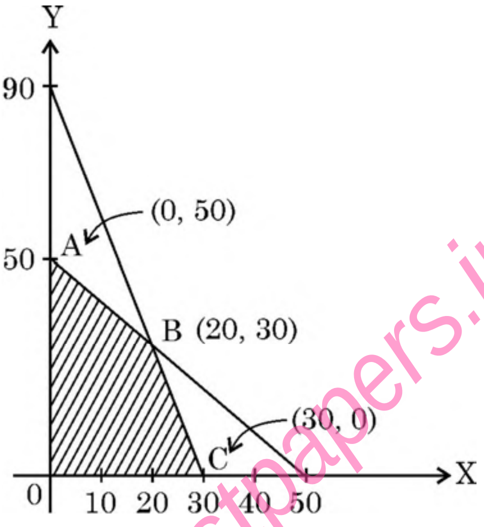
तो यादृच्छिक चर X का मान 2 होने की प्रायिकता है

(A) $\frac{1}{5}$
(B) $\frac{2}{5}$
(C) $\frac{4}{5}$
(D) 1

18. फलन $f(x) = kx - \sin x$ निरंतर वर्धमान है, यदि

(A) $k > 1$
(B) $k < 1$
(C) $k > -1$
(D) $k < -1$

16. The maximum value of Z = $4x + y$ for a L.P.P. whose feasible region is given below is :



(A) 50
(B) 110
(C) 120
(D) 170

17. The probability distribution of a random variable X is :

X	0	1	2	3	4
P(X)	0.1	k	2k	k	0.1

where k is some unknown constant.

The probability that the random variable X takes the value 2 is :

(A) $\frac{1}{5}$
(B) $\frac{2}{5}$
(C) $\frac{4}{5}$
(D) 1

18. The function $f(x) = kx - \sin x$ is strictly increasing for

(A) $k > 1$
(B) $k < 1$
(C) $k > -1$
(D) $k < -1$

अभिकथन – तर्क आधारित प्रश्न

प्रश्न संख्या **19** एवं **20** में एक अभिकथन (A) के बाद एक तर्क (R) दिया है । निम्न में से सही उत्तर चुनिए :

(A) अभिकथन (A) तथा तर्क (R) दोनों सत्य हैं । तर्क (R) अभिकथन (A) की पूरी व्याख्या करता है ।

(B) अभिकथन (A) तथा तर्क (R) दोनों सत्य हैं । तर्क (R) अभिकथन (A) की पूरी व्याख्या नहीं करता ।

(C) अभिकथन (A) सत्य है, परन्तु तर्क (R) असत्य है ।

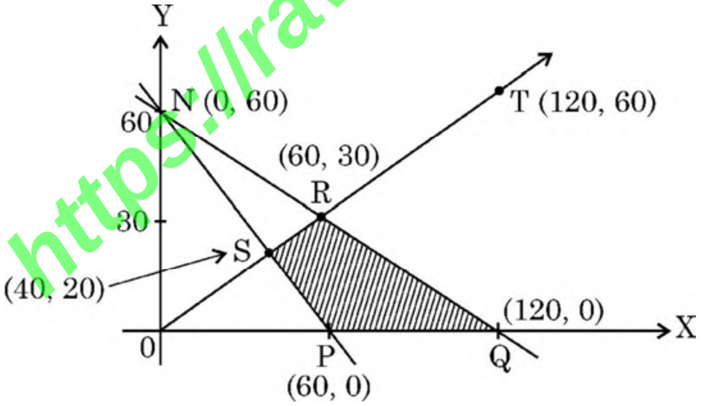
(D) अभिकथन (A) असत्य है जबकि तर्क (R) सत्य है ।

19. **अभिकथन (A) :** संबंध R = $\{(x, y) : (x + y)$ एक अभाज्य संख्या है तथा $x, y \in N\}$ एक स्वतुल्य संबंध नहीं है ।

तर्क (R) : सभी प्राकृत संख्याओं n के लिए, 2n एक भाज्य संख्या है ।

20. **अभिकथन (A) :** किसी LPP के लिए परिबद्ध सुसंगत क्षेत्र के कोणीय बिंदु दर्शाए गए हैं ।

Z = x + 2y का अधिकतम मान अनन्त बिंदुओं पर हैं ।



तर्क (R) : एक LPP जिसका सुसंगत क्षेत्र परिबद्ध हो, का इष्टतम हल कोणीय बिंदु पर होता है ।

ASSERTION-REASON BASED QUESTIONS

Questions No. **19** & **20**, are Assertion (A) and Reason (R) based questions carrying **1** mark each. Two statements are given, one labelled Assertion (A) and the other labelled Reason (R).

Select the correct answer from the codes (A), (B), (C) and (D) as given below :

(A) Both Assertion (A) and Reason (R) are true and the Reason (R) is the correct explanation of Assertion (A).

(B) Both Assertion (A) and Reason (R) are true and Reason (R) is not the correct explanation of the Assertion (A).

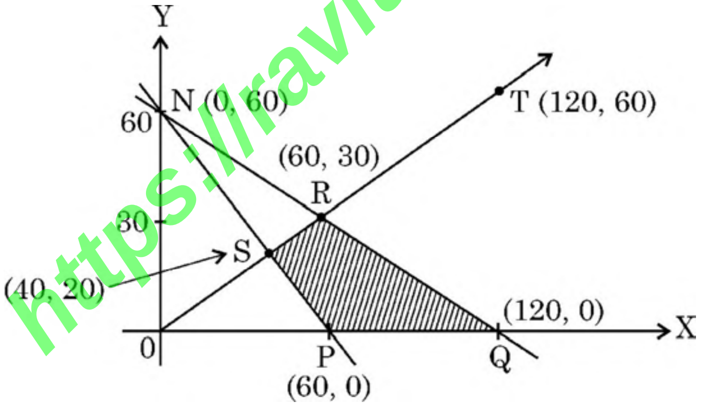
(C) Assertion (A) is true, but Reason (R) is false.

(D) Assertion (A) is false, but Reason (R) is true.

19. **Assertion (A) :** The relation $R = \{(x, y) : (x + y)$ is a prime number and $x, y \in N\}$ is not a reflexive relation.

Reason (R) : The number '2n' is composite for all natural numbers n.

20. **Assertion (A) :** The corner points of the bounded feasible region of a L.P.P. are shown below. The maximum value of $Z = x + 2y$ occurs at infinite points.



Reason (R) : The optimal solution of a LPP having bounded feasible region must occur at corner points.

खण्ड – ख

इस खण्ड में **5** अति लघु उत्तर वाले प्रश्न हैं, जिनमें प्रत्येक के **2** अंक हैं ।

21. (a) $\frac{-\pi}{2} < x < \frac{\pi}{2}$ के लिए $\tan^{-1}\left(\frac{\cos x}{1-\sin x}\right)$ को सरलतम रूप में व्यक्त कीजिए ।

अथवा

(b) $\tan^{-1}(1) + \cos^{-1}\left(-\frac{1}{2}\right) + \sin^{-1}\left(-\frac{1}{\sqrt{2}}\right)$ का मुख्य मान ज्ञात कीजिए ।

22. (a) यदि $y = \cos^3(\sec^2 2t)$ है, तो $\frac{dy}{dt}$ ज्ञात कीजिए ।

अथवा

(b) यदि $x^y = e^{x-y}$ है, तो सिद्ध कीजिए कि $\frac{dy}{dx} = \frac{\log x}{(1+\log x)^2}$.

23. वह अंतराल ज्ञात कीजिए, जिसमें फलन $f(x) = x^4 - 4x^3 + 10$ निरंतर ह्रासमान है ।

24. एक घन का आयतन 6 cm^3/s की दर से बढ़ रहा है । घन का पृष्ठीय क्षेत्रफल किस दर से बढ़ रहा है, जब इसके किनारे की लंबाई 8 cm है ?

25. ज्ञात कीजिए : $\int \frac{1}{x(x^2-1)}\, dx.$

खण्ड – ग

इस खण्ड में **6** लघु-उत्तर प्रकार के प्रश्न हैं, जिनमें प्रत्येक के **3** अंक हैं ।

26. दिया है कि $y = (\sin x)^x \cdot x^{\sin x} + a^x$ है, तो $\frac{dy}{dx}$ ज्ञात कीजिए ।

27. (a) मान ज्ञात कीजिए : $\int_0^{\frac{\pi}{4}} \frac{x\, dx}{1+\cos 2x+\sin 2x}$

अथवा

(b) ज्ञात कीजिए : $\int e^x \left[\frac{1}{(1+x^2)^{\frac{3}{2}}} + \frac{x}{\sqrt{1+x^2}}\right] dx$

SECTION – B

In this section there are **5** very short answer type questions of **2** marks each.

21. (a) Express $\tan^{-1}\left(\dfrac{\cos x}{1-\sin x}\right)$, where $\dfrac{-\pi}{2} < x < \dfrac{\pi}{2}$ in the simplest form.

OR

(b) Find the principal value of $\tan^{-1}(1) + \cos^{-1}\left(-\dfrac{1}{2}\right) + \sin^{-1}\left(-\dfrac{1}{\sqrt{2}}\right)$.

22. (a) If $y = \cos^3(\sec^2 2t)$, find $\dfrac{dy}{dt}$.

OR

(b) If $x^y = e^{x-y}$, prove that $\dfrac{dy}{dx} = \dfrac{\log x}{(1+\log x)^2}$.

23. Find the interval in which the function $f(x) = x^4 - 4x^3 + 10$ is strictly decreasing.

24. The volume of a cube is increasing at the rate of 6 cm^3/s. How fast is the surface area of cube increasing, when the length of an edge is 8 cm ?

25. Find : $\displaystyle\int \frac{1}{x(x^2-1)}\, dx$.

SECTION – C

In this section there are **6** short answer type questions of **3** marks each.

26. Given that $y = (\sin x)^x \cdot x^{\sin x} + a^x$, find $\dfrac{dy}{dx}$.

27. (a) Evaluate : $\displaystyle\int_0^{\frac{\pi}{4}} \frac{x\, dx}{1+\cos 2x+\sin 2x}$

OR

(b) Find : $\displaystyle\int e^x \left[\frac{1}{(1+x^2)^{\frac{3}{2}}} + \frac{x}{\sqrt{1+x^2}}\right] dx$

28. ज्ञात कीजिए : $\int \frac{3x + 5}{\sqrt{x^2 + 2x + 4}} \, dx$

29. (a) अवकल समीकरण $\frac{dy}{dx} = y \cot 2x$ का विशिष्ट हल ज्ञात कीजिए, दिया है कि $y\left(\frac{\pi}{4}\right) = 2$ ।

अथवा

(b) अवकल समीकरण $(x \, e^{\frac{y}{x}} + y) \, dx = x \, dy$ का विशिष्ट हल ज्ञात कीजिए, दिया है कि $y = 1$ है जब $x = 1$ है ।

30. निम्न रैखीय प्रोग्रामन समस्या को ग्राफ द्वारा हल कीजिए :

व्यवरोधों $x + y \leq 6$

$x \geq 2$

$y \leq 3$

$x, y \geq 0$

के अंतर्गत $Z = 2x + 3y$ का अधिकतमीकरण कीजिए ।

31. (a) 52 पत्तों की अच्छी प्रकार से फेंटी गई ताश की गड्डी में से एक पत्ता खो जाता है । शेष पत्तों में से यादृच्छया एक पत्ता निकाला जाता है, जो बादशाह वाला पत्ता पाया जाता है । खो गए पत्ते के बादशाह वाला पत्ता होने की प्रायिकता ज्ञात कीजिए ।

अथवा

(b) एक अभिनत पासे पर समसंख्या आने की प्रायिकता, विषम संख्या के आने की प्रायिकता से दुगुनी है । इस पासे को दो बार उछाला गया । छः आने की संख्या का प्रायिकता बंटन ज्ञात कीजिए । इस बंटन का माध्य भी ज्ञात कीजिए ।

28. Find : $\int \frac{3x+5}{\sqrt{x^2+2x+4}}\,dx$

29. (a) Find the particular solution of the differential equation $\frac{dy}{dx} = y \cot 2x$, given that $y\left(\frac{\pi}{4}\right) = 2$.

OR

(b) Find the particular solution of the differential equation

$$(xe^{\frac{y}{x}} + y)\,dx = x\,dy, \text{ given that } y = 1 \text{ when } x = 1.$$

30. Solve the following linear programming problem graphically :

Maximise $Z = 2x + 3y$

subject to the constraints :

$x + y \leq 6$

$x \geq 2$

$y \leq 3$

$x, y \geq 0$

31. (a) A card from a well shuffled deck of 52 playing cards is lost. From the remaining cards of the pack, a card is drawn at random and is found to be a King. Find the probability of the lost card being a King.

OR

(b) A biased die is twice as likely to show an even number as an odd number. If such a die is thrown twice, find the probability distribution of the number of sixes. Also, find the mean of the distribution.

खण्ड – घ

इस खण्ड में **चार** दीर्घ-उत्तर वाले प्रश्न हैं । प्रत्येक प्रश्न के **5** अंक हैं ।

32. (a) वक्र $y = x|x|$ का आलेख खींचिए । अत: इस वक्र, X-अक्ष तथा कोटियों $x = -2$ तथा $x = 2$ के बीच घिरे क्षेत्र का क्षेत्रफल समाकलन से ज्ञात कीजिए ।

अथवा

(b) समाकलन के प्रयोग से दीर्घवृत्त $9x^2 + 25y^2 = 225$, रेखाओं $x = -2$ तथा $x = 2$ और X-अक्ष के बीच घिरे क्षेत्र का क्षेत्रफल ज्ञात कीजिए ।

33. (a) माना $A = R - \{5\}$ तथा $B = R - \{1\}$ है । $f(x) = \dfrac{x-3}{x-5}$ द्वारा परिभाषित फलन $f : A \to B$ पर विचार कीजिए । दर्शाइए कि f एकैकी व आच्छादक है ।

अथवा

(b) जाँच कीजिए कि क्या सभी वास्तविक संख्याओं के समुच्चय R में परिभाषित संबंध

$S = \{(a, b) :$ जहाँ $a - b + \sqrt{2}$ एक अपरिमेय संख्या है $\}$

स्वतुल्य, सममित या संक्रामक है ।

34. यदि $A = \begin{bmatrix} 2 & 1 & -3 \\ 3 & 2 & 1 \\ 1 & 2 & -1 \end{bmatrix}$ है, तो A^{-1} ज्ञात कीजिए । अत: निम्न समीकरण निकाय का हल ज्ञात कीजिए ।

$2x + y - 3z = 13$

$3x + 2y + z = 4$

$x + 2y - z = 8$

35. (a) रेखा $\dfrac{x}{2} = \dfrac{2y-6}{4} = \dfrac{1-z}{-1}$ तथा इसके समांतर एक अन्य रेखा जो बिंदु $(4, 0, -5)$ से होकर जाती है, के बीच की दूरी ज्ञात कीजिए ।

अथवा

(b) यदि रेखाएँ $\dfrac{x-1}{-3} = \dfrac{y-2}{2k} = \dfrac{z-3}{2}$ तथा $\dfrac{x-1}{3k} = \dfrac{y-1}{1} = \dfrac{z-6}{-7}$ परस्पर लंबवत हैं, तो k का मान ज्ञात कीजिए । अत: उपरोक्त दोनों रेखाओं के लंबवत एक रेखा का सदिश समीकरण लिखिए, जो बिंदु $(3, -4, 7)$ से होकर जाती है ।

SECTION – D

In the section there are **4** long answer type questions of **5** marks each.

32. (a) Sketch the graph of $y = x|x|$ and hence find the area bounded by this curve, X-axis and the ordinates $x = -2$ and $x = 2$, using integration.

OR

(b) Using integration, find the area bounded by the ellipse $9x^2 + 25y^2 = 225$, the lines $x = -2$, $x = 2$, and the X-axis.

33. (a) Let A = R – {5} and B = R – {1}. Consider the function f : A → B, defined by $f(x) = \frac{x-3}{x-5}$. Show that f is one-one and onto.

OR

(b) Check whether the relation S in the set of real numbers R defined by S = {(a, b) : where $a - b + \sqrt{2}$ is an irrational number} is reflexive, symmetric or transitive.

34. If $A = \begin{bmatrix} 2 & 1 & -3 \\ 3 & 2 & 1 \\ 1 & 2 & -1 \end{bmatrix}$, find A^{-1} and hence solve the following system of equations :

$2x + y - 3z = 13$

$3x + 2y + z = 4$

$x + 2y - z = 8$

35. (a) Find the distance between the line $\frac{x}{2} = \frac{2y-6}{4} = \frac{1-z}{-1}$ and another line parallel to it passing through the point (4, 0, –5).

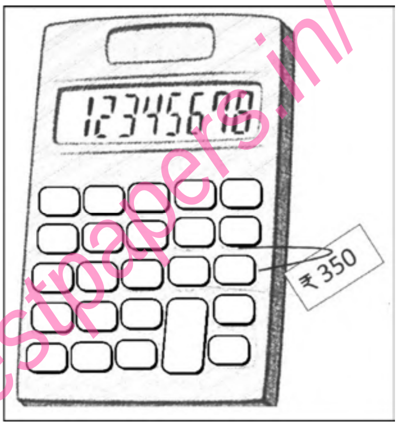
OR

(b) If the lines $\frac{x-1}{-3} = \frac{y-2}{2k} = \frac{z-3}{2}$ and $\frac{x-1}{3k} = \frac{y-1}{1} = \frac{z-6}{-7}$ are perpendicular to each other, find the value of k and hence write the vector equation of a line perpendicular to these two lines and passing through the point (3, –4, 7).

खण्ड – ङ

इस खण्ड में 3 प्रकरण आधारित प्रश्न हैं । प्रत्येक प्रश्न के **4** अंक हैं ।

36. एक स्टोर, कैल्कुलेटर ₹ 350 प्रति कैल्कुलेटर के भाव से बेच रहा है । मार्केट के एक सर्वे के अनुसार मूल्य (p) के घटाने पर बिकने वाले कैल्कुलेटरों की संख्या (x) बढ़ जाती है । मूल्य और बिकने वाली संख्या का संबंध, अर्थात् माँग फलन $p = 450 - \frac{1}{2}x$ द्वारा प्रदत्त है

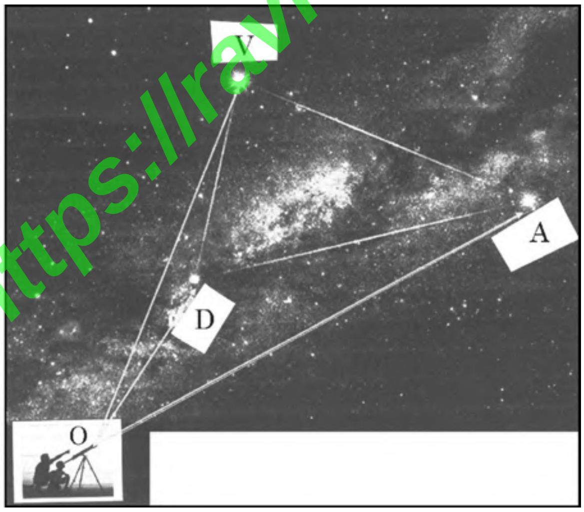


उपरोक्त के आधार पर निम्न प्रश्नों के उत्तर दीजिए :

(i) अधिकतम आय $R(x) = xp(x)$ प्राप्त करने के लिए कितनी इकाई (x) बेचने होंगे ? अपने उत्तर का सत्यापन कीजिए ।

(ii) अधिकतम आय के लिए एक कैल्कुलेटर के मूल्य को स्टोर को कितना घटाना होगा ?

37. एक खगोलीय केंद्र में एक प्रशिक्षक एक विशेष तारामंडल में सबसे चमकीले तीन सितारों को दर्शाता है । मान लें कि दूरबीन O (0, 0, 0) पर स्थित है तथा तीन सितारों की स्थितियाँ D, A तथा V पर इस प्रकार हैं कि उनके स्थिति-सदिश क्रमशः: $2\hat{i} + 3\hat{j} + 4\hat{k}$, $7\hat{i} + 5\hat{j} + 8\hat{k}$ तथा $-3\hat{i} + 7\hat{j} + 11\hat{k}$ हैं ।



SECTION – E

In this section, there are **3** case study based questions of **4** marks each.

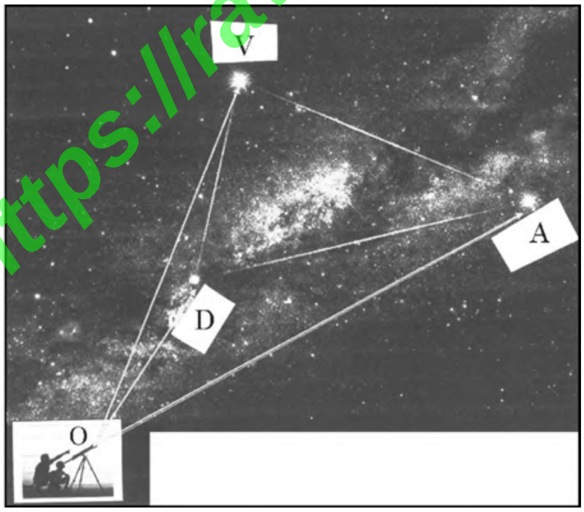
36. A store has been selling calculators at ₹ 350 each. A market survey indicates that a reduction in price (p) of calculator increases the number of units (x) sold. The relation between the price and quantity sold is given by the demand function $p = 450 - \frac{1}{2}x$.

Based on the above information, answer the following questions :

(i) Determine the number of units (x) that should be sold to maximise the revenue $R(x) = xp(x)$. Also, verify the result.

(ii) What rebate in price of calculator should the store give to maximise the revenue ?

37. An instructor at the astronomical centre shows three among the brightest stars in a particular constellation. Assume that the telescope is located at O(0, 0, 0) and the three stars have their locations at the points D, A and V having position vectors $2\hat{i} + 3\hat{j} + 4\hat{k}$, $7\hat{i} + 5\hat{j} + 8\hat{k}$ and $-3\hat{i} + 7\hat{j} + 11\hat{k}$ respectively.



उपरोक्त के आधार पर निम्न के उत्तर दीजिए :

(i) सितारा V, सितारे A से कितनी दूरी पर है ? 1

(ii) $\overrightarrow{DA}$ की दिशा में एक एकक-सदिश ज्ञात कीजिए । 1

(iii) $\angle VDA$ का माप ज्ञात कीजिए । 2

अथवा

(iii) सदिश $\overrightarrow{DV}$ का सदिश $\overrightarrow{DA}$ पर प्रक्षेप कितना है ? 2

38. रोहित, जसप्रीत और आलिया एक ही पद की तीन रिक्तियों के लिए साक्षात्कार के लिए उपस्थित हुए । रोहित के चुने जाने की प्रायिकता $\frac{1}{5}$ है, जसप्रीत के चुने जाने की प्रायिकता $\frac{1}{3}$ तथा आलिया के चुने जाने की प्रायिकता $\frac{1}{4}$ है । चयन की घटना एक दूसरे से स्वतंत्र है ।

उपरोक्त जानकारी के आधार पर निम्न प्रश्नों के उत्तर दें :

(i) इनमें से कम से कम एक के चुने जाने की प्रायिकता क्या है ? 1

(ii) $P(G \mid \overline{H})$ ज्ञात कीजिए जहाँ G, जसप्रीत के चुने जाने को दर्शाती है तथा $\overline{H}$ रोहित के न चुने जाने को दर्शाती है । 1

(iii) उनमें से केवल एक के चुने जाने की प्रायिकता ज्ञात कीजिए । 2

अथवा

(iii) उनमें से कोई दो के चुने जाने की प्रायिकता ज्ञात कीजिए । 2

Based on the above information, answer the following questions :

(i) How far is the star V from star A ? **1**

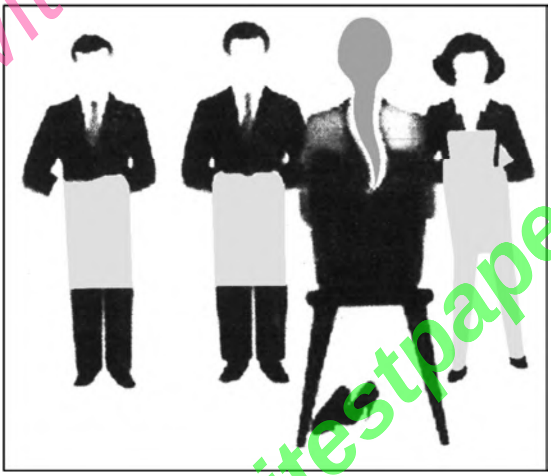
(ii) Find a unit vector in the direction of $\overrightarrow{DA}$. **1**

(iii) Find the measure of ∠VDA. **2**

OR

(iii) What is the projection of vector $\overrightarrow{DV}$ on vector $\overrightarrow{DA}$? **2**

38. Rohit, Jaspreet and Alia appeared for an interview for three vacancies in the same post. The probability of Rohit's selection is $\frac{1}{5}$, Jaspreet's selection is $\frac{1}{3}$ and Alia's selection is $\frac{1}{4}$. The event of selection is independent of each other.



Based on the above information, answer the following questions :

(i) What is the probability that at least one of them is selected ? **1**

(ii) Find $P(G \mid \overline{H})$ where G is the event of Jaspreet's selection and $\overline{H}$ denotes the event that Rohit is not selected. **1**

(iii) Find the probability that exactly one of them is selected. **2**

OR

(iii) Find the probability that exactly two of them are selected. **2**

All India 2024

CBSE Board Solved Paper

Time Allowed : 3 Hours *Maximum Marks : 80*

General Instructions:

(i) This question paper contains **38** questions. **All** questions are compulsory.

(ii) Question paper is divided into **Five** Sections - Sections **A, B, C, D** and **E**.

(iii) In Section **A** - Question Number **1** to **18** are Multiple Choise Questions (MCQ) type and Question Number **19** & **20** are Assertion-Reason based questions of **1** mark each.

(iv) In Section **B** - Question Number **21** to **25** are Very Short Answer (VSA) type questions of **2** marks each.

(v) In Section **C** - Question Number **26** to **31** are Short Answer (SA) type questions, carrying **3** marks each.

(vi) In Section **D** - Question Number **32** to **35** are Long Answer (LA) type questions carrying **5** marks each.

(vii) In Section **E** - Question Number **36** to **38** are case study based questions carrying **4** marks each where **2 VSA** type questions are of **1** mark each and **1 SA** type question is of **2** marks. Internal choice is provided in **2** marks question in each case study.

(viii) There is no overall choice. However, an internal choice has been provided in **2** questions in Section - **B**, **3** questions in Section - **C**, **2** questions in Section - **D** and **2** questions in Section - **E**.

(ix) Use of calculators is **NOT** allowed.

SECTION - A

This section consists of 20 multiple choice questions of 1 mark each.

1. If $\vec{a}$ and $\vec{b}$ are two vectors such that $|\vec{a}| = 1, |\vec{b}| = 2$ and $\vec{a}.\vec{b} = \sqrt{3}$, then the angle between $2\vec{a}$ and $-\vec{b}$ is :

(a) $\frac{\pi}{6}$ (b) $\frac{\pi}{3}$

(c) $\frac{5\pi}{6}$ (d) $\frac{11\pi}{6}$

2. The vectors $\vec{a} = 2\hat{i} - \hat{j} + \hat{k}, \vec{b} = \hat{i} - 3\hat{j} - 5\hat{k}$ and $\vec{c} = -3\hat{i} + 4\hat{j} + 4\hat{k}$ represents the sides of

(a) an equilaterlal triangle

(b) an obtuse - angled triangle

(c) an isosceles triangle

(d) a right - angled triangle

3. Let $\vec{a}$ be any vector such that $|\vec{a}|$ = a. The value of $|\vec{a} \times \hat{i}|^2 + |\vec{a} \times \hat{j}|^2 + |\vec{a} \times \hat{k}|^2$ is :

(a) a^2 (b) $2a^2$

(c) $3a^2$ (d) 0

4. If $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$ and $A^2 + 7I = kA$, then the value of k is :

(a) 1 (b) 2

(c) 5 (d) 7

5. Let $A = \begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{bmatrix}$ and $B = \frac{1}{3}\begin{bmatrix} -2 & 0 & 1 \\ 9 & 2 & -3 \\ 6 & 1 & \lambda \end{bmatrix}$. If AB = I, then the value of λ is:

(a) $\frac{-9}{4}$ (b) -2

(c) $\frac{-3}{2}$ (d) 0

6. Derivative of x^2 with respect to x^3, is:

(a) $\frac{2}{3x}$ (b) $\frac{3x}{2}$

(c) $\frac{2x}{3}$ (d) $6x^5$

7. The function $f(x) = |x| + |x - 2|$ is

(a) continuous, but not differentiable at $x = 0$ and $x = 2$.

(b) differentiable but not continuous at $x = 0$ and $x = 2$.

(c) continuous but not differentiable at $x = 0$ only.

(d) neither continuous nor differentiable at $x = 0$ and $x = 2$.

8. The value of $\int_0^{\pi} \tan^2\left(\frac{\theta}{3}\right) d\theta$ is:

(a) $\pi + \sqrt{3}$ (b) $3\sqrt{3} - \pi$

(c) $\sqrt{3} - \pi$ (d) $\pi - \sqrt{3}$

9. The integrating factor of the differential equation $\frac{dy}{dx} + \frac{2}{x}y = 0, x \neq 0$ is:

(a) $\frac{2}{x}$ (b) x^2

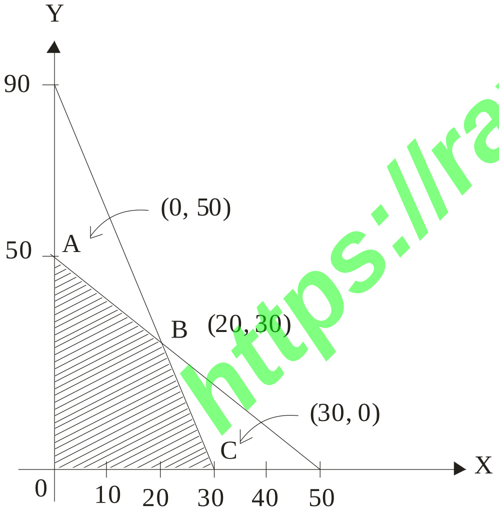
(c) $e^{\frac{2}{x}}$ (d) $e^{\log(2x)}$

10. The lines $\frac{1-x}{2} = \frac{y-1}{3} = \frac{z}{1}$ and $\frac{2x-3}{2p} = \frac{y}{-1} = \frac{z-4}{7}$ are perpendicular to each other for p equal to:

(a) $-\frac{1}{2}$ (b) $\frac{1}{2}$

(c) 2 (d) 3

11. The maximum value of $Z = 4x + y$ for a L. P. P. whose feasible region is given below is:



(a) 50 (b) 110

(c) 120 (d) 170

12. The probability distribution of a random variable X is:

X	0	1	2	3	4
P(X)	0.1	k	2k	k	0.1

where k is some unknown constant.

The probability that the random variable X takes the value 2 is:

(a) $\frac{1}{5}$ (b) $\frac{2}{5}$

(c) $\frac{4}{5}$ (d) 1

13. The function $f(x) = kx - \sin x$ is strictly increasing for

(a) $k > 1$ (b) $k < 1$

(c) $k > -1$ (d) $k < -1$

14. The Cartesian equation of a line passing through the point with position vector $\vec{a} = \hat{i} - \hat{j}$ and parallel to the line $\vec{r} = \hat{i} + \hat{k} + \mu(2\hat{i} - \hat{j})$, is

(a) $\frac{x-2}{1} = \frac{y+1}{0} = \frac{z}{1}$ (b) $\frac{x-1}{2} = \frac{y+1}{-1} = \frac{z}{0}$

(c) $\frac{x+1}{2} = \frac{y+1}{-1} = \frac{z}{0}$ (d) $\frac{x-1}{2} = \frac{y}{-1} = \frac{z-1}{0}$

15. If $\begin{bmatrix} a & c & 0 \\ b & d & 0 \\ 0 & 0 & 5 \end{bmatrix}$ is a scalar matrix, then the value of $a + 2b + 3c + 4d$ is:

(a) 0 (b) 5

(c) 10 (d) 25

16. Given that $A^{-1} = \frac{1}{7}\begin{bmatrix} 2 & 1 \\ -3 & 2 \end{bmatrix}$, matrix A is:

(a) $7\begin{bmatrix} 2 & -1 \\ 3 & 2 \end{bmatrix}$ (b) $\begin{bmatrix} 2 & -1 \\ 3 & 2 \end{bmatrix}$

(c) $\frac{1}{7}\begin{bmatrix} 2 & -1 \\ 3 & 2 \end{bmatrix}$ (d) $\frac{1}{49}\begin{bmatrix} 2 & -1 \\ 3 & 2 \end{bmatrix}$

17. If $A = \begin{bmatrix} 2 & 1 \\ -4 & -2 \end{bmatrix}$, then the value of $I - A + A^2 - A^3 + \ldots$ is:

(a) $\begin{bmatrix} -1 & -1 \\ 4 & 3 \end{bmatrix}$ (b) $\begin{bmatrix} 3 & 1 \\ -4 & -1 \end{bmatrix}$

(c) $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ (d) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

18. The integrating factor of the differential equation $(x + 2y^2)\frac{dy}{dx} = y (y > 0)$ is:

(a) $\frac{1}{x}$ (b) x

(c) y (d) $\frac{1}{y}$

Questions No. 19 & 20, are Assertion (A) and Reason (R) based questions carrying 1 marks each. Two statements are given, one labelled Assertion (A) and the other labelled Reason (R).

Select the correct answer from the codes (A), (B), (C) and (D) as given below:

(a) Both Assertion (A) and Reason (R) are true and the Reason (R) is the correct explanation of Assertion (A).

(b) Both Assertion (A) and Reason (R) are true and Reason (R) is not the correct explanation of the Assertion (A).

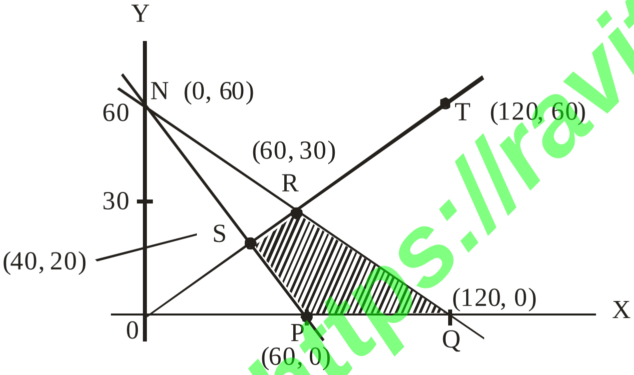
(c) Assertion (A) is true, but Reason (R) is false.

(d) Assertion (A) is false, but Reason (R) is true.

19. **Assertion (A) :** The relation $R = \{(x, y) : (x + y)$ is a prime number and $x, y \in N\}$ is not a reflexive relation.

Reason (R) : The number '2n' is composite for all natural numbers n.

20. **Assertion (A) :** The corner points of the bounded feasible region of a L. P. P. are shown below. The maximum value of Z = x + 2y occurs at infinite points.



Reason (R) : The optimal solution of a LPP having bounded feasible region must occur at corner points.

SECTION - B

In this section there are 5 very short answer type questions of 2 marks each.

21. (a) If $y = \cos^3(\sec^2 2t)$, find $\frac{dy}{dt}$.

OR

(b) If $x^y = e^{x-y}$, prove that $\frac{dy}{dx} = \frac{\log x}{(1+\log x)^2}$.

22. The volume of a cube is increasing at the rate of $6\text{cm}^{3}/\text{s}$. How fast is the surface area of cube increasing, when the length of an edge is 8 cm?

23. Show that the function f given by f(x) = sin x + cos x, is strictly decreasing in the interval $\left(\frac{\pi}{4}, \frac{5\pi}{4}\right)$.

24. (a) Express $\tan^{-1}\left(\frac{\cos x}{1-\sin x}\right)$, where $\frac{-\pi}{2} < x < \frac{\pi}{2}$ in the simplest form.

OR

(b) Find the principal value of

$$\tan^{-1}(1) + \cos^{-1}\left(-\frac{1}{2}\right) + \sin^{-1}\left(-\frac{1}{\sqrt{2}}\right).$$

25. Find : $\int \frac{2x}{(x^2+1)(x^2-4)}dx$.

SECTION - C

In this section there are 6 short answer type questions of 3 marks each.

26. Find $\frac{dy}{dx}$, if $y = (\cos x)^x + \cos^{-1}\sqrt{x}$ is given.

27. (a) Find the particular solution of the differential equation $\frac{dy}{dx} = y \cos 2x$, given that $y\left(\frac{\pi}{4}\right) = 2$.

OR

(b) Find the particular solution of the differential equation

$$\left(xe^{\frac{y}{x}} + y\right) dx = x\,dy,\text{ given that } y = 1 \text{ when } x = 1.$$

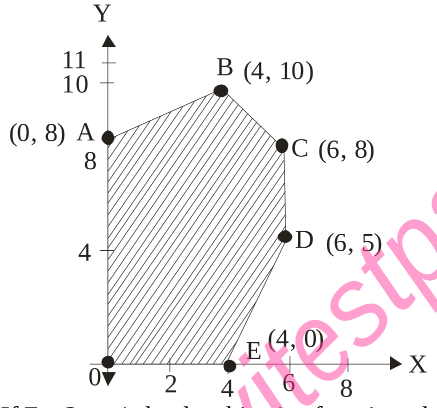
28. Find : $\int \sec^3 \theta d\theta$

29. (a) A card from a well shuffled deck of 52 playing cards is lost. From the remaining cards of the pack, a card is drawn at random and is found to be a King. Find the probability of the lost card being a King.

OR

(b) A biased die is twice as likely to show an even number as an odd number. If such a dies is thrown twice, find the probability distribution of the number of sixes. Also, find the mean of the distribution.

30. The corner points of the feasible region determined by the system of linear constraints are as shown in the following figure:



(i) If $Z = 3x - 4y$ be the objective function, then find the maximum value of Z.

(ii) If $Z = px + qy$ where $p, q > 0$ be the objective function. Find the condition on p and q so that maximum value of Z occurs at B(4, 10) and C(6, 8).

31. (a) Evaluate : $\int_0^{\frac{\pi}{4}} \frac{x dx}{1+\cos 2x+\sin 2x}$

OR

(b) Find : $\int e^x \left[\frac{1}{(1+x^2)^{\frac{3}{2}}}+\frac{x}{\sqrt{1+x^2}}\right]dx$

SECTION - D

In the section there are 4 long answer type questions of 5 marks each.

32. (a) Let $A = R - \{5\}$ and $B = R - \{1\}$. Consider the function $f : A \rightarrow B$, defined by $f(x) = \frac{x-3}{x-5}$. Show that f is one - one and onto.

OR

(b) Check whether the relation S in the set of real numbers R defined by $S = \{(a, b) :$ where $a - b + \sqrt{2}$ is an irrational number$\}$ is reflexive, symmetric or transitive.

33. (a) Find the distance between the line $\frac{x}{2}=\frac{2y-6}{4}=\frac{1-z}{-1}$ and another line parallel to it passing through the point (4, 0, –5).

OR

(b) If the line $\frac{x-1}{-3}=\frac{y-2}{2k}=\frac{z-3}{2}$ and $\frac{x-1}{3k}=\frac{y-1}{1}=\frac{z-6}{-7}$ are perpendicular to each other, find the value of k and hence write the vector equation of a line perpendicular to these two lines and passing through the point (3, –4, 7).

34. Find A^{-1}, if $A = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 3 & -1 \\ 1 & 0 & 1 \end{bmatrix}$. Hence, solve the following system of equations:

$x + 2y + z = 5$

$2x + 3y = 1$

$x - y + z = 8$

35. (a) Sketch the graph of $y = x|x|$ and hence find the area bounded by this curve, X - axis and the ordinates $x = -2$ and $x = 2$, using integration.

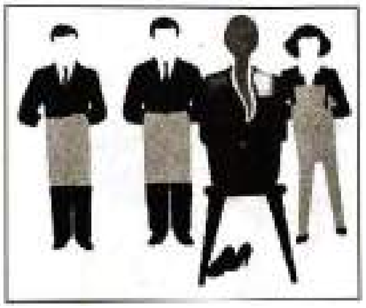
OR

(b) Using integration, find the area bounded by the ellipse $9x^2 + 25y^2 = 225$, the lines $x = -2$, $x = 2$, and the X - axis.

SECTION - E

In this section, there are 3 case study based question of 4 marks each.

36. Rohit, Jaspreet and Alia appeared for an interview for three vacancies in the same post. The probability of Rohit's selection is $\frac{1}{5}$, Jaspreet's selection is $\frac{1}{3}$ and Alia's selection is $\frac{1}{4}$. The event of selection is independent of each other.



Based on the above information, answer the following questions:

(i) What is the probability that at least one of them is selected? **1**

(ii) Find $P(G \mid \overline{H})$ where G is the event of Jaspreet's selection and $\overline{H}$ denotes the event that Rohit is not selected. **1**

(iii) Find the probability that exactly one of them is selected. **2**

OR

(iii) Find the probability that exactly two of them are selected. **2**

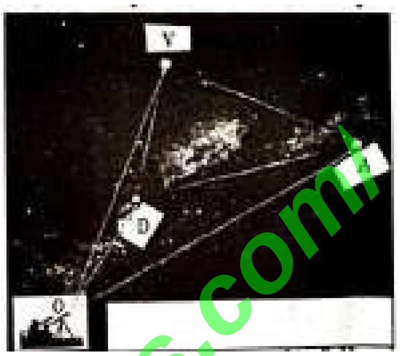
37. A store has been selling calculators at ₹ 350 each. A market survey indicated that a reduction in price (p) of calculator increases the number of units (x) sold. The relation between the price and quantity sold is given by the demand function $p = 450 - \frac{1}{2}x$.

Based on the above information, answer the following questions:

(i) Determine the number of units (x) that should be sold to maximise the revenue $R(x) = xp(x)$. Also, verify the result. **2**

(ii) What rebate in price of calculator should the store give to maximise the revenue? **2**

38. An instructor at the astronomical centre shows three among the brightest stars in a particular constellation. Assume that the telescope is located at O(0, 0, 0) and the three stars have their locations at the points D, A and V having position vectors $2\hat{i}+3\hat{j}+4\hat{k}, 7\hat{i}+5\hat{j}+8\hat{k}$ and $-3\hat{i}+7\hat{j}+11\hat{k}$ respectively.



Based on the above information, answer the following questions:

(i) How far is the star V from star A? **1**

(ii) Find a unit vector in the direction of $\overrightarrow{DA}$. **1**

(iii) Find the measure of $\angle VDA$. **2**

OR

(iii) What is the projection of vector $\overrightarrow{DV}$ on vector $\overrightarrow{DA}$? **2**

SECTION A

his section comprises multiple choice questions (MCQs) of 1 mark each.

. The principal value of $\cos^{-1}\left(-\frac{1}{\sqrt{2}}\right)$ is :

(A) $\frac{\pi}{4}$

(B) $-\frac{\pi}{4}$

(C) $\frac{3\pi}{4}$

(D) $\frac{2\pi}{3}$

2. The matrix $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 4 \end{bmatrix}$ is :

(A) an identity matrix (B) a scalar matrix

(C) a symmetric matrix (D) a skew-symmetric matrix

. If $[3x \quad 5].\begin{bmatrix} x \\ -15 \end{bmatrix} = 0$, then the value of x is :

(A) zero (B) 5

(C) $2\sqrt{5}$ (D) ± 5

. If $A = \begin{bmatrix} 1 & 2 \\ 4 & 2 \end{bmatrix}$ and $|3A| = k|A|$, then the value of k is :

(A) 3 (B) 9

(C) 6 (D) 27

. The area of a triangle with vertices (4, –1), (1, k) and (–2, –1) is 9 sq units. The value of k is :

(A) 8 (B) – 8

(C) 2 (D) 6

6. If $A = \begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix}$, then the value of $|\text{adj } A|$ is :

(A) 5 (B) 25

(C) – 5 (D) – 25

7. If $\frac{d}{dx}f(x) = 3x^2 - \frac{3}{x^4}$ such that f(1) = 0, then f(x) is :

(A) $6x + \frac{12}{x^5}$

(B) $x^4 - \frac{1}{x^3} + 2$

(C) $x^3 + \frac{1}{x^3} - 2$

(D) $x^3 + \frac{1}{x^3} + 2$

. If x = sin t and y = cos t, then $\frac{d^2y}{dx^2}$ at $t = \frac{\pi}{4}$ is :

(A) – 1 (B) 2

(C) $2\sqrt{2}$ (D) $-2\sqrt{2}$

. The area bounded by the line y = x, y-axis and the lines y = 0 and y = 4 is :

(A) 16 sq units (B) 8 sq units

(C) 4 sq units (D) 2 sq units

0. The maximum value of Z = 3x + 4y subject to the constraints $x + y \leq 1$, $x, y \geq 0$ is :

(A) 3 (B) 4

(C) 7 (D) 0

11. In an LPP, corner points of the feasible region determined by the system of linear constraints are (1, 1), (3, 0) and (0, 3). If $Z = ax + by$, where $a, b > 0$ is to be minimized, the condition on a and b, so that the minimum of Z occurs at (3, 0) and (1, 1), will be :

(A) $a = 2b$

(B) $a = \frac{b}{2}$

(C) $a = 3b$

(D) $a = b$

2. If $|\vec{a}| = 1$, $|\vec{b}| = 2$ and $\vec{a} \cdot \vec{b} = 2$, then the value of $|\vec{a} + \vec{b}|$ is :

(A) 9

(B) 3

(C) – 3

(D) 2

3. $\int \frac{\cot \sqrt{x}}{\sqrt{x}} dx$ is equal to :

(A) $\log \sin \sqrt{x} + C$

(B) $\frac{\log \sin \sqrt{x}}{2\sqrt{x}} + C$

(C) $2 \log \sin \sqrt{x} + C$

(D) $\frac{\log \sin \sqrt{x}}{\sqrt{x}} + C$

4. If the rate of change of volume of a sphere is twice the rate of change of its radius, then the surface area of the sphere is :

(A) 1 sq unit

(B) 2 sq units

(C) 3 sq units

(D) 4 sq units

15. If ‘m’ and ‘n’ are the degree and order respectively of the differential equation $1+\left(\frac{dy}{dx}\right)^3=\frac{d^2y}{dx^2}$, then the value of (m + n) is :

(A) 4

(B) 3

(C) 2

(D) 5

6. A coin is tossed three times. The probability of getting at least two heads is :

(A) $\frac{1}{2}$

(B) $\frac{3}{8}$

(C) $\frac{1}{8}$

(D) $\frac{1}{4}$

7. Two vectors $\vec{a}$ and $\vec{b}$ are such that $|\vec{a}\times\vec{b}| = \vec{a}\,.\,\vec{b}$. The angle between the two vectors is :

(A) 30° (B) 60°

(C) 45° (D) 90°

18. The integrating factor of the differential equation $\frac{dy}{dx}+y=\frac{1+y}{x}$ is :

(A) log x (B) – log x

(C) $e^x - x$ (D) $\frac{e^x}{x}$

Questions number ***19*** *and* ***20*** *are Assertion and Reason based questions. Two statements are given, one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer from the codes (A), (B), (C) and (D) as given below.*

(A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).

(B) Both Assertion (A) and Reason (R) are true, but Reason (R) is ***not*** the correct explanation of the Assertion (A).

(C) Assertion (A) is true, but Reason (R) is false.

(D) Assertion (A) is false, but Reason (R) is true.

9. *Assertion (A) :* $f(x) = [x]$, $x \in \mathbb{R}$, the greatest integer function is not differentiable at $x = 2$.

Reason (R) : The greatest integer function is not continuous at any integral value.

0. Consider the function $f : R \longrightarrow R$, defined as $f(x) = x^3$.

Assertion (A) : f(x) is a one-one function.

Reason (R) : f(x) is a one-one function, if co-domain = range.

SECTION B

'his section comprises 5 very short answer (VSA) type questions of 2 marks each.

1. If $e^y (x + 1) = 1$, prove that $\frac{dy}{dx} = -e^y$.

2. (a) Find the principal value of $\cos^{-1}\left(-\frac{1}{2}\right) + 2\sin^{-1}\left(\frac{1}{2}\right)$.

OR

(b) Prove that :

$$\tan^{-1}\sqrt{x} = \frac{1}{2}\cos^{-1}\left(\frac{1-x}{1+x}\right), x \in [0, 1]$$

23. A spherical balloon has a variable diameter $\frac{2}{3}(2t + 1)$. Find the rate of change of its volume with respect to t.

24. Find the angle between the lines

$\vec{r} = (3 + 2\lambda)\hat{i} - (2 - 2\lambda)\hat{j} + (6 + 2\lambda)\hat{k}$ and

$\vec{r} = (2\hat{j} - 5\hat{k}) + \mu(6\hat{i} + 3\hat{j} + 2\hat{k})$.

5. (a) Find the value of λ, if the points $(-1, -1, 2)$, $(2, 8, \lambda)$ and $(3, 11, 6)$ are collinear.

OR

(b) $\vec{a}$ and $\vec{b}$ are two co-initial vectors forming the adjacent sides of a parallelogram such that $|\vec{a}| = 10$, $|\vec{b}| = 2$ and $\vec{a} \cdot \vec{b} = 12$. Find the area of the parallelogram.

SECTION C

his section comprises 6 short answer (SA) type questions of 3 marks each.

6. Find the intervals in which the function f given by $f(x) = -2x^3 - 9x^2 - 12x + 1$ is :

(i) strictly increasing.

(ii) strictly decreasing.

7. (a) Find :

$$\int \sqrt{4x^2 - 4x + 10}\, dx$$

OR

(b) Evaluate :

$$\int_0^{\pi} \frac{x \sin x}{1 + \cos^2 x}\, dx$$

28. If $\hat{a}$, $\hat{b}$ and $\hat{c}$ are unit vectors such that $\hat{a}.\hat{b} = \hat{a}.\hat{c} = 0$ and the angle between $\hat{b}$ and $\hat{c}$ is $\frac{\pi}{6}$, then prove that $\hat{a} = \pm 2\,(\hat{b} \times \hat{c})$.

29. Solve the following LPP graphically :

Maximize Z = 2x + 3y

subject to the constraints $x + 4y \leq 8$

$$2x + 3y \leq 12$$

$$3x + y \leq 9$$

$$x \geq 0,\ y \geq 0.$$

0. (a) Four students of class XII are given a problem to solve independently. Their chances of solving the problem respectively are $\frac{1}{2}, \frac{1}{3}, \frac{2}{3}$ and $\frac{1}{5}$. Find the probability that at most one of them will solve the problem.

OR

(b) The probability distribution of a random variable X is given below :

X	1	2	4	2k	3k	5k
P(X)	$\frac{1}{2}$	$\frac{1}{5}$	$\frac{3}{25}$	$\frac{1}{10}$	$\frac{1}{25}$	$\frac{1}{25}$

Find k, if E(X) = 2·94 and also find $P(X \leq 4)$.

1. (a) Find the general solution of the differential equation

$$(2x^2 + y)\,dx = x\,dy.$$

OR

(b) For the differential equation $\frac{dy}{dx} - \frac{y}{x} + \text{cosec}\left(\frac{y}{x}\right) = 0$, find the particular solution, given that y = 0 when x = 1.

SECTION D

This section comprises 4 long answer (LA) type questions of 5 marks each.

32. (a) If $y = \cos x^2 + \cos^2 x + \cos^2 (x^2) + \cos (x^x)$, find $\frac{dy}{dx}$.

OR

(b) Find the intervals in which the function given by

$$f(x) = \frac{3}{10}x^4 - \frac{4}{5}x^3 - 3x^2 + \frac{36}{5}x + 11 \text{ is :}$$

(i) strictly increasing.

(ii) strictly decreasing.

3. If $A = \begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix}$, find BA and use this to solve the system of equations :

$y + 2z = 8,$

$x - y = -1,$

$2x + 3y + 4z = 20$

4. (a) Find the shortest distance between the lines l_1 and l_2 given by :

$$l_1 : \vec{r} = \hat{i} + 2\hat{j} - 4\hat{k} + \lambda(4\hat{i} + 6\hat{j} + 12\hat{k})$$

$$\text{and } l_2 : \vec{r} = 3\hat{i} + 3\hat{j} - 5\hat{k} + \mu(6\hat{i} + 9\hat{j} + 18\hat{k})$$

OR

(b) Show that the lines $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and $\frac{x-4}{5} = \frac{y-1}{2} = \frac{z}{1}$ intersect. Also, find their point of intersection.

35. Using integration, find the area of the region

$$\left\{(x, y) : 0 \le y \le x^2,\ \ 0 \le y \le x,\ \ 0 \le x \le 3\right\}.$$

SECTION E

This section comprises 3 case study based questions of 4 marks each.

Case Study – 1

36. A window is in the form of a rectangle surmounted by an equilateral triangle on its length. Let the rectangular part have length and breadth x and y metres respectively.

Based on the given information, answer the following questions :

(i) If the perimeter of the window is 12 m, find the relation between x and y. *1*

(ii) Using the expression obtained in (i), write an expression for the area of the window as a function of x only. *1*

(iii) (a) Find the dimensions of the rectangle that will allow maximum light through the window. (use expression obtained in (ii)) *2*

OR

(iii) (b) If it is given that the area of the window is 50 m^2, find an expression for its perimeter in terms of x. *2*

Case Study – 2

7. During the festival season, there was a mela organized by the Resident Welfare Association at a park, near the society. The main attraction of the mela was a huge swing installed at one corner of the park. The swing is traced to follow the path of a parabola given by $x^2 = y$.

Based on the above information, answer the following questions :

(i) Let $f : N \longrightarrow R$ is defined by $f(x) = x^2$. What will be the range ? *1*

(ii) Let $f : N \longrightarrow N$ is defined by $f(x) = x^2$. Check if the function is injective or not. *1*

(iii) (a) Let $f : \{1, 2, 3, 4 \ldots..\} \longrightarrow \{1, 4, 9, 16 \ldots..\}$ be defined by $f(x) = x^2$. Prove that the function is bijective. *2*

OR

(iii) (b) Let $f : \mathbb{R} \longrightarrow \mathbb{R}$ is defined by $f(x) = x^2$. Show that f is neither injective nor surjective. *2*

Case Study – 3

38. Two persons are competing for a position on the Managing Committee of an organisation. The probabilities that the first and the second person will be appointed are 0·5 and 0·6 respectively. Also, if the first person gets appointed, then the probability of introducing waste treatment plant is 0·7 and the corresponding probability is 0·4, if the second person gets appointed.

Based on the above information, answer the following questions :

(i) What is the probability that the waste treatment plant is introduced ? *2*

(ii) After the selection, if the waste treatment plant is introduced, what is the probability that the first person had introduced it ? *2*

12TH MATHAMATICS

Time allowed : 3hour **Maximum Marks = 80**

SECTION-A

(Question nos. 1 to 18 are Multiple choice Questions carrying 1 mark each)

1. A function $f: \mathbb{R} \rightarrow \mathbb{R}$ defined as $f(x) = x^2 - 4x + 5$ is :
(A) injective but not surjective.
(B) surjective but not injective.
(C) both injective and surjective.
(D) neither injective nor surjective.

2. If $A = \begin{bmatrix} a & c & -1 \\ b & 0 & 5 \\ 1 & -5 & 0 \end{bmatrix}$ is a skew-symmetric matrix, then the value of $2a - (b + c)$ is :
(A) 0
(B) 1
(C) -10
(D) 1

3. If A is a square matrix of order 3 such that the value of $|\text{adj} \cdot A| = 8$, then the value of $\left|A^T\right|$ is :
(A) $\sqrt{2}$
(B) $-\sqrt{2}$
(C) 8
(D) $2\sqrt{2}$

4. If inverse of matrix $\begin{bmatrix} 7 & -3 & -3 \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix}$ is the matrix $\begin{bmatrix} 1 & 3 & 3 \\ 1 & \lambda & 3 \\ 1 & 3 & 4 \end{bmatrix}$, then value of λ is :
(A) -4
(B) 1
(C) 3
(D) 4

5. If $[x \quad 2 \quad 0]\begin{bmatrix} 5 \\ -1 \\ x \end{bmatrix} = [3 \quad 1]\begin{bmatrix} -2 \\ x \end{bmatrix}$, then value of x is :
(A) -1
(B) 0
(C) 1
(D) 2

6. Find the matrix A^2, where $A = [a_{ij}]$ is a 2×2 matrix whose elements are given by $a_{ij} =$ maximum $(i, j) -$ minimum (i, j) :
(A) $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$
(B) $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$
(C) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$
(D) $\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$

7. If $xe^y = 1$, then the value of $\frac{dy}{dx}$ at $x = 1$ is :
(A) -1
(B) 1
(C) -e
(D) $-\frac{1}{e}$

8. Derivative of $e^{\sin^2 x}$ with respect to $\cos x$ is :
(A) $\sin xe^{\sin^2 x}$
(B) $\cos xe^{\sin^2 x}$

(C) $-2\cos\, x e^{\sin^2\, x}$

(D) $-2\sin^2\, x\cos\, x e^{\sin^2\, x}$

9. The function $f(x) = \frac{x}{2} + \frac{2}{x}$ has a local minima at x equal to :

(A) 2
(B) 1
(C) 0
(D) -2

10. Given a curve $y = 7x - x^3$ and x increases at the rate of 2 units per second. The rate at which the slope of the curve is changing, when $x = 5$ is :

(A) -60units/sec
(B) 60 units/sec
(C) -70 units/sec
(D) -140 units/sec

11. $\int \frac{1}{x(\log\, x)^2}\, dx$ is equal to :

(A) $2\log\,(\log\, x) + c$
(B) $-\frac{1}{\log\, x} + c$
(C) $\frac{(\log\, x)^3}{3} + c$
(D) $\frac{3}{(\log\, x)^3} + c$

12. The value of $\int_{-1}^{1} x|x|dx$ is :

(A) $\frac{1}{6}$
(B) $\frac{1}{3}$
(C) $-\frac{1}{6}$
(D) 0

13. Area of the region bounded by curve $y^2 = 4x$ and the X-axis between $x = 0$ and $x = 1$ is :

(A) $\frac{2}{3}$
(B) $\frac{8}{3}$
(C) 3
(D) $\frac{4}{3}$

14. The order of the differential equation $\frac{d^4y}{dx^4} - \sin\left(\frac{d^2y}{dx^2}\right) = 5$ is :

(A) 4
(B) 3
(C) 2
(D) not defined

15. The position vectors of points P and Q are $\vec{p}$ and $\vec{q}$ respectively. The point R divides line segment PQ in the ratio 3:1 and S is the mid-point of line segment PR. The position vector of S is :

(A) $\frac{\vec{p}+3\vec{q}}{4}$
(B) $\frac{\vec{p}+3\vec{q}}{8}$
(C) $\frac{5\vec{p}+3\vec{q}}{4}$
(D) $\frac{5\vec{p}+3\vec{q}}{8}$

16. The angle which the line $\frac{x}{1} = \frac{y}{-1} = \frac{z}{0}$ makes with the positive direction of Y-axis is :

(A) $\frac{5\pi}{6}$
(B) $\frac{3\pi}{4}$
(C) $\frac{5\pi}{4}$
(D) $\frac{7\pi}{4}$

17. The Cartesian equation of the line passing through the point $(1,-3,2)$ and parallel to the line : $\vec{r} = (2+\lambda)\hat{i} + \lambda\hat{j} + (2\lambda-1)\hat{k}$ is
    (A) $\frac{x-1}{2} = \frac{y+3}{0} = \frac{z-2}{-1}$
    (B) $\frac{x+1}{1} = \frac{y-3}{1} = \frac{z+2}{2}$
    (C) $\frac{x+1}{2} = \frac{y-3}{0} = \frac{z+2}{-1}$
    (D) $\frac{x-1}{1} = \frac{y+3}{1} = \frac{z-2}{2}$

18. If A and B are events such that $P(A/B) = P(B/A) \neq 0$, then :
    (A) $A \subset B$, but $A \neq B$
    (B) $A = B$
    (C) $A \cap B = \phi$
    (D) $P(A) = P(B)$

Assertion - Reason Based Questions

Direction : In questions numbers 19 and 20, two statements are given one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer from the following options :
(A) Both Assertion (A) and Reason (R) are true and the Reason (R) is the correct explanation of the Assertion (A).
(B) Both Assertion (A) and Reason (R) are true and Reason (R) is not the correct explanation of the Assertion (A).
(C) Assertion (A) is true, but Reason (R) is false.
(D) Assertion (A) is false, but Reason (R) is true.

19. **Assertion (A)** : Domain of $y = \cos^{-1}(x)$ is $[-1,1]$.
    Reason (R) : The range of the principal value branch of $y = \cos^{-1}(x)$ is $[0,\pi] - \left\{\frac{\pi}{2}\right\}$.

20. **Assertion (A) :** The vectors
    $\vec{a} = 6\hat{i} + 2\hat{j} - 8\hat{k}$
    $\vec{b} = 10\hat{i} - 2\hat{j} - 6\hat{k}$
    $\vec{c} = 4\hat{i} - 4\hat{j} + 2\hat{k}$
    represent the sides of a right angled triangle.
    Reason (R) : Three non-zero vectors of which none of two are collinear forms a triangle if their resultant is zero vector or sum of any two vectors is equal to the third.

SECTION-B

(Question nos. 21 to **25** are very short Answer type questions carrying **2** marks each)
Find value of k if

21. $\sin^{-1}\left[k\tan\left(2\cos^{-1}\frac{\sqrt{3}}{2}\right)\right] = \frac{\pi}{3}$.

22. (a) Verify whether the function f defined by

$$f(x) = \begin{cases} x\sin\left(\frac{1}{x}\right), & x \neq 0 \\ 0, & x = 0 \end{cases}$$

is continuous at $x = 0$ or not.

OR

(b) Check for differentiability of the function f defined by $f(x) = |x-5|$, at the point $x = 5$.

23. The area of the circle is increasing at a uniform rate of 2 cm^2/sec. How fast is the circumference of the circle increasing when the radius $r = 5$ cm ?

24. (a) Find: $\int \cos^3 xe^{\log \sin x} dx$

OR

(b) Find: $\int \frac{1}{5+4x-x^2}\, dx$

25. Find the vector equation of the line passing through the point $(2,3,-5)$ and making equal angles with the co-ordinate axes.

SECTION-C

(Question nos. 26 to 31 are short Answer type questions carrying **3** marks each)

26. (a) Find $\frac{dy}{dx}$, if $(\cos\ x)^y = (\cos\ y)^x$.

OR

(b) If $\sqrt{1-x^2} + \sqrt{1-y^2} = a(x-y)$, prove that $\frac{dy}{dx} = \sqrt{\frac{1-y^2}{1-x^2}}$.

27. If $x = a\sin^3\ \theta, y = b\cos^3\ \theta$, then find $\frac{d^2y}{dx^2}$ at $\theta = \frac{\pi}{4}$.

28. (a) Evaluate : $\int_0^{\pi} \frac{e^{\cos\ x}}{e^{\cos\ x}+e^{-co\ \ x}}\, dx$

OR

(b) Find: $\int \frac{2x+1}{(x+1)^2(x-1)} dx$

29. (a) Find the particular solution of the differential equation $\frac{dy}{dx} - 2xy = 3x^2e^{x^2}; y(0) = 5$.

OR

(b) Solve the following differential equation :
$x^2dy + y(x+y)dx = 0$

30. Find a vector of magnitude 4 units perpendicular to each of the vectors $2\hat{\imath} - \hat{\jmath} + \hat{k}$ and $\hat{\imath} + \hat{\jmath} - \hat{k}$ and hence verify your answer.

31. The random variable X has the following probability distribution where a and b are some constants :

X	1	2	3	4	5
P(X)	0.2	a	a	0.2	b

If the mean $E(X) = 3$, then find values of a and b and hence determine $P(X \geq 3)$.

SECTION-D

(Question nos. 32 to **35** are Long Answer type questions carrying 5 marks each)

32. (a) If $A = \begin{bmatrix} 1 & 2 & -3 \\ 2 & 0 & -3 \\ 1 & 2 & 0 \end{bmatrix}$, then find A^{-1} and hence solve the following system of equations :

$$x + 2y - 3z = 1$$
$$2x - 3z = 2$$
$$x + 2y = 3$$

OR

(b) Find the product of the matrices $\begin{bmatrix} 1 & 2 & -3 \\ 2 & 3 & 2 \\ 3 & -3 & -4 \end{bmatrix}\begin{bmatrix} -6 & 17 & 13 \\ 14 & 5 & -8 \\ -15 & 9 & -1 \end{bmatrix}$ and hence solve the system of linear equations:

$$x + 2y - 3z = -4$$
$$2x + 3y + 2z = 2$$
$$3x - 3y - 4z = 11$$

33. Find the area of the region bounded by the curve $4x^2 + y^2 = 36$ using integration.

34. (a) Find the co-ordinates of the foot of the perpendicular drawn from the point $(2,3,-8)$ to the line $\frac{4-x}{2} = \frac{y}{6} = \frac{1-z}{3}$.

Also, find the perpendicular distance of the given point from the line.

OR

(b) Find the shortest distance between the lines L_1 & L_2 given below :

L_1 : The line passing through $(2,-1,1)$ and parallel to $\frac{x}{1} = \frac{y}{1} = \frac{z}{3}$

$L_2 : \vec{r} = \hat{\imath} + (2\mu + 1)\hat{\jmath} - (\mu + 2)\hat{k}$.

35. Solve the following L.P.P. graphically :

Maximise $Z = 60x + 40y$

Subject to

$$x + 2y \leq 12$$
$$2x + y \leq 12$$
$$4x + 5y \geq 20$$
$$x, y \geq 0$$

SECTION-E

(Question nos. 36 to 38 are source based/case based/passage based/integrated units of assessment questions carrying 4 marks each)

36. (a) Students of a school are taken to a railway museum to learn about railways heritage and its history.

An exhibit in the museum depicted many rail lines on the track near the railway station. Let L be the set of all rail lines on the railway track and R be the relation on L defined by

R $= \{(l_1, l_2) : l_1 \text{ is parallel to } l_2\}$

On the basis of the above information, answer the following questions :

(i) Find whether the relation R is symmetric or not.

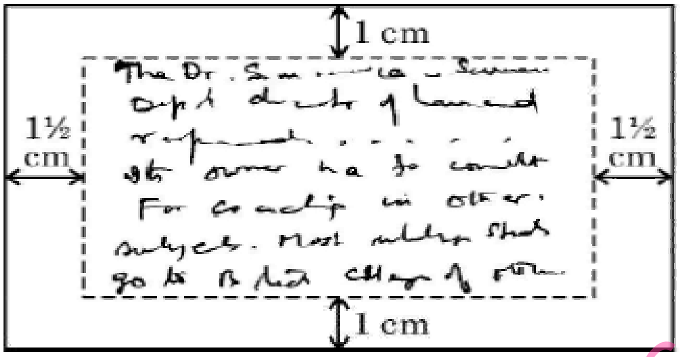
(ii) Find whether the relation R is transitive or not.

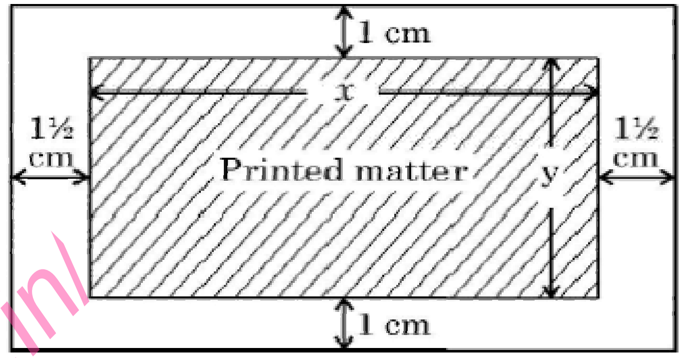
(iii) If one of the rail lines on the railway track is represented by the equation $y = 3x + 2$, then find the set of rail lines in R related to it.

OR

(b) Let S be the relation defined by $S = \{(l_1, l_2) : l_1 \text{ is perpendicular to } l_2\}$ check whether the relation S is symmetric and transitive.

37. A rectangular visiting card is to contain 24sq. cm. of printed matter. The margins at the top and bottom of the card are to be 1 cm and the margins on the left and right are to be 1 ½ cm as shown below :





On the basis of the above information, answer the following questions :
(i) Write the expression for the area of the visiting card in terms of x.
(ii) Obtain the dimensions of the card of minimum area.

38. A departmental store sends bills to charge its customers once a month. Past experience shows that 70% of its customers pay their first month bill in time. The store also found that the customer who pays the bill in time has the probability of 0.8 of paying in time next month and the customer who doesn't pay in time has the probability of 0.4 of paying in time the next month.
Based on the above information, answer the following questions:
(i) Let E_1 and E_2 respectively denote the event of customer paying or not paying the first month bill in time. Find $P(E_1), P(E_2)$.
(ii) Let A denotes the event of customer paying second month's bill in time, then find $P(A \mid E_1)$ and $P(A \mid E_2)$.
(iii) Find the probability of customer paying second month's bill in time.

OR

(iii) Find the probability of customer paying first month's bill in time if it is found that customer has paid the second month's bill in time.

12^{TH} MATHAMATICS

Time allowed : 3hour Maximum Marks = 80

SECTION A

Questions no. 1 to 18 are multiple choice questions (MCQs) and questions number 19 and 20 are Assertion-Reason based questions of 1 mark each.

1. If $A = [a_{ij}]$ is an identity matrix, then which of the following is true ?

(A) $a_{ij} = \begin{cases} 0, & \text{if } i = j \\ 1, & \text{if } i \neq j \end{cases}$

(B) $a_{ij} = 1, \forall i, j$

(C) $a_{ij} = 0, \forall i, j$

(D) $a_{ij} = \begin{cases} 0, & \text{if } i \neq j \\ 1, & \text{if } i = j \end{cases}$

2. Let R_+ denote the set of all non-negative real numbers. Then the function $f: R_+ \to R_+$ defined as $f(x) = x^2 + 1$ is :

(A) one-one but not onto

(B) onto but not one-one

(C) both one-one and onto

(D) neither one-one nor onto

3. Let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ be a square matrix such that adj $A = A$. Then, $(a + b + c + d)$ is equal to :

(A) 2 a

(B) 2 b

(C) 2 c

(D) 0

4. A function $f(x) = |1 - x + |x||$ is :

(A) discontinuous at $x = 1$ only

(B) discontinuous at $x = 0$ only

(C) discontinuous at $x = 0,1$

(D) continuous everywhere

5. If the sides of a square are decreasing at the rate of 1.5 cm/s, the rate of decrease of its perimeter is :

(A) 1.5 cm/s

(B) 6 cm/s

(C) 3 cm/s

(D) 2.25 cm/s

6. $\int_{-a}^{a} f(x)dx = 0$, if :

(A) $f(-x) = f(x)$

(B) $f(-x) = -f(x)$

(C) $f(a - x) = f(x)$

(D) $f(a - x) = -f(x)$

7. $x\log x\frac{dy}{dx} + y = 2\log x$ is an example of a :

(A) variable separable differential equation.

(B) homogeneous differential equation.

(C) first order linear differential equation.

(D) differential equation whose degree is not defined.

8. If $\vec{a} = 2\hat{\imath} - \hat{\jmath} + \hat{k}$ and $\vec{b} = \hat{\imath} + \hat{\jmath} - \hat{k}$, then $\vec{a}$ and $\vec{b}$ are:

(A) collinear vectors which are not parallel

(B) parallel vectors

(C) perpendicular vectors

(D) unit vectors

9. If α, β and γ are the angles which a line makes with positive directions of x, y and z axes respectively, then which of the following is **not** true?

(A) $\cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma = 1$
(B) $\sin^2 \alpha + \sin^2 \beta + \sin^2 \gamma = 2$
(C) $\cos 2\alpha + \cos 2\beta + \cos 2\gamma = -1$
(D) $\cos \alpha + \cos \beta + \cos \gamma = 1$

10. The restrictions imposed on decision variables involved in an objective function of a linear programming problem are called :
(A) feasible solutions
(B) constraints
(C) optimal solutions
(D) infeasible solutions

11. Let E and F be two events such that $P(E) = 0 \cdot 1, P(F) = 0 \cdot 3, P(E \cup F) = 0 \cdot 4$, then $P(F \mid E)$ is :
(A) 0.6
(B) $0 \cdot 4$
(C) 0.5
(D) 0

12. If A and B are two skew symmetric matrices, then $(AB + BA)$ is :
(A) a skew symmetric matrix
(B) a symmetric matrix
(C) a null matrix
(D) an identity matrix

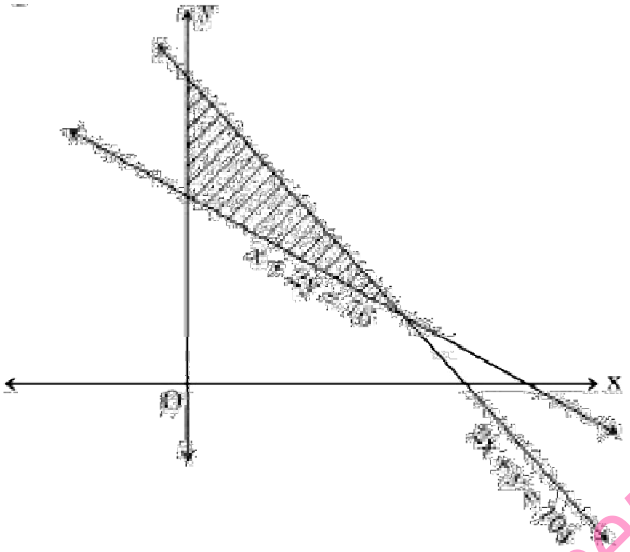
13. If $\begin{vmatrix} 1 & 3 & 1 \\ k & 0 & 1 \\ 0 & 0 & 1 \end{vmatrix} = \pm 6$, then the value of k is :
(A) 2
(B) -2
(C) ± 2
(D) ∓ 2

14. The derivative of 2^x w.r.t. 3^x is :
(A) $\left(\frac{3}{2}\right)^x \frac{\log 2}{\log 3}$
(B) $\left(\frac{2}{3}\right)^x \frac{\log 3}{\log 2}$
(C) $\left(\frac{2}{3}\right)^x \frac{\log 2}{\log 3}$
(D) $\left(\frac{3}{2}\right)^x \frac{\log 3}{\log 2}$

15. If $|\vec{a}| = 2$ and $-3 \leq k \leq 2$, then $|k\vec{a}| \in$:
(A) $[-6,4]$
(B) $[0,4]$
(C) $[4,6]$
(D) $[0,6]$

16. If a line makes an angle of $\frac{\pi}{4}$ with the positive directions of both x -axis and z -axis, then the angle which it makes with the positive direction of y -axis is :
(A) 0
(B) $\frac{\pi}{4}$
(C) $\frac{\pi}{2}$
(D) π

17. Of the following, which group of constraints represents the feasible region given below?



(A) $x + 2y \leq 76, 2x + y \geq 104, x, y \geq 0$
(B) $x + 2y \leq 76, 2x + y \leq 104, x, y \geq 0$
(C) $x + 2y \geq 76, 2x + y \leq 104, x, y \geq 0$
(D) $x + 2y \geq 76, 2x + y \geq 104, x, y \geq 0$

18. If $A = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 5 \end{bmatrix}$, then A^{-1} is :

(A) $\begin{bmatrix} \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{3} & 0 \\ 0 & 0 & \frac{1}{5} \end{bmatrix}$

(B) $30 \begin{bmatrix} \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{3} & 0 \\ 0 & 0 & \frac{1}{5} \end{bmatrix}$

(C) $\frac{1}{30} \begin{bmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 5 \end{bmatrix}$

(D) $\frac{1}{30} \begin{bmatrix} \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{3} & 0 \\ 0 & 0 & \frac{1}{5} \end{bmatrix}$

Questions number 19 and 20 are Assertion and Reason based questions. Two statements are given, one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer from the codes (A), (B), (C) and (D) as given below.
(A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).
(B) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A).
(C) Assertion (A) is true, but Reason (R) is false.
(D) Assertion (A) is false, but Reason (R) is true.

19. **Assertion** (**A**): Every scalar matrix is a diagonal matrix.
Reason (**R**): In a diagonal matrix, all the diagonal elements are 0.

20. **Assertion (A) :** Projection of $\vec{a}$ on $\vec{b}$ is same as projection of $\vec{b}$ on $\vec{a}$.
Reason (**R**) **:** Angle between $\vec{a}$ and $\vec{b}$ is same as angle between $\vec{b}$ and $\vec{a}$ numerically.

SECTION : B

Question no 21 to 25 are very short answer (VSA)type questions, carrying 2 mark each.

21. Evaluate: $\sec^2\left(\tan^{-1}\frac{1}{2}\right)+\mathrm{cosec}^2\left(\cot^{-1}\frac{1}{3}\right)$

22. (a) If $x=e^{x/y}$, prove that $\frac{dy}{dx}=\frac{\log x-1}{(\log x)^2}$

OR

(b) Check the differentiability of $f(x)=\begin{cases}x^2+1, & 0\le x<1\\ 3-x, & 1\le x\le 2\end{cases}$ at $x=1$

23. Evaluate : $\int_0^{\pi/2}\sin\ 2x\cos\ 3x dx$

OR

(b) Given $\frac{d}{dx}F(x)=\frac{1}{\sqrt{2x-x^2}}$ and $F(1)=0$, find $F(x)$.

24. Find the position vector of point C which divides the line segment joining points A and B having position vectors $\hat{i}+2\hat{j}-\hat{k}$ and $-\hat{i}+\hat{j}+\hat{k}$ respectively in the ratio 4: 1 externally. Further, find $|\overrightarrow{AB}|:|\overrightarrow{BC}|$.

25. Let $\vec{a}$ and $\vec{b}$ be two non-zero vectors.
Prove that $|\vec{a}\times\vec{b}|\le|\vec{a}||\vec{b}|$.
State the condition under which equality holds, i.e., $|\vec{a}\times\vec{b}|=|\vec{a}||\vec{b}|$.

SECTION : C

Question no 26 to 31 are short answer (SA) type question , carrying 3 mark each.

26. (a) If $x\cos(p+y)+\cos p\sin(p+y)=0$, prove that $\cos p\frac{dy}{dx}=-\cos^2(p+y)$, where p is a constant.

OR

(b) Find the value of a and b so that function f defined as :

$$f(x)=\begin{cases}\dfrac{x-2}{|x-2|}+a, & \text{if } x<2\\ a+b, & \text{if } x=2\\ \dfrac{x-2}{|x-2|}+b, & \text{if } x>2\end{cases}$$

is a continuous function.

27. (a) Find the intervals in which the function $f(x)=\frac{\log x}{x}$ is strictly increasing or strictly decreasing.

(b) Find the absolute maximum and absolute minimum values of the function f given by $f(x)=\frac{x}{2}+\frac{2}{x}$, on the interval [1,2].

28. Find : $\int\frac{x^2+1}{(x^2+2)(x^2+4)}dx$

29. Find : $\int\frac{2+\sin\ 2x}{1+\cos\ 2x}e^x dx$

OR

(b) Evaluate : $\int_0^{\pi/4} \frac{1}{\sin\ x+c \quad x} dx$

30. Solve the following linear programming problem graphically :
Maximise $z = 4x + 3y$, subject to the constraints

$$x + y \leq 800$$
$$2x + y \leq 1000$$
$$x \leq 400$$
$$x, y \geq 0$$

31. The chances of P, Q and R getting selected as CEO of a company are in the ratio 4: 1: 2 respectively. The probabilities for the company to increase its profits from the previous year under the new CEO, P, Q or R are 0.3,0.8 and 0.5 respectively. If the company increased the profits from the previous year, find the probability that it is due to the appointment of R as CEO.

SECTION D

Questions no. 32 to 35 are long answer (LA) type questions carrying 5 marks each.

32. A relation R on set $A = \{-4, -3, -2, -1, 0, 1, 2, 3, 4\}$ be defined as $R = \{(x, y): x + y$ is an integer divisible by 2$\}$. Show that R is an equivalence relation. Also, write the equivalence class [2].

33. (a) It is given that function $f(x) = x^4 - 62x^2 + ax + 9$ attains local maximum value at $x = 1$. Find the value of ' a ', hence obtain all other points where the given function f(x) attains local maximum or local minimum values.

OR

(b) The perimeter of a rectangular metallic sheet is 300 cm . It is rolled along one of its sides to form a cylinder. Find the dimensions of the rectangular sheet so that volume of cylinder so formed is maximum.

34. Using integration, find the area of the region enclosed between the circle $x^2 + y^2 = 16$ and the lines $x = -2$ and $x = 2$.

35. (a) Find the equation of the line passing through the point of intersection of the lines $\frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{3}$ and $\frac{x-1}{0} = \frac{y}{-3} = \frac{z-7}{2}$ and perpendicular to these given lines.

OR

(b) Two vertices of the parallelogram ABCD are given as $A(-1,2,1)$ and $B(1,-2,5)$. If the equation of the line passing through C and D is $\frac{x-4}{1} = \frac{y+7}{-2} = \frac{z-8}{2}$, then find the distance between sides AB and CD . Hence, find the area of parallelogram ABCD .

SECTION E

Questions no. 36 to 38 are case study based questions carrying 4 marks each .

36. Self-study helps students to build confidence in learning. It boosts the self-esteem of the learners. Recent surveys suggested that close to 50% learners were self-taught using internet resources and upskilled themselves.

A student may spend 1 hour to 6 hours in a day in upskilling self. The probability distribution of the number of hours spent by a student is given below :

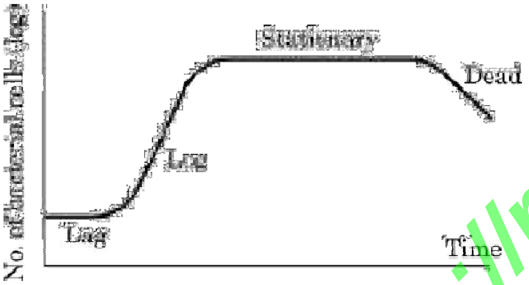
$$P(X = x) = \begin{cases} kx^2, & \text{for } x = 1,2,3 \\ 2kx, & \text{for } x = 4,5,6 \\ 0, & \text{otherwise} \end{cases}$$

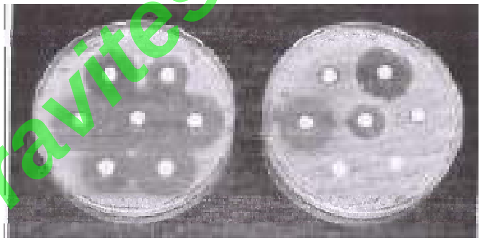
where x denotes the number of hours.
Based on the above information, answer the following questions :
(i) Express the probability distribution given above in the form of a probability distribution table.
(ii) Find the value of k .
(iii) (a) Find the mean number of hours spent by the student.

OR

(iii) (b) Find $P(1 < X < 6)$.

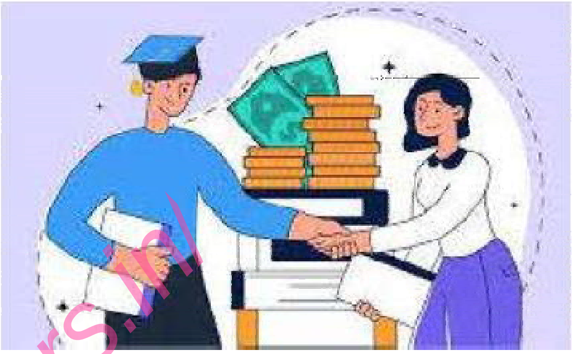
37. A bacteria sample of certain number of bacteria is observed to grow exponentially in a given amount of time. Using exponential growth model, the rate of growth of this sample of bacteria is calculated.





The differential equation representing the growth of bacteria is given as :
$\frac{dP}{dt} = kP$, where P is the population of bacteria at any time ' t '.
Based on the above information, answer the following questions:
(i) Obtain the general solution of the given differential equation and express it as an exponential function of ' t '.
(ii) If population of bacteria is 1000 at $t = 0$, and 2000 at $t = 1$, find the value of **k**.

38. A scholarship is a sum of money provided to a student to help him or her pay for education. Some students are granted scholarships based on their academic achievements, while others are rewarded based on their financial needs.



Every year a school offers scholarships to girl children and meritorious achievers based on certain criteria. In the session 2022 - 23, the school offered monthly scholarship of ₹ 3,000 each to some girl students and ₹ 4,000 each to meritorious achievers in academics as well as sports.

In all, 50 students were given the scholarships and monthly expenditure incurred by the school on scholarships was ₹ 1,80,000.

Based on the above information, answer the following questions :

(i) Express the given information algebraically using matrices.

(ii) Check whether the system of matrix equations so obtained is consistent or not.

(iii) (a) Find the number of scholarships of each kind given by the school, using matrices.

OR

(iii) (b) Had the amount of scholarship given to each girl child and meritorious student been interchanged, what would be the monthly expenditure incurred by the school?

12TH MATHAMATICS

Time allowed : 3hour Maximum Marks = 80

1. If the sum of all the elements of a 3 × 3 scalar matrix is 9 , then the product of all its elements is :
   (A) 0
   (B) 9
   (C) 27
   (D) 729
2. Let $f: R_+ \rightarrow [-5, \infty)$ be defined as $f(x) = 9x^2 + 6x - 5$, where R_+ is the set of all non-negative real numbers. Then, f is :
   (A) one-one
   (B) onto
   (C) bijective
   (D) neither one-one nor onto
3. If $\begin{vmatrix} -a & b & c \\ a & -b & c \\ a & b & -c \end{vmatrix} = kabc$, then the value of k is :
   (A) 0
   (B) 1
   (C) 2
   (D) 4
4. The number of points of discontinuity of $f(x) = \begin{cases} |x| + 3, & \text{if } x \leq -3 \\ -2x, & \text{if } -3 < x < 3 \\ 6x + 2, & \text{if } x \geq 3 \end{cases}$ is :
   (A) 0
   (B) 1
   (C) 2
   (D) infinite
5. The function $f(x) = x^3 - 3x^2 + 12x - 18$ is :
   (A) strictly decreasing on R
   (B) strictly increasing on R
   (C) neither strictly increasing nor strictly decreasing on **R**
   (D) strictly decreasing on $(-\infty, 0)$
6. $\int_0^{\pi/2} \frac{\sin x - \cos x}{1 + \sin x \cos x} dx$ is equal to :
   (A) π
   (B) Zero (0)
   (C) $\int_0^{\pi/2} \frac{2\sin x}{1 + \sin x \cos x} dx$
   (D) $\frac{\pi^2}{4}$
7. The differential equation $\frac{dy}{dx} = F(x, y)$ will not be a homogeneous differential equation, if $F(x, y)$ is :
   (A) $\cos x - \sin\left(\frac{y}{x}\right)$
   (B) $\frac{y}{x}$
   (C) $\frac{x^2 + y^2}{xy}$
   (D) $\cos^2\left(\frac{x}{y}\right)$
8. For any two vectors $\vec{a}$ and $\vec{b}$, which of the following statements is always true?
   (A) $\vec{a} \cdot \vec{b} \geq |\vec{a}||\vec{b}|$
   (B) $\vec{a} \cdot \vec{b} = |\vec{a}||\vec{b}|$
   (C) $\vec{a} \cdot \vec{b} \leq |\vec{a}||\vec{b}|$
   (D) $\vec{a} \cdot \vec{b} < |\vec{a}||\vec{b}|$

9. The coordinates of the foot of the perpendicular drawn from the point (0,1,2) on the x-axis are given by :
   (A) (1,0,0)
   (B) (2,0,0)
   (C) $(\sqrt{5}, 0, 0)$
   (D) (0,0,0)

10. The common region determined by all the constraints of a linear programming problem is called :
   (A) an unbounded region
   (B) an optimal region
   (C) a bounded region
   (D) a feasible region

11. Let E be an event of a sample space S of an experiment, then $P(S \mid E) =$
   (A) $P(S \cap E)$
   (B) $P(E)$
   (C) 1
   (D) 0

12. If $A = [a_{ij}]$ be a 3×3 matrix, where $a_{ij} = i - 3j$, then which of the following is false?
   (A) $a_{11} < 0$
   (B) $a_{12} + a_{21} = -6$
   (C) $a_{13} > a_{31}$
   (D) $a_{31} = 0$

13. The derivative of $\tan^{-1}(x^2)$ w.r.t. x is :
   (A) $\frac{x}{1+x^4}$
   (B) $\frac{2x}{1+x^4}$
   (C) $-\frac{2x}{1+x^4}$
   (D) $\frac{1}{1+x^4}$

14. The degree of the differential equation $(y'')^2 + (y')^3 = x\sin(y')$ is :
   (A) 1
   (B) 2
   (C) 3
   (D) not defined

15. The unit vector perpendicular to both vectors $\hat{i} + \hat{k}$ and $\hat{i} - \hat{k}$ is:
   (A) $2\hat{j}$
   (B) $\hat{j}$
   (C) $\frac{\hat{i}-\hat{k}}{\sqrt{2}}$
   (D) $\frac{\hat{i}+\hat{k}}{\sqrt{2}}$

16. Direction ratios of a vector parallel to line $\frac{x-1}{2} = -y = \frac{2z+1}{6}$ are :
   (A) $2, -1, 6$
   (B) 2,1,6
   (C) 2,1,3
   (D) $2, -1, 3$

17. If $F(x) = \begin{bmatrix} \cos x & -\sin x & 0 \\ \sin x & \cos x & 0 \\ 0 & 0 & 1 \end{bmatrix}$ and $[F(x)]^2 = F(kx)$, then the value of k is :
   (A) 1
   (B) 2
   (C) 0
   (D) -2

18. If a line makes an angle of 30° with the positive direction of x-axis, 120° with the positive direction of y-axis, then the angle which it makes with the positive direction of z -axis is :
   (A) 90°
   (B) 120°
   (C) 60°

(D) 0°

Questions number 19 and 20 are Assertion and Reason based questions. Two statements are given, one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer from the codes (A), (B), (C) and (D) as given below.

(A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).

(B) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A).

(C) Assertion (A) is true, but Reason (R) is false.

(D) Assertion (A) is false, but Reason (R) is true.

19. **Assertion (A) :** For any symmetric matrix A, $B'AB$ is a skew-symmetric matrix.
Reason (R) : A square matrix P is skew-symmetric if $P' = -P$.

20. **Assertion (A) :** For two non-zero vectors $\vec{a}$ and $\vec{b}$, $\vec{a} \cdot \vec{b} = \vec{b} \cdot \vec{a}$.
Reason (R) : For two non-zero vectors $\vec{a}$ and $\vec{b}$, $\vec{a} \times \vec{b} = \vec{b} \times \vec{a}$.

SECTION : B

21. (a) Find the value of $\tan^{-1}\left(-\frac{1}{\sqrt{3}}\right) + \cot^{-1}\left(\frac{1}{\sqrt{3}}\right) + \tan^{-1}\left[\sin\left(-\frac{\pi}{2}\right)\right]$.

OR

(b) Find the domain of the function $f(x) = \sin^{-1}(x^2 - 4)$. Also, find its range.

22. (a) If $f(x) = |\tan 2x|$, then find the value of $f'(x)$ at $x = \frac{\pi}{3}$.

OR

(b) If $y = \operatorname{cosec}(\cot^{-1} x)$, then prove that $\sqrt{1 + x^2}\frac{dy}{dx} - x = 0$.

23. If M and m denote the local maximum and local minimum values of the function $f(x) = x + \frac{1}{x} (x \neq 0)$ respectively, find the value of $(M - m)$.

24. Find : $\int \frac{e^{4x}-1}{e^{4x}+1} dx$

25. Show that $f(x) = e^x - e^{-x} + x - \tan^{-1} x$ is strictly increasing in its domain.

SECTION : C

26. (a) If $x = e^{\cos 3t}$ and $y = e^{\sin 3t}$, prove that $\frac{dy}{dx} = -\frac{y\log x}{x\log y}$.

OR

(b) Show that :

27. (a) Evaluate:

$$\int_{-2}^{2} \sqrt{\frac{2 - x}{2 + x}}\, dx$$

OR

(b) Find:

$$\int \frac{1}{x[(\log x)^2 - 3\log x - 4]} dx$$

28. (a) Find the particular solution of the differential equation given by $2xy + y^2 - 2x^2\frac{dy}{dx} = 0; y = 2$, when $x = 1$.

OR

(b) Find the general solution of the differential equation:

$$ydx = (x + 2y^2)dy$$

29. The position vectors of vertices of ΔABC are $A(2\hat{i} - \hat{j} + \hat{k})$, $B(\hat{i} - 3\hat{j} - 5\hat{k})$ and $C(3\hat{i} - 4\hat{j} - 4\hat{k})$. Find all the angles of $\triangle ABC$.

30. A pair of dice is thrown simultaneously. If X denotes the absolute difference of the numbers appearing on top of the dice, then find the probability distribution of X .

31. Find:

$$\int x^2 \cdot \sin^{-1}\left(x^{3/2}\right)dx$$

SECTION : D

32. (a) Show that a function $f: R \rightarrow R$ defined by $f(x) = \frac{2x}{1+x^2}$ is neither one-one nor onto. Further, find set A so that the given function $f: R \rightarrow A$ becomes an onto function.

OR

(b) A relation R is defined on $N \times N$ (where N is the set of natural numbers) as :

$$(a, b)R(c, d) \Leftrightarrow a - c = b - d$$

Show that R is an equivalence relation.

33. Find the equation of the line which bisects the line segment joining points A(2,3,4) and B(4,5,8) and is perpendicular to the lines $\frac{x-8}{3} = \frac{y+19}{-16} = \frac{z-10}{7}$ and $\frac{x-15}{3} = \frac{y-29}{8} = \frac{z-5}{-5}$.

34. (a) Solve the following system of equations, using matrices:

$$\frac{2}{x} + \frac{3}{y} + \frac{10}{z} = 4, \frac{4}{x} - \frac{6}{y} + \frac{5}{z} = 1, \frac{6}{x} + \frac{9}{y} - \frac{20}{z} = 2$$

where $x, y, z \neq 0$

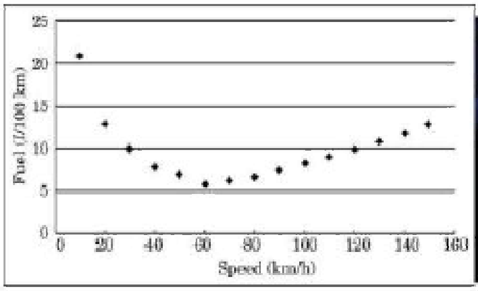
OR

(b) If $A = \begin{bmatrix} 1 & \cot x \\ -\cot x & 1 \end{bmatrix}$, show that $A'A^{-1} = \begin{bmatrix} -\cos 2x & -\sin 2x \\ \sin 2x & -\cos 2x \end{bmatrix}$.

35. If A_1 denotes the area of region bounded by $y^2 = 4x, x = 1$ and x -axis in the first quadrant and A_2 denotes the area of region bounded by $y^2 = 4x, x = 4$, find $A_1 : A_2$.

SECTION : E

36. Overspeeding increases fuel consumption and decreases fuel economy as a result of tyre rolling friction and air resistance. While vehicles reach optimal fuel economy at different speeds, fuel mileage usually decreases rapidly at speeds above 80 km/h.



The relation between fuel consumption F(l/100 km) and speed V(km/h) under some constraints is given as $F = \frac{V^2}{500} - \frac{V}{4} + 14.$

On the basis of the above information, answer the following questions:

(i) Find F, when V = 40 km/h.

(ii) Find $\frac{dF}{dV}$.

(iii) (a) Find the speed V for which fuel consumption F is minimum.

OR

(iii) (b) Find the quantity of fuel required to travel 600 km at the speed V at which $\frac{dF}{dV} = -0.01.$

37. The month of September is celebrated as the Rashtriya Poshan Maah across the country. Following a healthy and well-balanced diet is crucial in order to supply the body with the proper nutrients it needs. A balanced diet also keeps us mentally fit and promotes improved level of energy.

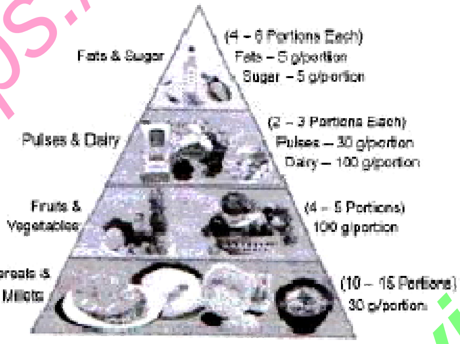


Figure-1

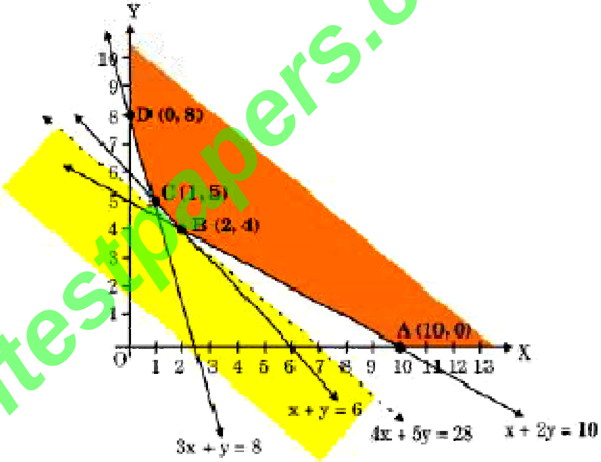


Figure-2

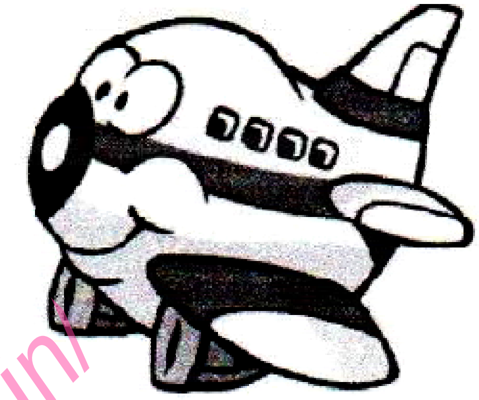
A dietician wishes to minimize the cost of a diet involving two types of foods, food X(xkg) and food Y(ykg) which are available at the rate of ₹16/kg and ₹20/kg respectively. The feasible region satisfying the constraints is shown in Figure-2.

On the basis of the above information, answer the following questions :

(i) Identify and write all the constraints which determine the given feasible region in Figure-2.

(ii) If the objective is to minimize cost Z = 16x + 20y, find the values of x and y at which cost is minimum. Also, find minimum cost assuming that minimum cost is possible for the given unbounded region.

38. Airplanes are by far the safest mode of transportation when the number of transported passengers are measured against personal injuries and fatality totals.



Previous records state that the probability of an airplane crash is 0.00001%. Further, there are 95% chances that there will be survivors after a plane crash. Assume that in case of no crash, all travellers survive.

12TH MATHAMATICS

Time allowed : 3hour Maximum Marks = 80

1. A function $f: R_+ \rightarrow R$ (where R_+ is the set of all non-negative real numbers) defined by $f(x) = 4x + 3$ is :
   (A) one-one but not onto
   (B) onto but not one-one
   (C) both one-one and onto
   (D) neither one-one nor onto
2. If a matrix has 36 elements, the number of possible orders it can have, is :
   (A) 13
   (B) 3
   (C) 5
   (D) 9
3. Which of the following statements is true for the function $f(x) = \begin{cases} x^2 + 3, & x \neq 0 \\ 1, & x = 0 \end{cases}$?
   (A) $f(x)$ is continuous and differentiable $\forall x \in \mathbb{R}$
   (B) $f(x)$ is continuous $\forall x \in \mathbb{R}$
   (C) $f(x)$ is continuous and differentiable $\forall x \in \mathbb{R} - \{0\}$
   (D) $f(x)$ is discontinuous at infinitely many points
4. Let $f(x)$ be a continuous function on $[a, b]$ and differentiable on (a, b). Then, this function $f(x)$ is strictly increasing in (a, b) if
   (A) $f'(x) < 0, \forall x \in (a, b)$
   (B) $f'(x) > 0, \forall x \in (a, b)$
   (C) $f'(x) = 0, \forall x \in (a, b)$
   (D) $f(x) > 0, \forall x \in (a, b)$
5. If $\begin{bmatrix} x + y & 2 \\ 5 & xy \end{bmatrix} = \begin{bmatrix} 6 & 2 \\ 5 & 8 \end{bmatrix}$, then the value of $\left(\frac{24}{x} + \frac{24}{y}\right)$ is :
   (A) 7
   (B) 6
   (C) 8
   (D) 18
6. $\int_a^b f(x)dx$ is equal to :
   (A) $\int_a^b f(a - x)dx$
   (B) $\int_a^b f(a + b - x)dx$
   (C) $\int_a^b f(x - (a + b))dx$
   (D) $\int_a^b f((a - x) + (b - x))dx$
7. Let θ be the angle between two unit vectors $\hat{a}$ and $\hat{b}$ such that $\sin\theta = \frac{3}{5}$ Then, $\hat{a} \cdot \hat{b}$ is equal to :
   (A) $\pm\frac{3}{5}$
   (B) $\pm\frac{3}{4}$
   (C) $\pm\frac{4}{5}$
   (D) $\pm\frac{4}{3}$
8. The integrating factor of the differential equation $(1 - x^2)\frac{dy}{dx} + xy = ax, -1 < x < 1$, is :
   (A) $\frac{1}{x^2 - 1}$
   (B) $\frac{1}{\sqrt{x^2 - 1}}$
   (C) $\frac{1}{1 - x^2}$
   (D) $\frac{1}{\sqrt{1 - x^2}}$
9. If the direction cosines of a line are $\sqrt{3}k, \sqrt{3}k, \sqrt{3}k$, then the value of k is :
   (A) ± 1

(B) $\pm\sqrt{3}$
(C) ± 3
(D) $\pm\frac{1}{3}$

10. A linear programming problem deals with the optimization of a/an
    (A) logarithmic function
    (B) linear function
    (C) quadratic function
    (D) exponential function

11. If $P(A \mid B) = P(A' \mid B)$, then which of the following statements is true?
    (A) $P(A) = P(A')$
    (B) $P(A) = 2P(B)$
    (C) $P(A \cap B) = \frac{1}{2}P(B)$
    (D) $P(A \cap B) = 2P(B)$

12. $\begin{vmatrix} x+1 & x-1 \\ x^2+x+1 & x^2-x+1 \end{vmatrix}$ is equal to :
    (A) $2x^3$
    (B) 2
    (C) 0
    (D) $2x^3 - 2$

13. The derivative of $\sin(x^2)$ w.r.t. x, at $x = \sqrt{\pi}$ is
    (A) 1
    (B) -1
    (C) $-2\sqrt{\pi}$
    (D) $2\sqrt{\pi}$

14. The order and degree of the differential equation $\left[1 + \left(\frac{dy}{dx}\right)^2\right]^3 = \frac{d^2y}{dx}$ respectively are :
    (A) 1,2
    (B) 2, 3
    (C) 2,1
    (D) 2,6

15. The vector with terminal point $A(2,-3,5)$ and initial point $B(3,-4,7)$ is:
    (A) $\hat{i} - \hat{j} + 2\hat{k}$
    (B) $\hat{i} + \hat{j} + 2\hat{k}$
    (C) $-\hat{i} - \hat{j} - 2\hat{k}$
    (D) $-\hat{i} + \hat{j} - 2\hat{k}$

16. The distance of point $P(a, b, c)$ from y -axis is :
    (A) b
    (B) b^2
    (C) $\sqrt{a^2 + c^2}$
    (D) $a^2 + c^2$

17. The number of corner points of the feasible region determined by constraints $x \geq 0, y \geq 0, x + y \geq 4$ is :
    (A) 0
    (B) 1
    (C) 2
    (D) 3

18. If A and B are two non-zero square matrices of same order such that $(A + B)^2 = A^2 + B^2$, then :
    (A) $AB = O$
    (B) $AB = -BA$
    (C) $BA = O$
    (D) $AB = BA$

Questions No. 19 & 20, are Assertion (A) and Reason (R) based questions carrying 1 mark each. Two statements are given, one labelled Assertion (A) and the other labelled Reason (R).
Select the correct answer from the codes (A), (B), (C) and (D) as given below :

(A) Both Assertion (A) and Reason (R) are true and the Reason (R) is the correct explanation of Assertion (A).
(B) Both Assertion (A) and Reason (R) are true and Reason (R) is not the correct explanation of the Assertion (A).
(C) Assertion (A) is true, but Reason (R) is false.
(D) Assertion (A) is false, but Reason (R) is true.

19. **Assertion (A) :** For matrix $A = \begin{bmatrix} 1 & \cos\theta & 1 \\ -\cos\theta & 1 & \cos\theta \\ -1 & -\cos\theta & 1 \end{bmatrix}$, where $\theta \in [0,2\pi]$, $|A| \in [2,4]$.

Reason (R): $\cos\theta \in [-1,1], \forall\theta \in [0,2\pi]$.

20. **Assertion (A) :** A line in space cannot be drawn perpendicular to x, y and z axes simultaneously.
Reason (R) : For any line making angles, α, β, γ with the positive directions of x, y and z axes respectively, $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$.

SECTION B

In this section there are 5 very short answer type questions of 2 marks each.

21. (A) Check whether the function $f(x) = x^2|x|$ is differentiable at $x = 0$ or not.

OR

(b) If $y = \sqrt{\tan\sqrt{x}}$, prove that $\sqrt{x}\frac{dy}{dx} = \frac{1+y^4}{4y}$

22. Show that the function $f(x) = 4x^3 - 18x^2 + 27x - 7$ has neither maxima nor minima.

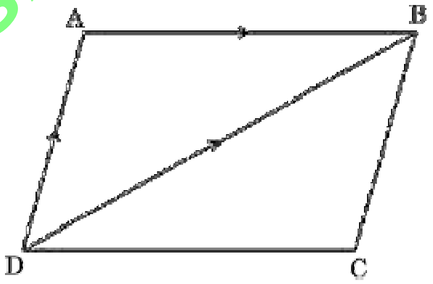
23. (a) Find: $\int x\sqrt{1+2x}dx$

OR

(b) Evaluate:
$\int_0^{\frac{\pi^2}{4}} \frac{\sin\sqrt{x}}{\sqrt{x}}dx$

24. If $\vec{a}$ and $\vec{b}$ are two non-zero vectors such that $(\vec{a}+\vec{b}) \perp \vec{a}$ and $(2\vec{a}+\vec{b}) \perp \vec{b}$, then prove that $|\vec{b}| = \sqrt{2}|\vec{a}|$.

25. In the given figure, ABCD is a parallelogram. If $\overrightarrow{AB} = 2\hat{i} - 4\hat{j} + 5\hat{k}$ and $\overrightarrow{DB} = 3\hat{i} - 6\hat{j} + 2\hat{k}$, then find $\overrightarrow{AD}$ and hence find the area of parallelogram ABCD.



SECTION : C

In this section there are 6 short answer type question of 3 marks each.

26. (A)A relation R on set $A = \{1,2,3,4,5\}$ is defined as $R = \{(x,y) : |x^2 - y^2| < 8\}$. Check whether the relation R is reflexive, symmetric and transitive.

OR

(B) A function f is defined from $R \to R$ as $f(x) = ax + b$, such that $f(1) = 1$ and $f(2) = 3$. Find function $f(x)$. Hence, check whether function $f(x)$ is one-one and onto or not.

27. (A) If $\sqrt{1-x^2} + \sqrt{1-y^2} = a(x-y)$, prove that $\frac{dy}{dx} = \sqrt{\frac{1-y^2}{1-x^2}}$.

OR

(B) If $y = (\tan\ x)^x$, then find $\frac{dy}{dx}$.

28. (B)Find:

$$\int \frac{x^2}{(x^2+4)(x^2+9)} dx$$

OR

(B)Evaluate : $\int_1^3 (|x-1| + |x-2| + |x-3|)dx$

29. Find the particular solution of the differential equation given by $x^2 \frac{dy}{dx} - xy = x^2 \cos^2\left(\frac{y}{2x}\right)$, given that when $x = 1, y = \frac{\pi}{2}$.

30. Solve the following linear programming problem graphically:
Maximise $z = 500x + 300y$,
subject to constraints

$$x + 2y \le 12$$
$$2x + y \le 12$$
$$4x + 5y \ge 20$$
$$x \ge 0, y \ge 0$$

31. E and F are two independent events such that $P(\bar{E}) = 0.6$ and $P(E \cup F) = 0 - 6$. Find $P(F)$ and $P(\bar{E} \cup \bar{F})$.

SECTION D

In this section are 4 long answer type question of 5 mark each.

32. (A) If $A = \begin{bmatrix} 1 & -2 & 0 \\ 2 & -1 & -1 \\ 0 & -2 & 1 \end{bmatrix}$, find A^{-1} and use it to solve the following system of equations:

$x - 2y = 10, 2x - y - z = 8, -2y + z = 7$

OR

(B) If $\begin{bmatrix} -1 & a & 2 \\ 1 & 2 & x \\ 3 & 1 & 1 \end{bmatrix}$ and $A^{-1} = \begin{bmatrix} 1 & -1 & 1 \\ -8 & 7 & -5 \\ b & y & 3 \end{bmatrix}$ find the value of $(a + x) - (b + y)$.

33. (a) Evaluate:

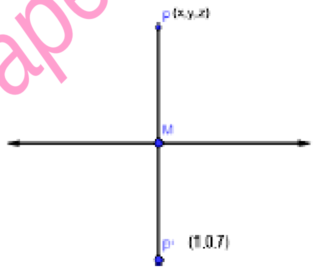
$$\int_0^{\frac{\pi}{4}} \frac{\sin x+\cos x}{9+16\sin 2x}dx$$

OR

(b) Evaluate: $\int_0^{\frac{\pi}{2}} \sin 2x\tan^{-1}(\sin x)dx$

34. Using integration, find the area of the ellipse $\frac{x^2}{16}+\frac{y^2}{4}=1$, included between the lines $x=-2$ and $x=2$.

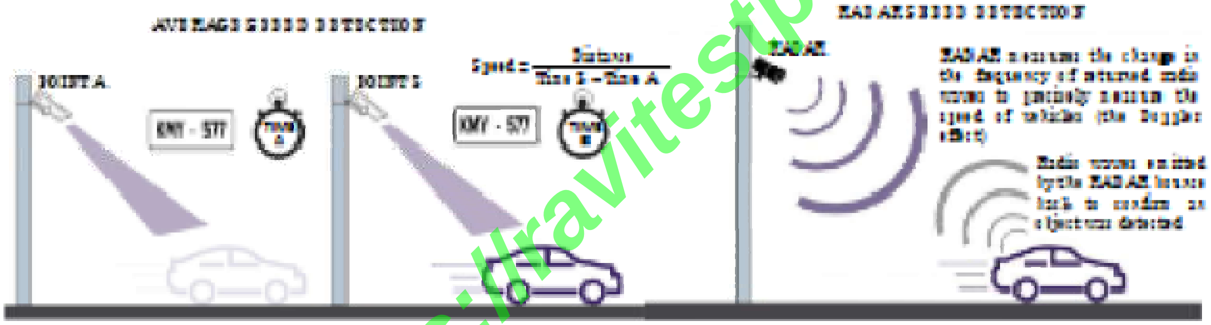
35. The image of point $P(x,y,z)$ with respect to line $\frac{x}{1}=\frac{y-1}{2}=\frac{z-2}{3}$ is $P'(1,0,7)$. Find the coordinates of point P.



SECTION : E

In this section there are 3 case study based question of 4 marks each.

36. The traffic police has installed Over Speed Violation Detection (OSVD) system at various locations in a city. These cameras can capture a speeding vehicle fron a distance of 300 m and even function in the dark.



A camera is installed on a pole at the height of 5 r . It detects a car travelling away from the pole at the speed of 20 m/s. At any point, x n away from the base of the pole, the angle of elevation of the speed camera from the car C is θ.
On the basis of the above information, answer the following questions:

(i) Express θ in terms of height of the camera installed on the pole and x. 1
(ii) Find $\frac{d\theta}{dx}$. 1
(iii) (a) Find the rate of change of angle of elevation with respect to time at an instant when the car is 50 m away from the pole. 2

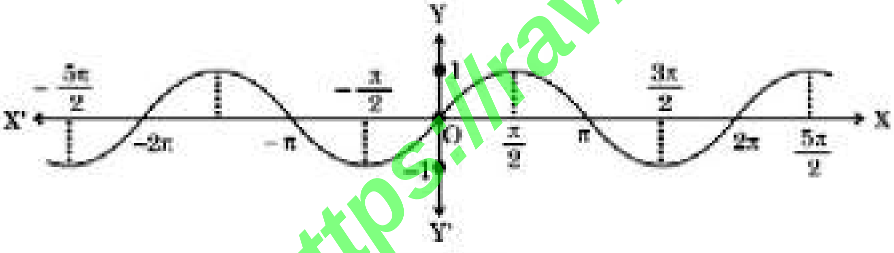
OR

(iii) (b) If the rate of change of angle of elevation with respect to tine of another car at a distance of 50 m from the base of the pole is $\frac{3}{101}$ rad/s, then find the speed of the car. 2

37. According to recent research, air turbulence has increased in various regions around the world due to climate change. Turbulence makes flights bumpy and often delays the flights.
Assume that, an airplane observes severe turbulence, moderate turbulence or light turbulence with equal probabilities. Further, the chance of an airplane reaching late to the destination are 55%, 37% and 17% due to severe, noderate and light turbulence respectively.

On the basis of the above information, answer the following questions:
(i) Find the probability that an airplane reached its destination late. 2
(ii) If the airplane reached its destination late, find the probability that it was due to moderate turbulence. 2

38. If a function f: X → Y defined asf(x) = y is one-one and onto, then we can define a unique function g: Y → X auch that g(y) = x, where x = X and y = f(x), y ∈ Y. Function g is called the inverse of function f.
The domain of sine function is R and function sine : R → R is neither one-one nor onto. The following graph show the sine function.
Let sine function be defined from set A to [−1,1] such that inverae of sine function exists, i.e., $\sin^{-1}$ x is defined fron [−1,1] to A .



y = sin x

On the basis of the above information, answer the following questions:
(i) If A is the interval other than principal value branch, give an exanple of one such interval. 1
(ii) If $\sin^{-1}$ (x) is defined from [−1,1] to its principal value branch, find the value of $\sin^{-1}\left(-\frac{1}{2}\right) - \sin^{-1}(1)$. 1
(iii) (a) Draw the graph of $\sin^{-1}$ x fron [−1,1] to its principal value branch. 2

OR

(iii) (b) Find the domain and range of $f(x) = 2\sin^{-1}(1 - x)$. 2

(i) ThisQuestionpapercontains **38**questions.**All**questionsare**compulsory**

(ii) ThisQuestion paperis dividedinto**five**Sections-**A,B, C, D**and **E.**

‘ii) In**SectionA**,Questionsno.**1**to**18**are**multiplechoicequestions(MCQs)**andQuestionsno.**19**and**20**are**Assertion-Reasonbased**questionsof**1markeach**.

v) In **Section B,** Questions no. **21** to **25** are **Very Short Answer (VSA)-type** questions, carrying **2 markseach**.

/) In**SectionC,**Questionsno. **26**to**31** are**ShortAnswer(SA)-type**questions,carrying**3markseach**.

/i) In**SectionD,**Questionsno. **32**to**35** are**LongAnswer(LA)-type**questions,carrying**5markseach**.

/ii) In**SectionE,**Questionsno.**36**to**38**are**Casestudy-basedquestions,**carrying**4markseach.**

/iii) Thereisnooverall choice.However,an internalchoicehasbeen providedin2 questionsinSectionB,3questionsin SectionC,2 questionsin SectionDand one subparteach in2questionsofSection E.

x) Useofcalculatorsis**not**allowed

SECTION A$\left[1\times20=20\right]$

If A is a square matrix of order 3, and $|adj\ A| = 729$, then $|A|$ is equal to

(a) 3 (b) 9 (c) ± 81 (d) None of these

Value of $tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right)$ is

) $tan^{-1}x$ (b) $2\ tan^{-1}x$ (c) $\frac{1}{2}tan^{-1}x$ (d) None of these

f $\begin{bmatrix} 2x+y & 4x \\ 5x-7 & 4x \end{bmatrix} = \begin{bmatrix} 7 & 7y-13 \\ y & x+6 \end{bmatrix}$, then find the value of x and y are

x= 3, y= 1 (b) x = 2, y = 3 (c) x = 2, y = 4 (d) x = 3, y = 3

4. If $\begin{vmatrix} 2x & 5 \\ 8 & x \end{vmatrix} = \begin{vmatrix} 6 & -2 \\ 7 & 3 \end{vmatrix}$ then the value of x is

(a) 3 (b) $\pm$ 3 (c) $\pm$ 6 (d) 6

5. If there are two values of p for which makes determinant,$\Delta = \begin{vmatrix} 1 & -2 & 5 \\ 2 & p & -1 \\ 0 & 4 & 2p \end{vmatrix} = 86$, then the sum of these number is

(a) 4 (b) 5 (c) -4 (d) 9

6. If A is a square matrix and A^2 = A, then $(I + A)^2$ – 3 A is equal to

(a) I (b) A (c) 2 A (d) 3 I

7. The function f(x) $= \frac{x}{2} + \frac{2}{x}$ has a local minima at x is equal to

(a) 2 (b) 1 (c) 0 (d) -2

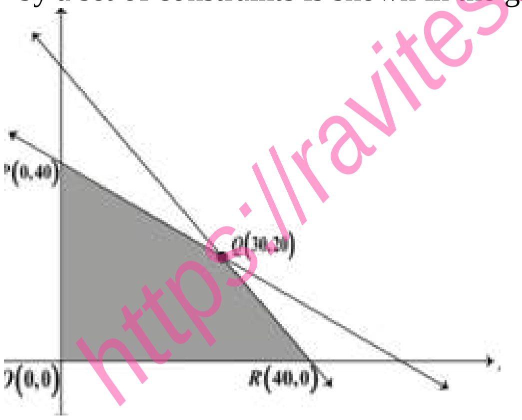
8. $\int \frac{\sqrt{\tan x}}{\sin x \cos x}$ dx is equal to

(a) $2\sqrt{\cot x}$ (b) $\frac{\sqrt{\tan x}}{2}$ (c) $\frac{2}{\sqrt{\tan x}}$ (d) $2\sqrt{\tan x}$

$\int_0^{\sqrt{5}}[x]$dx is

) $2\sqrt{5}$ (b) $2\sqrt{5} - 1$ (c) $2\sqrt{5} - 2$ (d) $2\sqrt{5} - 3$

. For the linear programming problem(**LPP**), the objective function is Z= x+y. The feasible region determined by a set of constraints is shown in the graph



Which of the following statements is true?

(a) Maximum value of Z is at P (40,0) (b) Maximum value of Z is at Q(30,20)

(c)Value 0f Z at Q (30,20) is less than value of Z at R (40,0)

(d) Value of Z at R(40,0) is less than value of Z at P(40,0)

11. The solution of differential equation tan y sec ^{2} x + tan x sec ^{2} y = 0 is

(a) tan x + tan y = C (b) tan x – tan y =C (c) tan x . tan y = C (d) $\frac{tanx}{\tan y}$ = C

12. Find the integrating factor of (1 $-$x^2) $\frac{dy}{dx}$ $-$xy= 1

(a) – x (b) $\frac{x}{1+x^2}$ (c) $\sqrt{1-x^2}$ (d) $\frac{1}{2}$ log (1 $-x^2$)

13. If $|\vec{a}|$ = 10 , $|\vec{b}|$ =2 and $\vec{a}$. $\vec{b}$ =12, then the value of $|\vec{a} x \vec{b}|$ is

(a) 5 (b) 10 (c) 14 (d) 16

14.If A and B are two events such that P(A) = 0.4, P(B) = 0.8 and P(B/A) =0.6

then P(AUB) is equal to

(a) 0.24 (b) 0.3 (c) 0.48 (d) 0.96

15. The lines $\frac{x}{1} = \frac{y}{2} = \frac{z}{3}$ and $\frac{x-1}{-2} = \frac{y+3}{-4} = \frac{z-5}{-6}$are

(a) parallel (b) Intersecting (c)skew (d) coincident

16.If$\vec{a}$ + $\vec{b}$ + $\vec{c}$ = $\vec{0}$,$|\vec{a}|$ = 3, $|\vec{b}|$ = 5, $|\vec{c}|$ = 7 then the angle between $\vec{a}$ and $\vec{b}$ is

(a) $\frac{\pi}{6}$ (b) $\frac{2\pi}{3}$ (c) $\frac{5\pi}{3}$ (d) $\frac{\pi}{3}$

17 .Area of the region bounded by the curve y = cos x between x = 0 and x = π is

(a) 1 sq units (b) 2 sq units (c) 3 sq units (d) 4 sq units

18. If the function f(x) = $\begin{cases} 4 \times 5^x & x < 0 \\ 8a + x & x \geq 0 \end{cases}$ is continuous , then the value of a is

(a) $\frac{1}{2}$ (b) 2 (c) 2 (d) 4

ASSERTION-REASON BASED QUESTIONS

Question numbers 19 and 20 are Assertion-Reason based questions carrying 1 mark each. Two statements are given, one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer from the options (A), (B), (C) and (D) as given below.

(a) Both (A) and (R) are true and (R) is the correct explanation of (A)

(b) Both (A) and (R) are true but (R) is not the correct explanation of (A).

(c) (A) is true but (R) is false

(d) (A) is false but (R) is true.

19 . **Assertion** (A): A function f : A →B can not be onto if n(A) < n(B)

Reason (R): A function f is onto if every element of co -domain has at least one pre image in the domain

20.**Assertion** (A) :If f(x) = $|\cos x|$, then f '$(\frac{\pi}{4})$ = - $\frac{1}{\sqrt{2}}$ and f ' $(\frac{3\pi}{4})$ = $\frac{1}{\sqrt{2}}$

Reason (R) : f(x) = $|\cos x| = \begin{cases} \cos x & x \epsilon \left(0, \frac{\pi}{2}\right) \\ -\cos x & x \epsilon \left(\frac{\pi}{2}, \pi\right) \end{cases}$

SECTION B **[2 X 5 = 10]**

21. Solve the equation cos (tan^{-1} x) = sin (cot$^{-1}\frac{3}{4}$)

22. Differentiate $\frac{x}{\sin x}$ with respect to sin x

23. Find $\frac{dy}{dx}$, if tan $^{-1}$ ($x^2 + y^2$)= a

OR

Find the derivative of $(\sin x)^{\cos x}$

24. Find the unit vector in the direction of the sum of the vectors

$\vec{a}$= 2$\hat{\imath}$+2 $\hat{\jmath}$ - 5 $\hat{k}$ and $\vec{b}$ = 2 $\hat{\imath}$ + $\hat{\jmath}$ +3 $\hat{k}$

OR

Find $|\vec{a} X \vec{b}|$, if $\vec{a}$ = 2$\hat{\imath}$+$\hat{\jmath}$ +3$\hat{k}$ and $\vec{b}$ = 3 $\hat{\imath}$+5$\hat{\jmath}$- 2 $\hat{k}$

25. **Find λ when the projection of $\vec{p}$ = λ î+ĵ+4k̂ and $\vec{q}$ = 2î + 6ĵ + 3k̂ is 4 units.**

SECTION C **[6 X 3 = 18]**

26. Sand is pouring from a pipe at the rate of 12 cm ^{3}/sec. The falling sand forms a cone on the ground in such a way that the height of the cone is always one sixth of the radius of the base. How fast is the height of the sand cone increasing with the height is 4 cm

27. Prove that f(x) = sin x + $\sqrt{3}$ cos x has maximum value at x = $\frac{\pi}{6}$

28. Find the vector equation of the line passing through the point (1,2,-4) and perpendicular to the two lines $\frac{x-8}{3} = \frac{y+19}{-16} = \frac{z-10}{7}$ and $\frac{x-15}{3} = \frac{y-29}{8} = \frac{z-5}{-5}$

OR

Find shortest distance between two lines $\vec{r} = \hat{\imath} + \hat{\jmath} + \lambda(2\,\hat{\imath} - \hat{\jmath} + \hat{k})$ and

$\vec{r} = 2\,\hat{\imath} + \hat{\jmath} - \hat{k} + \mu(3\,\hat{\imath} - 5\,\hat{\jmath} + 2\,\hat{k})$

29.Find $\int_{o}^{\frac{\pi}{4}} \log(1+\tan x)\,dx$

OR

Evaluate $\int \frac{sin^6x+cos^6x}{sin^2xcos^2x}$dx

30. Maximise Z = x+ y subject to the constraints

x+ 4 y ≤ 8 , 2 x + 3 y≤ 12 , 3 x + y ≤ 9, x ≥ 0 and y ≥ 0

31. A letter is known to have come either from ‘TATA NAGAR’ or from ‘ CALCUTTA’. On the envelop , just two consecutive letters TA is only visible. What is the probability that the letter came from ‘TATA NAGAR’

OR

Find the probability distribution of the maximum of the two scores obtained when a die is thrown twice.

SECTION D **[4 X 5 = 20]**

32. Find the area of the region bounded by the ellipse $\frac{x^2}{16} + \frac{y^2}{9} = 1$

OR

Find the area of the region bounded by the curve $y = \sqrt{16 - x^2}$ and X – axis

33.If A = $\begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}$ and B = $\begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix}$ then find BA and use this to solve the system of equations, x-y = 3 , 2x + 3y + 4z = 17 , y+ 2z = 7

34. If $(x-a)^2 + (y-b)^2 = c^2$, for some c> 0, prove that $\frac{\left[1+\left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}}}{\frac{d^2y}{dx^2}}$ is a constant independent of a and b

OR

If $y = e^{a\cos^{-1}x}$, -1 ≤ x ≤ 1 , show that $(1 - x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} - a^2y = 0$

35. Find the foot of the perpendicular from the point (2,3,-8) with respect to the line

$\frac{4-x}{2} = \frac{y}{6} = \frac{1-z}{3}$. Also find the perpendicular distance from the given point to the line

SECTION E [4 X 3 = 12]

36.The total cost C(n) of manufacturing n earphone sets per day in the house of Spark Electronic Limited is given by $C(n) = 400 + 4n + 0.0001n^2$ dollars.

Each earphone set is sold at $q= 10-0.0004n$ dollars , where $n \geq 0, q \geq 0$.

The daily profit in dollars is determined by the equation $P(n) = qn - C(n)$

Based on the above information answer the following questions:

(i) The marginal cost, M(n) is the change in total production cost that comes from making or producing one additional unit. It is determined by the instantaneous rate of change of the total cost . Find the marginal cost M(n) of 10 earphone sets.

(ii) What quantity of daily production maximizes the profit

37. A coach is training 3 players. He observes that the player A can hit a target 4 times in 5 shots, player B can hit 3 times in 4 shots and the player C can hit 2 times in 3 shots.

From this situation answer the following:

(i) Let the target is hit by A and B: the target is hit by B and, C: the target is hit by A and C. Then, what is the probability that A, B and, C all will hit,

(ii). Referring to (i), what is the probability that B, C will hit and A will lose?

(iii) With reference to the events mentioned in (i), what is the probability that 'any two of A, B and C will hit?

OR

(iv) What is the probability that 'none of them will hit the target'?

38.Sherlin and Danju are playing Ludo at home during Covid-19. While rolling the dice, Sherlin's sister Raji observed and noted the possible outcomes of the throw every time belongs to set {1,2,3,4,5,6}. Let A be the set of players while B be the set of all possible outcomes.

A = {S, D , B = {1,2,3,4,5,6}

(i) Verify that $R : B \rightarrow B$ be defined by R = {(x, y): $y is divisible by x$ } is an equivalence relation or not

(ii) Let R: $B \rightarrow B$ be defined by R={(1,1),(1,2), (2,2), (3,3), (4,4), (5,5),(6,6)}, then verify R isreflexive , symmetric or transitive

(iii) Raji wants to know the number of functions from A to B. How many number of functions are possible?

OR

(iv) Raji wants to know the number of relations possible from A to B. How many numbers of relations are possible?

……………………………………………..

Paper : 5

1. If B = $\begin{bmatrix} 5 & 2a & 1 \\ 0 & 2 & 1 \\ a & 3 & -1 \end{bmatrix}$ is the inverse of a 3 × 3 matrix A, then the sum of all values of a for which det (A) + 1 = 0, is

(a) 0
(b) - 1
(c) 1
(d) 2

2. If $A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$, then A^{2023} is equal to

(a) $\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$
(b) $\begin{bmatrix} 0 & 2023 \\ 0 & 0 \end{bmatrix}$
(c) $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$
(d) $\begin{bmatrix} 2023 & 0 \\ 0 & 2023 \end{bmatrix}$

3. Find the value of k for which $f(x) = \begin{cases} \frac{1-\cos}{8x^2} & , \text{ when } x \neq 0 \\ k & , \text{ when } x = 0 \end{cases}$ is continuous at $x = 0$.

(a) 1
(b) 2
(c) 3
(d) 4

4. If $\overline{a}$ and $\overline{b}$ are two collinear vectors, then which of the following are incorrect:

(a) $\overline{b} = \lambda\overline{a}$, for some scalar λ
(b) $\overline{a} = \pm\overline{b}$
(c) The respective components of $\overline{a}$ and $\overline{b}$ are proportional
(d) Both the vectors $\overline{a}$ and $\overline{b}$ have same direction, but different magnitudes.

5. Write the sum of the order and degree of the following differential equation: $\frac{d}{dx}\left\{\left(\frac{dy}{dx}\right)^3\right\} = 0$.

(a) 1
(b) 2
(c) 3
(d) 4

6. The maximum value of the object function $Z = 5x + 10y$ subject to the constraints $x + 2y \leq 120, x + y \geq 60, x - 2y \geq 0, x \geq 0, y \geq 0$ is

(a) 300
(b) 600
(c) 400
(d) None of these

7. The vector equation of the line passing through the point (–1, 5, 4) and perpendicular to the plane z = 0 is

(a) $\vec{r} = -\hat{i} + 5\hat{j} + 4\hat{k} + \lambda((\hat{i} + \hat{j})$
(b) $\vec{r} = -\hat{i} + 5\hat{j} + (4+\lambda)\hat{k}$
(c) $\vec{r} = \hat{i} - 5\hat{j} - 4\hat{k} + \lambda\hat{k}$
(d) $\vec{r} = \lambda\hat{k}$

8. Unit vector along $\overrightarrow{PQ}$, where coordinates of P and Q respectively are $(2,1,-1)$ and $(4,4,-7)$, is

(a) $2\hat{i} + 3\hat{j} - 6\hat{k}$

(b) $-2\hat{i}-3\hat{j}+6\hat{k}$

(c) $\frac{-2\hat{i}}{7}-\frac{3\hat{j}}{7}+\frac{6\hat{k}}{7}$

(d) $\frac{2\hat{i}}{7}+\frac{3\hat{j}}{7}-\frac{6\hat{k}}{7}$

9. $\int e^{5\log x}dx$ is equal to :

(a) $\frac{x^5}{5}+C$

(b) $\frac{x^6}{6}+C$

(c) $5x^4+C$

(d) $6x^5+C$

10. The number of values of k for which the system of equations

(k+1)x + 8y = 4k,

kx+(k+3)y = 3k–1

has no solution is

(a) 1

(b) 3

(c) 2

(d) infinite

11. Find the direction cosines of the line

$$\frac{x+2}{2}=\frac{2y-5}{3};z=-1$$

(a) $\frac{4}{\sqrt{29}},\frac{3}{\sqrt{29}},\frac{2}{\sqrt{29}}$

(b) $\frac{2}{\sqrt{157}},-\frac{3}{\sqrt{157}},\frac{12}{\sqrt{157}}$

(c) $\frac{2}{7},-\frac{3}{7},-\frac{6}{7}$

(d) $\frac{2}{7},-\frac{3}{7},\frac{6}{7}$

12. Unit vector parallel to the resultant of vectors $\vec{A}=4\hat{i}-3\hat{j}$ and $\vec{B}=8\hat{i}+8\hat{j}$ will be

(a) $\frac{24\hat{i}+5\hat{j}}{13}$

(b) $\frac{12\hat{i}+5\hat{j}}{13}$

(c) $\frac{6\hat{i}+5\hat{j}}{13}$

(d) None of these

13. If A is a square matrix of order 3, such that A (adj A) = 10 I, then |adj A| is equal to

(a) 1

(b) 10

(c) 100

(d) 101

14. Two groups are competing for the position on the Board of directors of a corporation. The probabilities that the first and the second groups will win are 0.6 and 0.4 respectively. Further, if the first group wins, the probability of introducing a new product is 0.7 and the corresponding probability is 0.3 if the second group wins. Find the probability that the new product introduced was by the second group.

(a) 0.22

(b) 0.11

(c) 0.33

(d) 0.45

15. The integrating Factor of the differential equation $(1-y^2)\frac{dx}{dy}+yx=ay(-1<y<1)$ is
(a) $\frac{1}{y^{2-1}}$
(b) $\frac{1}{\sqrt{y^2-1}}$
(c) $\frac{1}{1-y^2}$
(d) $\frac{1}{\sqrt{1-y^2}}$

16. Find the value of p if $(2\hat{\imath}+6\hat{\jmath}+27\hat{k})\times(\hat{\imath}+3\hat{\jmath}+p\hat{k})=0$
(a) $\frac{15}{2}$
(b) $\frac{23}{2}$
(c) $\frac{27}{2}$
(d) $\frac{9}{2}$

17. If A and B are two events such that $A \subset B$ and $P(B) \neq 0$, then which of the following is correct?
(a) P(A/B) = P(A)/P
(B) (b) P(A/B) < P(A)
(c) P(A/B) ≥ P(A)
(d) P(A/B) > P(B)

18. If for any two events A and B, $P(A)=\frac{4}{5}$ and $P(A\cap B)=\frac{7}{10}$, then P(B/A) is equal to
(a) $\frac{1}{10}$
(b) $\frac{1}{8}$
(c) $\frac{7}{8}$
(d) $\frac{17}{20}$

ASSERTION-REASON BASED QUESTIONS

In the following questions, a statement of Assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices.
(a) Both (A) and (R) are true and (R) is the correct explanation of (A).
(b) Both (A) and (R) are true but (R) is not the correct explanation of (A).
(c) (A) is true but (R) is false.
(d) (A) is false but (R) is true.

19. Assertion (A): $\int_2^8 \frac{\sqrt{10-x}}{\sqrt{x}+\sqrt{10-x}}dx=3.$
Reason (R): $\int_a^b f(x)dx=\int_a^b f(a+b-x)dx.$

20. Assertion : Line $\bar{r}=\overline{a_1}\ +\lambda\overline{b_1}$ and $\bar{r}\ \overline{a_2}$ a2+$\mu\overline{b_2}$ will intersect if $(\overline{b_1}\times\overline{b_2}).(\ \overline{a_2}-\overline{a_1})=0$
Reason: Two skew lines will intersect if distance between them is zero

Section –B

[This section comprises of very short answer type questions (VSA) of 2 marks each]

21. If f: A → B defined as $f(x)=x^2+2x+\frac{1}{1+(x+1)^2}$ is onto function, then set B is equal to

OR

21. If $A = \begin{bmatrix} 4 & 2 \\ -1 & 1 \end{bmatrix}$, prove that $(A - 2I)(A - 3I) = 0$.

22. Find the area of the region bounded by y^2=9 x,=2,x=4 and the x – axis in the first quadrant.

23. (a) Evaluate the following integrals:
$\int e^{2x} \sin(3x + 1) dx$

OR

(b) Evaluate :
$\sin^{-1}\left(\sin\frac{3\pi}{4}\right) + \cos^{-1}\left(\cos\frac{3\pi}{4}\right) + \tan^{-1}(1)$

24. The vector equation of a line AB is given by $\vec{r} = x_1(1 + \lambda)\hat{\imath} + y_1(1 + 2\lambda)\hat{\jmath} + z_1(1 + 3\lambda)\hat{k}$. The coordinates of A are (x_1, y_1, z_1) and $\vec{r}$ is the position vector a point (x, y, z) on AB.
(A) What is the equation of this line in cartesian form?
(B) If ’s coordinates are $(-2,5,-3)$, use the cartesian equation of the line to find the coordinates of **B**.
Show your steps.

25. Let X be a random variable which assumes values x_1, x_2, x_3, x_4 such that
$2P(X = x_1) = 3P(X = x_2) = P(X = x_3) = 5P(X = x_4)$.
Find the probability distribution of X.

Section – C

[This section comprises of short answer type questions (SA) of 3 marks each]

26. find
$$\int \frac{2x}{x^2 + 3x + 2} dx$$

27. Find the equation of the plane passing through the line of intersection of the planes $\vec{r} \cdot (\hat{\imath} + \hat{\jmath} + \hat{k}) = 6$ and $\vec{r} \cdot (2\hat{\imath} + 3\hat{\jmath} - 4\hat{k}) = -5$ and the point(1,1,1)

28. By using the properties of definite integrals, evaluate the integrals :
$$\int_0^{\frac{\pi}{4}} \log(1 + \tan x) dx$$

OR

28. Evaluate :
$\int_{\pi/6}^{\pi/3} \frac{dx}{1+\sqrt{\tan x}}$ is equal to

29. Find :
$$\int \left(\sqrt{x} - \frac{1}{\sqrt{x}}\right)^2 dx$$

OR

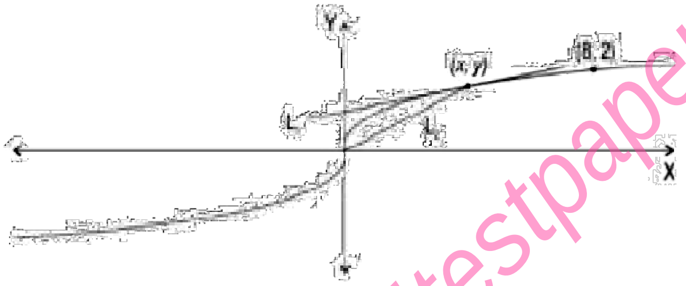
29

Evaluate $\int_{-2}^{2} \frac{x^2}{1+5^x} dx$

30. Minimize and maximize Z=600x+400y
Subject to the constraints
$x+2y \leq 12$
$2x+y \leq 12$
$4x+5y \leq 20$
$x \geq 0; y \geq 0$ by graphical method

OR

30. Shown below is a curve.



L_1 is the tangent to any point (x, y) on the curve.
L_2 is the line that connects the point (x, y) to the origin.
The slope of L_1 is one third of the slope of L_2.
Find the equation of the curve. Show your work.

31. Find the particular solution of the differential equation
$x\frac{dy}{dx} = y - \tan x\left(\frac{y}{x}\right)$, given that $y = \frac{\pi}{4}$ at $x = 1$.

Section –D

[This section comprises of long answer type questions (LA) of 5 marks each]

32. Find the area of the region in the first quadrant enclosed by x-axis, line $x = \sqrt{3}y$ and the circle $x^2 + y^2 = 4$.

33. A function $f: R \to R$ is defined as $f(x) = x^3 + 4$. Is it a bijection or not? In case it is a bijection, then find $f^{-1}(3)$.

OR

33. Prove that the function $f: N \to N$, defined by $(x)=x^2+x+1$ is one-one but not onto.

34. Find the inverse of the following matrix, using elementary transformations :

$$A = \begin{bmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$$

35 Find the area bounded by the circle $x^2 + y^2 = 16$ and the line $\sqrt{3}y = x$ in the first quadrant, using integration.

OR

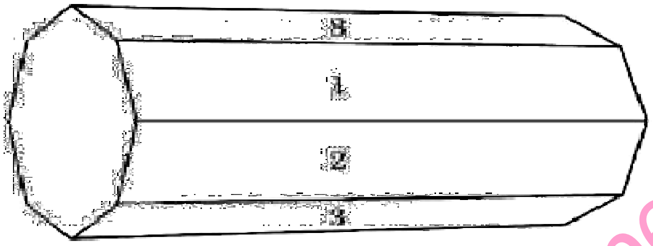
35. Find the equation of the plane that contains the point A(2, 1, – 1) and is perpendicular to the line of intersection of the planes 2x + y – z = 3 and x + 2y + z = 2. Also find the angle between the plane thus obtained and the y-axis.

Section –E

[This section comprises of 3 case- study/passage based questions of 4 marks each with sub parts. The first two case study questions have three sub parts (i), (ii), (iii) of marks 1,1,2 respectively. The third case study question has two sub parts of 2 marks each.)

Case study 1 :

36. An octagonal prism is a three-dimensional polyhedron bounded by two octagonal bases and eight rectangular side faces. It has 24 edges and 16 vertices.



The prism is rolled along the rectangular faces and number on the bottom face (touching the ground) is noted. Let X denote the number obtained on the bottom face and the following table give the probability distribution of X.

X:	1	2	3	4	5	6	7	8
P(X):	p	2p	2p	p	2p	p^2	$2p^2$	$7p^2 + p$

Based on the above information, answer the following questions :

36 (i) Find the value of p.

(ii) Find $P(X > 6)$.

(iii) (a) Find $P(X = 3m)$, where m is a natural number.

OR

(iii) (b) Find the mean E(X).

Case study 2 :

Case Study - 2

37. A building contractor undertakes a job to construct 4 flats on a plot along with parking area. Due to strike the probability of many construction workers not being present for the job is $0 \cdot 65$. The probability that many are not present and still the work gets completed on time is 0.35 The probability that work will be completed on time when all workers are present is 0.80

Let: E_1 : represent the event when many workers were not present for the job;

E_2 : represent the event when all workers were present; and

E : represent completing the construction work on time.

Based on the above information, answer the following questions :

37 (i) What is the probability that all the workers are present for the job ?

(ii) What is the probability that construction will be completed on time?

(iii) (a) What is the probability that many workers are not present given that the construction work is completed on time?

OR

(iii) (b) What is the probability that all workers were present given that the construction job was completed on time?

Case study 3

38. The use of electric vehicles will curb air pollution in the long run.

The use of electric vehicles is increasing every year and estimated electric vehicles in use at any time t is given by the function :

$$V(t) = \frac{1}{5}t^3 - \frac{5}{2}t^2 + 25t - 2$$

where t represents the time and $t = 1,2,3 \ldots$ corresponds to year 2001 , 2002,2003, ... respectively.

Based on the above information, answer the following questions :

(A) Can the above function be used to estimate number of vehicles in the year 2000 ? Justify.

(B) Prove that the function V(t) is an increasing function.

Paper : 4

1. If A is a symmetric matrix and B is a skew-symmetric matrix such that A + B = $\begin{bmatrix} 2 & 3 \\ 5 & -1 \end{bmatrix}$ then AB is equal to :

(a) $\begin{bmatrix} -4 & -1 \\ -1 & 4 \end{bmatrix}$

(b) $\begin{bmatrix} 4 & -2 \\ -1 & -4 \end{bmatrix}$

(c) $\begin{bmatrix} 4 & -2 \\ 1 & -4 \end{bmatrix}$

(d) $\begin{bmatrix} -4 & 2 \\ 1 & 4 \end{bmatrix}$

2. If $\begin{bmatrix} 2 & 0 \\ 5 & 4 \end{bmatrix} = P + Q$, where P is a symmetric and Q is a skew symmetric matrix, then Q is equal to

(a) $\begin{bmatrix} 2 & 5/2 \\ 5/2 & 4 \end{bmatrix}$

(b) $\begin{bmatrix} 0 & -5/2 \\ 5/2 & 0 \end{bmatrix}$

(c) $\begin{bmatrix} 0 & 5/2 \\ -5/2 & 0 \end{bmatrix}$

(d) $\begin{bmatrix} 2 & -5/2 \\ 5/2 & 4 \end{bmatrix}$

3. $f(x) = \begin{cases} 3x - 8 & if\, x \le 5 \\ 2k & if\, x > 5 \end{cases}$ *is continuous, find k*

(a) $\frac{2}{7}$

(b) $\frac{3}{7}$

(c) $\frac{4}{7}$

(d) $\frac{7}{2}$

4. Find the value of ‘p’ for which vectors $3\hat{\imath}+2\hat{\jmath}+9\hat{k}$ and $\hat{\imath}-2p\hat{\jmath}+3\hat{k}$ are parallel.

(a) $-\frac{1}{2}$

(b) $-\frac{1}{3}$

(c) $-\frac{1}{5}$

(d) $\frac{1}{6}$

5. Find the sum of the order and the degree of the following differential equation :

$$\frac{d^2y}{dx^2} + \sqrt[3]{\frac{dy}{dx}} + (1 + x) = 0$$

(a) 1

(b)2

(c) 3

(d) 4

6. The maximum value of z = 3x + 4y subject to the constraints x + y ≤ 40, x + 2y ≤ 60 x, y ≥ 0, is __________.

(a) 100

(b) 120

(c) 130

(d) 140

7. Find the area of parallelogram whose diagonals are determined by the vector
$\overline{a} = 3\hat{i} - \hat{j} - 2\hat{k}$ and $\overline{b} = -\hat{i} + 3\hat{j} - 3\hat{k}$
(a) $\frac{1}{2}\sqrt{266}$
(b) $\frac{1}{4}\sqrt{276}$
(c) $\frac{1}{3}\sqrt{268}$
(d) $\frac{1}{4}\sqrt{255}$

8. Direction cosines of the line $\frac{x-1}{2} = -y = \frac{z+1}{12}$ are:
(a) $\frac{2}{3}, \frac{-1}{3}, \frac{2}{3}$
(b) $\frac{2}{\sqrt{157}}, -\frac{3}{\sqrt{157}}, \frac{12}{\sqrt{157}}$
(c) $\frac{2}{7}, -\frac{3}{7}, -\frac{6}{7}$
(d) $\frac{2}{3}, -\frac{1}{3}, \frac{6}{3}$

9. $\int \frac{\sin(2x)}{\sin^2(x)+2\cos^2(x)} dx = ?$
(a) $-\log(1+\sin^2 x) + c$
(b) $\log|1+\cos^2 x| + c$
(c) $-\log|1+\cos^2 x| + c$
(d) $\log(1+\tan^2 x) + c$

10. Let A = $\begin{bmatrix} \cos a & -\sin a \\ \sin a & \cos a \end{bmatrix}$, $(a \in \mathrm{R})$ such that $A^{32} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$. Then a value of a is :
(a) $\frac{\pi}{32}$
(b) 0
(c) $\frac{\pi}{64}$
(d) $\frac{\pi}{16}$

11. The order and degree of the differential equation $\left[1 + \left(\frac{dy}{dx}\right)^2 + \sin\left(\frac{dy}{dx}\right)\right]^{3/4} = \frac{d^2y}{dx^2}$
(a) order = 2, degree = 3
(b) order = 2, degree = 4
(c) order = 2, degree = $\frac{3}{4}$
(d) order = 2, degree = not defined

12. The vectors $AB = 3i + 5j + 4k$ and $AC = 5i - 5j + 2k$ are side of a $\triangle ABC$. The length of the median through A is:
(a) $\sqrt{13}$ units
(b) $2\sqrt{5}$ units
(c) 5 units
(d) 10 units

13. If A is a square matrix of order 3 and |A| = 6 then the value of |adj A| is:
(a) 6
(b) 36
(c) 27
(d) 216

14. The random variable X has probability distribution P(X) of the following form.

$$P(X) = \begin{cases} k \text{ if } X = 0 \\ 2k \text{ if } X = 1 \\ 3k \text{ if } X = 2 \\ 0 \text{ otherwise} \end{cases}$$

(a) determine value of K
(a) 1/6
(b) ¼
(c) 1/5
(d) 1/7

15. The integrating factor for solving the differential equation $x\frac{dy}{dx} - y = 2x^2$ is :
(a) e^{-y}
(b) e^{-x}
(c) x
(d) $\frac{1}{x}$

16. Write the value of p for which $\vec{a} = 3\hat{i} + 2\hat{j} + 9\hat{k}$ and $\vec{b} = \hat{i} + p\hat{j} + 3\hat{k}$ are parallel vectors.
(a) $\frac{2}{3}$
(b) $\frac{1}{4}$
(c) $\frac{1}{8}$
(d) $\frac{1}{7}$

17. For what value of k is the following function continuous at = 2 ?

$$f(x) = \begin{cases} 2x + 1; & \text{if } x < 2 \\ k, & x = 2 \\ 3x - 1; & x > 2 \end{cases}$$

(a) 2
(b) 3
(c) 4
(d) 5

18. Two events A and B will be independent if
(a) $P(A' \cap B') = (1 - P(A))(1 - P(B))$
(b) $P(A) + P(B) = 1$
(c) $P(A) = P(B)$
(d) A and B are mutually exclusive

ASSERTION-REASON BASED QUESTIONS

In the following questions, a statement of Assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices.
(a) Both (A) and (R) are true and (R) is the correct explanation of (A).
(b) Both (A) and (R) are true but (R) is not the correct explanation of (A).
(c) (A) is true but (R) is false.
(d) (A) is false but (R) is true.

19. Assertion (A) If $0 < x < \frac{\pi}{2}$ then $sin^{-1}(cosx) + cos^{-1}(sinx) = \pi - 2x$
Reason (R) $cos^{-1}x = \frac{\pi}{2} - sin^{-1}x \forall x \in [0,1]$

20. Assertion (A) : The range of the function $f(x) = 2\sin^{-1}x + \frac{3\pi}{2}$, where $x \in [-1,1]$, is $\left[\frac{\pi}{2}, \frac{5\pi}{2}\right]$.
Reason (R) : The range of the principal value branch of $\sin^{-1}(x)$ is $[0, \pi]$.

Section –B

[This section comprises of very short answer type questions (VSA) of 2 marks each]

21. If $|\vec{a} \times \vec{b}|^2 + |\vec{a} \cdot \vec{b}|^2 = 400$ and $|\vec{a}| = 5$, then find the value of $|\vec{b}|$.

OR

21 Find all the possible vectors of magnitude $5\sqrt{3}$ which are equally inclined to the coordinate axes.

22. The sum of a matrix and its transpose is $\begin{bmatrix} 6 & -1 \\ -1 & 4 \end{bmatrix}$.
Find one such matrix for which this holds true.

23. Find the maximum area of an isosceles triangle inscribed in the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ with its vertex at one end of the major axis.

OR

23. Find the angle between unit vectors $\vec{a}$ and $\vec{b}$ so that $\sqrt{3}\vec{a}$ - $\vec{b}$ is also a unit vector.

24.Evalute :

$$\int \frac{dx}{x(logx - 2)(logx - 3)} =$$

25. A coin is tossed twice. The following table shows the probability distribution of number of tails :

X	0	1	2
P(X)	K	6 K	9 K

(a) Find the value of K.
(b) Is the coin tossed biased or unbiased? Justify your answer.

Section – C

[This section comprises of short answer type questions (SA) of 3 marks each]

26. Evaluate: $\int_0^{\pi/2} \frac{cosx}{(1+sin\ \ )(2+sin\ \ )} dx$

27. The anti-derivative of a function of the form $(3x - 1)f(x), \left(x \neq \frac{1}{3}\right)$, is given by $3x^4 - \frac{13}{3}x^3 + \frac{3}{2}x^2 + C$, where C is the constant of integration.
Find the value of f(6). Show your steps.

28.
Evaluate the integral:

$$\int_0^{\pi/2} \sqrt{sinx} \cdot cos^5 xdx$$

OR

28.
Find $\int \frac{2x}{x^2+3x+2} dx$

29. Solve the following differential equation : $x\frac{dy}{dx}$ +y –x + xy cot x = 0, x ≠ 0

OR

29. Solve the differential equation $(1 + y2)(1 + logx)dx + xdy = 0$, given that when $x = 1$ then $y = 1$.

30. Solve the following linear programming problem graphically :
Minimize : Z = 5x + 10y
subject to constraints :
$x + 2y \leq 120$,
$x + y \geq 60$,
$x - 2y \geq 0$,
$x \geq 0,\ y \geq 0$

OR

30.
Solve the following Linear Programming Problem graphically:
Minimize: $z = x + 2y$,
subject to the constraints:
$x + 2y \geq 100$,
$2x - y \leq 0$,
$2x + y \leq 200$,
$x, y \geq 0$.

31. If x = sin t, y = sin pt, prove that $(1 - x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} + p^2y = 0$

Section –D

[This section comprises of long answer type questions (LA) of 5 marks each]

32. A line l passes through point $(-1,3,-2)$ and is perpendicular to both the lines $\frac{x}{1} = \frac{y}{2} = \frac{z}{3}$ and $\frac{x+2}{-3} = \frac{y-1}{2} = \frac{z+1}{5}$. Find the vector equation of the line l. Hence, obtain its distance from origin.

33. Check whether the relation R defined in the set {1,2,3,4,5,6} as $R = \{(a,): b = a + 1\}$ is reflexive, symmetric or transitive

OR

33. Let f: $\mathbb{R} - \left\{-\frac{4}{3}\right\} \rightarrow \mathbb{R}$ be a function defined as f(x) $= \frac{4x}{3x+4}$. Show that f is a one-one and into function. Hence find f^{-1}

34. If A = $\begin{bmatrix} 1 & 3 & 4 \\ 2 & 1 & 2 \\ 5 & 1 & 1 \end{bmatrix}$ find A^{-1}.
Hence solve the system of equations
x + 3y + 4z = 8
2x + y + 2z = 5
and 5x + y + z = 7

35. Find the distance of the point P(– 2, – 4, 7) from the point of intersection Q of the line $\vec{r} = \left(3\hat{\imath} - 2\hat{\jmath} + 6\hat{k}\right) + \lambda \left(2\hat{\imath} - \hat{\jmath} + 2\hat{k}\right)$ and the plane $\vec{r} \cdot \left(3\hat{\imath} - 2\hat{\jmath} + 6\hat{k}\right)$ Also write the vector equation of the line PQ.

OR

35. Find the equations of the line passing through the points $A(2, -3, -1)$ and $B(8, -1, 2)$. Hence, find the coordinates of the points on this line which are at a distance of 14 units from point B.

Section –E

[This section comprises of 3 case- study/passage based questions of 4 marks each with sub parts. The first two case study questions have three sub parts (i), (ii), (iii) of marks 1,1,2 respectively. The third case study question has two sub parts of 2 marks each.)

36.

CASE STUDY 1:

An organization conducted bike race under two different categories - Boys and Girls. There were 28 participants in all. Among all of them, finally three from category 1 and two from category 2 were selected for the final race. Ravi forms two sets B and G with these participants for his college project.
Let $B = \{b_1, b_2, b_3\}$ and $G = \{g_1, g_2\}$, where B represents the set of Boys selected and G the set of Girls selected for the final race.

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Let $B = \{b_1, b_2, b_3\}$ and $G = \{g_1, g_2\}$, where B represents the set of Boys selected and G the set of Girls selected for the final race.

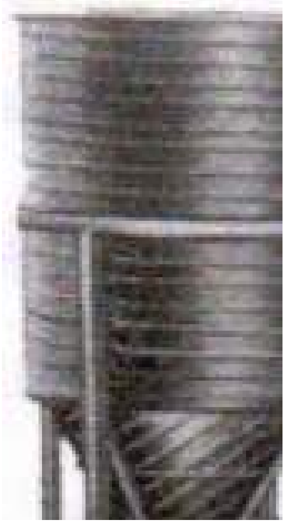
Based on the above information, answer the following questions :
(I) How many relations are possible from B to G ?
(II) Among all the possible relations from B to G, how many functions can be formed from B to G ?
(III) Let $R: B \rightarrow B$ be defined by $R = \{(x, y): x \text{ and } y \text{ are students of the same sex }\}$. Check if R is an equivalence relation.

Or

(III) A function f: B → G be defined by $f = \{(b_1, g_1), (b_2, g_2), (b_3, g_1)\}$ Check if f is bijective. Justify your answer.

Case Study - 2

37. A tank, as shown in the figure below, formed using a combination of a cylinder and a cone, offers better drainage as compared to a flat bottomed tank.



A tap is connected to such a tank whose conical part is full of water. Water is dripping out from a tap at the bottom at the uniform rate of $2\text{cm}^3/\text{s}$. The semi-vertical angle of the conical tank is 45°.

On the basis of given information, answer the following questions :

(i) Find the volume of water in the tank in terms of its radius r.

(ii) Find rate of change of radius at an instant when $r = 2\sqrt{2}$cm.

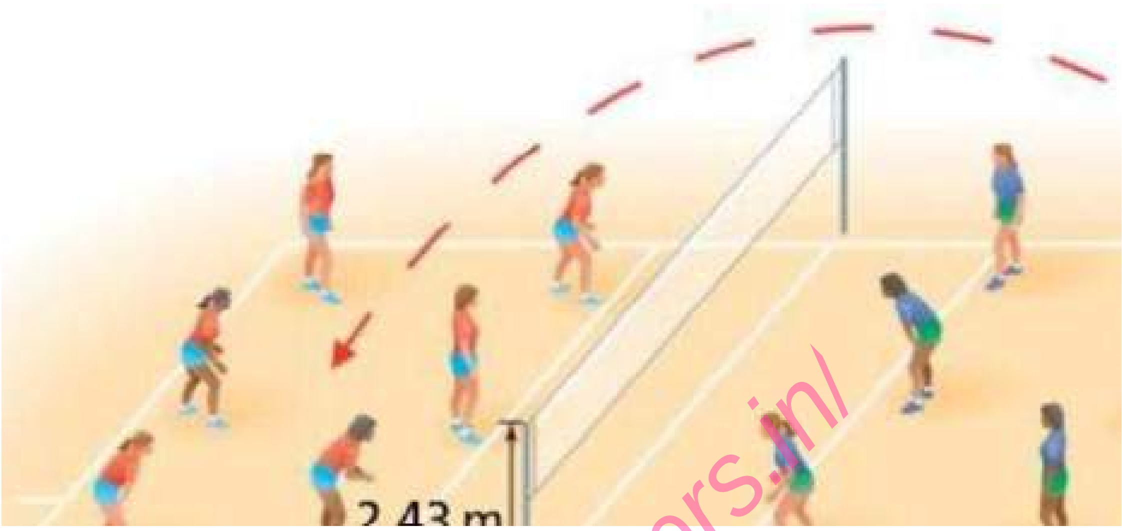
(iii) (a) Find the rate at which the wet surface of the conical tank is decreasing at an instant when radius $r = 2\sqrt{2}$cm.

OR

(iii) (b) Find the rate of change of height ' h ' at an instant when slant height is 4cm.

Case Study – 3

38. A volleyball player serves the ball which takes a parabolic path given by the equation $h(t) = -\frac{7}{2}t^2 + \frac{13}{2}t + 1$, where h(t) is the height of ball at any time t (in seconds), $(t \geq 0)$.



Based on the above information, answer the following questions :

(i) Is h(t) a continuous function? Justify.

(ii) Find the time at which the height of the ball is maximum.

Paper : 3

1. Let A = $\begin{bmatrix}\cos a & -\sin a\\ \sin a & \cos a\end{bmatrix}$, (a ∈ R) such that $A^{32} = \begin{bmatrix}0 & -1\\ 1 & 0\end{bmatrix}$. Then a value of a is :
(a) $\frac{\pi}{32}$
(b) 0
(c) $\frac{\pi}{64}$
(d) $\frac{\pi}{16}$

2. If A is a non-singular square matrix of order 3 such that $A^2 = 3A$, then value of |A| is
(a) – 3
(b) 3
(c) 9
(d) 27

3. Solution of the differential equation tany sec2x dx + tanx sec2ydy = 0 is :
(a) tanx + tany = k
(b) tanx – tany = k
(c) (tanx/tany)k
(d) tanx . tany = k

4. The function f(x) defined by $f(x) = \begin{cases}\frac{\sin^2 ax}{x^2}, & x \neq 0\\ 1, & x = 0\end{cases}$ is continuous at $x = 0$. The value(s) of a is/are
(a) 0
(b) ±1
(c) - 3
(d) ±2

5. If $\vec{a} = 2\hat{\imath} + 2\hat{\jmath} + 3\hat{k}, \vec{b} = -\hat{\imath} + 2\hat{\jmath} + \hat{k}$ and $\vec{c} = 3\hat{\imath} + \hat{\jmath}$ are such that $\vec{a} + \lambda\vec{b}$ is perpendicular to $\vec{c}$ then find the value of λ.
(a) 5
(b) 6
(c) 8
(d)10

6. If p and q are order and degree of differential equation $y^2\left(\frac{d^2y}{dx^2}\right)^2 + 3x\left(\frac{dy}{dx}\right)^{\frac{1}{3}} + x^2y^2 = \sin x$, then
(a) $p > q$
(b) $\frac{p}{q} = \frac{1}{2}$
(c) $p = q$
(d) $p < q$

7. The corner points of the feasible region determined by the system of linear constraints are (0,10), (5,5), (15,15), (0,20). Let z=px+qy where p,q>0. Condition on p and q so that the maximum of z occurs at both the points (15,15) and (0,20) is _________.
(a) q = 2p
(b) p = 2q
(c) p = q
(d) q = 3p

8. The two lines x = ay + b, z = cy + d; and x = a′y + b, z = c′y + d′ are perpendicular to each other, if
(a) $\frac{a}{a'} + \frac{c}{c'} = 1$

(b) $\frac{a}{a'}+\frac{c}{c'}$ = -1
(c) aa′ + cc′ = 1
(d) aa′ + cc′ = −1

9. $\int x^2 e^{x^3} dx$ equals
(a) $\frac{1}{3} e^{x^3}$ + c
(b) $\frac{1}{3} e^{x^4}$ + c
(c) $\frac{1}{2} e^{x^3}$ + c
(d) $\frac{1}{2} e^{x^2}$ + c

10. Let A = $\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$ and B = $\begin{pmatrix} a & 0 \\ 0 & b \end{pmatrix}$, a, b ϵ N , then
(a) There cannot exist any B such that AB = BA
(b) There exist more then one but finite number of B's such that AB = BA
(c) There exists exactly one B such that AB = BA
(d) There exist infinitely many B's such that AB = BA

11. If θ is the angle between two vectors $\vec{a}$ and $\vec{b}$, then $\vec{a}.\vec{b}$ ≥0 only when
(a) $0 < \theta < \frac{\pi}{2}$
(b) $0 \leq \theta \leq \frac{\pi}{2}$
(c) 0 < θ < n
(d) 0 ≤ θ ≤n

12. The solution set of the inequality $3x + 5y < 4$ is :
(a) an open half-plane not containing the origin.
(b) an open half-plane containing the origin.
(c) the whole XY-plane not containing the line 3x + 5y = 4.
(d) a closed half plane containing the origin.

13. If $\begin{vmatrix} 2 & 3 & 2 \\ x & x & x \\ 4 & 9 & 1 \end{vmatrix} + 3 = 0$, then find the value of x.
(a) 2
(b) 0
(c) - 1
(d) -3

14. An insurance company insured 3000 cyclists, 6000 scooter drivers and 9000 car drivers. The probability of an accident involving a cyclist, a scooter driver and a car driver are 0.3, 0.05 and 0.02 respectively. One of the insured persons meets with an accident. What is the probability that he is a cyclist?
(a) $\frac{15}{23}$
(b) $\frac{17}{23}$
(c) $\frac{13}{23}$
(d) $\frac{11}{23}$

15. The degree of the differential equation $\left(\frac{d^2y}{dx^2}\right)^2 - \left(\frac{dy}{dx}\right) = y^3$ is
(a) 1
(b) 2
(c) 3

(d) 6

16. Let $\vec{a} = \hat{i} + \hat{j} + \sqrt{2}\hat{k}, \vec{b} = b_1\hat{i} + b_2\hat{j} + \sqrt{2}\hat{k}$ and $\vec{c} = 5\hat{i} + \hat{j} + \sqrt{2}\hat{k}$ be three vectors such that the projection vector of $\vec{b}$ on $\vec{a}$ is $\vec{a}$. If $\vec{a} + \vec{b}$ is perpendicular to $\vec{c}$, then $|\vec{b}|$ is equal to:
(a) 6
(b) 4
(c) $\sqrt{22}$
(d) $\sqrt{32}$

17. The value of k for which function $f(x) = \begin{cases} \frac{kx}{|x|}, & \text{if } x < 0 \\ 3, & \text{if } x \geq 0 \end{cases}$ is continuous at $x = 0$ is :

(a) -3
(b) -2
(c)-5
(d) 0

18. If A and B are any two events such that $P(A) + P(B) - P(A \text{and} B) = P(A)$, then
(a) $P(B \mid A) = 1$
(b) $P(A \mid B) = 1$
(c) $P(B \mid A) = 0$
(d) $P(A \mid B) = 0$

ASSERTION-REASON BASED QUESTIONS
In the following questions, a statement of Assertion (A) is followed by a statement of Reason (R).
Choose the correct answer out of the following choices.
(a) Both (A) and (R) are true and (R) is the correct explanation of (A).
(b) Both (A) and (R) are true but (R) is not the correct explanation of (A).
(c) (A) is true but (R) is false.
(d) (A) is false but (R) is true.

19. Assertion (A) : If a line makes angles α, β, γ with positive direction of the coordinate axes, then $\sin^2\alpha + \sin^2\beta + \sin^2\gamma = 2$.
Reason (R) : The sum of squares of the direction cosines of a line is 1 .

20. Assertion (A) : $\int_2^8 \frac{\sqrt{10-x}}{\sqrt{x}+\sqrt{10-x}} dx = 3$
Reason (R): $\int_a^b f(x)dx = \int_a^b f(a+b-x)dx$

Section –B

[This section comprises of very short answer type questions (VSA) of 2 marks each]

21. (a) If $\frac{d}{dx}[F(x)] = \frac{\sec^4 x}{\operatorname{cosec}^4 x}$ and $F\left(\frac{\pi}{4}\right) = \frac{\pi}{4}$, then find F(x).

OR

21 (b) Find: $\int \frac{\log x}{(x+1)^2} dx$.

22. (a) If $f(x) = \begin{cases} x^2, & \text{if } x \geq 1 \\ x, & \text{if } x < 1 \end{cases}$, then show that f is not differentiable at $x = 1$.

23. If points $(2, -3)$, $(\lambda, -2)$ and $(0,5)$ are collinear, then find λ

OR

23 Evaluate: $\int \frac{dx}{x(x^3+8)}$

24. Find $\vec{a} \cdot (\vec{b} \times \vec{c})$, if $\vec{a} = 2\hat{i} + \hat{j} + 3\hat{k}, \vec{b} = -\hat{i} + 2\hat{j} + \hat{k}, \vec{c} = 3\hat{i} + \hat{j} + 2\hat{k}$

25. A pair of dice is thrown and the sum of the numbers appearing on the dice is observed to be 7 . Find the probability that the number 5 has appeared on atleast one die.

Section – C

[This section comprises of short answer type questions (SA) of 3 marks each]

26. Integrate the rational functions.

$$\int \frac{1}{x(x^4 - 1)} dx.$$

27. Suppose a girl throws a die. If she gets a 5 or 6, she tosses a coin three times and notes the number of heads. If she gets 1, 2, 3 or 4 she tosses a coin once and notes whether a head or tail is obtained. If she obtained exactly one head, what is the probability that she threw 1, 2,3 or 4 with the die.

28. Solve : $\int_0^{100\pi} \sqrt{1 - \cos 2x}\, dx$

OR

28. If the $\int \frac{5t}{\tan x-} dx = x + a \ln|\sin x - d \cos x| + k$, then a is equal to

29. Solve the differential equation : $(1 + x^2)\frac{dy}{dx} + 2xy - 4x^2 = 0$, subject to the initial condition $y(0) = 0$.

OR

29. Solve the following differential equation: $\sqrt{1 + x^2 + y^2 + x^2y^2} + xy\frac{dy}{dx} = 0$

30. Solve graphically the following linear programming problem :

Maximise $z = 6x + 3y$,

subject to the constraints

$4x + y \geq 80$

$3x + 2y \leq 150$

$x + 5y \geq 115$

$x \geq 0, y \geq 0$

OR

30. Solve the following linear programming problem by graphical method:

Maximize Z=3x+2y

subject to the in constraints

$x+2y\leq 10, 3x+y\leq 15, x, y\geq 0$

31. If $y = (\sec^{-1} x)^2$, $x > 0$, show that $x^2 (x^2 - 1)\frac{d^2y}{dx^2} + (2x^3 - x)\frac{dy}{dx} - 2 = 0$

Section –D

[This section comprises of long answer type questions (LA) of 5 marks each]

32. Sketch the graph of $y = |x + 3|$ and evaluate the area under the curve $y = |x + 3|$ above x-axis and between $x = -6$ *to* $x = 0$

33. In answering a question on a multiple choice test a student either knows the answer or guesses. Let 3/4 be the probability that he knows the answer and 1/4 be the probability that he guesses. Assuming that a student who guesses at the answer will be correct with probability 1/4 What is the probability that a student knows the answer

given that the answered it correctly?

OR

33. If N denotes the set of all natural numbers and R is the relation on N × N defined by $(a, b)R(c, d)$, if $ad(b + c) = bc(a + d)$. Show that R is an equivalence relation.

34. If $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 2 \\ 3 & 1 & 1 \end{bmatrix}$ find A^{-1}. Hence, solve the system of equations

$x + y + z = 6,$
$x + 2z = 7,$
$3x + y + z = 12.$

35. Find the coordinates of the foot of the perpendicular Q drawn from P(3, 2, 1) to the plane $2x - y + z + 1 = 0$. Also, find the distance PQ and the image of the point P treating this plane as a mirror.

OR

35. Find the vector equation of the line passing through (2, 3, 2) and parallel to the line $\vec{r} = (-2\hat{i}+3\hat{j}) + \mu\,(2\hat{i}-3\hat{j} + 6\hat{k})$ Also, find the distance between these two lines.

Section –E

[This section comprises of 3 case- study/passage based questions of 4 marks each with sub parts. The first two case study questions have three sub parts (i), (ii), (iii) of marks 1,1,2 respectively. The third case study question has two sub parts of 2 marks each.)

CASE STUDY 1 :

Solar Panels have to be installed carefully so that the tilt of the roof, and the direction to the sun, produce the largest possible electrical power in the solar panels.

A surveyor uses his instrument to determine the coordinates of the four corners of a roof where solar panels are to be mounted. In the picture , suppose the points are labelled counter clockwise from the roof corner nearest to the camera in units of meters P_1 (6,8,4) , P_2 (21,8,4), P_3 (21,16,10) and P_4 (6,16,10)

(i). What are the components to the two edge vectors defined by $\vec{A}$ = PV of P_2 − PV of P_1 and $\vec{B}$ = PV of P_4 − PV of P_1 ? (where PV stands for position vector)

(ii). Write the vector in standard notation with $\hat{i}, \hat{j}$ and $\hat{k}$ (where $\hat{i}, \hat{j}$ and $\hat{k}$ are the unit vectors along the three axes).

(iii)(a). What are the magnitudes of the vectors $\vec{A}$ and $\vec{B}$ and in what units?

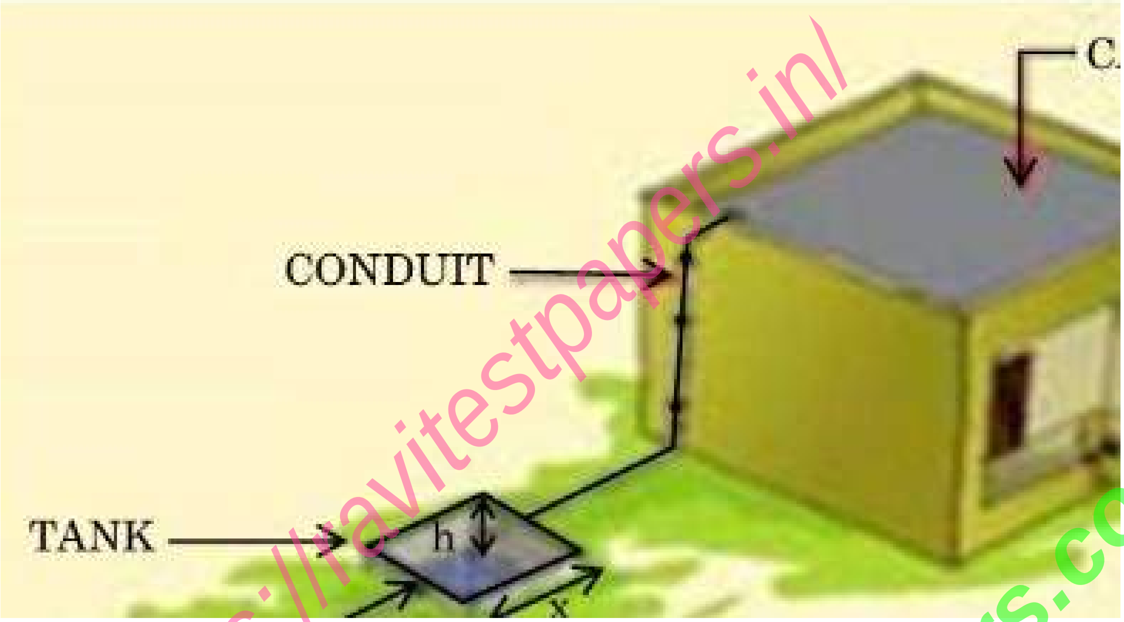
OR

(iii)(b)What are the components to the vector $\vec{N}$, perpendicular to $\vec{A}$ and $\vec{B}$ and the surface of the roof?

Case Study – 2

37. In order to set up a rain water harvesting system, a tank to collect rain water is to be dug. The tank should have a square base and a capacity of $250 m^3$. The cost of land is ₹ 5,000 per square metre and cost of digging increases with depth and for the whole tank, it is ₹ $40{,}000 h^2$, where h is the depth of the tank in metres. x is the side of the square base of the tank in metres.

ELEMENTS OF A TYPICAL RAIN WATER HARVESTING SYSTEM



Based on the above information, answer the following questions :

(i) Find the total cost C of digging the tank in terms of x.

(ii) Find $\frac{dC}{dx}$.

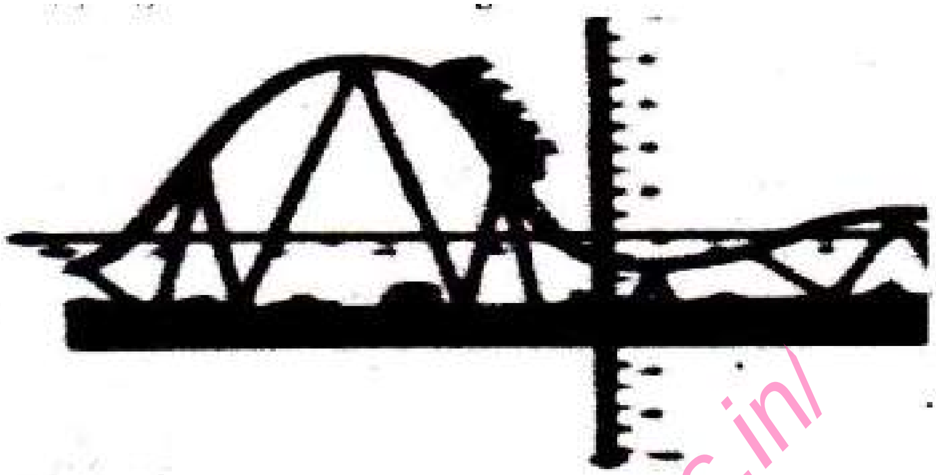
(iii) (a) Find the value of x for which cost C is minimum.

OR

(iii)(b) Check whether the cost function C(x) expressed in terms of x is increasing or not, where $x > 0$.

Case Study 3

38. The equation of the path traced by a roller-coaster is given by the polynomial $f(x) = a(x + 9)(x + 1)(x - 3)$. If the roller-coaster crosses y-axis at a point $(0, -1)$, answer the following :



(a) Find the value of ' a '.

(b) Find $f''(x)$ at $x = 1$.

Paper : 2

1. If P = [1 a 3 1 3 3 2 4 4] is the adjoint of a 3 × 3 matrix A and $|A|$ =4 , then a is a equal to
(a) 4
(b) 11
(c) 5
(d) 0

2. If A = [2 – 3 4], B = [3 2 2] , X = [1 2 3] and Y = [2 3 4] then AB + XY equals
(a) [28]
(b) [24]
(c) 28
(d) 24

3. If $(a, b), (c, d)$ and (e, f) are the vertices of $\triangle ABC$ and denotes the area of $\triangle ABC$, then $|a\ c\ e\ b\ d\ f\ 1\ 1\ 1\ |^2$ is equal to
(a) $2\Delta^2$
(b) $4\Delta^2$
(c) 2Δ
(d) 4Δ

4. For what value of k is the following function continuous at $x = 2$?

$$f(x) = \{2x + 1; \qquad x < 2\ k, x = 2\ 3x - 1;\ x > 2$$

(a) 2
(b) 4
(c) 3
(d) 5

5. Three vectors $\vec{a}, \vec{b}, \vec{c}$. Satisfy the condition $\vec{a} + \vec{b} + \vec{c} = 0$. Evaluate $= \vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}$, if $|\vec{a}| = 1; |\vec{b}| = 4; |\vec{c}| = 2$.
(a) $-\frac{11}{2}$
(b) $-\frac{21}{2}$
(c) $\frac{5}{2}$
(d) $-\frac{7}{2}$

6. The order and degree of the differential equation $\left[1 + \left(\frac{dx}{dy}\right)^3\right]^{\frac{7}{3}} = 7\left(\frac{d^2y}{dx^2}\right)$ are respectively.
(a) 2,3
(b) 3,2
(c) 2,2
(d) 3,3

7. The corner points of the feasible region determined by the following system of linear inequalities:
$2x + y \leq 10, x + 3y \leq 15, x, y \geqslant 0$ are (0,0), (5,0), (3,4) and (0,5). Let $Z = px + qy$, where $p, q > 0$. Condition on p and q so that the maximum of Z occurs at both (3,4) and (0,5) is
(a) $p = q$
(b) $p = 2q$
(c) $p = 3q$
(d) $q = 3p$

8. The vector equation of XY-plane is
(a) $\vec{r} \,.\, \hat{k} = 0$
(b) $\vec{r} \,.\, \hat{j} = 0$

(c) $\vec{r} \,.\, \hat{\imath} = 0$
(d) $\vec{r} \,.\, \vec{n} = 0$

9. $\int \frac{sin^2x - c \quad ^2x}{sin^2x cos^2x} dx$ is equal to
(a) $tanx + cotx + C$
(b) $tanx + cosecx + C$
(c) $-tanx + cotx + C$
(d) $tanx + secx + C$

10. If A is a 3×3 matrix such that |5·adjA|=5, then |A| is equal to
(a) $\pm\frac{1}{5}$
(b) ± 5
(c) ± 1
(d) $\pm\frac{1}{25}$ Solution 10

11. Equation of a line passing through point (a, b, c) and parallel to z-axis is
(a) $\frac{x-a}{1} = \frac{y-b}{0} = \frac{z-c}{0}$
(b) $\frac{x-a}{0} = \frac{y-b}{1} = \frac{z-c}{0}$
(c) $\frac{x-a}{0} = \frac{y-b}{1} = \frac{z-c}{1}$
(d) $\frac{x-a}{1} = \frac{y-b}{0} = \frac{z-c}{1}$

12. Unit vector along $\overrightarrow{PQ}$, where coordinates of P and Q respectively are $(2,1,-1)$ and $(4,4,-7)$, is
(a) $2\hat{\imath} + 3\hat{\jmath} - 6\hat{k}$
(b) $-2\hat{\imath} - 3\hat{\jmath} + 6\hat{k}$
(c) $\frac{-2\hat{\imath}}{7} - \frac{3\hat{\jmath}}{7} + \frac{6\hat{k}}{7}$
(d) $\frac{2\hat{\imath}}{7} + \frac{3\hat{\jmath}}{7} - \frac{6\hat{k}}{7}$

13. If $A = [1\ 0\ \ -1\ 7\ ]$, find k so that $A^2 - 8A - kl = O$, where I is a unit matrix and O is a null matrix of order 2.
(a) -3
(b) -7
(c) -9
(d) 11

14. For the married couple living in Jammu, the probability that a husband will vote in an election is 0.5 and the probability that his wife will vote is 0.4 . The probability that the husband votes, given that his wife also votes is 0.7 . Then the probability that husband and wife both will vote is
(a) 0.28
(b) 0.20
(c) 0.35
(d) 0.15

15. Write the sum of the order and degree of the following differential equation:

$$\frac{d}{dx} = \left\{ \left(\frac{dy}{dx} \right)^3 \right\} = 0$$

(a) 5

(b) 2
(c) 3
(d) 4

16. If is the angle between any two vectors $\vec{a}$ and $\vec{b}$ then $\left|\vec{a} \cdot \vec{b}\right| = \left|\vec{a} \times \vec{b}\right|$ when is equal to
(a) 0
(b) $\frac{}{4}$
(c) $\frac{}{2}$
(d)

17. Find the set of value of x for which $f(x) = cosx - x$ is decreasing in
(a) $(0, \infty)$
(b) $(-\infty, 0)$
(c) $(-\infty, \infty)$
(d) None of the above

18. Find the direction cosines of the vector $\hat{\imath} + 2\hat{\jmath} + 3\hat{k}$.
(a) $\frac{1}{\sqrt{14}}, \frac{2}{\sqrt{14}}$ and $\frac{3}{\sqrt{14}}$
(b) $\frac{9}{\sqrt{12}}, \frac{1}{\sqrt{12}}$ and $\frac{4}{\sqrt{12}}$
(c) $\frac{1}{\sqrt{17}}, \frac{2}{\sqrt{17}}$ and $\frac{3}{\sqrt{17}}$
(d) $\frac{9}{\sqrt{13}}, \frac{1}{\sqrt{13}}$ and $\frac{4}{\sqrt{13}}$

ASSERTION-REASON BASED QUESTIONS

In the following questions, a statement of Assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices.
(a) Both (A) and (R) are true and (R) is the correct explanation of (A).
(b) Both (A) and (R) are true but (R) is not the correct explanation of (A).
(c) (A) is true but (R) is false.
(d) (A) is false but (R) is true.

19. Assertion (A) lf $0 < x < \frac{\pi}{2}$ then $sin^{-1}(cosx) + cos^{-1}(sinx) = \pi - 2x$
Reason (R) $cos^{-1}x = \frac{\pi}{2} - sin^{-1}x \forall x \in [0,1]$

20. Assertion (A) : Two coins are tossed simultaneously. The probability of getting two heads, if it is known that at least one head comes up, is $\frac{1}{3}$.
Reason (R) : Let E and F be two events with a random experiment, then $P(F/E) = \frac{P(E \cap F)}{P(E)}$.

Section –B

[This section comprises of very short answer type questions (VSA) of 2 marks each]

21. Find the domain of the function y = $\cos^{-1}$ (|x - 1|). Show your steps.

OR

21. Consider a bijective function f :$R_+ \rightarrow (7, \infty)$ given by f(x) = $16x^2 + 24x + 7$, where R_+ is the set of all positive real numbers. Find the inverse function of f.

22. A particle moves along the curve $3y = ax^3 + 1$ such that at a point with x-coordinate 1, y-coordinate is changing twice as fast at x-coordinate. Find the value of a.

23. Find the cofactors of all the elements of $[1 \ \ -2 \ 4 \ 3 \ ]$

OR

23. If $x = sec\theta - cos\theta$ and $y = sec^n\theta - cos^n\theta$, then $\left(\frac{dy}{dx}\right)^2$ is equal to :

24. $\int \frac{10x^9+10^x log_{e^{10}} dx}{x^{10}+10^x}$

25. The probabilities of solving a specific problem independently by A and B are $\frac{1}{3}$ and $\frac{1}{5}$ respectively. If both try to solve the problem independently, find the probability that the problem is solved.

Section – C

[This section comprises of short answer type questions (SA) of 3 marks each]

26. Evaluate :

$$\int \frac{2x}{(x^2+1)(x^2+3)} dx$$

27. The anti-derivative of a function of the form $(3x-1)f(x), \left(x \neq \frac{1}{3}\right)$, is given by $3x^4 - \frac{13}{3}x^3 + \frac{3}{2}x^2 + C$, where C is the constant of integration.
Find the value of $f(6)$. Show your steps.

28. Evaluate :

$$\int \frac{1}{cos(x-a)cos(x-b)}$$

OR

28.
Solve: $\int e^{cot^{-1}x}\left(1 - \frac{x}{1+x^2}\right) dx$

29. Find the particular solution, satisfying the given condition, for the following differential equation : $\frac{dy}{dx} - \frac{y}{x} + cosec\left(\frac{y}{x}\right) = 0; y = 0$ when $x = 1$.

Or

29. Solve the differential equation: $(1+x^2)\frac{dy}{dx} + 2xy - 4x^2 = 0$, subject to the initial condition $y(0) = 0$.

30. Solve the following LPP graphically :
Minimise z = 5x + 7y
subject to the constraints
2x + y ≥ 8
x + 2y ≥ 10
x, y ≥ 0

OR

30. Solve the following linear programming problem graphically :
Minimise : $z = -3x + 4y$
subject to the constraints

$$x + 2y \leq 8, 3x + 2y \leq 12, x, y \geq 0.$$

31. If x $\sqrt{1 + \ y}$ + y $\sqrt{1 + \ x}$ = 0 and x ≠ y, prove that $\frac{dy}{dx}$ = - $\frac{1}{(x+1)^2}$

Section –D

[This section comprises of long answer type questions (LA) of 5 marks each]

32. Using the method of integration find the area of the region bounded by lines: 2x+y=4,3x−2y=6 and x−3y+5=0

33. Show that the relation R in the set $A = \{1,2,3,4,5\}$ given by $R = \{(a, b): |a - b|$ is even $\}$, is an equivalence relation. Show that all the elements of {1,3,5} are related to each other and all the elements of {2,4} are related to each other. But no element of {1,3, 5} is related to any element of {2,4}.

OR

33. Let $A = R - \{2\}$ and $B = R - \{1\}$. If $f: A \rightarrow B$ is a function defined by $f(x) = \frac{x-1}{x-2}$, show that f is one-one and onto.

34. Show that the matrix $A = [-8\ 5\ 2\ 4\ ]$ satisfies the equation satisfies the equation $A^2 + 4A - 42 = 0$ and hence find A^{-1}.

$$A = [(-8,5)(2,4)]$$

35. Find the vector equation of the plane passing through the intersection of the planes $\vec{r} \cdot (\hat{\imath} + \hat{\jmath} + \hat{k}) = 6$ and $\vec{r} \cdot (2\hat{\imath} + 3\hat{\jmath} + 4\hat{k}) = -5$, and the point (1,1,1).

OR

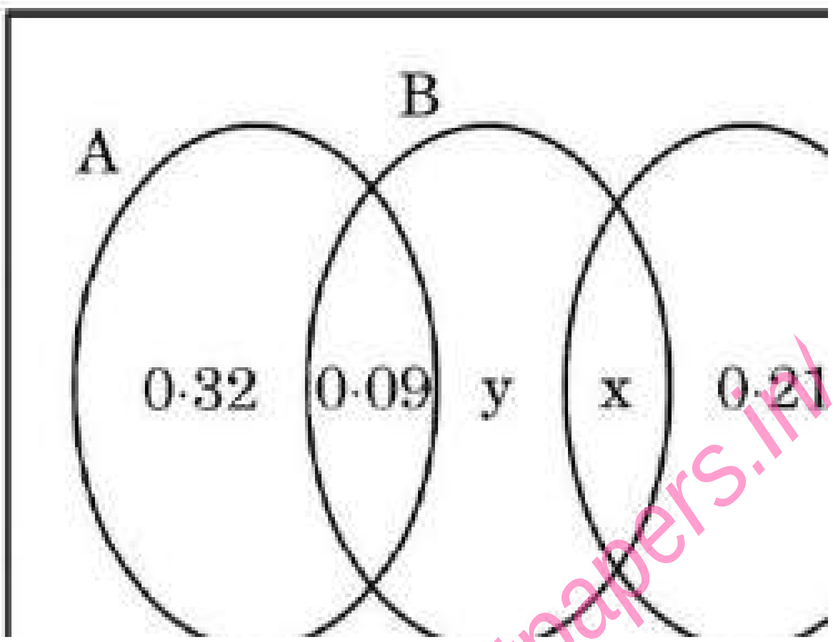
35 . Using integration, find the area lying above x-axis and included between the circle $x^2 + y^2 = 8x$ and inside the parabola $y^2 = 4x$.

Section –E

[This section comprises of 3 case- study/passage based questions of 4 marks each with sub parts. The first two case study questions have three sub parts (i), (ii), (iii) of marks 1,1,2 respectively. The third case study question has two sub parts of 2 marks each.)

Case study 1

The Venn diagram below represents the probabilities of three different types of Yoga, A, B and C performed by the people of a society. Further, it is given that probability of a member performing type C Yoga is 0.44.



On the basis of the above information, answer the following questions :

(i) Find the value of x.

(ii) Find the value of y.

(iii) (a) Find $P\left(\frac{C}{B}\right)$.

OR

(iii) (b) Find the probability that a randomly selected person of the society does Yoga of type A or B but not C.

Case study 2 :

36. Engine displacement is the measure of the cylinder volume swept by al the pistons of a piston engine. The piston moves inside the cylinder bore

The cylinder bore in the form of circular cylinder open at the top is to be made from a metal sheet of area $75\pi cm^2$.

Based on the above information, answer the following questions :

(i) If the radius of cylinder is $r cm$ and height is $h cm$, then write the volume V of cylinder in terms of radius r.

(ii) Find $\frac{dV}{dr}$.

(iii) (a) Find the radius of cylinder when its volume is maximum.

OR

(b) For maximum volume, $h > r$. State true or false and justify.

Case study 3

An equation involving derivatives of the dependent variable with respect to the independent variables is called a differential equation. A differential equation of the form $\frac{dy}{dx} = F(x,y)$ is said to be homogeneous if $F(x,y)$ is a homogeneous function of degree zero, whereas a function $F(x,y)$ is a homogenous function of degree n if $F(\lambda x, \lambda y) = \lambda^n F(x,y)$. To solve a homogeneous differential equation of the type $\frac{dy}{dx} = F(x,y) = g\left(\frac{y}{x}\right)$, we make the substitution $y = vx$ and then separate the variables.

Based on the above, answer the following questions :

(I) Show that $(x^2 - y^2)dx + 2xydy = 0$ is a differential equation of the type $\frac{dy}{dx} = g\left(\frac{y}{x}\right)$.

(II) Solve the above equation to find its general solution.

Paper 1

1. If A = $\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$, then the matrix A^{-50} when $\theta = \frac{\pi}{12}$ is equal to

(a) $\begin{bmatrix} \frac{1}{2} & \frac{\sqrt{3}}{2} \\ -\frac{\sqrt{3}}{2} & \frac{1}{2} \end{bmatrix}$

(b) $\begin{bmatrix} \frac{1}{2} & -\frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & \frac{1}{2} \end{bmatrix}$

(c) $\begin{bmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}$

(d) $\begin{bmatrix} \frac{\sqrt{3}}{2} & -\frac{1}{2} \\ \frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}$

2. If A is a square matrix such that $A^2 = A$, then $(I - A)^3 + A$ is equal to
(a) I
(b) 0
(c) I – A
(d) I + A

3. The area of a triangle formed by vertices O, A and B, where $\overrightarrow{OA} = \hat{i} + 2\hat{j} + 3\hat{k}$ and $\overrightarrow{OB} = -3\hat{i} - 2\hat{j} + \hat{k}$ is
(a) $3\sqrt{5}$ sq. units
(b) $5\sqrt{5}$ sq. units
(c) $6\sqrt{5}$ sq. units
(d) 4 sq. units

4. Determine the value of the constant ' k ' so that the function $f(x) = \{kx/|x|, x < 0$ and $3, x \geq 0$ is Continuous at $x = 0$

$f(x) = \begin{cases} \frac{kx}{|x|}, & \text{if } x < 0 \\ 3, & \text{if } x \geq 0 \end{cases}$ is
(a) -1
(b) -3
(c) 0
(d) -5

5. Let $\vec{a}, \vec{b}$ and $\vec{c}$ be three vectors such that $|\vec{a}| = 3|\vec{b}| = 4|\vec{c}| = 5$ and each one of them being perpendicular to the sum of the other two find $|\vec{a} + \vec{b} + \vec{c}|$.
(a) $5\sqrt{2}$
(b) $7\sqrt{2}$
(c) $35\sqrt{2}$
(d) $3\sqrt{2}$

6. Find the order and degree of $\frac{d^2y}{dx^2} = \left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}$

(a) 1

(b) 2

(c) 3

(d) 4

7. The maximum value of Z = 3x + 4y subject to the constraints : x+ y ≤ 4, x ≥ 0 , y ≥ 0 is :

(a) 0

(b) 12

(c) 16

(d) 18

8. Which of the following is CLOSEST to the area under the parabola given by y = 4x2, bounded by the x-axis, and the lines x = (-1) and x = (-2)?

(a) 6 sq units

(b) 8 sq units

(c) 9 sq units

(d) 12 sq units

9. For any integer n, the value of $\int_0^\pi e^{\sin^2 x} \cos^3(2n+1)x dx$ is

(a) -1

(b) 0

(c) 1

(d)2

10. Let A = $\begin{vmatrix} 5 & 5a & a \\ 0 & a & 5a \\ 0 & 0 & 5 \end{vmatrix}$. IF $|A^2|$ =25 , then |a| equals

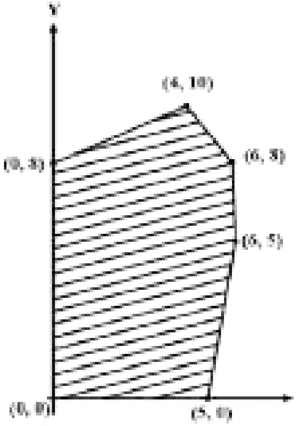
(a) 1/5

(b) 5

(c) 5^2

(d) 1

11. The feasible solution for a LPP is shown in Fig. Let Z =3x−4y be the Objective function, Minimum of Z occurs at Objective function, Minimum of Z occurs at



objective function. Minimum of Z occurs at
(a). (0, 0)
(b). (0, 8)
(c). (5, 0)
(d). (4, 10)

12. If $\vec{a} = \hat{\imath} + 2\hat{\jmath} + 2\hat{k}$, $|\vec{b}| = 5$ and the angle between $\vec{a}$ and $\vec{b}$ is $\frac{\pi}{6}$, then the area of the triangle formed by these two vectors as two sides is :
(a) $\frac{15}{2}$
(b) 15
(c) $\frac{15}{4}$
(d) $\frac{15\sqrt{3}}{2}$

13. Given that A is a square matrix of order 3 and | A | = - 4, then | adj A | is equal to :
(a) -4
(b) 4
(c) -16
(d) 16

14. In a test, an examinee either guesses or copies or knows the answer to a multiple choice question with four choices. The probability that he makes a guess is $\frac{1}{3}$ and the probability that he copies the answer is $\frac{1}{6}$. The probability that his answer is correct given that he copied it is $\frac{1}{8}$. The probability that he knew the answer to the question given that he correctly answered it is $\frac{4k}{29}$.Find the value of k
(a) $\frac{24}{29}$
(b) $\frac{23}{29}$
(c) $\frac{21}{29}$
(d) $\frac{19}{29}$

15. The general solution of the differential equation $x\frac{dy}{dx} + 2y = x^2$ is y = ___________
(a) $\frac{x^2}{4} + Cx^{-2}$
(b) $\frac{x^2}{4}$
(c) $\frac{x^2}{4} + C$
(d) $\frac{x^{-2}}{4} + Cx^2$

16. For what value of ’a’ the vectors $2\hat{\imath} - 3\hat{\jmath} + 4\hat{k}$ and $a\hat{\imath} + 6\hat{\jmath} - 8\hat{k}$ are collinear
(a) -4
(b) 6
(c) 7
(d)9

17. Determine the value of k for which the following function is continuous at $x = 3$.

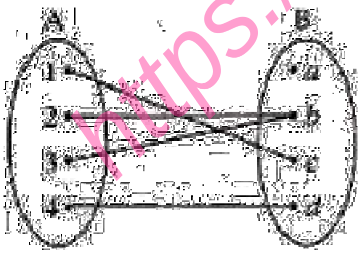
$f(x) = \frac{x^2 - 9}{x - 3}, x \neq 3$
$f(x) = k, x = 3$
(a) 2
(b) 4
(c) 6
(d) 8

18. Find the value of ' p ' for which the vectors $\vec{a} = 3\hat{i} + 2\hat{j} + 9\hat{k}$ and $\vec{b} = \hat{i} + p\hat{j} + 3\hat{k}$ are parallel.
(a) $\frac{2}{3}$
(b) $\frac{1}{3}$
(c) $\frac{1}{5}$
(d) $\frac{3}{5}$

ASSERTION-REASON BASED QUESTIONS

In the following questions, a statement of Assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices.
(a) Both (A) and (R) are true and (R) is the correct explanation of (A).
(b) Both (A) and (R) are true but (R) is not the correct explanation of (A).
(c) (A) is true but (R) is false.
(d) (A) is false but (R) is true.

19. ASSERTION: A mapping shown in the arrow diagram, the function $f:A\to$, is injective



REASON: A function $f:A\to B$ is said to be onto if every element of B has a pre-image in A

20. Assertion (A): The domain of the function $\sec^{-1}2x$ is $\left(-\infty, -\frac{1}{2}\right] \cup \left[\frac{1}{2}, \infty\right)$
Reason (R): $\sec^{-1}(-2) = -\frac{\pi}{4}$

Section –B

[This section comprises of very short answer type questions (VSA) of 2 marks each]

21. Find the value of $\sin^{-1}\left[\sin\left(\frac{13\pi}{7}\right)\right]$.

Or

21

Find $\frac{dy}{dx}$ at $t = \frac{2\pi}{3}$ when $x = 10(t - \sin t)$ and $y = 12(1 - \cos t)$.

22. If $A = \begin{bmatrix} 1 & 2 \\ 4 & 2 \end{bmatrix}$, then find the value of k if $|2A| = k|A|$

23. If $\vec{a} = \hat{i} - \hat{j} + 7\hat{k}$ and $\vec{b} = 5\hat{i} - \hat{j} + \lambda\hat{k}$, then find the value of λ so that the vectors $\vec{a} + \vec{b}$ and $\vec{a} - \vec{b}$ are orthogonal.

OR

23. The position vectors of the points P, Q and R are $\hat{p}, \hat{q}$ and $\hat{r}$ respectively. A vector $\vec{v} = k(\hat{q} + \hat{r})$ is such that $\hat{p} \cdot \vec{v} = \hat{q} \cdot \vec{v}$, where k is a scalar. Prove that $(\hat{p} - \hat{r}) \cdot (\hat{p} - \hat{q}) = 0$

24.

$$\int \frac{e^x(1+x)}{\cos^2(xe^x)}dx =$$

25. Two balls are drawn at random from a bag containing 2 white , 3 red , 5 green and 4 black balls, one by one without, replacement. Find the probability that both the balls are of different colours.

Section – C

[This section comprises of short answer type questions (SA) of 3 marks each]

26. Integrate $\int \frac{\cos2x - \cos2}{\cos x - \cos\alpha}dx$

27.

A random variable X has the following probability distribution:

X:	0	1	2	3	4	5	6	7
P(X):	0	k	2k	2k	3k	k^2	$2k^2$	$7k^2 + k$

Determine k and P (X < 3)

28. Evaluate $\int_{-1}^{2}|x^3 - x|\,dx$

OR

28. By using the properties of definite integrals, evaluate the integrals :

$$\int_0^4 |x - 1|\,dx$$

29. Find the particular solution of the following differential equation:

$$\frac{dy}{dx} = 1 + x^2 + y^2 + x^2y^2, \text{ given that } y = 1 \text{ when } x = 0$$

OR

29. Solve the differential equation : $xdy - ydx = \sqrt{x^2 + y^2}dx$, given that $y = 0$ when $x = 1$.

30. A cottage industry manufactures pedestal lamps and wooden shades.

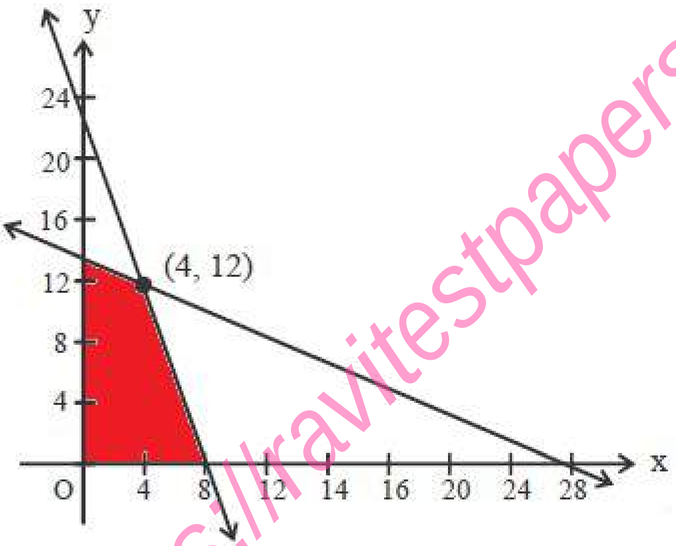
Both the products require machine time as well as craftsman time in the making. The number of hour(s) required for producing 1 unit of each and the corresponding profit is given in the following table :

Item	Machine time	Craftsman time	Profit (in Rs.)
Pedestal lamp	1.5 hours	3 hours	30
Wooden shades	3 hours	1 hour	20

In a day, the factory has availability of not more than 42 hours of machine time and 24 hours of craftsman time. Assuming that all items manufactured are sold, how should the manufacturer schedule his daily production in order to maximise the profit ? Formulate it as an LPP and solve it graphically.

OR

30. Let number of pedestal lamps = x
number of wooden shades = y
Maximize Profit Z = 30x + 20y



getting corners points & values of Z

(0, 0)	0
(8, 0)	240
(4, 12)	360
(0, 14)	280

Maximum profit = Rs 360 where x = 4, y = 12

31. If y = $(\sin^{-1}x)^2$, prove that $(1 - x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} = 0$

Section –D

[This section comprises of long answer type questions (LA) of 5 marks each]

32. Using the method of integration, find the area of the region bounded by the lines $3x - 2y + 1 = 0, 2x + 3y - 21 = 0$ and $x - 5y + 9 = 0$.

33. Let L be the set of all lines in XY plane and R be the relation in L defined as R = {(L_1, L_2): L_1 is parallel to L_2}. Show that R is an equivalence relation. Find the set of all lines related to the line y = 2x + 4.

OR

33. Show that the function f: R → R defined by $f(x) = \frac{x}{x^2+1}, \forall x \in R$ is neither one-one nor onto.

34. Using matrices, solve the following system of linear equations:

$2x + 3y + 10z = 4,\ 4x - 6y + 5z = 1$ *and* $6x + 9y - 20z = 2$

35. Find the vector and cartesian equations of the plane passing through the points (2, 5, – 3), (– 2, – 3, 5) and (5, 3, – 3). Also, find the point of intersection of this plane with the line passing through points (3, 1, 5) and (– 1, – 3, – 1).

OR

35. Find the vector equation of a line passing through the point (2,3,2) and parallel to the line $\vec{r} = (-2\hat{i} + 3\hat{j}) + \lambda(2\hat{i} - 3\hat{j} + 6\hat{k})$. Also find the distance between these lines.

Section –E

[This section comprises of 3 case- study/passage based questions of 4 marks each with sub parts. The first two case study questions have three sub parts (i), (ii), (iii) of marks 1,1,2 respectively. The third case study question has two sub parts of 2 marks each.)

Case study : 1

36.

Let f(x) be a real valued function. Then its

- Left Hand Derivative (L.H.D.) : $Lf'(a) = \lim_{h\to 0} \frac{f(a-h)-f(a)}{-h}$

- Right Hand Derivative (R.H.D.) : $Rf'(a) = \lim_{h\to 0} \frac{f(a+h)-f(a)}{h}$

Also, a function f(x) is said to be differentiable at $x = a$ if its L.H.D. and R.H.D. at $x = a$ exist and both are equal.

For the function $f(x) = \begin{cases} |x-3|, x \geq 1 \\ \frac{x^2}{4} - \frac{3x}{2} + \frac{13}{4}, x < 1 \end{cases}$ answer the following questions :

(i) What is R.H.D. of f(x) at $x = 1$?

(ii) What is L.H.D. of f(x) at $x = 1$?

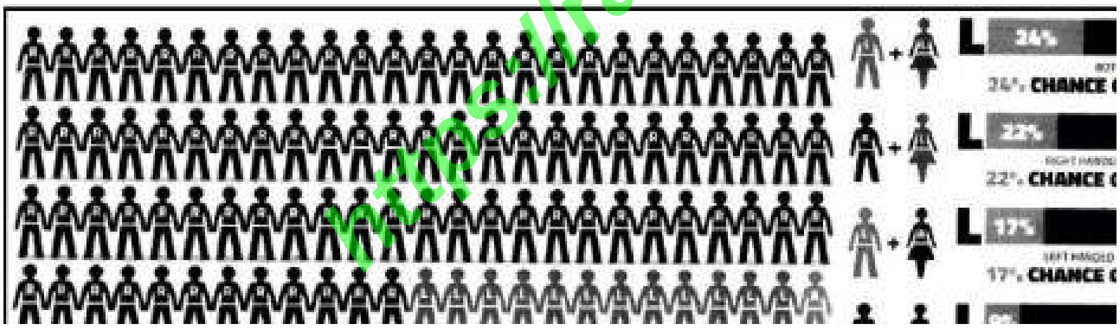
(iii) (a) Check if the function f(x) is differentiable at $x = 1$.

OR

(iii) (b) Find $f'(2)$ and $f'(-1)$. Solution

Case study 2

37.Recent studies suggest that roughly 12% of the world population is left handed.



Depending upon the parents, the chances of having a left handed child are as follows :

A : When both father and mother are left handed :
Chances of left handed child is 24%.
B : When father is right handed and mother is left handed :
Chances of left handed child is 22%.
C : When father is left handed and mother is right handed :

Chances of left handed child is 17%.

D : When both father and mother are right handed :

Chances of left handed child is 9%.

Assuming that $P(A) = P(B) = P(C) = P(D) = \frac{1}{4}$ and L denotes the event that child is left handed.

Based on the above information, answer the following questions :

(i) Find $P(L/C)$

(ii) Find $P(\overline{L}/A)$

(iii) (a) Find $P(A/L)$

OR

(b) Find the probability that a randomly selected child is left handed given that exactly one of the parents is left handed.

Case study 3

A shopkeeper sells three types of flower seeds A1, A2, A3. They are sold in the form of a mixture, where the proportions of these seeds are 4: 4: 2, respectively. The germination rates of the three types of seeds are 45%, 60% and 35% respectively.

Based on the above information :

(a) Calculate the probability that a randomly chosen seed will germinate;

(b) Calculate the probability that the seed is of type A2, given that a randomly chosen seed germinates.

12TH MATHAMATICS

Time allowed : 3hour Maximum Marks = 80

1. If $A = \begin{bmatrix} \cos x & -\sin x \\ \sin x & \cos x \end{bmatrix}$, then the value of x, for which A is an identity matrix, is
   (A) $\frac{\pi}{2}$
   (B) π
   (C) 0
   (D) $\frac{3\pi}{2}$

2. If the matrix $A = \begin{bmatrix} 0 & 5 & -7 \\ a & 0 & 3 \\ b & -3 & 0 \end{bmatrix}$ is a skew-symmetric matrix, then the values of ' a ' and ' b ' are :
   (A) $a = 5, b = 3$
   (B) $a = 5, b = -7$
   (C) $a = -5, b = -7$
   (D) $a = -5, b = 7$

3. If $\begin{vmatrix} x+2 & x-4 \\ x-2 & x+3 \end{vmatrix} = \begin{vmatrix} 6 & -2 \\ 1 & 3 \end{vmatrix}$, then the value of x is :
   (A) 1
   (B) 2
   (C) -2
   (D) -1

4. If $\begin{bmatrix} 8 & 14 \\ 9 & 7 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 3 & 1 \end{bmatrix} X$, then matrix X is :
   (A) $\begin{bmatrix} 3 & 7 \\ 2 & 0 \end{bmatrix}$
   (B) $\begin{bmatrix} 2 & 0 \\ 7 & 3 \end{bmatrix}$
   (C) $\begin{bmatrix} 2 & 0 \\ 3 & 7 \end{bmatrix}$
   (D) $\begin{bmatrix} 2 & 0 \\ -3 & 7 \end{bmatrix}$

5. The value of k, for which $f(x) = \begin{cases} \frac{\sqrt{3}\cos x + \sin x}{3x + \frac{\pi}{2}}, & x \neq -\frac{\pi}{3} \\ k, & x = -\frac{\pi}{3} \end{cases}$ is continuous at $x = -\frac{\pi}{3}$, is :
   (A) $\frac{2}{3}$
   (B) $-\frac{2}{3}$
   (C) $\frac{3}{2}$
   (D) 6

6. Let the vectors $\vec{a}$ and $\vec{b}$ be such that $|\vec{a}| = \sqrt{3}$ and $|\vec{b}| = \frac{2}{\sqrt{3}}$, then $\vec{a} \times \vec{b}$ is a unit vector, if the angle between $\vec{a}$ and $\vec{b}$ is :
   (A) $\frac{\pi}{3}$
   (B) $\frac{\pi}{4}$
   (C) $\frac{\pi}{6}$
   (D) $\frac{\pi}{2}$

7. If $\vec{a} = 2\hat{i} - 2\hat{j} + \hat{k}, \vec{b} = \hat{i} + 2\hat{j} - 3\hat{k}$ and $\vec{c} = 2\hat{i} - \hat{j} + 4\hat{k}$, then the projection of $(\vec{c} - \vec{b})$ along $\vec{a}$ is:
   (A) 15
   (B) 5

(C) $\frac{2}{3}$
(D) 1

8. The angle between the lines $\frac{x+1}{2}=\frac{2-y}{-5}=\frac{z}{4}$ and $\frac{x-3}{1}=\frac{y-7}{2}=\frac{5-z}{3}$ is :
(A) $\frac{\pi}{4}$
(B) $\frac{\pi}{2}$
(C) $\frac{\pi}{3}$
(D) $\frac{\pi}{6}$

9. The Cartesian equations of a line are given as

$$6x-2=3y+1=2z-2$$

The direction ratios of the line are:
(A) $2,-1,3$
(B) $1,-2,-3$
(C) $1,2,3$
(D) $3,1,2$

10. The solution set of the inequation $2x+3y<6$ is :
(A) open half-plane not containing origin
(B) whole xy-plane except the points lying on the line $2x+3y=6$
(C) open half-plane containing origin
(D) half-plane containing the origin and the points lying on the line $2x+3y=6$

11. The maximum value of the objective function $z=3x+5y$ subject to the constraints $x\geq 0, y\geq 0$ and $4x+3y\leq 12$ is :
(A) 15
(B) 29
(C) 9
(D) 20

12. If the points $A(3,-2)$, $B(k,2)$ and $C(8,8)$ are collinear, then the value of k is :
(A) 2
(B) -3
(C) 5
(D) -4

13. If $\vec{a},\vec{b}$ and $\vec{c}$ are unit vectors such that $\vec{a}+\vec{b}+\vec{c}=\vec{0}$, then $(\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a})$ is equal to :
(A) $\frac{3}{2}$
(B) $\frac{1}{2}$
(C) $-\frac{1}{2}$
(D) $-\frac{3}{2}$

14. $\int\frac{\cos 2x}{\sin^2 x\cos^2 x}dx$ is equal to :
(A) $\cot x+\tan x+c$
(B) $-\cot x+\tan x+c$
(C) $\cot x-\tan x+c$
(D) $-\cot x-\tan x+c$

15. The solution of the differential equation $\frac{dy}{dx} = 1 - x + y - xy$ is :

(A) $\log|1+y| = x - \frac{x^2}{2} + c$

(B) $\log|1+y| = -x + \frac{x^2}{2} + c$

(C) $e^y = x - \frac{x^2}{2} + c$

(D) $e^{(1+y)} = -x + \frac{x^2}{2} + c$

16. The degree of the differential equation $x\left(\frac{d^2y}{dx^2}\right)^3 + y\left(\frac{dy}{dx}\right)^4 + y^5 = 0$ is :

(A) 2
(B) 3
(C) 4
(D) 5

17. The integrating factor of the differential equation $\frac{dy}{dx} + y\tan x = 2x + x^2\tan x$ is :

(A) $e^{\sec x}$
(B) $\sec x + \tan x$
(C) $\sec x$
(D) $\cos x$

18. The probabilities of A, B and C solving a problem are $\frac{1}{3}, \frac{1}{5}$ and $\frac{1}{6}$ respectively. The probability that the problem is solved, is :

(A) $\frac{4}{9}$
(B) $\frac{5}{9}$
(C) $\frac{1}{90}$
(D) $\frac{1}{3}$

19. Questions number 19 and 20 are Assertion and Reason based questions. Two statements are given, one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer from the codes (A), (B), (C) and (D) as given below.

(A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).
(B) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A).
(C) Assertion (A) is true, but Reason (R) is false.
(D) Assertion (A) is false, but Reason (R) is true.

Assertion (A) : $\sec^{-1}\left(\frac{2}{\sqrt{3}}\right) = \frac{\pi}{6}$

Reason (R): $\cos\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2}$

20. **Assertion (A) :** If the side of a square is increasing at the rate of 0.2 cm/s, then the rate of increase of its perimeter is 0.8 cm/s.
Reason (R) : Perimeter of a square = 4 (side).

SECTION B

This section comprises of Very Short Answer (VSA) type questions of 2 marks each.

21. (a) Find the value of $\tan^{-1}(1) + \cos^{-1}\left(-\frac{1}{2}\right) + \sin^{-1}\left(-\frac{1}{2}\right)$.

OR

(b) Find the domain of the function $y = \cos^{-1}(x^2 - 4)$.

22. (a)Differentiate $\cot^{-1}\left(\sqrt{1+x^2} + x\right)$ w.r.t. x.

(b) If $(\cos\ x)^y = (\cos\ y)^x$, find $\frac{dy}{dx}$.

23. Find the intervals on which the function $f(x) = 10 - 6x - 2x^2$ is
(a) strictly increasing (b) strictly decreasing.

24. Show that of all rectangles inscribed in a given circle, the square has the maximum area.

25. Find : $\int\ \operatorname{cosec}^3\ (3x + 1)\cot\ (3x + 1)dx$

SECTION C

This section comprises of Short Answer (SA) type questions of 3 marks each.

26. If $x = a\cos\ \theta$ and $y = b\sin\ \theta$, then prove that $\frac{d^2y}{dx^2} = -\frac{b^4}{a^2y^3}$.

27. Find: $\int\ \frac{2x}{(x^2+1)(x^2+3)}dx$

28. (a) Evaluate: $\int_{-6}^{6}\ |x + 2|dx$
(b) Find :

29. (a) Find the particular solution of the differential equation $2xy\frac{dy}{dx} = x^2 + 3y^2$, given that $y(1) = 0$.
(b) Solve the differential equation $\frac{dy}{dx} + 2y\tan\ x = \sin\ x$, given that $y = 0$, when $x = \frac{\pi}{3}$.

30. The corner points of the feasible region determined by the system of linear constraints are A(0,40), B(20, 40), C(60, 20) and D(60,0). The objective function of the L.P.P. is $z = 4x + 3y$. Find the point of the feasible region at which the value of objective function is maximum and the point at which the value is minimum. Hence, find the maximum and the minimum values.

31. (a) A card is randomly drawn from a well-shuffled pack of 52 playing cards. Events A and B are defined as under :
A : Getting a card of diamond
B : Getting a queen
Determine whether the events A and B are independent or not.

OR

(b) Find the probability distribution of the number of doublets in three throws of a pair of dice.

SECTION : D

This section comprises of Long Answer (LA) type question of 5 marks each.

32. Let $A = \{x \mid x \in Z, 0 \leq x \leq 12\}$. Show that the relation $R = \{(a, b): a, b \in A, (a - b) \text{ is divisible by } 4\}$ is an equivalence relation. Find the set of elements related to 2.

OR

(b) Let $A = \mathbb{R} - \{4\}$ and $B = \mathbb{R} - \{1\}$ and let function $f: A \rightarrow B$ be defined as $f(x) = \frac{x-3}{x-4}$ for $\forall x \in A$. Show that f is one-one and onto.

33. Using matrices, solve the following system of linear equations :

$$3x + 4y + 2z = 8; 2y - 3z = 3; x - 2y + 6z = -2$$

34. Using integration, find the area of the region bounded by the curve $y = x^2, x = -1, x = 1$ and the x -axis.

35. (a) Write the vector equations of the following lines and hence find the shortest distance between them :

$$\frac{x+1}{2} = \frac{y+1}{-6} = \frac{z+1}{1} \text{ and } \frac{x-3}{1} = \frac{y-5}{-2} = \frac{z-7}{1}$$

OR

(b) Find the length and the coordinates of the foot of the perpendicular drawn from the point P(5,9,3) to the line $\frac{x-1}{2}=\frac{y-2}{3}=\frac{z-3}{4}$. Also, find the coordinates of the image of the point P in the given line.

SECTION E

This section comprises of 3 case-study based questions of 4 marks each.

36. Case Study - 1

The relation between the height of the plant (y in cm) with respect to exposure to sunlight is governed by the relation $y = 4x - \frac{1}{2}x^2$, where x is the number of days it is exposed to sunlight.

Based on the above, answer the following questions:

(i) Find the rate of growth of the plant with respect to sunlight. 1

(ii) What is the number of days it will take for the plant to grow to the maximum height? 2

(iii) What is the maximum height of the plant? 1

37.

Case Study - 2

A cricket match is organised between two clubs P and Q for which a team from each club is chosen. Remaining players of club P and club Q are respectively sitting along the lines AB and CD , where the points are $A(3,4,0), B(5,3,3), C(6,-4,1)$ and $D(13,-5,-4)$.

Based on the above, answer the following questions:

(i) Write the direction ratios of vector $\overrightarrow{AB}$. 1

(ii) Write a unit vector in the direction of $\overrightarrow{CD}$. 1

(iii) (a) Find the angle between vectors $\overrightarrow{AB}$ and $\overrightarrow{CD}$. 1

OR

(iii) (b) Write a vector perpendicular to both $\overrightarrow{AB}$ and $\overrightarrow{CD}$. 2

38.

Case Study - 3

A coach is training 3 players. He observes that player A can hit a target 4 times in 5 shots, player B can hit 3 times in 4 shots and player C can hit 2 times in 3 shots.

Based on the above, answer the following questions :

(i) Find the probability that all three players miss the target. 1

(ii) Find the probability that all of them hit the target.

(iii) (a) Find the probability that only one of them hits the target. 1

OR

(iii) (b) Find the probability that exactly two of them hit the target. 2

General Instructions :

Read the following instructions very carefully and strictly follow them:

(i) This Question paper contains 38 questions. All questions are compulsory.

(ii) This Question paper is divided into five Sections - A, B, C, D and E.

(iii) In Section A, Questions no. 1 to 18 are multiple choice questions (MCQs) and Questions no. 19 and 20 are Assertion-Reason based questions of 1 mark each.

(iv) In Section B, Questions no. 21 to 25 are Very Short Answer (VSA)-type questions, carrying 2 marks each.

(v) In Section C, Questions no. 26 to 31 are Short Answer (SA)-type questions, carrying 3 marks each.

(vi) In Section D, Questions no. 32 to 35 are Long Answer (LA)-type questions, carrying 5 marks each.

(vii) In Section E, Questions no. 36 to 38 are Case study-based questions, carrying 4 marks each.

(viii) There is no overall choice. However, an internal choice has been provided in 2 questions in Section B, 3 questions in Section C, 2 questions in Section D and one subpart each in 2 questions of Section E.

(ix) Use of calculators is not allowed

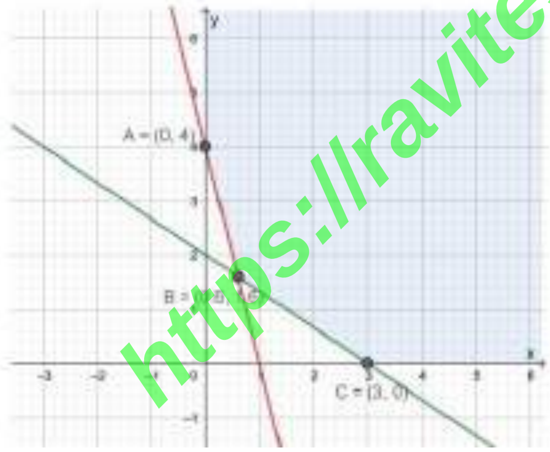
SECTION-A [1× 20= 20]

(This section comprises of multiple choice questions (MCQs) of 1 mark each)

Select the correct option (Question 1 - Question 18):

1. If $|adj\ A| = 144$, where A is a square matrix of order 3×3 , then $|A| =$
   a)12 b)-12 c)± 12 d)16
2. If $\begin{bmatrix} 5 & 2x+3 \\ 3x-1 & x \end{bmatrix}$ is a symmetric matrix , then value of x is :
   a) 4 b) 3 c) 2 d) 1
3. The interval, in which function $y = x^3 + 6x^2 + 6$ is increasing is :
   a) $(-\infty, -4) \cup (0, \infty)$ b)$(-\infty, 4)$ c) $(-4,0)$
   d) $(-\infty, 0) \cup (4, \infty)$
4. If B is a non singular matrix of order 3 such that $B^2 = 2B$, then the value of $|B| =$
   a) -2 b) 2 c) 4 d) 8
5. The integrating factor of the differential equation $x\frac{dy}{dx} - y = x^2 e^x$ is :
   a) x b) $\frac{1}{x}$ c) $-x$ d) e^{-x}
6. If the points $A(3, -2)$, $B(k, 2)$ and $C(8,8)$ are collinear , then the value of k is :
   a) 2 b) -3 c) 5 d) -4

7. If order of matrix A is 2×3 , of matrix B is 3×2 and of matrix C is 3×3 , then which one of the following is **not** defined ?
   a) $C(A + B')$ b) $C(A + B')'$ c) BAC d) $CB + A'$
8. If A and B are two events such that $P(A) = 0.2$, $P(B) = 0.4$ and $P(A \cup B) = 0.5$, then the value of $P(A/B)$ is :
   a) 0.1 b) 0.25 c) 0.5 d) 0.08
9. A unit vector perpendicular to the two vectors $\vec{a} = -2\hat{\imath} + 2\hat{\jmath} - 3\hat{k}$ and $\vec{b} = \hat{\imath} - \hat{\jmath} + \hat{k}$ is given by :
   a) $\hat{\imath} - \hat{\jmath}$ b) $\hat{\imath} + \hat{k}$ c) $-\frac{1}{\sqrt{2}}\hat{\imath} - \frac{1}{\sqrt{2}}\hat{\jmath}$ d) $\frac{1}{\sqrt{2}}\hat{\imath} - \frac{1}{\sqrt{2}}\hat{\jmath}$
10. What is the value of $\frac{projection\ of\ \vec{a}\ \text{on}\ \vec{b}}{projection\ of\ \vec{b}\ \text{on}\ \vec{a}}$ for vectors $\vec{a} = 2\hat{\imath} - 3\hat{\jmath} - 6\hat{k}$ and $\vec{b} = 2\hat{\imath} - 2\hat{\jmath} + \hat{k}$
    a) $\frac{3}{7}$ b) $\frac{7}{3}$ c) $\frac{4}{3}$ d) $\frac{4}{7}$
11. The feasible region , for the constraints $x \geq 0$, $y \geq 0$ and $x + y \leq 2$ lies in :
    a) IV quadrant b) III quadrant c) II quadrant d) I quadrant
12. $\int \frac{\cos 2x}{\sin^2 x cos^2 x} dx$ is equal to :
    a) $cotx + tanx + c$ b) $-cotx + tanx + c$
    c) $cotx - tanx + c$ d) $-cotx - tanx + c$
13. If $f(x)$ is an odd function , then $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} f(x) \cos^3 x\, dx$ equals :
    a) $2 \int_0^{\frac{\pi}{2}} f(x) \cos^3 x\, dx$ b) 0
    c) $2 \int_0^{\frac{\pi}{2}} f(x)\, dx$ d) $2 \int_0^{\frac{\pi}{2}} 2 \cos^3 x\, dx$
14. The degree and order of differential equation $y''^2 + \log(y') = x^5$ respectively are :
    a) not defined , 5 b) not defined , 2
    c) 5 , not defined d) 2,2
15. If $y = \cot^{-1} x$, $x < 0$, then
    a) $\frac{\pi}{2} < y \leq \pi$ b) $\frac{\pi}{2} < y < \pi$ c) $-\frac{\pi}{2} < y < 0$ d) $-\frac{\pi}{2} \leq y < 0$
16. The corner points of the shaded unbounded feasible region of an LPP are (0, 4), (0.6, 1.6) and (3, 0) as shown in the figure. The minimum value of the objective function Z = 4x + 6y occurs at



    a) (0.6 ,1.6) only b) (3,0) only c) (0.6 , 1.6) and (3,0) only
    d) at every point of the line-segment joining the points (0.6, 1.6) and (3, 0)
17. The function $f(x) = [x]$, where $[x]$ is the greatest integer function that is less than or equal to x , is continuous at :
    a) 4 b) -2 c) 1.5 d) 1

18. The area (in sq.units) of the region bounded by the curve $y = x, x - axis$, $x = 0$ and $x = 2$ is :

a) $\frac{3}{2}$ b) $\frac{1}{2}\log 2$ c) 2 d) 4

ASSERTION-REASON BASED QUESTIONS

(Question numbers 19 and 20 are Assertion-Reason based questions carrying 1 mark each. Two statements are given, one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer from the options (A), (B), (C) and (D) as given below.)

(A) Both (A) and (R) are true and (R) is the correct explanation of (A).
(B) Both (A) and (R) are true but (R) is not the correct explanation of (A).
(C) (A) is true but (R) is false.
(D) (A) is false but (R) is true.

19. ASSERTION (A) : The function $f(x) = |x - 6|cosx$ is differentiable in $R - \{6\}$.
REASON (R) : If a function f is continuous at a point c then it is also differentiable at that point .

20. ASSERTION (A) : $\sec^{-1}\left(\frac{2}{\sqrt{3}}\right) = \frac{\pi}{6}$
REASON (R) : $cos\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2}$

SECTION B [2 × 5 =10]

(This section comprises of 5 very short answer (VSA) type questions of 2 marks each.)

21. Find the domain of $\cos^{-1}(3x - 2)$.

22. Given that $f(x) = \frac{\log x}{x}$, find the point of local maximum of $f(x)$.

23. (a) If $y\sqrt{1 - x^2} + x\sqrt{1 - y^2} = 1$, then prove that $\frac{dy}{dx} = -\sqrt{\frac{1-y^2}{1-x^2}}$.

OR

23 . (b) Find the derivative of $\tan^{-1} x$ with respect to $logx$ (where $x \in (1, \infty)$)

24. a) Find $|\vec{x}|$ if $(\vec{x} - \vec{a}).(\vec{x} + \vec{a}) = 12$, where $\vec{a}$ is a unit vector.

OR

24. b) If $\vec{a} = \hat{\imath} - \hat{\jmath} + 7\hat{k}$ and $\vec{b} = 5\hat{\imath} - \hat{\jmath} + \lambda\hat{k}$, find value of λ so that the vectors $\vec{a} + \vec{b}$ and $\vec{a} - \vec{b}$ are orthogonal.

25. The two co initial sides of a parallelogram are $2\,\hat{\imath} - 4\hat{\jmath} - 5\hat{k}$ and $2\hat{\imath} + 2\hat{\jmath} + 3\hat{k}$. Find the diagonals and use them to find the area of the parallelogram .

SECTION C [3 × 6 =18]

(This section comprises of 6 short answer (SA) type questions of 3 marks each.)

26. Find the intervals in which the function given by $f(x) = \sin 3x$, $x \in \left[0, \frac{\pi}{2}\right]$ is
a) increasing b) decreasing

27. A kite is flying at a height of 3 metres and 5 metres of string is out. If the kite is moving away horizontally at the rate of 200 cm/s, find the rate at which the string is being released.

28. a) If $\vec{a} \times \vec{b} = \vec{c} \times \vec{d}$ and $\vec{a} \times \vec{c} = \vec{b} \times \vec{d}$, show that $\vec{a} - \vec{d}$ is parallel to $\vec{b} - \vec{c}$.

OR

28. b) Find the value(s) of $\boldsymbol{a}$ so that the following lines are skew :
$\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-a}{4}$, $\frac{x-4}{5} = \frac{y-1}{2} = z$

29. a) Evaluate $\int \sqrt{\frac{x}{1-x^3}}\,dx$, $x \in (0,1)$

OR

29. b) Evaluate $\int_0^1 x(1-x)^n\,dx$ where $n \in N$

30. The corner points of the feasible region determined by the system of linear constraints in an LPP are $(0,10), (5,5), (15,15)$ and $(0,20)$. Let $z = px + q$,$p, q > 0$ be the objective function , then find the relation between p and q so that the maximum of z occurs at both the points $(15,15)$ and $(0,20)$.

31. a) A coin is biased so that the head is three times as likely to occur as tail . If the coin is tossed twice , find the probability distribution of number of tails . Also find the mean of the distribution.

OR

31. b) Let A and B be the events such that $P(A) = \frac{1}{2}$, $P(B) = \frac{7}{12}$ and $P(\text{not A or not B}) = \frac{1}{4}$. Find whether A and B are

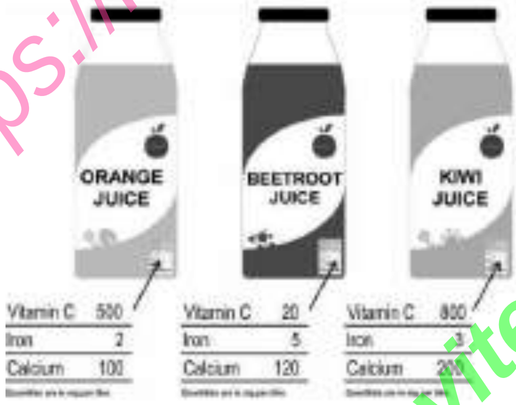
(i) mutually exclusive (ii) independent

SECTION D [5 × 4 =20]

(This section comprises of 4 long answer (LA) type questions of 5 marks each.)

32. Using integration , find the area of the region bounded by the curve $y = x^2, x = -1, x = 1$ and the $x - axis$.

33. Sravan is a nutritionist. He wants to create a mixture of orange juice, beetroot juice and kiwi juice that can provide 1860 mg of vitamin C, 22 mg of iron and 760 mg of calcium. The quantity of each nutrient per litre of juice is shown below.



Using the matrix method, find how many litres of each juice Sravan should add into the mixture.

34. a) If $(ax + b)e^{\frac{y}{x}} = x$, then show that $x^3\frac{d^2y}{dx^2} = \left(x\frac{dy}{dx} - y\right)^2$.

OR

34. b) If $(x-a)^2 + (y-b)^2 = c^2$, for some $c > 0$, prove that $\frac{\left[1+\left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}}}{\frac{d^2y}{dx^2}}$ is a constant independent of a and b.

35. a) Find the coordinates of the image of the point $(1,6,3)$ with respect to the line $\vec{r} = (\hat{j} + 2\hat{k}) + \lambda(\hat{i} + 2\hat{j} + 3\hat{k})$, where λ is a scalar . Also find the distance of the image from the y –axis.

OR

35. (b) An aeroplane is flying along the line $\vec{r} = \lambda(\hat{i} - \hat{j} + \hat{k})$ where λ is a scalar and another aeroplane is flying along the line $\vec{r} = \hat{i} - \hat{j} + \mu(-2\hat{j} + \hat{k})$ where μ is a scalar. At what points on the lines should they reach, so that the distance between them is the shortest? Find the shortest possible distance between them.

SECTION E [4 × 3 =12]

(This section comprises of 3 case-study/passage-based questions of 4 marks each with subparts. The first two case study questions have three subparts (i), (ii), (iii) of marks 1, 1, 2 respectively. The third case study question has two subparts of 2 marks each)

36. Students of a school are taken to a railway museum to learn about railways heritage and its history .

An exhibit in the museum depicted many rail lines on the track near the railway station . Let L be the set of all rail lines on the railway track and R be the relation on L defined by

R $=\{(l_1, l_2): l_1$ is parallel to $l_2\}$

On the basis of above information , answer the following questions :

(i) Find whether the relation R is symmetric or not.

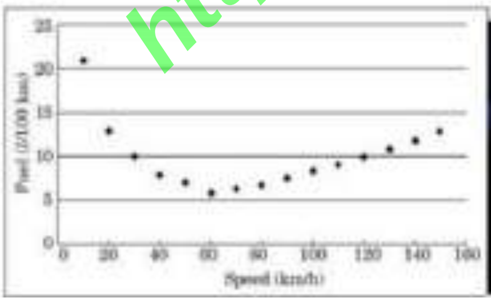
(ii) Find whether the relation R is transitive or not .

(iii) a) If one of the rail lines on the railway track is represented by the equation $y = 3x + 2$, then find the set of rail lines in R related to it .

OR

(iii) b) Let S be the relation defined by S $=\{(l_1, l_2): l_1$ is perpendicular to $l_2\}$. Check whether the relation S is symmetric and transitive .

37. Over speeding increases fuel consumption and decreases fuel economy as a result of tyre rolling friction and air resistance. While vehicles reach optimal fuel economy at different speeds , fuel mileage usually decreases rapidly at speeds above 80 km/h



The relation between fuel consumption F(l/100km) and speed V (km/h) under some constraints is given as $F = \frac{V^2}{500} - \frac{V}{4} + 14$.
On the basis of the above information , answer the following questions :

(i) Find F, when V = 40 km/h.

(ii) Find $\frac{dF}{dV}$.

(iii) a) Find the speed V for which fuel consumption F is minimum .

OR

(iii) b) Find the quantity of fuel required to travel 600 km at the speed V at which $\frac{dF}{dV} = -0.01$.

38. A Shopkeeper sells three types of flower seeds A_1, A_2 and A_3 . They are sold as a mixture where the proportions are $4:4:2$ respectively . The germination rates of the three types of seeds are $45\%, 60\%$ and 35% .
Based on the above information , answer the following questions :

(i) What is the probability of a randomly chosen seed to germinate ?

(ii) What is the probability that a randomly selected seed is of type A_1 , given that it germinates ?

****

4. **In Section B**, Questions no. **21 to 25** are **Very Short Answer (VSA)**-type questions, carrying 2 marks each.
5. **In Section C**, Questions no. **26 to 31** are **Short Answer (SA)-type questions**, carrying 3 marks each.
6. **In Section D**, Questions no. **32 to 35** are **Long Answer (LA)-type questions**, carrying 5 marks each.
7. **In Section E**, Questions no. **36 to 38** are **Case study-based questions**, carrying 4 marks each.
8. There is no overall choice. However, **an internal choice has been provided in 2 questions in Section B, 3 questions in Section C, 2 questions in Section D and one subpart each in 2 questions of Section E.**
9. Use of calculators is not allowed.

SECTION-A [1 ×20 =20]

(This section comprises of multiple-choice questions (MCQs) of 1 mark each)
Select the correct option (Question 1 - Question 18):

Q1. For which of the given values of x and y, the following pair of matrices are equal?

$$\begin{bmatrix}3x+7 & 5\\ y+1 & 2-3x\end{bmatrix}, \begin{bmatrix}0 & y-2\\ 8 & 4\end{bmatrix}$$

(a) $x = \frac{-1}{3}, y = 7$ (b) no such x and y possible (c) $y = 7, x = \frac{-2}{3}$ (d) $x = \frac{-1}{3}, y = \frac{-2}{3}$

Q2. Assume $\boldsymbol{X}, \boldsymbol{Y}, \boldsymbol{Z}, \boldsymbol{W}$ and P are matrices of order $2 \times n, 3 \times k, 2 \times p, n \times 3$ and $\boldsymbol{p} \times \boldsymbol{k}$, respectively. Then the restriction on $\boldsymbol{n}, \boldsymbol{k}$ and $\boldsymbol{p}$ so that $\boldsymbol{PY} + \boldsymbol{WY}$ will be defined are:
(a) $k = 3, p = n$(b) $\boldsymbol{k}$ is arbitrary, $\boldsymbol{p} = 2$ (c) p is arbitrary, $k = 3$ (d) $k = 2, p = 3$

Q3. The interval in which the function y=x^2e^{-x} is increasing is:
(a) (-∞, ∞) (b)(-2, 0) (c) (2, ∞) (d) (0, 2)

Q4. A and B are two matrices such that $AB = A$ and $BA = B$ then B^2 is
(a) A (b) B (c) 0 (d) I

Q5. The general solution of the differential equation $\log\left(\frac{dy}{dx}\right) + x = 0$ is
(a) $y = e^{-x} + c$ (b) $y = -e^{-x} + c$ (c) $y = e^{x} + c$ (d) $y = -e^{x} + c$

Q6. If A is a square matrix of order 3 such that $|A| = -5$ then value of $|-AA'|$ is
(a) 125 (b) - 125 (c) 25 (d) – 25

Q7. If A, B are symmetric matrices of same order, then AB – BA is a
(a) Skew symmetric matrix (b) Symmetric matrix (c) Zero matrix (d) Identity matrix

Q8. Two events A and B will be independent, if
(a) A and B are mutually exclusive
(b) P(A′B′) = [1 – P(A)] [1 – P(B)]
(c) P(A) = P(B)
(d) P(A) + P(B) = 1

Q9. A vector in the direction of vector $\hat{\imath} - 2\hat{\jmath} + 2\hat{k}$ that has magnitude 15 is
(a) $-5\hat{\imath} - 10\hat{\jmath} - 10\hat{k}$ (b) $5\hat{\imath} + 10\hat{\jmath} + 10\hat{k}$ (c) $-5\hat{\imath} + 10\hat{\jmath} + 10\hat{k}$ (d) $5\hat{\imath} - 10\hat{\jmath} + 10\hat{k}$

0. If $|\vec{a}| = 3, |\vec{b}| = 4$ and $|\vec{a} + \vec{b}| = 5$, then $|\vec{a} - \vec{b}| =$
(A) 3 (B) 4 (C) 5 (D) 8

Q11. The region represented by graph of the inequality 2x + 3y > 6 is
(a) half plane that contains the origin
(b) half plane that neither contains the origin nor the points on the line 2x + 3y=6
(c) whole XOY-plane excluding the points on the line 2x + 3y = 6
(d) entire XOY plane

Q12. $\int e^x \sec x\,(1 + \tan x)\,dx = \cdots$
(a) $e^x \cos x + c$ (b) $e^x \sec x + c$ (c) $e^x \sin x + c$ (d) $e^x \tan x + c$

Q13. $\int_0^{2\pi} \operatorname{cosec}^7 x dx =$
(a) 0 (b) 1 (c) 4 (d) 2π

Q14. The number of arbitrary constants in the particular solution of a differential equation of third order is /are
(a) 3 (b) 2 (c) 1 (d) 0

Q15. If $\cos\left[\tan^{-1}\left\{\cot\left(\sin^{-1}\frac{1}{2}\right)\right\}\right] = \cdots$
(a) 1 (b) 0 (c) $\frac{1}{\sqrt{2}}$ (d) $\frac{1}{2}$

Q16. The corner points of the feasible region in the graphical representation of a linear programming problem are (2,72), (15,20) and (40,15). If z = 18x+9y be the objective function, then:
(a) z is maximum at (2,72), minimum at (15,20)
(b) z is maximum at (15,20), minimum at (40,15)
(c) z is maximum at (40,15), minimum at (15,20)
(d) z is maximum at (40,15), minimum at (2,72)

Q17. If x = t^2, y= t^3, then $\frac{d^2y}{dx^2} =$
(a) $\frac{3}{2}$ (b) $\frac{3}{4t}$ (c) $\frac{3}{2t}$ (d) $\frac{3t}{2}$

Q18. The area bounded by the line y = x, x-axis and lines x = – 1 to x = 2, is
a) 0 sq. units b) $\frac{1}{2}$ sq units c) $\frac{3}{2}$ sq units d) $\frac{5}{2}$ sq units

ASSERTION – REASON BASED QUESTIONS

Directions: Each of these questions contains two statements, Assertion and Reason. Each of these questions also has four alternative choices, only one of which is the correct answer. You have to select one of the codes (a), (b), (c) and (d) given below.

(a) Assertion is correct, reason is correct; reason is a correct explanation for assertion.
(b) Assertion is correct, reason is correct; reason is not a correct explanation for assertion
(c) Assertion is correct, reason is incorrect
(d) Assertion is incorrect, reason is correct.

Q19. Consider the function $f(x)=\begin{cases} x^2 - 5x + 6\ x - 3\ , \text{for } x \neq 3 \\ k, \quad \text{for } x = 3 \end{cases}$ is continuous at x = 3

Assertion (A): The value of k is 4

Reason (R): If f(x) is continuous at a point a then $\lim_{x \to a} f(x) = f(a)$

Q20. Assertion (A) : If $y = tan^{-1}\left(\frac{\cos x + sinx}{\cos x - sinx}\right)$, $-\frac{\pi}{4} < x < \frac{\pi}{4}$ then $\frac{dy}{dx} = 1$

Reason(R) : $\frac{\cos x + sinx}{\cos x - sinx} = \tan(x + \frac{\pi}{4})$

SECTION B
VERY SHORT ANSWER TYPE QUESTIONS(VSA)
(Each question carries 2 marks)

Q21. Evaluate : $\sin^{-1}\left(\sin\frac{3\pi}{4}\right) + \cos^{-1}\left(\cos\frac{3\pi}{4}\right) + \tan^{-1}(1)$

Q22. Function f is defined as $f(x) = \begin{cases} 2x + 2, & \text{if} \quad x < 2 \\ k, & \text{if} \quad x = 2 \\ 3x, & \text{if} \quad x > 2 \end{cases}$ Find the value of k for which the function f is continuous at x = 2.

OR

If $x = a(\theta - \sin\theta)$ and $y = a(1 + \cos\theta)$, find $\frac{dy}{dx}$ at $\theta = \frac{\pi}{3}$

Q23. Find the point on the curve $y^2 = 8x$ for which the abscissa and ordinate change at the same rate

Q24. Find the projection of the vector $\vec{a} = 2\hat{\imath} + 3\hat{\jmath} + 2\hat{k}$ on the vector $\vec{b} = \hat{\imath} + 2\hat{\jmath} + \hat{k}$.

OR

If $\vec{a} = 4\hat{\imath} - \hat{\jmath} + \hat{k}$ and $\vec{b} = 2\hat{\imath} - 2\hat{\jmath} + \hat{k}$, then find a unit vector along the vector $\vec{a} \times \vec{b}$.

Q25. If $\vec{a}, \vec{b}, \vec{c}$ are unit vectors such that $\vec{a} + \vec{b} + \vec{c} = \vec{0}$, then write the value of $\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}$.

SECTION C
SHORT ANSWER TYPE QUESTIONS(SA)
(Each question carries 3 marks)

Q26. Find the intervals in which the function $f(x) = \frac{3}{2}x^4 - 4x^3 - 45x^2 + 51$ is strictly increasing and strictly decreasing

Q27. The volume of a cube is increasing at the rate of 9 cm^3/s. How fast is its surface area increasing when the length of an edge is 10 cm?

Q28. Evaluate: $\int_{-1}^{2} |x^3 - x| dx$

OR

Find $\int \frac{x+3}{\sqrt{5-4x-2x^2}} dx$

Q29. Let $\vec{a} = 4\hat{\imath} + 5\hat{\jmath} - \hat{k}, \vec{b} = \hat{\imath} - 4\hat{\jmath} + 5\hat{k}$ and $\vec{c} = 3\hat{\imath} + \hat{\jmath} - \hat{k}$. Find a vector $\vec{d}$ which is perpendicular to both $\vec{c}$ and $\vec{b}$ and $\vec{d} \cdot \vec{a} = 21$.

OR

Show that the lines $\frac{x+1}{3} = \frac{y+3}{5} = \frac{z+5}{7}$ and $\frac{x-2}{1} = \frac{y-4}{3} = \frac{z-6}{5}$ intersect. Also find their point of intersection.

Q30. Consider the following Linear Programming Problem: Minimize $Z = x + 2y$ Subject to $2x + y \geq 3, x + 2y \geq 6, x, y \geq 0$. Show graphically that the minimum of Z occurs at more than two points.

Q31. A problem is given to three students whose probabilities of solving it are $\frac{1}{3}, \frac{1}{4}$ and $\frac{1}{6}$ respectively. If the events of solving the problem are independent, find probabilities that at least one of them solves it.

OR

A man is known to speak truth 3 out of 4 times. He throws a die and reports that it is a six. Find the probability that it is actually a six.

SECTION D

LONG ANSWER TYPE QUESTIONS(LA)

(Each question carries 5 marks)

Q32. Use product $\begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{bmatrix} \begin{bmatrix} -2 & 0 & 1 \\ 9 & 2 & -3 \\ 6 & 1 & -2 \end{bmatrix}$ to solve the system of equations:

$x - y + 2z = 1, \; 2y - 3z = 1, \; 3x - 2y + 4z = 2$

Q33. Draw the graph of y=|x+1| and find the area bounded by it with x- axis, x= −4 and x=2.

Q34. If x and y are connected parametrically by the equations and $x = \frac{\sin^3 t}{\sqrt{\cos 2t}}, y = \frac{\cos^3 t}{\sqrt{\cos 2t}}$ find $\frac{dy}{dx}$

OR

Determine a, b, c so that $f(x) = \begin{cases} \frac{\sin(a+1)x + \sin x}{x}, & x < 0 \\ c, & x = 0 \\ \frac{\sqrt{x + bx^2} - \sqrt{x}}{bx^{3/2}}, & x > 0 \end{cases}$ is continuous at x = 0.

Q35. Find the shortest distance between the lines.

$$\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4} \quad and \quad \frac{x-2}{3} = \frac{y-4}{4} = \frac{z-5}{5}$$

OR

Find the vector equation of the line passing through the point (1, 2, – 4) and perpendicular to the two lines:

$$\frac{x-8}{3} = \frac{y+19}{-16} = \frac{z-10}{7} \quad and \quad \frac{x-15}{3} = \frac{y-29}{8} = \frac{z-5}{-5}$$

SECTION E

(3 case study questions carry 4 marks each)

Q36. Vani and Mani are playing Ludo at home while it was raining outside. While rolling the dice Vani's brother Varun observed and noted the possible outcomes of the throw every time belongs to the set $\{1,2,3,4,5,6\}$. Let A be the set of players while B be the set of all possible outcomes.A = $\{$ Vani, Mani $\}, B = \{1,2,3,4,5,6\}$.

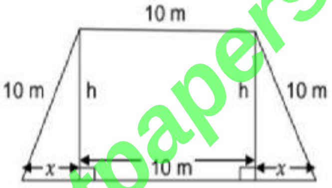
Answer the following questions:

a. Let $R: B \to B$, be defined by $R = \{(x,y) : y$ is divisible by $x\}$.Verify that whether R is reflexive, symmetric and transitive. (2marks)

b. Is it possible to define an onto function from A to B? Justify. (1mark)

c. Which kind of relation is R defined on B given by
R= $\{(1,2),(2,2),(1,3),(3,4),(3,1),(4,3),(5,5)\}$? (1mark)
OR
Find the number of possible relations from A to B .

Q37. Read the following passage and answer the questions given below: The front gate of a building is in the shape of a trapezium as shown below. Its three sides other than base are 10m each. The height of the gate is h meter. On the basis of this information and figure given below answer the following questions:



I. Find area **A** of the gate expressed as a function of **x**.

II. Find value of $\frac{dA}{dx}$.

III. Find x and show that area is maximum

OR
Find maximum area of trapezium.

Q38. Case-Study 3:

A biased die is tossed and respective probabilities for various faces to turn up are the following :

Face	1	2	3	4	5	6
Probability	$0 \cdot 1$	$0 \cdot 24$	$0 \cdot 19$	$0 \cdot 18$	$0 \cdot 15$	K

Based on the above information, answer the following questions:

(a) What is the value of K ? 2 marks

(b) If a face showing an even number has turned up, then what is the probability that it is the face with 2 or 4 ? 2marks

END OF PAPER

SECTION – A

This section has **20** multiple choice questions of **1** mark each.

1. A function $f : \mathbb{R} \rightarrow \mathbb{R}$ defined as $f(x) = x^2 - 4x + 5$ is :

   (A) injective but not surjective. (B) surjective but not injective.

   (C) both injective and surjective. (D) neither injective nor surjective.

2. If $A = \begin{bmatrix} a & c & -1 \\ b & 0 & 5 \\ 1 & -5 & 0 \end{bmatrix}$ is a skew-symmetric matrix, then the value of $2a - (b + c)$ is :

   (A) 0 (B) 1

   (C) –10 (D) 10

3. If A is a square matrix of order 3 such that the value of $|\text{adj} \cdot A| = 8$, then the value of $|A^T|$ is :

   (A) $\sqrt{2}$ (B) $-\sqrt{2}$

   (C) 8 (D) $2\sqrt{2}$

4. If inverse of matrix $\begin{bmatrix} 7 & -3 & -3 \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix}$ is the matrix $\begin{bmatrix} 1 & 3 & 3 \\ 1 & \lambda & 3 \\ 1 & 3 & 4 \end{bmatrix}$, then value of λ is :

   (A) –4 (B) 1

   (C) 3 (D) 4

5. If $\begin{bmatrix} x & 2 & 0 \end{bmatrix} \begin{bmatrix} 5 \\ -1 \\ x \end{bmatrix} = \begin{bmatrix} 3 & 1 \end{bmatrix} \begin{bmatrix} -2 \\ x \end{bmatrix}$, then value of x is :

   (A) –1 (B) 0

   (C) 1 (D) 2

6. Find the matrix A^2, where $A = [a_{ij}]$ is a 2×2 matrix whose elements are given by a_{ij} = maximum (i, j) – minimum (i, j) :

(A) $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ (B) $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

(C) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ (D) $\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$

7. If $xe^y = 1$, then the value of $\frac{dy}{dx}$ at $x = 1$ is :

(A) -1 (B) 1

(C) $-e$ (D) $-\frac{1}{e}$

8. Derivative of $e^{\sin^2 x}$ with respect to $\cos x$ is :

(A) $\sin x \, e^{\sin^2 x}$ (B) $\cos x \, e^{\sin^2 x}$

(C) $-2 \cos x \, e^{\sin^2 x}$ (D) $-2 \sin^2 x \cos x \, e^{\sin^2 x}$

9. The function $f(x) = \frac{x}{2} + \frac{2}{x}$ has a local minima at x equal to :

(A) 2 (B) 1

(C) 0 (D) –2

10. Given a curve $y = 7x - x^3$ and x increases at the rate of 2 units per second. The rate at which the slope of the curve is changing, when $x = 5$ is :

(A) –60 units/sec (B) 60 units/sec

(C) –70 units/sec (D) –140 units/sec

11. $\int \frac{1}{x(\log x)^2} dx$ is equal to :

(A) $2 \log (\log x) + c$ (B) $-\frac{1}{\log x} + c$

(C) $\frac{(\log x)^3}{3} + c$ (D) $\frac{3}{(\log x)^3} + c$

12. The value of $\int_{-1}^{1} x\,|x|\,dx$ is :

(A) $\frac{1}{6}$ (B) $\frac{1}{3}$

(C) $-\frac{1}{6}$ (D) 0

13. Area of the region bounded by curve $y^2 = 4x$ and the X-axis between $x = 0$ and $x = 1$ is :

(A) $\frac{2}{3}$ (B) $\frac{8}{3}$

(C) 3 (D) $\frac{4}{3}$

14. The order of the differential equation $\frac{d^4y}{dx^4} - \sin\left(\frac{d^2y}{dx^2}\right) = 5$ is :

(A) 4 (B) 3

(C) 2 (D) not defined

15. The position vectors of points P and Q are $\vec{p}$ and $\vec{q}$ respectively. The point R divides line segment PQ in the ratio 3 : 1 and S is the mid-point of line segment PR. The position vector of S is :

(A) $\frac{\vec{p} + 3\vec{q}}{4}$ (B) $\frac{\vec{p} + 3\vec{q}}{8}$

(C) $\frac{5\vec{p} + 3\vec{q}}{4}$ (D) $\frac{5\vec{p} + 3\vec{q}}{8}$

16. The angle which the line $\frac{x}{1} = \frac{y}{-1} = \frac{z}{0}$ makes with the positive direction of Y-axis is :

(A) $\frac{5\pi}{6}$ (B) $\frac{3\pi}{4}$

(C) $\frac{5\pi}{4}$ (D) $\frac{7\pi}{4}$

17. The Cartesian equation of the line passing through the point (1, –3, 2) and parallel to the line :

$\vec{r} = (2 + \lambda)\hat{i} + \lambda\hat{j} + (2\lambda - 1)\hat{k}$ is

(A) $\frac{x-1}{2} = \frac{y+3}{0} = \frac{z-2}{-1}$ (B) $\frac{x+1}{1} = \frac{y-3}{1} = \frac{z+2}{2}$

(C) $\frac{x+1}{2} = \frac{y-3}{0} = \frac{z+2}{-1}$ (D) $\frac{x-1}{1} = \frac{y+3}{1} = \frac{z-2}{2}$

18. If A and B are events such that P(A/B) = P(B/A) $\neq$ 0, then :

(A) $A \subset B$, but $A \neq B$ (B) $A = B$

(C) $A \cap B = \phi$ (D) $P(A) = P(B)$

Assertion – Reason Based Questions

Direction : In questions numbers **19** and **20**, two statements are given one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer from the following options :

(A) Both Assertion (A) and Reason (R) are true and the Reason (R) is the correct explanation of the Assertion (A).

(B) Both Assertion (A) and Reason (R) are true and Reason (R) is not the correct explanation of the Assertion (A).

(C) Assertion (A) is true, but Reason (R) is false.

(D) Assertion (A) is false, but Reason (R) is true.

19. **Assertion (A) :** Domain of $y = \cos^{-1}(x)$ is $[-1, 1]$.

Reason (R) : The range of the principal value branch of $y = \cos^{-1}(x)$ is $[0, \pi] - \left\{\frac{\pi}{2}\right\}$.

20. **Assertion (A)** : The vectors

$$\vec{a} = 6\hat{i} + 2\hat{j} - 8\hat{k}$$

$$\vec{b} = 10\hat{i} - 2\hat{j} - 6\hat{k}$$

$$\vec{c} = 4\hat{i} - 4\hat{j} + 2\hat{k}$$

represent the sides of a right angled triangle.

Reason (R) : Three non-zero vectors of which none of two are collinear forms a triangle if their resultant is zero vector or sum of any two vectors is equal to the third.

SECTION – B

This section has **5** Very Short Answer questions of **2** marks each.

21. Find value of k if

$$\sin^{-1}\left[k\tan\left(2\cos^{-1}\frac{\sqrt{3}}{2}\right)\right] = \frac{\pi}{3}.$$

22. (a) Verify whether the function f defined by

$$f(x) = \begin{cases} x\sin\left(\frac{1}{x}\right) & , \ x \neq 0 \\ 0 & , \ x = 0 \end{cases}$$

is continuous at $x = 0$ or not.

OR

(b) Check for differentiability of the function f defined by $f(x) = |x - 5|$, at the point $x = 5$.

23. The area of the circle is increasing at a uniform rate of 2 cm^2/sec. How fast is the circumference of the circle increasing when the radius r = 5 cm ?

24. (a) Find : $\int \cos^3 x \, e^{\log \sin x} \, dx$

OR

(b) Find : $\int \frac{1}{5 + 4x - x^2} \, dx$

25. Find the vector equation of the line passing through the point (2, 3, –5) and making equal angles with the co-ordinate axes.

SECTION – C

There are **6** short answer questions in this section. Each is of **3** marks.

26. (a) Find $\frac{dy}{dx}$, if $(\cos x)^y = (\cos y)^x$.

OR

(b) If $\sqrt{1 - x^2} + \sqrt{1 - y^2} = a(x - y)$, prove that $\frac{dy}{dx} = \sqrt{\frac{1 - y^2}{1 - x^2}}$.

27. If $x = a \sin^3 \theta$, $y = b \cos^3 \theta$, then find $\frac{d^2y}{dx^2}$ at $\theta = \frac{\pi}{4}$.

28. (a) Evaluate : $\int_0^{\pi} \frac{e^{\cos x}}{e^{\cos x} + e^{-\cos x}} \, dx$

OR

(b) Find : $\int \frac{2x + 1}{(x + 1)^2 \, (x - 1)} \, dx$

29. (a) Find the particular solution of the differential equation

$$\frac{dy}{dx} - 2xy = 3x^2 e^{x^2} ; y(0) = 5.$$

OR

(b) Solve the following differential equation :

$$x^2 \, dy + y(x + y) \, dx = 0$$

30. Find a vector of magnitude 4 units perpendicular to each of the vectors $2\hat{i} - \hat{j} + \hat{k}$ and $\hat{i} + \hat{j} - \hat{k}$ and hence verify your answer.

31. The random variable X has the following probability distribution where a and b are some constants :

X	1	2	3	4	5
P(X)	0.2	a	a	0.2	b

If the mean E(X) = 3, then find values of a and b and hence determine $P(X \geq 3)$.

SECTION – D

There are **4** long answer questions in this section. Each question is of **5** marks.

32. (a) If $A = \begin{bmatrix} 1 & 2 & -3 \\ 2 & 0 & -3 \\ 1 & 2 & 0 \end{bmatrix}$, then find A^{-1} and hence solve the following system of equations :

$$x + 2y - 3z = 1$$

$$2x - 3z = 2$$

$$x + 2y = 3$$

OR

(b) Find the product of the matrices $\begin{bmatrix} 1 & 2 & -3 \\ 2 & 3 & 2 \\ 3 & -3 & -4 \end{bmatrix} \begin{bmatrix} -6 & 17 & 13 \\ 14 & 5 & -8 \\ -15 & 9 & -1 \end{bmatrix}$ and hence solve the system of linear equations :

$x + 2y - 3z = -4$

$2x + 3y + 2z = 2$

$3x - 3y - 4z = 11$

33. Find the area of the region bounded by the curve $4x^2 + y^2 = 36$ using integration.

34. (a) Find the co-ordinates of the foot of the perpendicular drawn from the point $(2, 3, -8)$ to the line $\frac{4-x}{2} = \frac{y}{6} = \frac{1-z}{3}$.

Also, find the perpendicular distance of the given point from the line.

OR

(b) Find the shortest distance between the lines L_1 & L_2 given below :

L_1 : The line passing through $(2, -1, 1)$ and parallel to $\frac{x}{1} = \frac{y}{1} = \frac{z}{3}$

$L_2 : \vec{r} = \hat{i} + (2\mu + 1)\hat{j} - (\mu + 2)\hat{k}$.

35. Solve the following L.P.P. graphically :

Maximise $Z = 60x + 40y$

Subject to $x + 2y \leq 12$

$2x + y \leq 12$

$4x + 5y \geq 20$

$x, y \geq 0$

SECTION – E

In this section there are **3** case study questions of **4** marks each.

36. (a) Students of a school are taken to a railway museum to learn about railways heritage and its history.

An exhibit in the museum depicted many rail lines on the track near the railway station. Let L be the set of all rail lines on the railway track and R be the relation on L defined by

$R = \{(l_1, l_2) : l_1 \text{ is parallel to } l_2\}$

On the basis of the above information, answer the following questions :

(i) Find whether the relation R is symmetric or not.

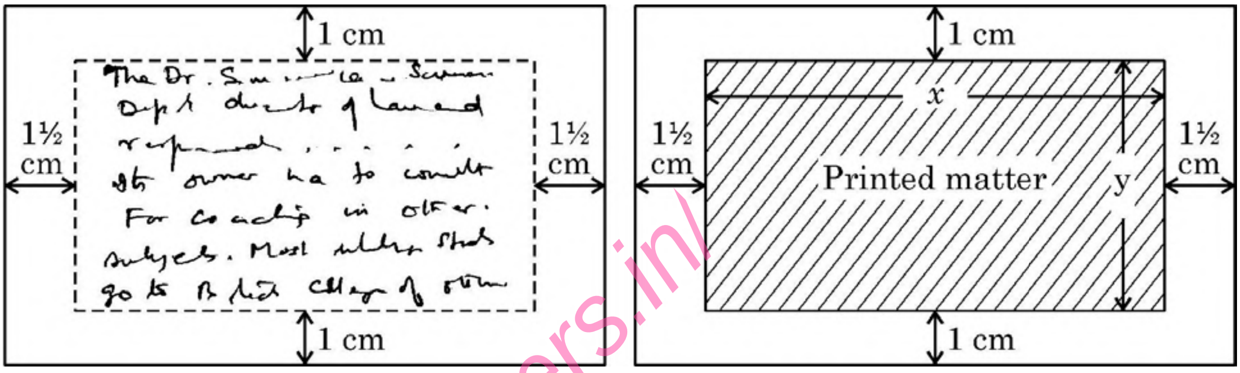
(ii) Find whether the relation R is transitive or not.

(iii) If one of the rail lines on the railway track is represented by the equation $y = 3x + 2$, then find the set of rail lines in R related to it.

OR

(b) Let S be the relation defined by $S = \{(l_1, l_2) : l_1 \text{ is perpendicular to } l_2\}$ check whether the relation S is symmetric and transitive.

37. A rectangular visiting card is to contain 24 sq.cm. of printed matter. The margins at the top and bottom of the card are to be 1 cm and the margins on the left and right are to be 1½ cm as shown below :



On the basis of the above information, answer the following questions :

(i) Write the expression for the area of the visiting card in terms of x.

(ii) Obtain the dimensions of the card of minimum area.

38. A departmental store sends bills to charge its customers once a month. Past experience shows that 70% of its customers pay their first month bill in time. The store also found that the customer who pays the bill in time has the probability of 0.8 of paying in time next month and the customer who doesn't pay in time has the probability of 0.4 of paying in time the next month.

Based on the above information, answer the following questions :

(i) Let E_1 and E_2 respectively denote the event of customer paying or not paying the first month bill in time.
Find $P(E_1)$, $P(E_2)$.

(ii) Let A denotes the event of customer paying second month's bill in time, then find $P(A|E_1)$ and $P(A|E_2)$.

(iii) Find the probability of customer paying second month's bill in time.

OR

(iii) Find the probability of customer paying first month's bill in time if it is found that customer has paid the second month's bill in time.

Series &RQPS/S

Set – 3

प्रश्न-पत्र कोड
Q.P. Code **65/S/3**

अनुक्रमांक
Roll No.

परीक्षार्थी प्रश्न-पत्र कोड को उत्तर-पुस्तिका के मुख-पृष्ठ पर अवश्य लिखें ।
Candidates must write the Q.P. Code on the title page of the answer-book.

- *कृपया जाँच कर लें कि इस प्रश्न-पत्र में मुद्रित पृष्ठ **23** हैं ।*
- *कृपया जाँच कर लें कि इस प्रश्न-पत्र में **38** प्रश्न हैं ।*
- *प्रश्न-पत्र में दाहिने हाथ की ओर दिए गए प्रश्न-पत्र कोड को परीक्षार्थी उत्तर-पुस्तिका के मुख-पृष्ठ पर लिखें ।*
- ***कृपया प्रश्न का उत्तर लिखना शुरू करने से पहले, उत्तर-पुस्तिका में प्रश्न का क्रमांक अवश्य लिखें ।***
- *इस प्रश्न-पत्र को पढ़ने के लिए 15 मिनट का समय दिया गया है । प्रश्न-पत्र का वितरण पूर्वाह्न में 10.15 बजे किया जाएगा । 10.15 बजे से 10.30 बजे तक छात्र केवल प्रश्न-पत्र को पढ़ेंगे और इस अवधि के दौरान वे उत्तर-पुस्तिका पर कोई उत्तर नहीं लिखेंगे ।*
- *Please check that this question paper contains **23** printed pages.*
- *Please check that this question paper contains **38** questions.*
- *Q.P. Code given on the right hand side of the question paper should be written on the title page of the answer-book by the candidate.*
- ***Please write down the serial number of the question in the answer-book before attempting it.***
- *15 minute time has been allotted to read this question paper. The question paper will be distributed at 10.15 a.m. From 10.15 a.m. to 10.30 a.m., the students will read the question paper only and will not write any answer on the answer-book during this period.*

गणित
MATHEMATICS

निर्धारित समय : 3 घण्टे *अधिकतम अंक : 80*

Time allowed : 3 hours *Maximum Marks : 80*

General Instructions :

Read the following instructions very carefully and strictly follow them :

(i) *This question paper contains* ***38*** *questions.* ***All*** *questions are* ***compulsory****.*

(ii) *This question paper is divided into* ***five*** *Sections –* ***A, B, C, D*** *and* ***E****.*

(iii) *In* ***Section A,*** *Questions no.* ***1*** *to* ***18*** *are Multiple Choice Questions (MCQs) and questions number* ***19*** *and* ***20*** *are Assertion-Reason based questions of* ***1*** *mark each.*

v) *In* ***Section B,*** *Questions no.* ***21*** *to* ***25*** *are Very Short Answer (VSA) type questions, carrying* ***2*** *marks each.*

) *In* ***Section C,*** *Questions no.* ***26*** *to* ***31*** *are Short Answer (SA) type questions, carrying* ***3*** *marks each.*

i) *In* ***Section D,*** *Questions no.* ***32*** *to* ***35*** *are Long Answer (LA) type questions carrying* ***5*** *marks each.*

ii) *In* ***Section E,*** *Questions no.* ***36*** *to* ***38*** *are case study based questions carrying* ***4*** *marks each.*

iii) *There is no overall choice. However, an internal choice has been provided in 2 questions in Section B, 3 questions in Section C, 2 questions in Section D and 2 questions in Section E.*

x) *Use of calculator is* ***not*** *allowed.*

SECTION A

his section comprises 20 Multiple Choice Questions (MCQs) carrying ***1*** *mark each. 20×1=20*

. If $A = \begin{bmatrix} 1 & 3 & 3 \\ 1 & 3 & 4 \\ 1 & 4 & 3 \end{bmatrix}$, then the value of $\det(A^{-1})$ is :

(A) -1 (B) 1

(C) 0 (D) 2

2. The greatest integer function defined by $f(x) = [x]$, $1 < x < 3$ is not differentiable at x =

(A) 0 (B) 1

(C) 2 (D) $\frac{3}{2}$

3. If the radius of a circle is increasing at the rate of 0.5 cm/s, then the rate of increase of its circumference is :

(A) $\frac{2\pi}{3}$ cm/s (B) π cm/s

(C) $\frac{4\pi}{3}$ cm/s (D) 2π cm/s

. $\int \frac{x-3}{(x-1)^3} e^x \, dx$ is equal to :

(A) $\frac{2\,e^x}{(x-1)^3} + C$ (B) $\frac{-2\,e^x}{(x-1)^2} + C$

(C) $\frac{e^x}{(x-1)} + C$ (D) $\frac{e^x}{(x-1)^2} + C$

. The area (in sq. units) of the region bounded by the curve $y = x$, x-axis, $x = 0$ and $x = 2$ is :

(A) $\frac{3}{2}$ (B) $\frac{1}{2} \log 2$

(C) 2 (D) 4

. What is the value of $\frac{\text{projection of } \vec{a} \text{ on } \vec{b}}{\text{projection of } \vec{b} \text{ on } \vec{a}}$

for vectors $\vec{a} = 2\hat{i} - 3\hat{j} - 6\hat{k}$ and $\vec{b} = 2\hat{i} - 2\hat{j} + \hat{k}$?

(A) $\frac{3}{7}$ (B) $\frac{7}{3}$

(C) $\frac{4}{3}$ (D) $\frac{4}{7}$

7. If $f(x) = x^x$, then $f'(e)$ is equal to :

(A) 0 (B) 2

(C) e^e (D) $2e^e$

8. The function f given by $f(x) = x^3 - 3x^2 + 3x,\ x \in \mathbb{R}$ is increasing on :

(A) $[1, \infty)$ (B) $(1, \infty)$

(C) $(-\infty, \infty)$ (D) $(-\infty, 1)$

9. If $\vec{a}$ and $\vec{b}$ are two vectors such that $\vec{a}\cdot\vec{b} > 0$ and $|\vec{a}\cdot\vec{b}| = |\vec{a}\times\vec{b}|$, then the angle between $\vec{a}$ and $\vec{b}$ is :

(A) $\frac{\pi}{4}$ (B) $\frac{\pi}{3}$

(C) $\frac{2\pi}{3}$ (D) $\frac{3\pi}{4}$

0. For two matrices A and B, given that $A^{-1} = \frac{1}{4}B$, then inverse of $(4A)$ is :

(A) 4B (B) B

(C) $\frac{1}{4}B$ (D) $\frac{1}{16}B$

1. $\int_{\pi/6}^{\pi/3} \log(\tan x)\, dx$ is equal to :

(A) $\frac{\pi}{2}$ (B) $\frac{\pi}{4}$

(C) 0 (D) $\frac{\pi}{12}$

12. If X, Y and XY are matrices of order 2×3, $m \times n$ and 2×5 respectively, then number of elements in matrix Y is :

(A) 6 (B) 10

(C) 15 (D) 35

13. The number of discontinuities of the function f given by

$$f(x) = \begin{cases} x+2, & \text{if} \quad x<0 \\ e^{x}, & \text{if} \quad 0 \le x \le 1 \\ 2-x, & \text{if} \quad x>1 \end{cases}$$

is :

(A) 0 (B) 1

(C) 2 (D) 3

4. The integrating factor of the differential equation $(x^2 - 1)\frac{dy}{dx} + 2xy = \frac{1}{x^2 - 1}$ is :

(A) e^{2x} (B) $e^{x^2 - 1}$

(C) $\log(x^2 - 1)$ (D) $x^2 - 1$

5. Let $y = f\left(\frac{1}{x}\right)$ and $f'(x) = x^3$. What is the value of $\frac{dy}{dx}$ at $x = \frac{1}{2}$?

(A) $-\frac{1}{64}$ (B) $-\frac{1}{32}$

(C) -32 (D) -64

6. The vectors $\vec{a} = 2\hat{i} - 4\hat{j} + \lambda\hat{k}$ and $\vec{b} = 3\hat{i} - 6\hat{j} + \hat{k}$ are collinear if value of λ is :

(A) -30 (B) 30

(C) $\frac{3}{2}$ (D) $\frac{2}{3}$

17. The vector equation of the line passing through the origin and perpendicular to the lines $\frac{x}{1} = \frac{y}{0} = \frac{z}{0}$ and $\frac{x}{0} = \frac{y}{0} = \frac{z}{1}$ is :

(A) $\vec{r} = \lambda\hat{i}$ (B) $\vec{r} = \lambda\hat{j}$

(C) $\vec{r} = \lambda\hat{k}$ (D) $\vec{r} = \lambda(\hat{i} + \hat{k})$

18. If $y = \log\sqrt{\sec\sqrt{x}}$, then the value of $\frac{dy}{dx}$ at $x = \frac{\pi^2}{16}$ is :

(A) $\frac{1}{\pi}$ (B) π

(C) $\frac{1}{2}$ (D) $\frac{1}{4}$

*uestions number **19** and **20** are Assertion and Reason based questions. Two statements re given, one labelled Assertion (A) and the other labelled Reason (R). Select the orrect answer from the codes (A), (B), (C) and (D) as given below.*

(A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).

(B) Both Assertion (A) and Reason (R) are true, but Reason (R) is ***not*** the correct explanation of the Assertion (A).

(C) Assertion (A) is true, but Reason (R) is false.

(D) Assertion (A) is false, but Reason (R) is true.

9. *Assertion (A) :* $\cos^{-1}\left(\cos\frac{13\pi}{6}\right)$ is equal to $\frac{\pi}{6}$.

Reason (R) : The range of the principal value branch of the function $y = \cos^{-1} x$ is $[0, \pi]$.

0. *Assertion (A) :* If R and S are two events such that $P(R \mid S) = 1$ and $P(S) > 0$, then $S \subset R$.

Reason (R) : If two events A and B are such that $P(A \cap B) = P(B)$, then $A \subset B$.

SECTION B

This section comprises Very Short Answer (VSA) type questions of 2 marks each.

21. Find the value of

$$\tan^{-1}\left(\tan\frac{3\pi}{5}\right) + \cos^{-1}\left(\cos\frac{13\pi}{6}\right) + \sin^{-1}\left(-\frac{1}{2}\right).$$

22. (a) Find :

$$\int \frac{x^3 - 1}{x^3 - x}\, dx$$

OR

(b) Evaluate :

$$\int_{-4}^{0} |x+2|\, dx$$

3. (a) If $y = (\sin^{-1} x)^2$, then find $(1 - x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx}$.

OR

(b) If $y^x = x^y$, then find $\frac{dy}{dx}$.

4. Given that $f(x) = \frac{\log x}{x}$, find the point of local maximum of f(x).

5. Find the Cartesian equation of the line passing through the origin, perpendicular to y-axis and making equal acute angles with x and z axes.

SECTION C

his section comprises Short Answer (SA) type questions of 3 marks each.

26. (a) Find :

$$\int \frac{dx}{\cos x \sqrt{\cos 2x}}$$

OR

(b) Find :

$$\int \frac{5x-3}{\sqrt{1+4x-2x^2}}\,dx$$

27. (a) If $y = e^{a\cos^{-1}x}$, then show that $(1-x^2)\dfrac{d^2y}{dx^2} - x\dfrac{dy}{dx} - a^2y = 0$.

OR

(b) Find $\dfrac{dy}{dx}$, if $y = x^{\cos x} - 2^{\sin x}$.

8. It is known that 20% of the students in a school have above 90% attendance and 80% of the students are irregular. Past year results show that 80% of students who have above 90% attendance and 20% of irregular students get ‘A’ grade in their annual examination. At the end of a year, a student is chosen at random from the school and is found to have an ‘A’ grade. What is the probability that the student is irregular ?

9. Find the general solution of the differential equation $\dfrac{dy}{dx} = xy \log x \log y$.

0. Find all vectors of magnitude $8\sqrt{14}$ units that are perpendicular to the vectors $2\hat{i} - \hat{k}$ and $2\hat{j} + 3\hat{k}$.

1. (a) Find a matrix A such that

$$A\begin{bmatrix} 4 & 0 \\ -1 & -2 \end{bmatrix} = \begin{bmatrix} 17 & 10 \\ 0 & -16 \end{bmatrix}.$$

Also, find A^{-1}.

OR

(b) Given a square matrix A of order 3 such that $A^2 = \begin{bmatrix} 3 & -4 & 4 \\ 0 & -1 & 0 \\ -2 & 2 & -3 \end{bmatrix}$,

show that $A^3 = A^{-1}$.

SECTION D

This section comprises Long Answer (LA) type questions of 5 marks each.

32. Prove that the relation R in the set of integers Z defined as

R = {(a, b) : 2 divides (a + b)} is an equivalence relation. Also, determine [3].

33. (a) Find the shortest distance between the lines $\frac{x-8}{3} = \frac{y+9}{-16} = \frac{z-10}{7}$ and

$$\frac{x-15}{3} = \frac{y-29}{8} = \frac{z-5}{-5}.$$

OR

(b) Find the point of intersection of the lines

$\vec{r} = \hat{i} - \hat{j} + 6\hat{k} + \lambda(3\hat{i} - \hat{k})$, and

$\vec{r} = -3\hat{j} + 3\hat{k} + \mu(\hat{i} + 2\hat{j} - \hat{k})$.

Also, find the vector equation of the line passing through the point of intersection of the given lines and perpendicular to both the lines.

34. Solve the following linear programming problem graphically :

Minimise $Z = 6x + 7y$

subject to constraints

$x + 2y \geq 240$

$3x + 4y \leq 620$

$2x + y \geq 180$

$x, y \geq 0.$

35. (a) Using integration, find the area of the region bounded by the curve $y = \sqrt{4 - x^2}$, the lines $x = -\sqrt{2}$ and $x = \sqrt{3}$ and the x-axis.

OR

(b) Using integration, evaluate the area of the region bounded by the curve $y = x^2$, the lines y = 1 and y = 3 and the y-axis.

~~~

SECTION E

This section comprises 3 case study based questions of 4 marks each.

Case Study – 1

6. An architect is developing a plot of land for a commercial complex. When asked about the dimensions of the plot, he said that if the length is decreased by 25 m and the breadth is increased by 25 m, then its area increases by 625 m^2. If the length is decreased by 20 m and the breadth is increased by 10 m, then its area decreases by 200 m^2.

On the basis of the above information, answer the following questions :

(i) Formulate the linear equations in x and y to represent the given information.

(ii) Find the dimensions of the plot of land by matrix method.
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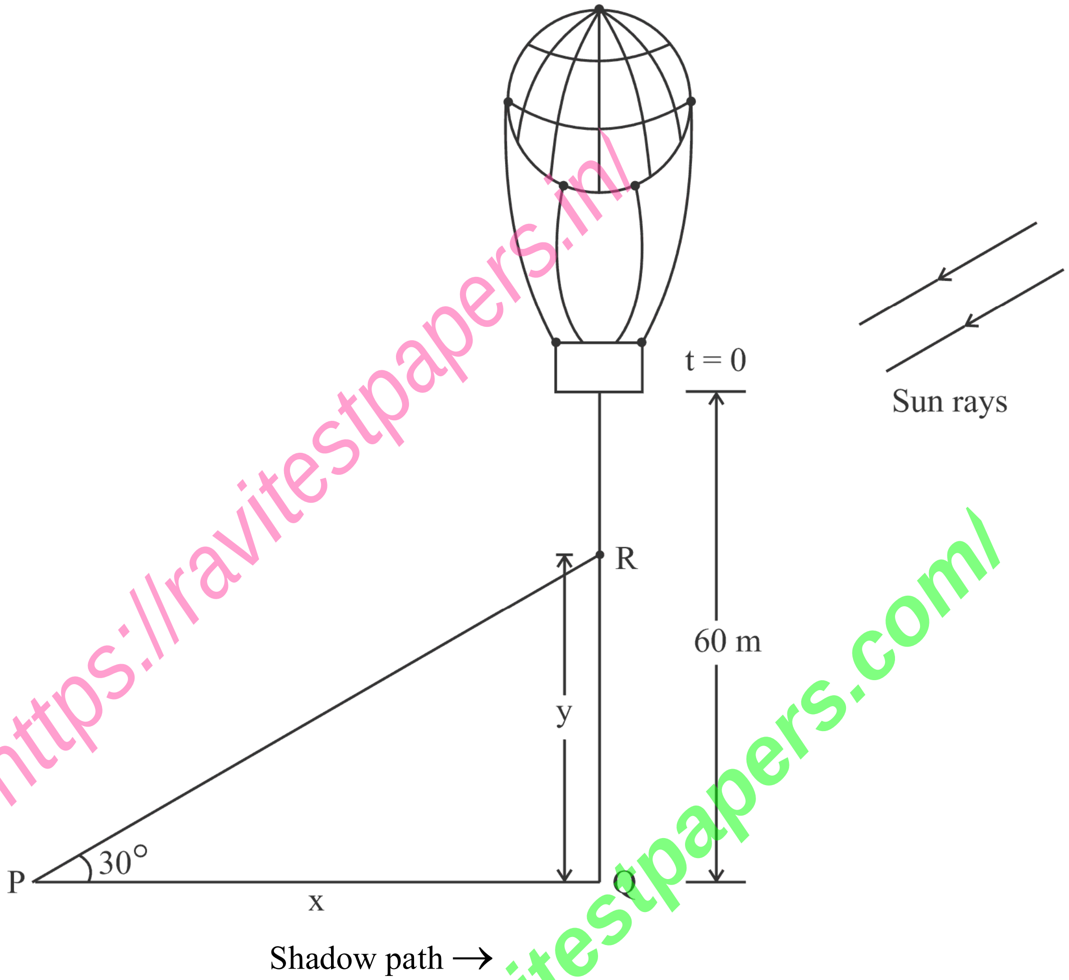
Case Study – 2

37. A sandbag is dropped from a balloon at a height of 60 metres.



When the angle of elevation of the sun is 30°, the position of the sandbag is given by the equation $y = 60 - 4.9\ t^2$, where y is the height of the sandbag above the ground and t is the time in seconds.

On the basis of the above information, answer the following questions :

(i) Find the relation between x and y, where x is the distance of the shadow at P from the point Q and y is the height of the sandbag above the ground.

(ii) After how much time will the sandbag be 35 metres above the ground ?

(iii) (a) Find the rate at which the shadow of the sandbag is travelling along the ground when the sandbag is at a height of 35 metres.

OR

(iii) (b) How fast is the height of the sandbag decreasing when 2 seconds have elapsed ?

Series EF1GH/C **SET~2**

प्रश्न–पत्र कोड
Q.P. Code **65/C/2**

रोल नं.
Roll No.

परीक्षार्थी प्रश्न-पत्र कोड को उत्तर-पुस्तिका के मुख-पृष्ठ पर अवश्य लिखें ।
Candidates must write the Q.P. Code on the title page of the answer-book.

गणित

MATHEMATICS

*

निर्धारित समय : 3 घण्टे
Time allowed : 3 hours

अधिकतम अंक : 80
Maximum Marks : 80

नोट / NOTE :

(i) *कृपया जाँच कर लें कि इस प्रश्न-पत्र में मुद्रित पृष्ठ* ***23*** *हैं ।*
Please check that this question paper contains ***23*** *printed pages.*

(ii) *प्रश्न-पत्र में दाहिने हाथ की ओर दिए गए प्रश्न-पत्र कोड को परीक्षार्थी उत्तर-पुस्तिका के मुख-पृष्ठ पर लिखें ।*
Q.P. Code given on the right hand side of the question paper should be written on the title page of the answer-book by the candidate.

(iii) *कृपया जाँच कर लें कि इस प्रश्न-पत्र में* ***38*** *प्रश्न हैं ।*
Please check that this question paper contains ***38*** *questions.*

(iv) ***कृपया प्रश्न का उत्तर लिखना शुरू करने से पहले, उत्तर-पुस्तिका में प्रश्न का क्रमांक अवश्य लिखें ।***
Please write down the serial number of the question in the answer-book before attempting it.

(v) *इस प्रश्न-पत्र को पढ़ने के लिए 15 मिनट का समय दिया गया है । प्रश्न-पत्र का वितरण पूर्वाह्न में 10.15 बजे किया जाएगा । 10.15 बजे से 10.30 बजे तक छात्र केवल प्रश्न-पत्र को पढ़ेंगे और इस अवधि के दौरान वे उत्तर-पुस्तिका पर कोई उत्तर नहीं लिखेंगे ।*
15 minute time has been allotted to read this question paper. The question paper will be distributed at 10.15 a.m. From 10.15 a.m. to 10.30 a.m., the students will read the question paper only and will not write any answer on the answer-book during this period.

सामान्य निर्देश :

निम्नलिखित निर्देशों को बहुत सावधानी से पढ़िए और उनका सख़्ती से पालन कीजिए :

(i) *इस प्रश्न-पत्र में* ***38*** *प्रश्न हैं ।* ***सभी*** *प्रश्न* ***अनिवार्य*** *हैं ।*

(ii) *यह प्रश्न-पत्र* ***पाँच*** *खण्डों में विभाजित है –* ***क, ख, ग, घ*** *एवं* ***ङ*** *।*

(iii) ***खण्ड क*** *में प्रश्न संख्या* ***1*** *से* ***18*** *तक बहुविकल्पीय तथा प्रश्न संख्या* ***19*** *एवं* ***20*** *अभिकथन एवं तर्क आधारित* ***एक-एक*** *अंक के प्रश्न हैं ।*

(iv) ***खण्ड ख*** *में प्रश्न संख्या* ***21*** *से* ***25*** *तक अति लघु-उत्तरीय (VSA) प्रकार के* ***दो-दो*** *अंकों के प्रश्न हैं ।*

(v) ***खण्ड ग*** *में प्रश्न संख्या* ***26*** *से* ***31*** *तक लघु-उत्तरीय (SA) प्रकार के* ***तीन-तीन*** *अंकों के प्रश्न हैं ।*

(vi) ***खण्ड घ*** *में प्रश्न संख्या* ***32*** *से* ***35*** *तक दीर्घ-उत्तरीय (LA) प्रकार के* ***पाँच-पाँच*** *अंकों के प्रश्न हैं ।*

(vii) ***खण्ड ङ*** *में प्रश्न संख्या* ***36*** *से* ***38*** *प्रकरण अध्ययन आधारित* ***चार-चार*** *अंकों के प्रश्न हैं ।*

(viii) *प्रश्न-पत्र में समग्र विकल्प नहीं दिया गया है । यद्यपि, खण्ड ख के 2 प्रश्नों में, खण्ड ग के 3 प्रश्नों में, खण्ड घ के 2 प्रश्नों में तथा खण्ड ङ के 2 प्रश्नों में आंतरिक विकल्प का प्रावधान दिया गया है ।*

(ix) *कैल्कुलेटर का उपयोग* ***वर्जित*** *है ।*

खण्ड क

इस खण्ड में बहुविकल्पीय प्रश्न हैं, जिनमें प्रत्येक प्रश्न 1 अंक का है ।

1. $\sin^{-1}(2x\sqrt{1-x^2})$ के सापेक्ष $\tan^{-1}\left(\frac{x}{\sqrt{1-x^2}}\right)$ का अवकलज है :

(a) $-\frac{1}{4}$

(b) $\frac{1}{2}$

(c) 2

(d) $-\frac{1}{2}$

General Instructions :

Read the following instructions very carefully and strictly follow them :

(i) *This question paper contains* ***38*** *questions.* ***All*** *questions are* ***compulsory****.*

(ii) *This question paper is divided into* ***five*** *Sections –* ***A, B, C, D*** *and* ***E****.*

(iii) *In* ***Section A,*** *Questions no.* ***1*** *to* ***18*** *are multiple choice questions (MCQs) and questions number* ***19*** *and* ***20*** *are Assertion-Reason based questions of* ***1*** *mark each.*

(iv) *In* ***Section B,*** *Questions no.* ***21*** *to* ***25*** *are very short answer (VSA) type questions, carrying* ***2*** *marks each.*

(v) *In* ***Section C,*** *Questions no.* ***26*** *to* ***31*** *are short answer (SA) type questions, carrying* ***3*** *marks each.*

(vi) *In* ***Section D,*** *Questions no.* ***32*** *to* ***35*** *are long answer (LA) type questions carrying* ***5*** *marks each.*

(vii) *In* ***Section E,*** *Questions no.* ***36*** *to* ***38*** *are case study based questions carrying* ***4*** *marks each.*

(viii) *There is no overall choice. However, an internal choice has been provided in 2 questions in Section B, 3 questions in Section C, 2 questions in Section D and 2 questions in Section E.*

(ix) *Use of calculators is* ***not*** *allowed.*

SECTION A

This section comprises multiple choice questions (MCQs) of 1 mark each.

1. Derivative of $\tan^{-1}\left(\frac{x}{\sqrt{1-x^2}}\right)$ with respect to $\sin^{-1}(2x\sqrt{1-x^2})$ is :

(a) $-\frac{1}{4}$ (b) $\frac{1}{2}$

(c) 2 (d) $-\frac{1}{2}$

2. यह दिया गया है कि $X\begin{bmatrix}3 & 2\\1 & -1\end{bmatrix} = \begin{bmatrix}4 & 1\\2 & 3\end{bmatrix}$ है । तो आव्यूह X है :

(a) $\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}$ (b) $\begin{bmatrix}0 & -1\\1 & 1\end{bmatrix}$

(c) $\begin{bmatrix}1 & 1\\1 & -1\end{bmatrix}$ (d) $\begin{bmatrix}1 & -1\\1 & -1\end{bmatrix}$

3. आव्यूह $\begin{bmatrix}4 & 3 & 2\\2 & -1 & 0\\1 & 2 & 3\end{bmatrix}$ में दूसरी पंक्ति और तीसरे स्तम्भ में स्थित अवयव के सहखंड का मान है :

(a) 5 (b) – 5 (c) – 11 (d) 11

4. अवकल समीकरण $(1 + y^2)(1 + \log x)\,dx + x\,dy = 0$ का हल है :

(a) $\tan^{-1} y + \log |x| + \dfrac{(\log |x|)^2}{2} = C$

(b) $\tan^{-1} y - \log |x| + \dfrac{(\log |x|)^2}{2} = C$

(c) $\tan^{-1} y - \log |x| - \dfrac{(\log |x|)^2}{2} = C$

(d) $\tan^{-1} y + \log |x| - \dfrac{(\log |x|)^2}{2} = C$

5. यदि ABCD एक समांतर चतुर्भुज है और AC तथा BD इसके विकर्ण हैं, तो $\overrightarrow{AC} + \overrightarrow{BD}$ है :

(a) $2\overrightarrow{DA}$ (b) $2\overrightarrow{AB}$ (c) $2\overrightarrow{BC}$ (d) $2\overrightarrow{BD}$

6. यदि $x = a \cos \theta + b \sin \theta$, $y = a \sin \theta - b \cos \theta$ है, तो निम्नलिखित में से कौन-सा सही है ?

(a) $y^2 \dfrac{d^2y}{dx^2} - x\dfrac{dy}{dx} + y = 0$ (b) $y^2 \dfrac{d^2y}{dx^2} + x\dfrac{dy}{dx} + y = 0$

(c) $y^2 \dfrac{d^2y}{dx^2} + x\dfrac{dy}{dx} - y = 0$ (d) $y^2 \dfrac{d^2y}{dx^2} - x\dfrac{dy}{dx} - y = 0$

2. It is given that $X\begin{bmatrix} 3 & 2 \\ 1 & -1 \end{bmatrix} = \begin{bmatrix} 4 & 1 \\ 2 & 3 \end{bmatrix}$. Then matrix X is :

(a) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ (b) $\begin{bmatrix} 0 & -1 \\ 1 & 1 \end{bmatrix}$

(c) $\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$ (d) $\begin{bmatrix} 1 & -1 \\ 1 & -1 \end{bmatrix}$

3. The value of the cofactor of the element of second row and third column in the matrix $\begin{bmatrix} 4 & 3 & 2 \\ 2 & -1 & 0 \\ 1 & 2 & 3 \end{bmatrix}$ is :

(a) 5 (b) -5 (c) -11 (d) 11

4. Solution of the differential equation $(1 + y^2)(1 + \log x)\, dx + x\, dy = 0$ is :

(a) $\tan^{-1} y + \log |x| + \dfrac{(\log |x|)^2}{2} = C$

(b) $\tan^{-1} y - \log |x| + \dfrac{(\log |x|)^2}{2} = C$

(c) $\tan^{-1} y - \log |x| - \dfrac{(\log |x|)^2}{2} = C$

(d) $\tan^{-1} y + \log |x| - \dfrac{(\log |x|)^2}{2} = C$

5. If ABCD is a parallelogram and AC and BD are its diagonals, then $\vec{AC} + \vec{BD}$ is :

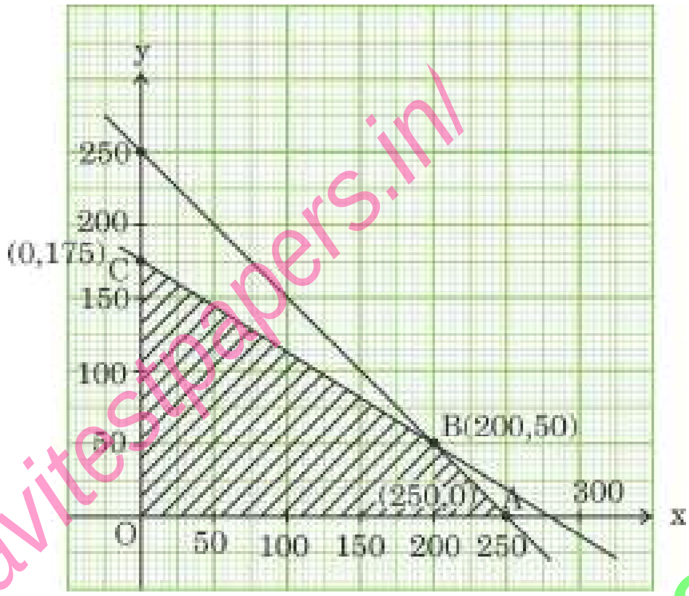
(a) $2\vec{DA}$ (b) $2\vec{AB}$ (c) $2\vec{BC}$ (d) $2\vec{BD}$

6. If $x = a \cos \theta + b \sin \theta$, $y = a \sin \theta - b \cos \theta$, then which one of the following is true ?

(a) $y^2 \dfrac{d^2y}{dx^2} - x\dfrac{dy}{dx} + y = 0$ (b) $y^2 \dfrac{d^2y}{dx^2} + x\dfrac{dy}{dx} + y = 0$

(c) $y^2 \dfrac{d^2y}{dx^2} + x\dfrac{dy}{dx} - y = 0$ (d) $y^2 \dfrac{d^2y}{dx^2} - x\dfrac{dy}{dx} - y = 0$

7. एक LPP के परिबद्ध सुसंगत क्षेत्र के कोणीय बिंदु O(0, 0), A(250, 0), B(200, 50) और C(0, 175) हैं । यदि उद्देश्य फलन $Z = 2ax + by$ का अधिकतम मान बिंदुओं A(250, 0) और B(200, 50) पर है, तो a और b के बीच का संबंध होगा :



(a) 2a = b (b) 2a = 3b (c) a = b (d) a = 2b

8. एक परिवार में 2 बच्चे हैं और बड़ा बच्चा एक लड़की है । दोनों बच्चों के लड़कियाँ होने की प्रायिकता है :

(a) $\frac{1}{4}$ (b) $\frac{1}{8}$ (c) $\frac{1}{2}$ (d) $\frac{3}{4}$

9. यदि आव्यूह $A = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$ और $A^2 = kA$ है, तो k का मान होगा :

(a) 1 (b) – 2 (c) 2 (d) – 1

10. बिंदु (1, – 2, 3) से गुज़रने वाली और सदिश $3\hat{i} - 2\hat{j} + 4\hat{k}$ के समांतर रेखा का सदिश समीकरण है :

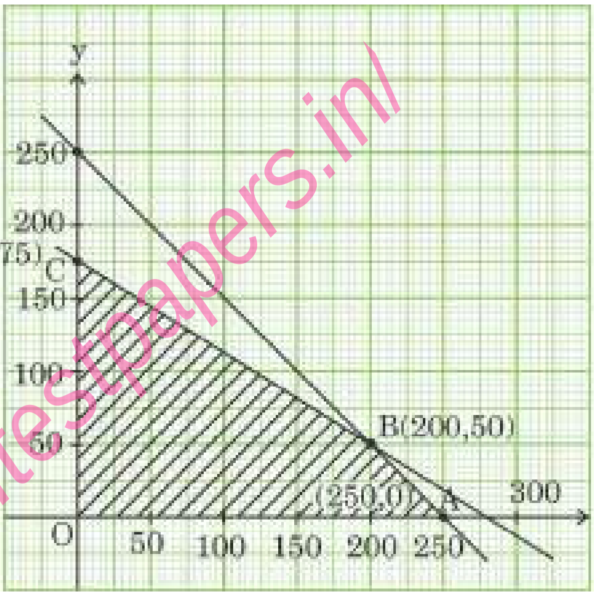
(a) $\vec{r} = (-\hat{i} + 2\hat{j} + 3\hat{k}) + \lambda(3\hat{i} - 2\hat{j} + 4\hat{k})$

(b) $\vec{r} = (-3\hat{i} + 2\hat{j} - 4\hat{k}) + \lambda(\hat{i} - 2\hat{j} + 3\hat{k})$

(c) $\vec{r} = (\hat{i} - 2\hat{j} + 3\hat{k}) + \lambda(3\hat{i} - 2\hat{j} + 4\hat{k})$

(d) $\vec{r} = (3\hat{i} - 2\hat{j} + 4\hat{k}) + \lambda(\hat{i} - 2\hat{j} + 3\hat{k})$

7. The corner points of the bounded feasible region of an LPP are O(0, 0), A(250, 0), B(200, 50) and C(0, 175). If the maximum value of the objective function $Z = 2ax + by$ occurs at the points A(250, 0) and B(200, 50), then the relation between a and b is :



(a) $2a = b$
(b) $2a = 3b$
(c) $a = b$
(d) $a = 2b$

8. A family has 2 children and the elder child is a girl. The probability that both children are girls is :

(a) $\frac{1}{4}$
(b) $\frac{1}{8}$
(c) $\frac{1}{2}$
(d) $\frac{3}{4}$

9. If matrix $A = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$ and $A^2 = kA$, then the value of k is :

(a) 1
(b) -2
(c) 2
(d) -1

10. The vector equation of a line which passes through the point $(1, -2, 3)$ and is parallel to the vector $3\hat{i} - 2\hat{j} + 4\hat{k}$ is :

(a) $\vec{r} = (-\hat{i} + 2\hat{j} + 3\hat{k}) + \lambda(3\hat{i} - 2\hat{j} + 4\hat{k})$
(b) $\vec{r} = (-3\hat{i} + 2\hat{j} - 4\hat{k}) + \lambda(\hat{i} - 2\hat{j} + 3\hat{k})$
(c) $\vec{r} = (\hat{i} - 2\hat{j} + 3\hat{k}) + \lambda(3\hat{i} - 2\hat{j} + 4\hat{k})$
(d) $\vec{r} = (3\hat{i} - 2\hat{j} + 4\hat{k}) + \lambda(\hat{i} - 2\hat{j} + 3\hat{k})$

11. यदि $\begin{bmatrix} 3 & 2 \\ 1 & x \end{bmatrix} \begin{bmatrix} x \\ 1 \end{bmatrix} = \begin{bmatrix} 14 \\ 8 \end{bmatrix}$ है, तो x है :

(a) $\frac{16}{3}$ (b) – 3

(c) – 4 (d) 4

12. यदि A, कोटि 3 का एक वर्ग आव्यूह है और $|A| = 6$ है, तो $|\text{adj } A|$ का मान है :

(a) 6 (b) 36

(c) 27 (d) 216

13. $\int_0^{\pi/6} \sin 3x \, dx$ का मान है :

(a) $-\frac{\sqrt{3}}{2}$ (b) $-\frac{1}{3}$

(c) $\frac{\sqrt{3}}{2}$ (d) $\frac{1}{3}$

14. यदि $\vec{a}$, $\vec{b}$ और $(\vec{a} + \vec{b})$ सभी मात्रक सदिश हैं और $\vec{a}$ तथा $\vec{b}$ के बीच का कोण θ है, तो θ का मान होगा :

(a) $\frac{2\pi}{3}$ (b) $\frac{5\pi}{6}$ (c) $\frac{\pi}{3}$ (d) $\frac{\pi}{6}$

15. $\int \frac{\cos 2x}{\sin^2 x \, . \cos^2 x} dx$ बराबर है :

(a) tan x – cot x + C (b) – cot x – tan x + C

(c) cot x + tan x + C (d) tan x – cot x – C

16. अर्ध-तल $2x + y - 4 \le 0$ में स्थित बिंदु है :

(a) (0, 8) (b) (1, 1)

(c) (5, 5) (d) (2, 2)

11. If $\begin{bmatrix} 3 & 2 \\ 1 & x \end{bmatrix} \begin{bmatrix} x \\ 1 \end{bmatrix} = \begin{bmatrix} 14 \\ 8 \end{bmatrix}$, then x is :

(a) $\frac{16}{3}$ (b) -3

(c) -4 (d) 4

12. If A is a square matrix of order 3 and $|A| = 6$, then the value of $|\text{adj } A|$ is :

(a) 6 (b) 36

(c) 27 (d) 216

13. The value of $\int_0^{\pi/6} \sin 3x \, dx$ is :

(a) $-\frac{\sqrt{3}}{2}$ (b) $-\frac{1}{3}$

(c) $\frac{\sqrt{3}}{2}$ (d) $\frac{1}{3}$

14. If $\vec{a}$, $\vec{b}$ and $(\vec{a} + \vec{b})$ are all unit vectors and θ is the angle between $\vec{a}$ and $\vec{b}$, then the value of θ is :

(a) $\frac{2\pi}{3}$ (b) $\frac{5\pi}{6}$ (c) $\frac{\pi}{3}$ (d) $\frac{\pi}{6}$

15. $\int \frac{\cos 2x}{\sin^2 x \cdot \cos^2 x} dx$ is equal to

(a) $\tan x - \cot x + C$ (b) $-\cot x - \tan x + C$

(c) $\cot x + \tan x + C$ (d) $\tan x - \cot x - C$

16. The point which lies in the half-plane $2x + y - 4 \leq 0$ is :

(a) (0, 8) (b) (1, 1)

(c) (5, 5) (d) (2, 2)

17. माना कि दो बिंदुओं P और Q के स्थिति सदिश क्रमशः $\vec{a} - 2\vec{b}$ और $2\vec{a} + \vec{b}$ हैं । P और Q को मिलाने वाले रेखाखण्ड को 3 : 2 के अनुपात में बाह्यत: विभाजित करने वाले बिंदु का स्थिति सदिश है :

(a) $4\vec{a} + 7\vec{b}$

(b) $\dfrac{8\vec{a} + 7\vec{b}}{5}$

(c) $4\vec{a} - 7\vec{b}$

(d) $\vec{a} + 4\vec{b}$

18. अवकल समीकरण $\left(\dfrac{d^2y}{dx^2}\right)^2 + \left(\dfrac{dy}{dx}\right)^3 + x^4 = 0$ की कोटि और घात में अंतर है :

(a) 1 (b) 2 (c) – 1 (d) 0

प्रश्न संख्या ***19*** *और* ***20*** *अभिकथन एवं तर्क आधारित प्रश्न हैं और प्रत्येक प्रश्न का 1 अंक है । दो कथन दिए गए हैं जिनमें एक को अभिकथन (A) तथा दूसरे को तर्क (R) द्वारा अंकित किया गया है । इन प्रश्नों के सही उत्तर नीचे दिए गए कोडों (a), (b), (c) और (d) में से चुनकर दीजिए ।*

(a) अभिकथन (A) और तर्क (R) दोनों सही हैं और तर्क (R), अभिकथन (A) की सही व्याख्या करता है ।

(b) अभिकथन (A) और तर्क (R) दोनों सही हैं और तर्क (R), अभिकथन (A) की सही व्याख्या ***नहीं*** करता है ।

(c) अभिकथन (A) सही है, परन्तु तर्क (R) ग़लत है ।

(d) अभिकथन (A) ग़लत है, परन्तु तर्क (R) सही है ।

19. *अभिकथन (A) :* $\cot^{-1}(\sqrt{3})$ का मुख्य मान $\dfrac{\pi}{6}$ है ।

तर्क (R) : $\cot^{-1} x$ का प्रांत $\mathbb{R} - \{-1, 1\}$ है ।

20. *अभिकथन (A) :* शीर्षों A(0, 0, 0), B(3, 4, 5), C(8, 8, 8) और D(5, 4, 3) से बना चतुर्भुज एक समचतुर्भुज है ।

तर्क (R) : ABCD एक समचतुर्भुज है, यदि AB = BC = CD = DA, AC ≠ BD है ।

17. Let P and Q be two points with position vectors $\vec{a} - 2\vec{b}$ and $2\vec{a} + \vec{b}$ respectively. The position vector of a point which divides the join of P and Q externally in the ratio 3 : 2 is :

(a) $4\vec{a} + 7\vec{b}$

(b) $\dfrac{8\vec{a} + 7\vec{b}}{5}$

(c) $4\vec{a} - 7\vec{b}$

(d) $\vec{a} + 4\vec{b}$

18. The difference of the order and the degree of the differential equation $\left(\dfrac{d^2y}{dx^2}\right)^2 + \left(\dfrac{dy}{dx}\right)^3 + x^4 = 0$ is :

(a) 1 (b) 2 (c) – 1 (d) 0

*Questions number **19** and **20** are Assertion and Reason based questions carrying 1 mark each. Two statements are given, one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer from the codes (a), (b), (c) and (d) as given below.*

(a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).

(b) Both Assertion (A) and Reason (R) are true and Reason (R) is ***not*** the correct explanation of the Assertion (A).

(c) Assertion (A) is true, but Reason (R) is false.

(d) Assertion (A) is false, but Reason (R) is true.

19. *Assertion (A) :* The principal value of $\cot^{-1}(\sqrt{3})$ is $\dfrac{\pi}{6}$.

Reason (R) : Domain of $\cot^{-1} x$ is $\mathbb{R} - \{-1, 1\}$.

20. *Assertion (A) :* Quadrilateral formed by vertices A(0, 0, 0), B(3, 4, 5), C(8, 8, 8) and D(5, 4, 3) is a rhombus.

Reason (R) : ABCD is a rhombus if AB = BC = CD = DA, AC ≠ BD.

खण्ड ख

इस खण्ड में अति लघु-उत्तरीय (VSA) प्रकार के प्रश्न हैं, जिनमें प्रत्येक के 2 अंक हैं ।

21. वह अंतराल ज्ञात कीजिए, जिसमें फलन $x^3 - 12x^2 + 36x + 17$ निरंतर वर्धमान है ।

22. वह बिंदु ज्ञात कीजिए, जिनमें फलन $f(x) = \dfrac{4 + x^2}{4x - x^3}$ असंतत है ।

23. (क) यदि $\vec{a}$, $\vec{b}$ और $\vec{c}$ तीन सदिश इस प्रकार हैं कि $|\vec{a}| = 7$, $|\vec{b}| = 24$, $|\vec{c}| = 25$ और $\vec{a} + \vec{b} + \vec{c} = \vec{0}$ है, तो $\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}$ का मान ज्ञात कीजिए ।

अथवा

(ख) यदि एक रेखा x-अक्ष, y-अक्ष और z-अक्ष के साथ क्रमशः α, β और γ कोण बनाती है, तो सिद्ध कीजिए कि $\sin^2 \alpha + \sin^2 \beta + \sin^2 \gamma = 2$ है ।

24. (क) सरल कीजिए :

$$\tan^{-1}\left(\frac{\cos x}{1 - \sin x}\right)$$

अथवा

(ख) सिद्ध कीजिए कि $f(x) = [x]$ द्वारा प्रदत्त महत्तम पूर्णांक फलन $f : \mathbb{R} \to \mathbb{R}$ न तो एकैकी है और न ही आच्छादक है ।

25. सदिशों $\vec{a} = \hat{i} - 2\hat{j} + 3\hat{k}$ और $\vec{b} = 2\hat{i} + 3\hat{j} - 5\hat{k}$ के लिए सत्यापित कीजिए कि $\vec{a}$ और $\vec{a} \times \vec{b}$ के बीच का कोण $\frac{\pi}{2}$ होता है ।

खण्ड ग

इस खण्ड में लघु-उत्तरीय (SA) प्रकार के प्रश्न हैं, जिनमें प्रत्येक के 3 अंक हैं ।

26. ज्ञात कीजिए :

$$\int \frac{e^x}{\sqrt{5 - 4e^x - e^{2x}}}\, dx$$

SECTION B

This section comprises very short answer (VSA) type questions of 2 marks each.

21. Find the interval in which the function $x^3 - 12x^2 + 36x + 17$ is strictly increasing.

22. Find the points at which the function $f(x) = \dfrac{4 + x^2}{4x - x^3}$ is discontinuous.

23. (a) If $\vec{a}$, $\vec{b}$ and $\vec{c}$ are three vectors such that $|\vec{a}| = 7$, $|\vec{b}| = 24$, $|\vec{c}| = 25$ and $\vec{a} + \vec{b} + \vec{c} = \vec{0}$, then find the value of $\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}$.

OR

(b) If a line makes angles α, β and γ with x-axis, y-axis and z-axis respectively, then prove that $\sin^2 \alpha + \sin^2 \beta + \sin^2 \gamma = 2$.

24. (a) Simplify :

$$\tan^{-1}\left(\frac{\cos x}{1 - \sin x}\right)$$

OR

(b) Prove that the greatest integer function $f : \mathbb{R} \rightarrow \mathbb{R}$, given by $f(x) = [x]$, is neither one-one nor onto.

25. For the vectors $\vec{a} = \hat{i} - 2\hat{j} + 3\hat{k}$ and $\vec{b} = 2\hat{i} + 3\hat{j} - 5\hat{k}$, verify that the angle between $\vec{a}$ and $\vec{a} \times \vec{b}$ is $\frac{\pi}{2}$.

SECTION C

This section comprises short answer (SA) type questions of 3 marks each.

26. Find :

$$\int \frac{e^x}{\sqrt{5 - 4e^x - e^{2x}}}\, dx$$

27. ज्ञात कीजिए :

$$\int \frac{2x^2+1}{x^2(x^2+4)}\,dx$$

28. (क) दो थैलों में से थैले A में 2 सफ़ेद और 3 लाल गेंदें हैं और थैले B में 4 सफ़ेद और 5 लाल गेंदें हैं । यादृच्छया एक गेंद को एक थैले में से निकाला गया और पाया गया कि यह लाल है । प्रायिकता ज्ञात कीजिए कि इसे थैले B में से निकाला गया था ।

अथवा

(ख) 50 व्यक्तियों के समूह में से 20 सदैव सच बोलते हैं । इस समूह में से यादृच्छया 2 व्यक्तियों को चुना गया (बिना प्रतिस्थापना के) । चुने गए उन व्यक्तियों की संख्या का प्रायिकता बंटन ज्ञात कीजिए जो सदैव सच बोलते हैं ।

29. (क) अवकल समीकरण $\frac{dy}{dx} = \frac{xy}{x^2+y^2}$ का विशिष्ट हल ज्ञात कीजिए, दिया गया है कि जब $x = 0$ है, तो $y = 1$ है ।

अथवा

(ख) अवकल समीकरण $(1 + x^2)\frac{dy}{dx} + 2xy = \frac{1}{1+x^2}$ का विशिष्ट हल ज्ञात कीजिए, दिया गया है कि जब $x = 1$ है, तो $y = 0$ है ।

30. (क) मान ज्ञात कीजिए :

$$\int_0^{\pi/2} \frac{x \sin x \cos x}{\sin^4 x + \cos^4 x}\,dx$$

अथवा

(ख) मान ज्ञात कीजिए :

$$\int_1^3 (|x-1| + |x-2|)\,dx$$

31. निम्नलिखित रैखिक प्रोग्रामन समस्या को आलेखीय विधि से हल कीजिए :
निम्नलिखित व्यवरोधों के अंतर्गत,

$z = 10x + 15y$ का अधिकतमीकरण कीजिए

$$3x + 2y \le 50$$
$$x + 4y \ge 20$$
$$x \ge 8,\ \ y \ge 0$$

27. Find :

$$\int \frac{2x^2 + 1}{x^2(x^2 + 4)}\, dx$$

28. (a) Out of two bags, bag A contains 2 white and 3 red balls and bag B contains 4 white and 5 red balls. One ball is drawn at random from one of the bags and is found to be red. Find the probability that it was drawn from bag B.

OR

(b) Out of a group of 50 people, 20 always speak the truth. Two persons are selected at random from the group (without replacement). Find the probability distribution of number of selected persons who always speak the truth.

29. (a) Find the particular solution of the differential equation $\frac{dy}{dx} = \frac{xy}{x^2 + y^2}$, given that y = 1 when x = 0.

OR

(b) Find the particular solution of the differential equation $(1 + x^2)\frac{dy}{dx} + 2xy = \frac{1}{1 + x^2}$, given that y = 0 when x = 1.

30. (a) Evaluate :

$$\int_0^{\pi/2} \frac{x \sin x \cos x}{\sin^4 x + \cos^4 x}\, dx$$

OR

(b) Evaluate :

$$\int_1^3 (|x - 1| + |x - 2|)\, dx$$

31. Solve the following Linear Programming Problem graphically:
Maximise z = 10x + 15y
subject to the constraints :

$$3x + 2y \leq 50$$
$$x + 4y \geq 20$$
$$x \geq 8, \;\; y \geq 0$$

खण्ड घ

इस खण्ड में दीर्घ-उत्तरीय (LA) प्रकार के प्रश्न हैं, जिनमें प्रत्येक के 5 अंक हैं ।

32. (क) दर्शाइए कि रेखाएँ $\frac{x+1}{3} = \frac{y+3}{5} = \frac{z+5}{7}$ और $\frac{x-2}{1} = \frac{y-4}{3} = \frac{z-6}{5}$ प्रतिच्छेदी रेखाएँ हैं । इनका प्रतिच्छेदन बिन्दु भी ज्ञात कीजिए ।

अथवा

(ख) रेखा युग्मों $\frac{x-1}{2} = \frac{y+1}{3} = z$ और $\frac{x+1}{5} = \frac{y-2}{1}$; $z = 2$ के बीच की न्यूनतम दूरी ज्ञात कीजिए ।

33. (क) दर्शाइए कि वास्तविक संख्याओं के समुच्चय $\mathbb{R}$ में

$$S = \{(a, b) : a \leq b^3,\ a \in \mathbb{R},\ b \in \mathbb{R}\}$$

द्वारा परिभाषित संबंध S न तो स्वतुल्य है, न सममित है और न ही संक्रामक है ।

अथवा

(ख) माना कि समुच्चय A = {1, 2, 3, 4, 5, 6, 7} में संबंध R इस प्रकार परिभाषित है R = {(a, b) : a और b दोनों या तो विषम हैं या सम हैं} दर्शाइए कि R एक तुल्यता संबंध है । अत:, तुल्यता वर्ग [1] के अवयव ज्ञात कीजिए ।

34. माना $A = \begin{bmatrix} 1 & 1 & 1 \\ 6 & 7 & 8 \\ 6 & 7 & -8 \end{bmatrix}$ है, तो A^{-1} ज्ञात कीजिए, अत: निम्नलिखित रैखिक समीकरण निकाय को हल कीजिए :

$$x + y + z = 5000$$
$$6x + 7y + 8z = 35800$$
$$6x + 7y - 8z = 7000$$

35. समाकलन के उपयोग से उस त्रिभुज ABC से घिरे क्षेत्र का क्षेत्रफल ज्ञात कीजिए जिसकी भुजाएँ $4x - y + 5 = 0$, $x + y - 5 = 0$ और $x - 4y + 5 = 0$ रेखाओं द्वारा निरूपित है ।

SECTION D

This section comprises long answer type questions (LA) of 5 marks each.

32. (a) Show that the lines $\frac{x+1}{3} = \frac{y+3}{5} = \frac{z+5}{7}$ and $\frac{x-2}{1} = \frac{y-4}{3} = \frac{z-6}{5}$ intersect. Also find their point of intersection.

OR

(b) Find the shortest distance between the pair of lines $\frac{x-1}{2} = \frac{y+1}{3} = z$ and $\frac{x+1}{5} = \frac{y-2}{1}$; $z = 2$.

33. (a) Show that the relation S in set $\mathbb{R}$ of real numbers defined by

$$S = \{(a, b) : a \leq b^3,\ a \in \mathbb{R},\ b \in \mathbb{R}\}$$

is neither reflexive, nor symmetric, nor transitive.

OR

(b) Let R be the relation defined in the set $A = \{1, 2, 3, 4, 5, 6, 7\}$ by $R = \{(a, b) : \text{both a and b are either odd or even}\}$. Show that R is an equivalence relation. Hence, find the elements of equivalence class [1].

34. Let $A = \begin{bmatrix} 1 & 1 & 1 \\ 6 & 7 & 8 \\ 6 & 7 & -8 \end{bmatrix}$, find A^{-1} and hence solve the following system of linear equations :

$$x + y + z = 5000$$
$$6x + 7y + 8z = 35800$$
$$6x + 7y - 8z = 7000$$

35. Using integration, find the area of the region bounded by the triangle ABC when its sides are given by the lines $4x - y + 5 = 0$, $x + y - 5 = 0$ and $x - 4y + 5 = 0$.

खण्ड ङ

इस खण्ड में 3 प्रकरण अध्ययन आधारित प्रश्न हैं जिनमें प्रत्येक के 4 अंक हैं ।

प्रकरण अध्ययन – 1

36. एक हाउसिंग सोसाइटी अपने निवासियों के लिए तैराकी हेतु एक पूल (तालाब) बनाना चाहती है । इसके लिए उन्हें एक वर्गाकार भूमि खरीदनी है और इस गहराई तक खोदना है कि इस पूल की क्षमता 250 घन मीटर हो जाए । भूमि की कीमत ₹ 500 प्रति वर्ग मीटर है । खोदने की कीमत में गहराई की अधिकता के अनुसार वृद्धि होती जाती है तथा पूरे पूल की लागत ₹ 4000 $(\text{गहराई})^2$ है ।

मान लीजिए कि वर्गाकार प्लॉट की भुजा x मीटर और गहराई h मीटर है ।

उपर्युक्त सूचना के आधार पर, निम्न प्रश्नों के उत्तर दीजिए :

(i) लागत फलन C(h) को h के पदों में लिखिए । *1*

(ii) क्रांतिक बिंदु ज्ञात कीजिए । *1*

SECTION E

This section comprises 3 case study based questions of 4 marks each.

Case Study – 1

36. A housing society wants to commission a swimming pool for its residents. For this, they have to purchase a square piece of land and dig this to such a depth that its capacity is 250 cubic metres. Cost of land is ₹ 500 per square metre. The cost of digging increases with the depth and cost for the whole pool is ₹ 4000 $(\text{depth})^2$.

Suppose the side of the square plot is x metres and depth is h metres.
On the basis of the above information, answer the following questions :

(i) Write cost C(h) as a function in terms of h. *1*

(ii) Find critical point. *1*

(iii) (क) द्वितीय अवकलज परीक्षण द्वारा h का वह मान ज्ञात कीजिए, जिसके लिए पूल बनाने की लागत न्यूनतम हो । पूल बनाने की न्यूनतम लागत क्या है ? *2*

अथवा

(iii) (ख) प्रथम अवकलज परीक्षण से पूल की ऐसी गहराई ज्ञात कीजिए कि पूल बनाने की लागत न्यूनतम हो । न्यूनतम लागत के लिए x और h के बीच का संबंध भी ज्ञात कीजिए । *2*

प्रकरण अध्ययन – 2

37. एक समूह क्रियाकलाप की कक्षा में 10 विद्यार्थी हैं जिनकी आयु 16, 17, 15, 14, 19, 17, 16, 19, 16 और 15 वर्ष हैं । एक विद्यार्थी को यादृच्छया इस प्रकार चुना गया कि प्रत्येक विद्यार्थी के चुने जाने की संभावना समान है और चुने गए विद्यार्थी की आयु को लिखा गया ।

उपर्युक्त सूचना के आधार पर, निम्न प्रश्नों के उत्तर दीजिए :

(i) प्रायिकता ज्ञात कीजिए कि चुने गए विद्यार्थी की आयु एक भाज्य संख्या है । *1*

(ii) माना X चुने हुए विद्यार्थी की आयु है, तो X का क्या मान हो सकता है ? *1*

(iii) (क) यादृच्छया चर X का प्रायिकता बंटन ज्ञात कीजिए तथा माध्य आयु ज्ञात कीजिए । *2*

अथवा

(iii) (a) Use second derivative test to find the value of h for which cost of constructing the pool is minimum. What is the minimum cost of construction of the pool ? *2*

OR

(iii) (b) Use first derivative test to find the depth of the pool so that cost of construction is minimum. Also, find relation between x and h for minimum cost. *2*

Case Study – 2

37. In a group activity class, there are 10 students whose ages are 16, 17, 15, 14, 19, 17, 16, 19, 16 and 15 years. One student is selected at random such that each has equal chance of being chosen and age of the student is recorded.

On the basis of the above information, answer the following questions :

(i) Find the probability that the age of the selected student is a composite number. *1*

(ii) Let X be the age of the selected student. What can be the value of X ? *1*

(iii) (a) Find the probability distribution of random variable X and hence find the mean age. *2*

OR

(iii) (ख) एक यादृच्छया चुने गए विद्यार्थी की आयु 15 वर्ष से अधिक पाई गई । प्रायिकता ज्ञात कीजिए कि उसकी आयु एक अभाज्य संख्या है । 2

प्रकरण अध्ययन – 3

38. एक कृषि संस्थान में, वैज्ञानिक बीजों की किस्मों को अलग-अलग वातावरणों में उगाने का प्रयोग करते हैं जिससे कि स्वस्थ पौधे उगें और अधिक उपज प्राप्त हो ।

एक वैज्ञानिक ने अवलोकन किया कि एक विशेष बीज अंकुरित होने के बाद बहुत तेज़ी से बढ़ रहा है । उसने बीज के अंकुरण के बाद से ही पौधे की वृद्धि को रिकॉर्ड किया था और उसने कहा कि इस वृद्धि को फलन $f(x) = \frac{1}{3}x^3 - 4x^2 + 15x + 2, \; 0 \le x \le 10$ से परिभाषित किया जा सकता है, जहाँ x दिनों की वह संख्या है जिनमें पौधा सूर्य के प्रकाश से उजागर था ।

उपर्युक्त सूचना के आधार पर, निम्न प्रश्नों के उत्तर दीजिए :

(i) इस फलन f(x) के क्रांतिक बिंदु कौन-से हैं ? 2

(ii) द्वितीय अवकलज परीक्षण का प्रयोग करके, फलन का न्यूनतम मान ज्ञात कीजिए । 2

(iii) (b) A student was selected at random and his age was found to be greater than 15 years. Find the probability that his age is a prime number. *2*

Case Study – 3

38. In an agricultural institute, scientists do experiments with varieties of seeds to grow them in different environments to produce healthy plants and get more yield.

A scientist observed that a particular seed grew very fast after germination. He had recorded growth of plant since germination and he said that its growth can be defined by the function

$$f(x) = \frac{1}{3}x^3 - 4x^2 + 15x + 2, \ \ 0 \le x \le 10$$

where x is the number of days the plant is exposed to sunlight.

On the basis of the above information, answer the following questions :

(i) What are the critical points of the function f(x) ? *2*

(ii) Using second derivative test, find the minimum value of the function. *2*

Series : WXYZ/S **SET~2**

प्रश्न-पत्र कोड
Q.P. Code **65/S/2**

रोल नं.
Roll No. ○○○○○○○○

परीक्षार्थी प्रश्न-पत्र कोड को उत्तर-पुस्तिका के मुख-पृष्ठ पर अवश्य लिखें।
Candidates must write the Q.P. Code on the title page of the answer-book.

गणित
MATHEMATICS

*नेर्धारित समय : **3** घण्टे* *अधिकतम अंक : **80***
*Time allowed : **3** hours* *Maximum Marks : **80***

नोट

) कृपया जाँच कर लें कि इस प्रश्न-पत्र में मुद्रित पृष्ठ **23** हैं।

I) प्रश्न-पत्र में दाहिने हाथ की ओर दिए गए प्रश्न-पत्र कोड को परीक्षार्थी उत्तर-पुस्तिका के मुख-पृष्ठ पर लिखें।

II) कृपया जाँच कर लें कि इस प्रश्न-पत्र में **38** प्रश्न हैं।

V) **कृपया प्रश्न का उत्तर लिखना शुरू करने से पहले, उत्तर-पुस्तिका में यथा स्थान पर प्रश्न का क्रमांक अवश्य लिखें।**

) इस प्रश्न-पत्र को पढ़ने के लिए 15 मिनट का समय दिया गया है। प्रश्न-पत्र का वितरण पूर्वाह्न में 10.15 बजे किया जाएगा। 10.15 बजे से 10.30 बजे तक परीक्षार्थी केवल प्रश्न-पत्र को पढ़ेंगे और इस अवधि के दौरान वे उत्तर-पुस्तिका पर कोई उत्तर नहीं लिखेंगे।

NOTE

(I) Please check that this question paper contains **23** printed pages.

(II) Q.P. Code given on the right hand side of the question paper should be written on the title page of the answer-book by the candidate.

(III) Please check that this question paper contains **38** questions.

(IV) **Please write down the Serial Number of the question in the answer-book at the given place before attempting it.**

(V) 15 minute time has been allotted to read this question paper. The question paper will be distributed at 10.15 a.m. From 10.15 a.m. to 10.30 a.m., the candidates will read the question paper only and will not write any answer on the answer-book during this period. #

सामान्य निर्देश :

निम्नलिखित निर्देशों को बहुत सावधानी से पढ़िए और उनका सख़्ती से पालन कीजिए :

(i) इस प्रश्न-पत्र में **38** प्रश्न हैं। **सभी** प्रश्न **अनिवार्य** हैं।

(ii) यह प्रश्न-पत्र **पाँच** खण्डों में विभाजित है – **क, ख, ग, घ** एवं **ङ**।

(iii) **खण्ड क** में प्रश्न संख्या **1** से **18** तक बहुविकल्पीय (MCQ) तथा प्रश्न संख्या **19** एवं **20** अभिकथन एवं तर्क आधारित **1** अंक के प्रश्न हैं।

v) **खण्ड ख** में प्रश्न संख्या **21** से **25** तक अति लघु-उत्तरीय (VSA) प्रकार के **2** अंकों के प्रश्न हैं।

) **खण्ड ग** में प्रश्न संख्या **26** से **31** तक लघु-उत्तरीय (SA) प्रकार के **3** अंकों के प्रश्न हैं।

i) **खण्ड घ** में प्रश्न संख्या **32** से **35** तक दीर्घ-उत्तरीय (LA) प्रकार के **5** अंकों के प्रश्न हैं।

ii) **खण्ड ङ** में प्रश्न संख्या **36** से **38** तक प्रकरण अध्ययन आधारित **4** अंकों के प्रश्न हैं।

iii) प्रश्न-पत्र में समग्र विकल्प नहीं दिया गया है। यद्यपि, खण्ड ख के 2 प्रश्नों में, खण्ड ग के 3 प्रश्नों में, खण्ड घ के 2 प्रश्नों में तथा खण्ड ङ के 2 प्रश्नों में आंतरिक विकल्प का प्रावधान दिया गया है।

x) कैल्कुलेटर का उपयोग **वर्जित** है।

खण्ड क

ा खण्ड में बहुविकल्पीय प्रश्न (MCQ) हैं, जिनमें प्रत्येक प्रश्न 1 अंक का है।

. $f(x) = \cos^{-1}(2x)$ का प्रान्त है :

(A) $[-1, 1]$ (B) $\left[0, \frac{1}{2}\right]$

(C) $[-2, 2]$ (D) $\left[-\frac{1}{2}, \frac{1}{2}\right]$

General Instructions :

Read the following instructions very carefully and strictly follow them :

(i) *This question paper contains* ***38*** *questions.* ***All*** *questions are* ***compulsory****.*

(ii) *This question paper is divided into* ***five*** *Sections –* ***A, B, C, D*** *and* ***E****.*

(iii) *In* ***Section A,*** *Questions no.* ***1*** *to* ***18*** *are multiple choice questions (MCQs) and questions number* ***19*** *and* ***20*** *are Assertion-Reason based questions of* ***1*** *mark each.*

v) *In* ***Section B,*** *Questions no.* ***21*** *to* ***25*** *are very short answer (VSA) type questions, carrying* ***2*** *marks each.*

) *In* ***Section C,*** *Questions no.* ***26*** *to* ***31*** *are short answer (SA) type questions, carrying* ***3*** *marks each.*

i) *In* ***Section D,*** *Questions no.* ***32*** *to* ***35*** *are long answer (LA) type questions carrying* ***5*** *marks each.*

ii) *In* ***Section E,*** *Questions no.* ***36*** *to* ***38*** *are case study based questions carrying* ***4*** *marks each.*

iii) *There is no overall choice. However, an internal choice has been provided in 2 questions in Section B, 3 questions in Section C, 2 questions in Section D and 2 questions in Section E.*

x) *Use of calculator is* ***not*** *allowed.*

SECTION A

his section comprises multiple choice questions (MCQs) of 1 mark each.

. The domain of $f(x) = \cos^{-1}(2x)$ is :

(A) $[-1, 1]$ (B) $\left[0, \frac{1}{2}\right]$

(C) $[-2, 2]$ (D) $\left[-\frac{1}{2}, \frac{1}{2}\right]$

2. यदि $[2x \quad 3]\begin{bmatrix} x \\ -8 \end{bmatrix} = 0$ है, तो x का मान है :

(A) शून्य (B) 3

(C) $2\sqrt{3}$ (D) $\pm 2\sqrt{3}$

3. एक व्युत्क्रमणीय आव्यूह X के लिए, यदि $X^2 = I$ है, तो X^{-1} बराबर है :

(A) X (B) X^2

(C) I (D) O

. एक त्रिभुज जिसके शीर्ष (3, 0), (0, k) तथा (– 3, 0) हैं, का क्षेत्रफल 9 वर्ग इकाई है । k का मान है :

(A) 9 (B) – 9

(C) 3 (D) 6

. सारणिक $\begin{vmatrix} \cos 75^\circ & \sin 75^\circ \\ \sin 15^\circ & \cos 15^\circ \end{vmatrix}$ का मान है :

(A) 1 (B) शून्य

(C) $\frac{1}{2}$ (D) $\frac{\sqrt{3}}{2}$

. $\sin^{-1} x$ के सापेक्ष $\sin^{-1}(2x^2 - 1)$ का अवकलज है :

(A) $\frac{2}{x}$ (B) 2

(C) $\frac{\sqrt{1-x^2}}{\sqrt{1-4x^2}}$ (D) $1 - x^2$

2. If $[2x \quad 3] \begin{bmatrix} x \\ -8 \end{bmatrix} = 0$, then the value of x is :

(A) zero

(B) 3

(C) $2\sqrt{3}$

(D) $\pm 2\sqrt{3}$

3. For a non-singular matrix X, if $X^2 = I$, then X^{-1} is equal to :

(A) X

(B) X^2

(C) I

(D) O

. The area of a triangle with vertices (3, 0), (0, k) and (– 3, 0) is 9 sq units. The value of k is :

(A) 9

(B) – 9

(C) 3

(D) 6

. The value of the determinant $\begin{vmatrix} \cos 75^{\circ} & \sin 75^{\circ} \\ \sin 15^{\circ} & \cos 15^{\circ} \end{vmatrix}$ is :

(A) 1

(B) zero

(C) $\frac{1}{2}$

(D) $\frac{\sqrt{3}}{2}$

. The derivative of $\sin^{-1}(2x^2 - 1)$ with respect to $\sin^{-1} x$ is :

(A) $\frac{2}{x}$

(B) 2

(C) $\frac{\sqrt{1-x^2}}{\sqrt{1-4x^2}}$

(D) $1 - x^2$

7. यदि A कोटि n का एक तत्समक आव्यूह है, तो A (Adj A) है एक :

(A) तत्समक आव्यूह

(B) पंक्ति आव्यूह

(C) शून्य आव्यूह

(D) विषम सममित आव्यूह

. वक्र $x = y^2$, y-अक्ष तथा रेखाओं $y = 3$ तथा $y = 4$ द्वारा घिरे क्षेत्र का क्षेत्रफल है :

(A) $\frac{74}{3}$ वर्ग इकाई (B) $\frac{37}{3}$ वर्ग इकाई

(C) 74 वर्ग इकाई (D) 37 वर्ग इकाई

. एक LPP में, रैखिक निकाय व्यवरोधों द्वारा बने सुसंगत क्षेत्र के कोनीय बिंदु (1, 1), (3, 0) तथा (0, 3) हैं। यदि $Z = ax + by$, जहाँ $a, b > 0$ का न्यूनतमीकरण करना हो और Z का न्यूनतम मान (3, 0) तथा (1, 1) पर हो, तो a तथा b के बीच का संबंध होगा :

(A) $a = 2b$

(B) $a = \frac{b}{2}$

(C) $a = 3b$

(D) $a = b$

0. यदि $\frac{d}{dx} f(x) = 3x^2 - \frac{3}{x^4}$ इस प्रकार है कि $f(1) = 0$ है, तो f(x) है :

(A) $6x + \frac{12}{x^5}$

(B) $x^4 - \frac{1}{x^3} + 2$

(C) $x^3 + \frac{1}{x^3} - 2$

(D) $x^3 + \frac{1}{x^3} + 2$

7. If A is an identity matrix of order n, then A (Adj A) is a/an :

(A) identity matrix

(B) row matrix

(C) zero matrix

(D) skew symmetric matrix

8. The area bounded by the curve $x = y^2$, y-axis and the lines y = 3 and y = 4 is :

(A) $\frac{74}{3}$ sq units   (B) $\frac{37}{3}$ sq units

(C) 74 sq units   (D) 37 sq units

. In an LPP, corner points of the feasible region determined by the system of linear constraints are (1, 1), (3, 0) and (0, 3). If Z = ax + by, where a, b > 0 is to be minimized, the condition on a and b, so that the minimum of Z occurs at (3, 0) and (1, 1), will be :

(A) a = 2b

(B) $a = \frac{b}{2}$

(C) a = 3b

(D) a = b

0. If $\frac{d}{dx} f(x) = 3x^2 - \frac{3}{x^4}$ such that f(1) = 0, then f(x) is :

(A) $6x + \frac{12}{x^5}$

(B) $x^4 - \frac{1}{x^3} + 2$

(C) $x^3 + \frac{1}{x^3} - 2$

(D) $x^3 + \frac{1}{x^3} + 2$

11. व्यवरोधों $x + y \le 1,\ x,\ y \ge 0$ के अंतर्गत $Z = 3x + 4y$ का अधिकतम मान है :

(A) 3 (B) 4

(C) 7 (D) 0

12. $\int \frac{\tan^2 \sqrt{x}}{\sqrt{x}}\, dx$ बराबर है :

(A) $\sec\sqrt{x} + C$ (B) $2\sqrt{x} \tan x - x + C$

(C) $2\,(\tan\sqrt{x} - \sqrt{x}\,) + C$ (D) $2 \tan\sqrt{x} - x + C$

3. एक सिक्का 3 बार उछाला गया। कम-से-कम दो बार चित आने की प्रायिकता है :

(A) $\frac{1}{2}$

(B) $\frac{3}{8}$

(C) $\frac{1}{8}$

(D) $\frac{1}{4}$

4. यदि $|\vec{a}| = 1$, $|\vec{b}| = 2$ तथा $\vec{a} \cdot \vec{b} = 2$ है, तो $|\vec{a} + \vec{b}|$ का मान है :

(A) 9

(B) 3

(C) – 3

(D) 2

11. The maximum value of $Z = 3x + 4y$ subject to the constraints $x + y \leq 1$, $x, y \geq 0$ is :

(A) 3 (B) 4

(C) 7 (D) 0

12. $\int \frac{\tan^2 \sqrt{x}}{\sqrt{x}}\,dx$ is equal to :

(A) $\sec\sqrt{x} + C$ (B) $2\sqrt{x}\tan x - x + C$

(C) $2(\tan\sqrt{x} - \sqrt{x}) + C$ (D) $2\tan\sqrt{x} - x + C$

3. A coin is tossed three times. The probability of getting at least two heads is :

(A) $\frac{1}{2}$

(B) $\frac{3}{8}$

(C) $\frac{1}{8}$

(D) $\frac{1}{4}$

4. If $|\vec{a}| = 1$, $|\vec{b}| = 2$ and $\vec{a} \cdot \vec{b} = 2$, then the value of $|\vec{a} + \vec{b}|$ is :

(A) 9

(B) 3

(C) -3

(D) 2

15. यदि एक गोले के आयतन के परिवर्तन की दर उसकी त्रिज्या के परिवर्तन की दर से दुगुनी है, तो गोले का पृष्ठीय क्षेत्रफल है :

(A) 1 वर्ग इकाई

(B) 2 वर्ग इकाई

(C) 3 वर्ग इकाई

(D) 4 वर्ग इकाई

6. अवकल समीकरण $\frac{dy}{dx} = 2x \, . e^{x^2 + y}$ का व्यापक हल है :

(A)	$e^{x^2 + y} = C$	(B)	$e^{x^2} + e^{-y} = C$
(C)	$e^{x^2} = e^{y} + C$	(D)	$e^{x^2 - y} = C$

7. यदि 'm' तथा 'n' क्रमश: अवकल समीकरण $1 + \left(\frac{dy}{dx}\right)^3 = \frac{d^2y}{dx^2}$ की घात तथा कोटि हैं, तो $(m + n)$ का मान है :

(A) 4

(B) 3

(C) 2

(D) 5

18. दो सदिश $\vec{a}$ तथा $\vec{b}$ इस प्रकार हैं कि $| \vec{a} \times \vec{b} | = \vec{a} \, . \, \vec{b}$ है। दोनों सदिशों के बीच का कोण है :

(A)	30°	(B)	60°
(C)	45°	(D)	90°

15. If the rate of change of volume of a sphere is twice the rate of change of its radius, then the surface area of the sphere is :

(A) 1 sq unit

(B) 2 sq units

(C) 3 sq units

(D) 4 sq units

6. The general solution of the differential equation $\frac{dy}{dx} = 2x \cdot e^{x^2 + y}$ is :

(A) $e^{x^2 + y} = C$ (B) $e^{x^2} + e^{-y} = C$

(C) $e^{x^2} = e^{y} + C$ (D) $e^{x^2 - y} = C$

7. If 'm' and 'n' are the degree and order respectively of the differential equation $1 + \left(\frac{dy}{dx}\right)^3 = \frac{d^2y}{dx^2}$, then the value of (m + n) is :

(A) 4

(B) 3

(C) 2

(D) 5

8. Two vectors $\vec{a}$ and $\vec{b}$ are such that $|\vec{a} \times \vec{b}| = \vec{a} \cdot \vec{b}$. The angle between the two vectors is :

(A) 30° (B) 60°

(C) 45° (D) 90°

*प्रश्न संख्या **19** और **20** अभिकथन एवं तर्क आधारित प्रश्न हैं। दो कथन दिए गए हैं जिनमें एक को अभिकथन (A) तथा दूसरे को तर्क (R) द्वारा अंकित किया गया है। इन प्रश्नों के सही उत्तर नीचे दिए गए कोड (A), (B), (C) और (D) में से चुनकर दीजिए।*

(A) अभिकथन (A) और तर्क (R) दोनों सही हैं और तर्क (R), अभिकथन (A) की सही व्याख्या करता है।

(B) अभिकथन (A) और तर्क (R) दोनों सही हैं, परन्तु तर्क (R), अभिकथन (A) की सही व्याख्या ***नहीं*** करता है।

(C) अभिकथन (A) सही है, परन्तु तर्क (R) ग़लत है।

(D) अभिकथन (A) ग़लत है, परन्तु तर्क (R) सही है।

9. फलन f : R $\longrightarrow$ R पर विचार कीजिए, जिसे $f(x) = x^3$ के रूप में परिभाषित किया गया है।

अभिकथन (A) : f(x) एक एकैकी फलन है।

तर्क (R) : यदि फलन का सहप्रान्त इसके परिसर के समान हो, तो f(x) एकैकी फलन होता है।

0. *अभिकथन (A) :* महत्तम पूर्णांक फलन $f(x) = [x]$, $x \in \mathbb{R}$ में, $x = 2$ पर अवकलनीय नहीं है।

तर्क (R) : महत्तम पूर्णांक फलन किसी भी पूर्णांकीय मान पर संतत नहीं होता।

खण्ड ख

खण्ड में अति लघु-उत्तरीय (VSA) प्रकार के 5 प्रश्न हैं, जिनमें प्रत्येक के 2 अंक हैं।

1. वक्र $\sqrt{x} + \sqrt{y} = 1$ के लिए, बिंदु $\left(\frac{1}{9}, \frac{1}{9}\right)$ पर $\frac{dy}{dx}$ का मान ज्ञात कीजिए।

2. (क) $\cos^{-1}\left(-\frac{1}{2}\right) + 2\sin^{-1}\left(\frac{1}{2}\right)$ का मुख्य मान ज्ञात कीजिए।

अथवा

(ख) सिद्ध कीजिए कि :

$$\tan^{-1}\sqrt{x} = \frac{1}{2}\cos^{-1}\left(\frac{1-x}{1+x}\right), \; x \in [0, 1]$$

*Questions number **19** and **20** are Assertion and Reason based questions. Two statements are given, one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer from the codes (A), (B), (C) and (D) as given below.*

(A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).

(B) Both Assertion (A) and Reason (R) are true, but Reason (R) is ***not*** the correct explanation of the Assertion (A).

(C) Assertion (A) is true, but Reason (R) is false.

(D) Assertion (A) is false, but Reason (R) is true.

9. Consider the function $f : R \longrightarrow R$, defined as $f(x) = x^3$.

Assertion (A) : f(x) is a one-one function.

Reason (R) : f(x) is a one-one function, if co-domain = range.

0. *Assertion (A) :* $f(x) = [x]$, $x \in \mathbb{R}$, the greatest integer function is not differentiable at $x = 2$.

Reason (R) : The greatest integer function is not continuous at any integral value.

SECTION B

his section comprises 5 very short answer (VSA) type questions of 2 marks each.

1. For the curve $\sqrt{x} + \sqrt{y} = 1$, find the value of $\frac{dy}{dx}$ at the point $\left(\frac{1}{9}, \frac{1}{9}\right)$.

2. (a) Find the principal value of $\cos^{-1}\left(-\frac{1}{2}\right) + 2\sin^{-1}\left(\frac{1}{2}\right)$.

OR

(b) Prove that :

$$\tan^{-1}\sqrt{x} = \frac{1}{2}\cos^{-1}\left(\frac{1-x}{1+x}\right), x \in [0, 1]$$

23. (क) यदि बिंदु $(-1, -1, 2)$, $(2, 8, \lambda)$ तथा $(3, 11, 6)$ संरेख हैं, तो λ का मान ज्ञात कीजिए।

अथवा

(ख) $\vec{a}$ तथा $\vec{b}$ दो सह-प्रारंभिक सदिश (co-initial vectors) हैं जो एक समांतर चतुर्भुज की संलग्न भुजाएँ बनाते हैं और $|\vec{a}| = 10$, $|\vec{b}| = 2$ तथा $\vec{a} \cdot \vec{b} = 12$ है। समांतर चतुर्भुज का क्षेत्रफल ज्ञात कीजिए।

4. 13 m लंबी एक सीढ़ी दीवार के सहारे झुकी है। सीढ़ी का नीचे का सिरा, भूमि के अनुदिश दीवार से दूर 2 m/s की दर से खींचा जाता है। दीवार पर इसकी ऊँचाई किस दर से घट रही है, जबकि सीढ़ी के नीचे का सिरा दीवार से 12 m दूर है ?

5. बिंदु $(1, 2, -3)$ से होकर जाने वाली उस रेखा का सदिश समीकरण ज्ञात कीजिए जो दी गई दोनों रेखाओं $\frac{x-8}{3} = \frac{y+16}{-16} = \frac{x-10}{7}$ तथा $\frac{x-15}{3} = \frac{y-29}{-8} = \frac{z-5}{-5}$ के लंबवत हो।

खण्ड ग

खण्ड में लघु-उत्तरीय (SA) प्रकार के 6 प्रश्न हैं, जिनमें प्रत्येक के 3 अंक हैं।

6. (क) मान ज्ञात कीजिए :

$$I = \int_2^4 (|x-2| + |x-3| + |x-4|)\, dx$$

अथवा

(ख) ज्ञात कीजिए :

$$\int \frac{dx}{(x+2)(x^2+1)}$$

27. वक्र $y = -x^3 + 3x^2 + 9x - 30$ की प्रवणता (ढाल) का अधिकतम मान ज्ञात कीजिए।

23. (a) Find the value of λ, if the points (–1, –1, 2), (2, 8, λ) and (3, 11, 6) are collinear.

OR

(b) $\vec{a}$ and $\vec{b}$ are two co-initial vectors forming the adjacent sides of a parallelogram such that $|\vec{a}| = 10$, $|\vec{b}| = 2$ and $\vec{a} \cdot \vec{b} = 12$. Find the area of the parallelogram.

4. A ladder 13 m long is leaning against the wall. The bottom of the ladder is pulled along the ground away from the wall at the rate of 2 m/s. How fast is the height on the wall decreasing when the foot of the ladder is 12 m away from the wall ?

5. Determine the vector equation of a line passing through the point (1, 2, – 3) and perpendicular to both the given lines

$$\frac{x-8}{3} = \frac{y+16}{-16} = \frac{x-10}{7} \text{ and } \frac{x-15}{3} = \frac{y-29}{-8} = \frac{z-5}{-5}.$$

SECTION C

his section comprises 6 short answer (SA) type questions of 3 marks each.

6. (a) Evaluate :

$$I = \int_2^4 (|x-2|+|x-3|+|x-4|)\,dx$$

OR

(b) Find :

$$\int \frac{dx}{(x+2)(x^2+1)}$$

27. Find the maximum slope of the curve $y = -x^3 + 3x^2 + 9x - 30$.

28. (क) अवकल समीकरण $x^2 \dfrac{dy}{dx} = x^2 + xy + y^2$ का व्यापक हल ज्ञात कीजिए।

अथवा

(ख) अवकल समीकरण $\dfrac{dy}{dx} - 3y \cot x = \sin 2x$ का विशिष्ट हल ज्ञात कीजिए, दिया गया है कि $y = 2$ है जब $x = \dfrac{\pi}{2}$ है।

9. यदि $\hat{a}$, $\hat{b}$ तथा $\hat{c}$ मात्रक सदिश हैं तथा $\hat{a} . \hat{b} = \hat{a} . \hat{c} = 0$ है, $\hat{b}$ तथा $\hat{c}$ के बीच का कोण $\dfrac{\pi}{6}$ है, तो सिद्ध कीजिए कि $\hat{a} = \pm 2 (\hat{b} \times \hat{c})$.

0. (क) कक्षा XII के चार विद्यार्थियों को एक समस्या स्वतंत्र रूप से हल करने के लिए दी गई है। उनकी समस्या को हल कर पाने की संभावनाएँ क्रमश: $\dfrac{1}{2}, \dfrac{1}{3}, \dfrac{2}{3}$ तथा $\dfrac{1}{5}$ हैं। उनमें से अधिकतम एक द्वारा समस्या हल कर पाने की प्रायिकता ज्ञात कीजिए।

अथवा

(ख) एक यादृच्छिक चर X का प्रायिकता बंटन नीचे दिया गया है :

X	1	2	4	2k	3k	5k
P(X)	$\frac{1}{2}$	$\frac{1}{5}$	$\frac{3}{25}$	$\frac{1}{10}$	$\frac{1}{25}$	$\frac{1}{25}$

यदि $E(X) = 2{\cdot}94$ है, तो k ज्ञात कीजिए और $P(X \leq 4)$ भी ज्ञात कीजिए।

1. निम्नलिखित रैखिक प्रोग्रामन समस्या को ग्राफ द्वारा हल कीजिए :

व्यवरोधों $x + 4y \leq 8$

$2x + 3y \leq 12$

$3x + y \leq 9$

$x \geq 0, y \geq 0$

के अंतर्गत $Z = 2x + 3y$ का अधिकतमीकरण कीजिए।

28. (a) Find the general solution of the differential equation

$$x^2 \frac{dy}{dx} = x^2 + xy + y^2.$$

OR

(b) Find the particular solution of the differential equation

$$\frac{dy}{dx} - 3y \cot x = \sin 2x, \text{ given that } y = 2 \text{ when } x = \frac{\pi}{2}.$$

9. If $\hat{a}$, $\hat{b}$ and $\hat{c}$ are unit vectors such that $\hat{a} \cdot \hat{b} = \hat{a} \cdot \hat{c} = 0$ and the angle between $\hat{b}$ and $\hat{c}$ is $\frac{\pi}{6}$, then prove that $\hat{a} = \pm 2 (\hat{b} \times \hat{c})$.

0. (a) Four students of class XII are given a problem to solve independently. Their chances of solving the problem respectively are $\frac{1}{2}, \frac{1}{3}, \frac{2}{3}$ and $\frac{1}{5}$. Find the probability that at most one of them will solve the problem.

OR

(b) The probability distribution of a random variable X is given below :

X	1	2	4	2k	3k	5k
P(X)	$\frac{1}{2}$	$\frac{1}{5}$	$\frac{3}{25}$	$\frac{1}{10}$	$\frac{1}{25}$	$\frac{1}{25}$

Find k, if E(X) = 2·94 and also find $P(X \leq 4)$.

1. Solve the following LPP graphically :

Maximize $Z = 2x + 3y$

subject to the constraints $x + 4y \leq 8$

$$2x + 3y \leq 12$$

$$3x + y \leq 9$$

$$x \geq 0, y \geq 0.$$

खण्ड घ

इस खण्ड में 4 दीर्घ-उत्तरीय (LA) प्रकार के प्रश्न हैं, जिनमें प्रत्येक के 5 अंक हैं।

32. यदि $A = \begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}$ है, तो A^{-1} ज्ञात कीजिए। A^{-1} के प्रयोग से दिए गए समीकरणों के नीचे दिए गए निकाय को हल कीजिए :

$2x - 3y + 5z = 11;$

$3x + 2y - 4z = -5;$

$x + y - 2z = -3$

3. (क) नीचे दी गई रेखाओं l_1 तथा l_2 में न्यूनतम दूरी ज्ञात कीजिए :

$$l_1 : \vec{r} = \hat{i} + 2\hat{j} - 4\hat{k} + \lambda(4\hat{i} + 6\hat{j} + 12\hat{k})$$

$$\text{और } l_2 : \vec{r} = 3\hat{i} + 3\hat{j} - 5\hat{k} + \mu(6\hat{i} + 9\hat{j} + 18\hat{k})$$

अथवा

(ख) दर्शाइए कि रेखाएँ $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ तथा $\frac{x-4}{5} = \frac{y-1}{2} = \frac{z}{1}$ प्रतिच्छेदी रेखाएँ हैं। रेखाओं का प्रतिच्छेदन बिंदु भी ज्ञात कीजिए।

4. (क) यदि $y = \cos x^2 + \cos^2 x + \cos^2 (x^2) + \cos (x^x)$ है, तो $\frac{dy}{dx}$ ज्ञात कीजिए।

अथवा

(ख) उन अंतरालों को ज्ञात कीजिए जिनमें दिया गया फलन :

$$f(x) = \frac{3}{10}x^4 - \frac{4}{5}x^3 - 3x^2 + \frac{36}{5}x + 11$$

(i) निरंतर वर्धमान है।

(ii) निरंतर ह्रासमान है।

35. समाकलन के प्रयोग से क्षेत्र $\left\{(x, y) : 0 \le y \le x^2,\ 0 \le y \le x,\ 0 \le x \le 3\right\}$ का क्षेत्रफल ज्ञात कीजिए।

SECTION D

This section comprises 4 long answer (LA) type questions of 5 marks each.

32. If $A = \begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}$, find A^{-1}. Using A^{-1}, solve the given system of equations :

$2x - 3y + 5z = 11$

$3x + 2y - 4z = -5$

$x + y - 2z = -3$.

3. (a) Find the shortest distance between the lines l_1 and l_2 given by :

$$l_1 : \vec{r} = \hat{i} + 2\hat{j} - 4\hat{k} + \lambda(4\hat{i} + 6\hat{j} + 12\hat{k})$$

and $l_2 : \vec{r} = 3\hat{i} + 3\hat{j} - 5\hat{k} + \mu(6\hat{i} + 9\hat{j} + 18\hat{k})$

OR

(b) Show that the lines $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and $\frac{x-4}{5} = \frac{y-1}{2} = \frac{z}{1}$ intersect. Also, find their point of intersection.

4. (a) If $y = \cos x^2 + \cos^2 x + \cos^2 (x^2) + \cos (x^x)$, find $\frac{dy}{dx}$.

OR

(b) Find the intervals in which the function given by

$f(x) = \frac{3}{10}x^4 - \frac{4}{5}x^3 - 3x^2 + \frac{36}{5}x + 11$ is :

(i) strictly increasing.

(ii) strictly decreasing.

35. Using integration, find the area of the region

$\left\{(x, y) : 0 \le y \le x^2, \ 0 \le y \le x, \ 0 \le x \le 3\right\}$.

खण्ड ङ

इस खण्ड में 3 प्रकरण-अध्ययन आधारित प्रश्न हैं, जिनमें प्रत्येक के 4 अंक हैं।

प्रकरण अध्ययन – 1

36. एक खिड़की एक आयत के रूप में है जिसके ऊपर लंबाई पर एक समबाहु त्रिभुज अध्यारोपित है। माना आयताकार भाग की लंबाई x मीटर तथा चौड़ाई y मीटर है।

उपर्युक्त सूचना के आधार पर, निम्नलिखित प्रश्नों के उत्तर दीजिए :

(i) यदि खिड़की का परिमाप 12 m है, तो x तथा y के बीच संबंध ज्ञात कीजिए। *1*

(ii) (i) में प्राप्त व्यंजक के प्रयोग से, खिड़की के क्षेत्रफल का केवल x के फलन के रूप में व्यंजक लिखिए। *1*

(iii) (क) आयत की वह विमाएँ ज्ञात कीजिए जिनसे खिड़की से अधिक-से-अधिक प्रकाश आ सके। ((ii) में प्राप्त व्यंजक का प्रयोग कीजिए) *2*

अथवा

(iii) (ख) यदि यह दिया गया हो कि खिड़की का क्षेत्रफल 50 m^2 है, तो खिड़की के परिमाप का x के पदों में व्यंजक ज्ञात कीजिए। *2*

प्रकरण अध्ययन – 2

7. त्योहारों के मौसम में, एक सोसाइटी के आवासीय कल्याण संघ ने साथ वाले पार्क में एक मेले का आयोजन किया। मेले का मुख्य आकर्षण, पार्क के एक कोने में लगा झूला था जो झूलते समय फलन $x^2 = y$ का परवलय पथ पूरा करता था।

उपर्युक्त सूचना के आधार पर, निम्नलिखित प्रश्नों के उत्तर दीजिए :

(i) माना $f : N \to R,\ f(x) = x^2$ द्वारा परिभाषित है। इसका परिसर क्या होगा ? *1*

(ii) माना $f : N \to N,\ f(x) = x^2$ द्वारा परिभाषित है, तो जाँच कीजिए कि फलन एकैकी है या नहीं। *1*

(iii) (क) माना $f : \{1, 2, 3, 4 .....\} \longrightarrow \{1, 4, 9, 16 .....\}$ में $f(x) = x^2$ द्वारा परिभाषित है, तो सिद्ध कीजिए कि फलन एकैकी-आच्छादी है। *2*

अथवा

(iii) (ख) माना $f : \mathbb{R} \longrightarrow \mathbb{R}$ में $f(x) = x^2$ द्वारा परिभाषित है, तो दर्शाइए कि f न तो एकैकी है और न ही आच्छादी है। *2*

SECTION E

This section comprises 3 case study based questions of 4 marks each.

Case Study – 1

36. A window is in the form of a rectangle surmounted by an equilateral triangle on its length. Let the rectangular part have length and breadth x and y metres respectively.

Based on the given information, answer the following questions :

(i) If the perimeter of the window is 12 m, find the relation between x and y. *1*

(ii) Using the expression obtained in (i), write an expression for the area of the window as a function of x only. *1*

(iii) (a) Find the dimensions of the rectangle that will allow maximum light through the window. (use expression obtained in (ii)) *2*

OR

(iii) (b) If it is given that the area of the window is 50 m^2, find an expression for its perimeter in terms of x. *2*

Case Study – 2

7. During the festival season, there was a mela organized by the Resident Welfare Association at a park, near the society. The main attraction of the mela was a huge swing installed at one corner of the park. The swing is traced to follow the path of a parabola given by $x^2 = y$.

Based on the above information, answer the following questions :

(i) Let $f : N \longrightarrow R$ is defined by $f(x) = x^2$. What will be the range ? *1*

(ii) Let $f : N \longrightarrow N$ is defined by $f(x) = x^2$. Check if the function is injective or not. *1*

(iii) (a) Let $f : \{1, 2, 3, 4 .....\} \longrightarrow \{1, 4, 9, 16 .....\}$ be defined by $f(x) = x^2$. Prove that the function is bijective. *2*

OR

(iii) (b) Let $f : \mathbb{R} \longrightarrow \mathbb{R}$ is defined by $f(x) = x^2$. Show that f is neither injective nor surjective. *2*

प्रकरण अध्ययन – 3

38. एक संगठन की प्रबंध समिति के पद के लिए दो व्यक्ति प्रतिस्पर्धा कर रहे हैं। पहले तथा दूसरे व्यक्ति के चयन की प्रायिकता क्रमश: 0·5 तथा 0·6 है। इसके अतिरिक्त, यदि पहले व्यक्ति का चयन होता है, तो इसके द्वारा अपशिष्ट उपचार संयंत्र शुरू करने की प्रायिकता 0·7 है, जबकि दूसरे व्यक्ति का चयन होता है, तो उसकी संबंधित प्रायिकता 0·4 है।

उपर्युक्त सूचना के आधार पर, निम्नलिखित प्रश्नों के उत्तर दीजिए :

(i) अपशिष्ट उपचार संयंत्र के शुरू होने की प्रायिकता क्या है ? *2*

(ii) चयन के बाद, यदि अपशिष्ट उपचार संयंत्र शुरू हो गया है तो इसकी क्या प्रायिकता है कि पहले व्यक्ति ने इसे शुरू किया है ? *2*

Case Study – 3

38. Two persons are competing for a position on the Managing Committee of an organisation. The probabilities that the first and the second person will be appointed are 0·5 and 0·6 respectively. Also, if the first person gets appointed, then the probability of introducing waste treatment plant is 0·7 and the corresponding probability is 0·4, if the second person gets appointed.

Based on the above information, answer the following questions :

(i) What is the probability that the waste treatment plant is introduced ? *2*

(ii) After the selection, if the waste treatment plant is introduced, what is the probability that the first person had introduced it ? *2*

All India 2025

CBSE Board Solved Paper

Time Allowed : 3 Hours *Maximum Marks : 80*

General Instructions:

- (i) This question paper contains **38** questions. **All** questions are compulsory.
- (ii) Question paper is divided into **Five** Sections - Sections **A, B, C, D** and **E**.
- (iii) In Section **A** - Question Number **1** to **18** are Multiple Choice Questions (MCQs) type and Question Number **19** & **20** are Assertion-Reason Based Questions of **1** mark each.
- (iv) In Section **B** - Question Number **21** to **25** are Very Short Answer (VSA) type questions, carrying **2** marks each.
- (v) In Section **C** - Question Number **26** to **31** are Short Answer (SA) type questions, carrying **3** marks each.
- (vi) In Section **D** - Question Number **32** to **35** are Long Answer (LA) type questions, carrying **5** marks each.
- (vii) In Section **E** - Question Number **36** to **38** are Case Study Based Questions, carrying **4** marks each where **2 VSA** type questions are of **1** mark each and **1 SA** type question is of **2** marks. Internal choice is provided in **2** marks question in each case study.
- (viii) There is no overall choice. However, an internal choice has been provided in **2** questions in Section - **B**, **3** questions in Section - **C**, **2** questions in Section - **D** and **2** questions in Section - **E**.
- (ix) Use of calculators is **NOT** allowed.

SECTION - A

This section consists of 20 multiple choice questions of 1 mark each.

1. If $f(x) = \begin{cases} \dfrac{\sin^2 ax}{x^2}, & x \neq 0 \\ 1, & x = 0 \end{cases}$ is continuous at $x = 0$, then the value of a is :

(a) 1 (b) – 1

(c) ±1 (d) 0

2. The principal value of $\cot^{-1}\left(-\frac{1}{\sqrt{3}}\right)$ is :

(a) $-\frac{\pi}{3}$ (b) $-\frac{2\pi}{3}$

(c) $\frac{\pi}{3}$ (d) $\frac{2\pi}{3}$

3. If A and B are two square matrices of the same order, then $(A + B)(A - B)$ is equal to :

(a) $A^2 - AB + BA - B^2$ (b) $A^2 + AB - BA - B^2$

(c) $A^2 - AB - BA - B^2$ (d) $A^2 - B^2 + AB + BA$

4. If A = $[a_{ij}]$ is a 3 × 3 diagonal matrix such that $a_{11} = 1$, $a_{22} = 5$ and $a_{33} = -2$, then | A | is :

(a) 0 (b) – 10

(c) 10 (d) 1

5. If A = $\begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix}$, then A^3 is :

(a) $3\begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix}$ (b) $\begin{bmatrix} 125 & 0 & 0 \\ 0 & 125 & 0 \\ 0 & 0 & 125 \end{bmatrix}$

(c) $\begin{bmatrix} 15 & 0 & 0 \\ 0 & 15 & 0 \\ 0 & 0 & 15 \end{bmatrix}$ (d) $\begin{bmatrix} 5^3 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix}$

6. If $\begin{vmatrix} 2x & 5 \\ 12 & x \end{vmatrix} = \begin{vmatrix} 6 & -5 \\ 4 & 3 \end{vmatrix}$, then the value of x is :

(a) 3 (b) 7

(c) ± 7 (d) ± 3

7. If P (A ∪ B) = 0·9 and P (A ∩ B) = 0·4, then $P(\overline{A}) + P(\overline{B})$ is:

(a) 0·3 (b) 1

(c) 1·3 (d) 0·7

8. If a matrix A is both symmetric and skew-symmetric, then A is a:

(a) diagonal matrix (b) zero matrix

(c) non-singular matrix (d) scalar matrix

9. The slope of the curve $y = -x^3 + 3x^2 + 8x - 20$ is maximum at :

(a) (1, – 10) (b) (1, 10)

(c) (10, 1) (d) (– 10, 1)

10. The area of the region enclosed between the curve $y = x\,|\,x\,|$, x-axis, $x = -2$ and $x = 2$ is :

(a) $\frac{8}{3}$ (b) $\frac{16}{3}$

(c) 0 (d) 8

11. $\int \frac{\cos 2x}{\sin^2 x \cos^2 x} dx$ is equal to :

(a) $\cot x + \tan x + C$

(b) $-(\cot x + \tan x) + C$

(c) $-\cot x + \tan x + C$

(d) $\cot x - \tan x + C$

12. If $\int_0^a \frac{1}{1+4x^2} dx = \frac{\pi}{8}$, then the value of '$a$' is :

(a) $\frac{1}{4}$ (b) $\frac{1}{2}$

(c) $\frac{1}{8}$ (d) 4

13. If $f(x) = \{[x], x \in R\}$ is the greatest integer function, then the correct statement is :

(a) f is continuous but not differentiable at $x = 2$.

(b) f is neither continuous nor differentiable at $x = 2$.

(c) f is continuous as well as differentiable at $x = 2$.

(d) f is not continuous but differentiable at $x = 2$.

14. The integrating factor of the differential equation

$$\frac{dx}{dy} = \frac{x \log x}{\frac{2}{x}\log x - y} \text{ is :}$$

(a) $\frac{1}{8x}$ (b) e

(c) $e^{\log x}$ (d) $\log x$

15. Let $\vec{a}$ be a position vector whose tip is the point (2, – 3). If $\vec{AB} = \vec{a}$, where coordinates of A are (– 4, 5), then the coordinates of B are :

(a) (– 2, – 2) (b) (2, – 2)

(c) (– 2, 2) (d) (2, 2)

16. The respective values of $|\vec{a}|$ and $|\vec{b}|$, if given

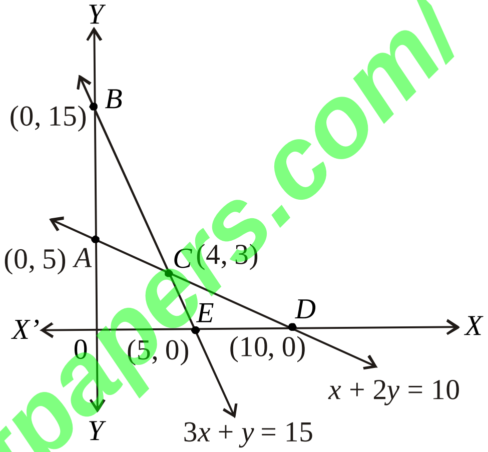
$(\vec{a} - \vec{b}).(\vec{a} + \vec{b}) = 512$ and $|\vec{a}| = 3|\vec{b}|$, are:

(a) 48 and 16 (b) 3 and 1

(c) 24 and 8 (d) 6 and 2

17. For a Linear Programming Problem (LPP), the given objective function $Z = 3x + 2y$ is subject to constraints:

$x + 2y \le 10, 3x + y \le 15, x, y \ge 0$



The correct feasible region is :

(a) ABC

(b) $AOEC$

(c) CED

(d) Open unbounded region BCD

18. The sum of the order and degree of the differential equation

$$\left[1+\left(\frac{dy}{dx}\right)^2\right]^3 = \frac{d^2y}{dx^2} \text{ is :}$$

(a) 2 (b) $\frac{5}{2}$

(c) 3 (d) 4

Questions No. 19 & 20, are Assertion (A) and Reason (R) based questions carrying 1 marks each. Two statements are given, one labelled Assertion (A) and the other labelled Reason (R).

Select the correct answer from the codes (A), (B), (C) and (D) as given below:

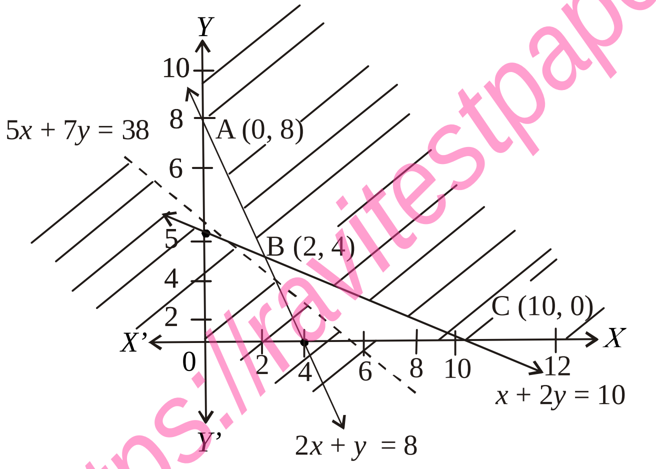
(a) Both Assertion (A) and Reason (R) are true and the Reason (R) is the correct explanation of Assertion (A).

(b) Both Assertion (A) and Reason (R) are true and Reason (R) is not the correct explanation of the Assertion (A).

(c) Assertion (A) is true, but Reason (R) is false.

(d) Assertion (A) is false, but Reason (R) is true.

19. **Assertion (A):** The shaded portion of the graph represents the feasible region for the given Linear Programming Problem (LPP).



Min $Z = 50x + 70y$

Subject to constraints

$2x + y \geq 8, x + 2y \geq 10, x, y \geq 0$

$Z = 50x + 70y$ has a minimum value = 380 at B (2, 4).

Reason (R): The region representing $50x + 70y < 380$ does not have any point common with the feasible region.

20. **Assertion (A):** Let $A = \{x \in R : -1 \leq x \leq 1\}$. If $f : A \to A$ be defined as : $f(x) = x^2$, then f is not an onto function.

Reason (R): If $y = -1 \in A$, then $x = \pm\sqrt{-1} \notin A$.

SECTION - B

In this section there are 5 very short answer type questions of 2 marks each.

21. Find the domain of $\sec^{-1}(2x + 1)$.

22. The radius of a cylinder is decreasing at a rate of 2 cm/s and the altitude is increasing at the rate of 3 cm/s. Find the rate of change of volume of this cylinder when its radius is 4 cm and altitude is 6 cm.

23. (a) Find a vector of magnitude 5 which is perpendicular to both the vectors $3\hat{i} - 2\hat{j} + \hat{k}$ and $4\hat{i} + 3\hat{j} - 2\hat{k}$.

OR

(b) Let $\vec{a}$, $\vec{b}$ and $\vec{c}$ be three vectors such that $\vec{a}\,.\,\vec{b} = \vec{a}\,.\,\vec{c}$ and $\vec{a} \times \vec{b} = \vec{a} \times \vec{c}$, $\vec{a} \neq 0$. Show that $\vec{b} = \vec{c}$.

24. A man needs to hang two lanterns on a straight wire whose end points have coordinates A (4, 1, – 2) and B (6, 2, – 3). Find the coordinates of the points where he hangs the lanterns such that these points trisect the wire AB.

25. (a) Differentiate $\dfrac{\sin x}{\sqrt{\cos x}}$ with respect to x.

OR

(b) If $y = 5 \cos x - 3 \sin x$, prove that $\dfrac{d^2y}{dx^2} + y = 0$.

SECTION - C

In this section there are 6 short answer type questions of 3 marks each.

26. Show that $f(x) = \tan^{-1}(\sin x + \cos x)$ is an increasing function in $\left[0, \dfrac{\pi}{4}\right]$.

27. (a) The probability that a student buys a colouring book is 0·7 and that she buys a box of colours is 0·2. The probability that she buys a colouring book, given that she buys a box of colours, is 0·3. Find the probability that the student :

(i) Buys both the colouring book and the box of colours.

(ii) Buys a box of colours given that she buys the colouring book.

OR

(b) A person has a fruit box that contains 6 apples and 4 oranges. He picks out a fruit three times, one after replacing the previous one in the box. Find:

(i) The probability distribution of the number of oranges he draws.

(ii) The expectation of the random variable (number of oranges).

28. Find the particular solution of the differential equation

$$\frac{dy}{dx} - \frac{y}{x} + \text{cosec}\left(\frac{y}{x}\right) = 0; \text{ given that } y = 0, \text{ when } x = 1.$$

29. (a) Find : $\displaystyle\int \frac{2x}{(x^2+3)(x^2-5)}\,dx$

OR

(b) Evaluate : $\int_{1}^{4} (|x-2|+|x-4|)\, dx$

30. In the Linear Programming Problem (LPP), find the point/points giving maximum value for $Z = 5x + 10y$

subject to constraints

$x + 2y \le 120, x + y \ge 60, x - 2y \ge 0, x, y \ge 0$

31. (a) If $\vec{a} + \vec{b} + \vec{c} = \vec{0}$ such that $|\vec{a}| = 3$, $|\vec{b}| = 5$, $|\vec{c}| = 7$, then find the angle between $\vec{a}$ and $\vec{b}$.

OR

(b) If $\vec{a}$ and $\vec{b}$ are unit vectors inclined with each other at an angle θ, then prove that $\frac{1}{2}|\vec{a} - \vec{b}| = \sin\frac{\theta}{2}$.

SECTION - D

In the section there are 4 long answer type questions of 5 marks each.

32. Draw a rough sketch of the curve $y = \sqrt{x}$. Using integration, find the area of the region bounded by the curve $y = \sqrt{x}$, $x = 4$ and x-axis, in the first quadrant.

33. An amount of ₹ 10,000 is put into three investments at the rate of 10%, 12% and 15% per annum. The combined annual income of all three investment is ₹ 1,310, however the combined annual income of the first and the second investments is ₹ 190 short of the income from the third. Use matrix method and find the investment amount in each at the beginning of the year.

34. (a) Find the foot of the perpendicular drawn from the point (1, 1, 4) on the line $\frac{x+2}{5} = \frac{y+1}{2} = \frac{-z+1}{-3}$.

OR

(b) Find the point on the line $\frac{x-1}{3} = \frac{y+1}{2} = \frac{z-4}{3}$ at a distance of $2\sqrt{2}$ units from the point (– 1, – 1, 2).

35. (a) For a positive constant 'a', differentiate $a^{t+\frac{1}{t}}$ with respect to $\left(t+\frac{1}{t}\right)^{a}$, where t is a non-zero real number.

OR

(b) Find $\frac{dy}{dx}$ if $y^x + x^y + x^x = a^b$, where a and b are constants.

SECTION - E

In this section, there are 3 case study based question of 4 marks each.

Case Study - 1

36. A gardener wanted to plant vegetables in his garden. Hence he bought 10 seeds of brinjal plant, 12 seeds of cabbage plant and 8 seeds of radish plant. The shopkeeper assured him of germination probabilities of brinjal, cabbage and radish to be 25%, 35% and 40% respectively. But before he could plant the seeds, they got mixed up in the bag and he had to sow them randomly.

Radish

Cabbage

Brinjal

Based upon the above information, answer the following questions:

(i) Calculate the probability of a randomly chosen seed to germinate.

(ii) What is the probability that it is a cabbage seed, given that the chosen seed germinates?

Case Study - 2

37. A carpenter needs to make a wooden cuboidal box, closed from all sides, which has a square base and fixed volume. Since he is short of the paint required to paint the box on completion, he wants the surface area to be minimum.

On the basis of the above information, answer the following questions:

(i) Taking length = breadth = x m and height = y m, express the surface area (S) of the box in terms of x and its volume (V), which is constant.

(ii) Find $\frac{dS}{dx}$.

(iii) Find a relation between x and y such that the surface area (S) is minimum.

OR

If surface area (S) is constant, the volume (V) $= \frac{1}{4}(Sx - 2x^3)$, x being the edge of base. Show that volume (V) is maximum for $x = \sqrt{\frac{S}{6}}$.

Case Study - 3

38. Let A be the set of 30 students of class XII in a school. Let $f : \text{A} \rightarrow \text{N}$, N is set of natural numbers such that function $f(x)$ = Roll Number of student x.

On the basis of the given information, answer the following:

(i) Is f a bijective function?

(ii) Give reasons to support your answer to (i).

(iii) Let R be a relation defined by the teacher to plan the seating arrangement of students in pairs, where $R = \{(x, y) : x, y$ are Roll numbers of students such that $y = 3x\}$. List the elements of R. Is the relation R reflexive, symmetric and transitive? Justify your answer.

OR

Let R be a relation defined by $R = \{(x, y) : x, y$ are Roll numbers of students such that $y = x^3\}$.

List the elements of R. Is R a function ? Justify your answer.

Series EF1GH/C **SET~1**

प्रश्न–पत्र कोड
Q.P. Code **65/C/1**

रोल नं.
Roll No.

परीक्षार्थी प्रश्न-पत्र कोड को उत्तर-पुस्तिका के मुख-पृष्ठ पर अवश्य लिखें ।
Candidates must write the Q.P. Code on the title page of the answer-book.

गणित
MATHEMATICS

*

निर्धारित समय : 3 घण्टे | अधिकतम अंक : 80
Time allowed : 3 hours | Maximum Marks : 80

नोट / NOTE :

(i) *कृपया जाँच कर लें कि इस प्रश्न-पत्र में मुद्रित पृष्ठ **23** हैं ।*
*Please check that this question paper contains **23** printed pages.*

(ii) *प्रश्न-पत्र में दाहिने हाथ की ओर दिए गए प्रश्न-पत्र कोड को परीक्षार्थी उत्तर-पुस्तिका के मुख-पृष्ठ पर लिखें ।*
Q.P. Code given on the right hand side of the question paper should be written on the title page of the answer-book by the candidate.

(iii) *कृपया जाँच कर लें कि इस प्रश्न-पत्र में **38** प्रश्न हैं ।*
*Please check that this question paper contains **38** questions.*

(iv) ***कृपया प्रश्न का उत्तर लिखना शुरू करने से पहले, उत्तर-पुस्तिका में प्रश्न का क्रमांक अवश्य लिखें ।***
Please write down the serial number of the question in the answer-book before attempting it.

(v) *इस प्रश्न-पत्र को पढ़ने के लिए 15 मिनट का समय दिया गया है । प्रश्न-पत्र का वितरण पूर्वाह्न में 10.15 बजे किया जाएगा । 10.15 बजे से 10.30 बजे तक छात्र केवल प्रश्न-पत्र को पढ़ेंगे और इस अवधि के दौरान वे उत्तर-पुस्तिका पर कोई उत्तर नहीं लिखेंगे ।*
15 minute time has been allotted to read this question paper. The question paper will be distributed at 10.15 a.m. From 10.15 a.m. to 10.30 a.m., the students will read the question paper only and will not write any answer on the answer-book during this period.

सामान्य निर्देश :

निम्नलिखित निर्देशों को बहुत सावधानी से पढ़िए और उनका सख़्ती से पालन कीजिए :

(i) *इस प्रश्न-पत्र में **38** प्रश्न हैं । **सभी** प्रश्न **अनिवार्य** हैं ।*

(ii) *यह प्रश्न-पत्र **पाँच** खण्डों में विभाजित है – **क, ख, ग, घ** एवं **ङ** ।*

(iii) ***खण्ड क** में प्रश्न संख्या **1** से **18** तक बहुविकल्पीय तथा प्रश्न संख्या **19** एवं **20** अभिकथन एवं तर्क आधारित **एक-एक** अंक के प्रश्न हैं ।*

(iv) ***खण्ड ख** में प्रश्न संख्या **21** से **25** तक अति लघु-उत्तरीय (VSA) प्रकार के **दो-दो** अंकों के प्रश्न हैं ।*

(v) ***खण्ड ग** में प्रश्न संख्या **26** से **31** तक लघु-उत्तरीय (SA) प्रकार के **तीन-तीन** अंकों के प्रश्न हैं ।*

(vi) ***खण्ड घ** में प्रश्न संख्या **32** से **35** तक दीर्घ-उत्तरीय (LA) प्रकार के **पाँच-पाँच** अंकों के प्रश्न हैं ।*

(vii) ***खण्ड ङ** में प्रश्न संख्या **36** से **38** प्रकरण अध्ययन आधारित **चार-चार** अंकों के प्रश्न हैं ।*

(viii) *प्रश्न-पत्र में समग्र विकल्प नहीं दिया गया है । यद्यपि, खण्ड ख के 2 प्रश्नों में, खण्ड ग के 3 प्रश्नों में, खण्ड घ के 2 प्रश्नों में तथा खण्ड ङ के 2 प्रश्नों में आंतरिक विकल्प का प्रावधान दिया गया है ।*

(ix) *कैल्कुलेटर का उपयोग **वर्जित** है ।*

खण्ड क

इस खण्ड में बहुविकल्पीय प्रश्न हैं, जिनमें प्रत्येक प्रश्न 1 अंक का है ।

1. यदि A, कोटि 3 का एक वर्ग आव्यूह है और $|A| = 6$ है, तो $|\text{adj } A|$ का मान है :

(a) 6 (b) 36

(c) 27 (d) 216

2. $\int_0^{\pi/6} \sin 3x \, dx$ का मान है :

(a) $-\frac{\sqrt{3}}{2}$ (b) $-\frac{1}{3}$

(c) $\frac{\sqrt{3}}{2}$ (d) $\frac{1}{3}$

General Instructions :

Read the following instructions very carefully and strictly follow them :

(i) *This question paper contains* ***38*** *questions.* ***All*** *questions are* ***compulsory****.*

(ii) *This question paper is divided into* ***five*** *Sections –* ***A, B, C, D*** *and* ***E****.*

(iii) *In* ***Section A,*** *Questions no.* ***1*** *to* ***18*** *are multiple choice questions (MCQs) and questions number* ***19*** *and* ***20*** *are Assertion-Reason based questions of* ***1*** *mark each.*

(iv) *In* ***Section B,*** *Questions no.* ***21*** *to* ***25*** *are very short answer (VSA) type questions, carrying* ***2*** *marks each.*

(v) *In* ***Section C,*** *Questions no.* ***26*** *to* ***31*** *are short answer (SA) type questions, carrying* ***3*** *marks each.*

(vi) *In* ***Section D,*** *Questions no.* ***32*** *to* ***35*** *are long answer (LA) type questions carrying* ***5*** *marks each.*

(vii) *In* ***Section E,*** *Questions no.* ***36*** *to* ***38*** *are case study based questions carrying* ***4*** *marks each.*

(viii) *There is no overall choice. However, an internal choice has been provided in 2 questions in Section B, 3 questions in Section C, 2 questions in Section D and 2 questions in Section E.*

(ix) *Use of calculators is* ***not*** *allowed.*

SECTION A

This section comprises multiple choice questions (MCQs) of 1 mark each.

1. If A is a square matrix of order 3 and $|A| = 6$, then the value of $|\text{adj } A|$ is :

(a) 6 (b) 36

(c) 27 (d) 216

2. The value of $\int_{0}^{\pi/6} \sin 3x \, dx$ is :

(a) $-\frac{\sqrt{3}}{2}$ (b) $-\frac{1}{3}$

(c) $\frac{\sqrt{3}}{2}$ (d) $\frac{1}{3}$

3. यदि $\vec{a}$, $\vec{b}$ और $(\vec{a} + \vec{b})$ सभी मात्रक सदिश हैं और $\vec{a}$ तथा $\vec{b}$ के बीच का कोण θ है, तो θ का मान होगा :

(a) $\frac{2\pi}{3}$ (b) $\frac{5\pi}{6}$ (c) $\frac{\pi}{3}$ (d) $\frac{\pi}{6}$

4. सदिश $\hat{i}$ का सदिश $\hat{i} + \hat{j} + 2\hat{k}$ पर प्रक्षेप है :

(a) $\frac{1}{\sqrt{6}}$ (b) $\sqrt{6}$ (c) $\frac{2}{\sqrt{6}}$ (d) $\frac{3}{\sqrt{6}}$

5. एक परिवार में 2 बच्चे हैं और बड़ा बच्चा एक लड़की है । दोनों बच्चों के लड़की होने की प्रायिकता है :

(a) $\frac{1}{4}$ (b) $\frac{1}{8}$ (c) $\frac{1}{2}$ (d) $\frac{3}{4}$

6. रेखा $\frac{x+3}{3} = \frac{4-y}{2} = \frac{z+8}{6}$ के समांतर और बिंदु $(2, -4, 5)$ से गुज़रने वाली रेखा का सदिश समीकरण है :

(a) $\vec{r} = (-2\hat{i} + 4\hat{j} - 5\hat{k}) + \lambda(3\hat{i} + 2\hat{j} + 6\hat{k})$

(b) $\vec{r} = (2\hat{i} - 4\hat{j} + 5\hat{k}) + \lambda(3\hat{i} - 2\hat{j} + 6\hat{k})$

(c) $\vec{r} = (2\hat{i} - 4\hat{j} + 5\hat{k}) + \lambda(3\hat{i} + 2\hat{j} + 6\hat{k})$

(d) $\vec{r} = (-2\hat{i} + 4\hat{j} - 5\hat{k}) + \lambda(3\hat{i} - 2\hat{j} - 6\hat{k})$

7. x के किस मान के लिए, सारणिक $\begin{vmatrix} 2x & -3 \\ 5 & x \end{vmatrix}$ और $\begin{vmatrix} 10 & 1 \\ -3 & 2 \end{vmatrix}$ समान हैं ?

(a) ± 3 (b) -3 (c) ± 2 (d) 2

8. आव्यूह $\begin{bmatrix} 4 & 3 & 2 \\ 2 & -1 & 0 \\ 1 & 2 & 3 \end{bmatrix}$ में दूसरी पंक्ति और तीसरे स्तम्भ में स्थित अवयव के सहखंड का मान है :

(a) 5 (b) -5 (c) -11 (d) 11

3. If $\vec{a}$, $\vec{b}$ and $(\vec{a} + \vec{b})$ are all unit vectors and θ is the angle between $\vec{a}$ and $\vec{b}$, then the value of θ is :

(a) $\frac{2\pi}{3}$ (b) $\frac{5\pi}{6}$ (c) $\frac{\pi}{3}$ (d) $\frac{\pi}{6}$

4. The projection of vector $\hat{i}$ on the vector $\hat{i} + \hat{j} + 2\hat{k}$ is :

(a) $\frac{1}{\sqrt{6}}$ (b) $\sqrt{6}$ (c) $\frac{2}{\sqrt{6}}$ (d) $\frac{3}{\sqrt{6}}$

5. A family has 2 children and the elder child is a girl. The probability that both children are girls is :

(a) $\frac{1}{4}$ (b) $\frac{1}{8}$ (c) $\frac{1}{2}$ (d) $\frac{3}{4}$

6. The vector equation of a line which passes through the point (2, – 4, 5) and is parallel to the line $\frac{x+3}{3} = \frac{4-y}{2} = \frac{z+8}{6}$ is :

(a) $\vec{r} = (-2\hat{i} + 4\hat{j} - 5\hat{k}) + \lambda(3\hat{i} + 2\hat{j} + 6\hat{k})$

(b) $\vec{r} = (2\hat{i} - 4\hat{j} + 5\hat{k}) + \lambda(3\hat{i} - 2\hat{j} + 6\hat{k})$

(c) $\vec{r} = (2\hat{i} - 4\hat{j} + 5\hat{k}) + \lambda(3\hat{i} + 2\hat{j} + 6\hat{k})$

(d) $\vec{r} = (-2\hat{i} + 4\hat{j} - 5\hat{k}) + \lambda(3\hat{i} - 2\hat{j} - 6\hat{k})$

7. For which value of x, are the determinants $\begin{vmatrix} 2x & -3 \\ 5 & x \end{vmatrix}$ and $\begin{vmatrix} 10 & 1 \\ -3 & 2 \end{vmatrix}$ equal ?

(a) ± 3 (b) – 3 (c) ± 2 (d) 2

8. The value of the cofactor of the element of second row and third column in the matrix $\begin{bmatrix} 4 & 3 & 2 \\ 2 & -1 & 0 \\ 1 & 2 & 3 \end{bmatrix}$ is :

(a) 5 (b) – 5 (c) – 11 (d) 11

9. अवकल समीकरण $\left(\frac{d^2y}{dx^2}\right)^2 + \left(\frac{dy}{dx}\right)^3 + x^4 = 0$ की कोटि और घात में अंतर है :

(a) 1 (b) 2 (c) -1 (d) 0

10. यदि आव्यूह $A = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$ और $A^2 = kA$ है, तो k का मान होगा :

(a) 1 (b) -2 (c) 2 (d) -1

11. $\int \frac{\cos 2x}{\sin^2 x \cdot \cos^2 x} dx$ बराबर है :

(a) $\tan x - \cot x + C$ (b) $-\cot x - \tan x + C$

(c) $\cot x + \tan x + C$ (d) $\tan x - \cot x - C$

12. अवकल समीकरण $(3x^2 + y) \frac{dx}{dy} = x$ का समाकलन गुणक है :

(a) $\frac{1}{x}$ (b) $\frac{1}{x^2}$ (c) $\frac{2}{x}$ (d) $-\frac{1}{x}$

13. अर्ध-तल $2x + y - 4 \leq 0$ में स्थित बिंदु है :

(a) (0, 8) (b) (1, 1)

(c) (5, 5) (d) (2, 2)

14. यदि $(\cos x)^y = (\cos y)^x$ है, तो $\frac{dy}{dx}$ बराबर है :

(a) $\frac{y \tan x + \log (\cos y)}{x \tan y - \log (\cos x)}$

(b) $\frac{x \tan y + \log (\cos x)}{y \tan x + \log (\cos y)}$

(c) $\frac{y \tan x - \log (\cos y)}{x \tan y - \log (\cos x)}$

(d) $\frac{y \tan x + \log (\cos y)}{x \tan y + \log (\cos x)}$

9. The difference of the order and the degree of the differential equation $\left(\frac{d^2y}{dx^2}\right)^2 + \left(\frac{dy}{dx}\right)^3 + x^4 = 0$ is :

(a) 1 (b) 2 (c) – 1 (d) 0

10. If matrix $A = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$ and $A^2 = kA$, then the value of k is :

(a) 1 (b) – 2 (c) 2 (d) – 1

11. $\int \frac{\cos 2x}{\sin^2 x \cdot \cos^2 x} dx$ is equal to

(a) $\tan x - \cot x + C$ (b) $-\cot x - \tan x + C$

(c) $\cot x + \tan x + C$ (d) $\tan x - \cot x - C$

12. The integrating factor of the differential equation $(3x^2 + y)\frac{dx}{dy} = x$ is

(a) $\frac{1}{x}$ (b) $\frac{1}{x^2}$ (c) $\frac{2}{x}$ (d) $-\frac{1}{x}$

13. The point which lies in the half-plane $2x + y - 4 \leq 0$ is :

(a) (0, 8) (b) (1, 1)

(c) (5, 5) (d) (2, 2)

14. If $(\cos x)^y = (\cos y)^x$, then $\frac{dy}{dx}$ is equal to :

(a) $\frac{y \tan x + \log(\cos y)}{x \tan y - \log(\cos x)}$

(b) $\frac{x \tan y + \log(\cos x)}{y \tan x + \log(\cos y)}$

(c) $\frac{y \tan x - \log(\cos y)}{x \tan y - \log(\cos x)}$

(d) $\frac{y \tan x + \log(\cos y)}{x \tan y + \log(\cos x)}$

15. यह दिया गया है कि $X\begin{bmatrix}3 & 2\\1 & -1\end{bmatrix}=\begin{bmatrix}4 & 1\\2 & 3\end{bmatrix}$ है । तो आव्यूह X है :

(a) $\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}$ (b) $\begin{bmatrix}0 & -1\\1 & 1\end{bmatrix}$

(c) $\begin{bmatrix}1 & 1\\1 & -1\end{bmatrix}$ (d) $\begin{bmatrix}1 & -1\\1 & -1\end{bmatrix}$

16. यदि ABCD एक समांतर चतुर्भुज है और AC तथा BD इसके विकर्ण हैं, तो $\overrightarrow{AC}+\overrightarrow{BD}$ है :

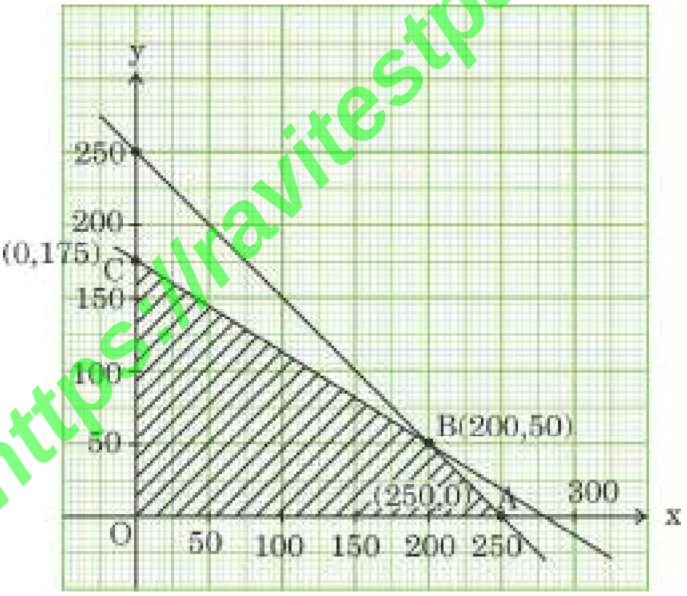
(a) $2\overrightarrow{DA}$ (b) $2\overrightarrow{AB}$ (c) $2\overrightarrow{BC}$ (d) $2\overrightarrow{BD}$

17. यदि $x = a\cos\theta + b\sin\theta$, $y = a\sin\theta - b\cos\theta$ है, तो निम्नलिखित में से कौन-सा सही है ?

(a) $y^2\frac{d^2y}{dx^2} - x\frac{dy}{dx} + y = 0$ (b) $y^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$

(c) $y^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} - y = 0$ (d) $y^2\frac{d^2y}{dx^2} - x\frac{dy}{dx} - y = 0$

18. एक LPP के परिबद्ध सुसंगत क्षेत्र के कोणीय बिंदु O(0, 0), A(250, 0), B(200, 50) और C(0, 175) हैं । यदि उद्देश्य फलन Z = 2ax + by का अधिकतम मान बिंदुओं A(250, 0) और B(200, 50) पर है, तो a और b के बीच का संबंध होगा :



(a) 2a = b (b) 2a = 3b (c) a = b (d) a = 2b

15. It is given that $X \begin{bmatrix} 3 & 2 \\ 1 & -1 \end{bmatrix} = \begin{bmatrix} 4 & 1 \\ 2 & 3 \end{bmatrix}$. Then matrix X is :

(a) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

(b) $\begin{bmatrix} 0 & -1 \\ 1 & 1 \end{bmatrix}$

(c) $\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$

(d) $\begin{bmatrix} 1 & -1 \\ 1 & -1 \end{bmatrix}$

16. If ABCD is a parallelogram and AC and BD are its diagonals, then $\overrightarrow{AC} + \overrightarrow{BD}$ is :

(a) $2\overrightarrow{DA}$

(b) $2\overrightarrow{AB}$

(c) $2\overrightarrow{BC}$

(d) $2\overrightarrow{BD}$

17. If $x = a\cos\theta + b\sin\theta$, $y = a\sin\theta - b\cos\theta$, then which one of the following is true ?

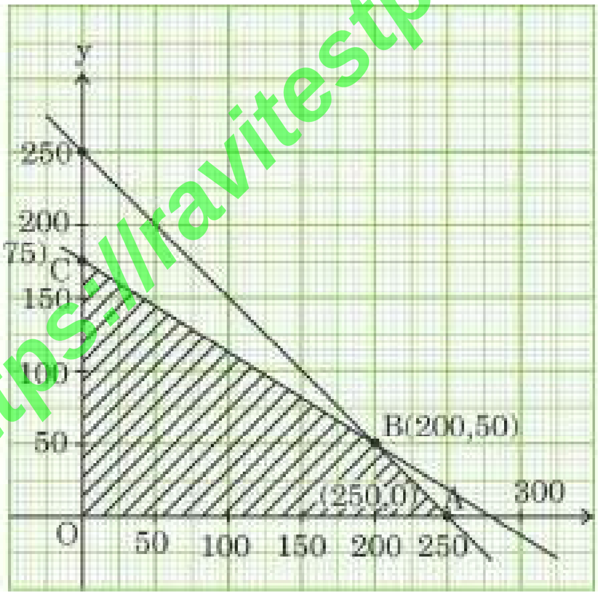
(a) $y^2\frac{d^2y}{dx^2} - x\frac{dy}{dx} + y = 0$

(b) $y^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$

(c) $y^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} - y = 0$

(d) $y^2\frac{d^2y}{dx^2} - x\frac{dy}{dx} - y = 0$

18. The corner points of the bounded feasible region of an LPP are O(0, 0), A(250, 0), B(200, 50) and C(0, 175). If the maximum value of the objective function $Z = 2ax + by$ occurs at the points A(250, 0) and B(200, 50), then the relation between a and b is :



(a) 2a = b

(b) 2a = 3b

(c) a = b

(d) a = 2b

*प्रश्न संख्या **19** और **20** अभिकथन एवं तर्क आधारित प्रश्न हैं और प्रत्येक प्रश्न का 1 अंक है । दो कथन दिए गए हैं जिनमें एक को अभिकथन (A) तथा दूसरे को तर्क (R) द्वारा अंकित किया गया है । इन प्रश्नों के सही उत्तर नीचे दिए गए कोडों (a), (b), (c) और (d) में से चुनकर दीजिए ।*

(a) अभिकथन (A) और तर्क (R) दोनों सही हैं और तर्क (R), अभिकथन (A) की सही व्याख्या करता है ।

(b) अभिकथन (A) और तर्क (R) दोनों सही हैं और तर्क (R), अभिकथन (A) की सही व्याख्या ***नहीं*** करता है ।

(c) अभिकथन (A) सही है, परन्तु तर्क (R) ग़लत है ।

(d) अभिकथन (A) ग़लत है, परन्तु तर्क (R) सही है ।

19. *अभिकथन (A) :* $\cot^{-1}(\sqrt{3})$ का मुख्य मान $\frac{\pi}{6}$ है ।

तर्क (R) : $\cot^{-1} x$ का प्रांत $\mathbb{R} - \{-1, 1\}$ है ।

20. *अभिकथन (A) :* शीर्षों A(0, 0, 0), B(3, 4, 5), C(8, 8, 8) और D(5, 4, 3) से बना चतुर्भुज एक समचतुर्भुज है ।

तर्क (R) : ABCD एक समचतुर्भुज है, यदि $AB = BC = CD = DA,\ AC \neq BD$ है ।

खण्ड ख

इस खण्ड में अति लघु-उत्तरीय (VSA) प्रकार के प्रश्न हैं, जिनमें प्रत्येक के 2 अंक हैं ।

21. यदि तीन शून्येतर सदिश $\vec{a}$, $\vec{b}$ और $\vec{c}$ ऐसे हैं कि $\vec{a} \cdot \vec{b} = \vec{a} \cdot \vec{c}$ और $\vec{a} \times \vec{b} = \vec{a} \times \vec{c}$ है, तो दर्शाइए कि $\vec{b} = \vec{c}$ ।

22. (क) सरल कीजिए :

$$\tan^{-1}\left(\frac{\cos x}{1 - \sin x}\right)$$

अथवा

(ख) सिद्ध कीजिए कि f(x) = [x] द्वारा प्रदत्त महत्तम पूर्णांक फलन $f : \mathbb{R} \to \mathbb{R}$ न तो एकैकी है और न ही आच्छादक है ।

*Questions number **19** and **20** are Assertion and Reason based questions carrying 1 mark each. Two statements are given, one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer from the codes (a), (b), (c) and (d) as given below.*

(a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).

(b) Both Assertion (A) and Reason (R) are true and Reason (R) is ***not*** the correct explanation of the Assertion (A).

(c) Assertion (A) is true, but Reason (R) is false.

(d) Assertion (A) is false, but Reason (R) is true.

19. *Assertion (A) :* The principal value of $\cot^{-1}(\sqrt{3})$ is $\frac{\pi}{6}$.

Reason (R) : Domain of $\cot^{-1} x$ is $\mathbb{R} - \{-1, 1\}$.

20. *Assertion (A) :* Quadrilateral formed by vertices A(0, 0, 0), B(3, 4, 5), C(8, 8, 8) and D(5, 4, 3) is a rhombus.

Reason (R) : ABCD is a rhombus if AB = BC = CD = DA, AC ≠ BD.

SECTION B

This section comprises very short answer (VSA) type questions of 2 marks each.

21. If three non-zero vectors are $\vec{a}$, $\vec{b}$ and $\vec{c}$ such that $\vec{a} \cdot \vec{b} = \vec{a} \cdot \vec{c}$ and $\vec{a} \times \vec{b} = \vec{a} \times \vec{c}$, then show that $\vec{b} = \vec{c}$.

22. (a) Simplify :

$$\tan^{-1}\left(\frac{\cos x}{1 - \sin x}\right)$$

OR

(b) Prove that the greatest integer function $f : \mathbb{R} \to \mathbb{R}$, given by $f(x) = [x]$, is neither one-one nor onto.

23. फलन f इस प्रकार परिभाषित है

$$f(x) = \begin{cases} 2x + 2, & \text{यदि} \quad x < 2 \\ k, & \text{यदि} \quad x = 2 \\ 3x, & \text{यदि} \quad x > 2 \end{cases}$$

k का वह मान ज्ञात कीजिए, जिसके लिए फलन f, $x = 2$ पर संतत है ।

24. वह अंतराल ज्ञात कीजिए जिसमें फलन $f(x) = x^4 - 4x^3 + 4x^2 + 15$, निरंतर वर्धमान है ।

25. (क) यदि $\vec{a}$, $\vec{b}$ और $\vec{c}$ तीन सदिश इस प्रकार हैं कि $|\vec{a}| = 7$, $|\vec{b}| = 24$, $|\vec{c}| = 25$ और $\vec{a} + \vec{b} + \vec{c} = \vec{0}$ है, तो $\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}$ का मान ज्ञात कीजिए ।

अथवा

(ख) यदि एक रेखा x-अक्ष, y-अक्ष और z-अक्ष के साथ क्रमशः α, β और γ कोण बनाती है, तो सिद्ध कीजिए कि $\sin^2 \alpha + \sin^2 \beta + \sin^2 \gamma = 2$ है ।

खण्ड ग

इस खण्ड में लघु-उत्तरीय (SA) प्रकार के प्रश्न हैं, जिनमें प्रत्येक के 3 अंक हैं ।

26. (क) मान ज्ञात कीजिए :

$$\int_0^{\pi/2} \frac{x \sin x \cos x}{\sin^4 x + \cos^4 x} \, dx$$

अथवा

(ख) मान ज्ञात कीजिए :

$$\int_1^3 (|x-1| + |x-2|) \, dx$$

23. Function f is defined as

$$f(x) = \begin{cases} 2x + 2, & \text{if} \quad x < 2 \\ k, & \text{if} \quad x = 2 \\ 3x, & \text{if} \quad x > 2 \end{cases}$$

Find the value of k for which the function f is continuous at x = 2.

24. Find the intervals in which the function $f(x) = x^4 - 4x^3 + 4x^2 + 15$, is strictly increasing.

25. (a) If $\vec{a}$, $\vec{b}$ and $\vec{c}$ are three vectors such that $|\vec{a}| = 7$, $|\vec{b}| = 24$, $|\vec{c}| = 25$ and $\vec{a} + \vec{b} + \vec{c} = \vec{0}$, then find the value of $\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}$.

OR

(b) If a line makes angles α, β and γ with x-axis, y-axis and z-axis respectively, then prove that $\sin^2 \alpha + \sin^2 \beta + \sin^2 \gamma = 2$.

SECTION C

This section comprises short answer (SA) type questions of 3 marks each.

26. (a) Evaluate :

$$\int_0^{\pi/2} \frac{x \sin x \cos x}{\sin^4 x + \cos^4 x} \, dx$$

OR

(b) Evaluate :

$$\int_1^3 (|x - 1| + |x - 2|) \, dx$$

27. (क) अवकल समीकरण $\frac{dy}{dx} = \frac{xy}{x^2 + y^2}$ का विशिष्ट हल ज्ञात कीजिए, दिया गया है कि जब $x = 0$ है, तो $y = 1$ है ।

अथवा

(ख) अवकल समीकरण $(1 + x^2)\frac{dy}{dx} + 2xy = \frac{1}{1 + x^2}$ का विशिष्ट हल ज्ञात कीजिए, दिया गया है कि जब $x = 1$ है, तो $y = 0$ है ।

28. (क) दो थैलों में से थैले A में 2 सफ़ेद और 3 लाल गेंदें हैं और थैले B में 4 सफ़ेद और 5 लाल गेंदें हैं । यादृच्छया एक गेंद को एक थैले में से निकाला गया और पाया गया कि यह लाल है । प्रायिकता ज्ञात कीजिए कि इसे थैले B में से निकाला गया था ।

अथवा

(ख) 50 व्यक्तियों के समूह में से 20 सदैव सच बोलते हैं । इस समूह में से यादृच्छया 2 व्यक्तियों को चुना गया (बिना प्रतिस्थापना के) । चुने गए उन व्यक्तियों की संख्या का प्रायिकता बंटन ज्ञात कीजिए जो सदैव सच बोलते हैं ।

29. ज्ञात कीजिए :

$$\int \frac{\cos \theta}{\sqrt{3 - 3\sin \theta - \cos^2 \theta}} \, d\theta$$

30. निम्नलिखित रैखिक प्रोग्रामन समस्या को आलेखीय विधि से हल कीजिए :

निम्न व्यवरोधों के अंतर्गत,

$z = 3x + 8y$ का न्यूनतमीकरण कीजिए :

$$3x + 4y \geq 8$$
$$5x + 2y \geq 11$$
$$x \geq 0, y \geq 0$$

31. ज्ञात कीजिए :

$$\int \frac{2x^2 + 1}{x^2(x^2 + 4)} \, dx$$

27. (a) Find the particular solution of the differential equation $\frac{dy}{dx} = \frac{xy}{x^2 + y^2}$, given that y = 1 when x = 0.

OR

(b) Find the particular solution of the differential equation $(1 + x^2)\frac{dy}{dx} + 2xy = \frac{1}{1 + x^2}$, given that y = 0 when x = 1.

28. (a) Out of two bags, bag A contains 2 white and 3 red balls and bag B contains 4 white and 5 red balls. One ball is drawn at random from one of the bags and is found to be red. Find the probability that it was drawn from bag B.

OR

(b) Out of a group of 50 people, 20 always speak the truth. Two persons are selected at random from the group (without replacement). Find the probability distribution of number of selected persons who always speak the truth.

29. Find :

$$\int \frac{\cos\theta}{\sqrt{3 - 3\sin\theta - \cos^2\theta}}\, d\theta$$

30. Solve the following Linear Programming Problem graphically :

Minimise z = 3x + 8y

subject to the constraints

$$3x + 4y \geq 8$$
$$5x + 2y \geq 11$$
$$x \geq 0, y \geq 0$$

31. Find :

$$\int \frac{2x^2 + 1}{x^2(x^2 + 4)}\, dx$$

खण्ड घ

इस खण्ड में दीर्घ-उत्तरीय (LA) प्रकार के प्रश्न हैं, जिनमें प्रत्येक के 5 अंक हैं ।

32. यदि आव्यूह $A = \begin{bmatrix} 3 & 2 & 1 \\ 4 & 1 & 3 \\ 1 & 1 & 1 \end{bmatrix}$ है, तो A^{-1} ज्ञात कीजिए, अत: निम्नलिखित रैखिक समीकरण निकाय को हल कीजिए :

$$3x + 2y + z = 2000$$
$$4x + y + 3z = 2500$$
$$x + y + z = 900$$

33. (क) दर्शाइए कि रेखाएँ $\frac{x+1}{3} = \frac{y+3}{5} = \frac{z+5}{7}$ और $\frac{x-2}{1} = \frac{y-4}{3} = \frac{z-6}{5}$ प्रतिच्छेदी रेखाएँ हैं । इनका प्रतिच्छेदन बिन्दु भी ज्ञात कीजिए ।

अथवा

(ख) रेखा युग्मों $\frac{x-1}{2} = \frac{y+1}{3} = z$ और $\frac{x+1}{5} = \frac{y-2}{1}$; $z = 2$ के बीच की न्यूनतम दूरी ज्ञात कीजिए ।

34. त्रिभुज ABC का क्षेत्रफल समाकलन विधि के प्रयोग से ज्ञात कीजिए जो कि रेखाओं जिनके समीकरण $5x - 2y - 10 = 0$, $x - y - 9 = 0$ और $3x - 4y - 6 = 0$ हैं, से घिरा हुआ है ।

35. (क) दर्शाइए कि वास्तविक संख्याओं के समुच्चय $\mathbb{R}$ में

$$S = \{(a, b) : a \le b^3,\ a \in \mathbb{R},\ b \in \mathbb{R}\}$$

द्वारा परिभाषित संबंध S न तो स्वतुल्य है, न सममित है और न ही संक्रामक है ।

अथवा

(ख) माना कि समुच्चय A = {1, 2, 3, 4, 5, 6, 7} में संबंध R इस प्रकार परिभाषित है R = {(a, b) : a और b दोनों या तो विषम हैं या सम हैं} दर्शाइए कि R एक तुल्यता संबंध है । अत:, तुल्यता वर्ग [1] के अवयव ज्ञात कीजिए ।

SECTION D

This section comprises long answer type questions (LA) of 5 marks each.

32. If matrix $A = \begin{bmatrix} 3 & 2 & 1 \\ 4 & 1 & 3 \\ 1 & 1 & 1 \end{bmatrix}$, find A^{-1} and hence solve the following system of linear equations :

$$3x + 2y + z = 2000$$
$$4x + y + 3z = 2500$$
$$x + y + z = 900$$

33. (a) Show that the lines $\frac{x+1}{3} = \frac{y+3}{5} = \frac{z+5}{7}$ and $\frac{x-2}{1} = \frac{y-4}{3} = \frac{z-6}{5}$ intersect. Also find their point of intersection.

OR

(b) Find the shortest distance between the pair of lines $\frac{x-1}{2} = \frac{y+1}{3} = z$ and $\frac{x+1}{5} = \frac{y-2}{1}$; $z = 2$.

34. Find the area of the triangle ABC bounded by the lines represented by the equations $5x - 2y - 10 = 0$, $x - y - 9 = 0$ and $3x - 4y - 6 = 0$, using integration method.

35. (a) Show that the relation S in set $\mathbb{R}$ of real numbers defined by

$$S = \{(a, b) : a \leq b^3,\ a \in \mathbb{R},\ b \in \mathbb{R}\}$$

is neither reflexive, nor symmetric, nor transitive.

OR

(b) Let R be the relation defined in the set A = {1, 2, 3, 4, 5, 6, 7} by R = {(a, b) : both a and b are either odd or even}. Show that R is an equivalence relation. Hence, find the elements of equivalence class [1].

खण्ड ङ

इस खण्ड में 3 प्रकरण अध्ययन आधारित प्रश्न हैं जिनमें प्रत्येक के 4 अंक हैं ।

प्रकरण अध्ययन – 1

36. एक समूह क्रियाकलाप की कक्षा में 10 विद्यार्थी हैं जिनकी आयु 16, 17, 15, 14, 19, 17, 16, 19, 16 और 15 वर्ष हैं । एक विद्यार्थी को यादृच्छया इस प्रकार चुना गया कि प्रत्येक विद्यार्थी के चुने जाने की संभावना समान है और चुने गए विद्यार्थी की आयु को लिखा गया ।

उपर्युक्त सूचना के आधार पर, निम्न प्रश्नों के उत्तर दीजिए :

(i) प्रायिकता ज्ञात कीजिए कि चुने गए विद्यार्थी की आयु एक भाज्य संख्या है । *1*

(ii) माना X चुने हुए विद्यार्थी की आयु है, तो X का क्या मान हो सकता है ? *1*

(iii) (क) यादृच्छया चर X का प्रायिकता बंटन ज्ञात कीजिए तथा माध्य आयु ज्ञात कीजिए । *2*

अथवा

(iii) (ख) एक यादृच्छया चुने गए विद्यार्थी की आयु 15 वर्ष से अधिक पाई गई । प्रायिकता ज्ञात कीजिए कि उसकी आयु एक अभाज्य संख्या है । *2*

SECTION E

This section comprises 3 case study based questions of 4 marks each.

Case Study – 1

36. In a group activity class, there are 10 students whose ages are 16, 17, 15, 14, 19, 17, 16, 19, 16 and 15 years. One student is selected at random such that each has equal chance of being chosen and age of the student is recorded.

On the basis of the above information, answer the following questions :

(i) Find the probability that the age of the selected student is a composite number. *1*

(ii) Let X be the age of the selected student. What can be the value of X ? *1*

(iii) (a) Find the probability distribution of random variable X and hence find the mean age. *2*

OR

(iii) (b) A student was selected at random and his age was found to be greater than 15 years. Find the probability that his age is a prime number. *2*

प्रकरण अध्ययन – 2

37. एक हाउसिंग सोसाइटी अपने निवासियों के लिए तैराकी हेतु एक पूल (तालाब) बनाना चाहती है । इसके लिए उन्हें एक वर्गाकार भूमि खरीदनी है और इस गहराई तक खोदना है कि इस पूल की क्षमता 250 घन मीटर हो जाए । भूमि की कीमत ₹ 500 प्रति वर्ग मीटर है । खोदने की कीमत में गहराई की अधिकता के अनुसार वृद्धि होती जाती है तथा पूरे पूल की लागत ₹ 4000 $(\text{गहराई})^2$ है ।

मान लीजिए कि वर्गाकार प्लॉट की भुजा x मीटर और गहराई h मीटर है ।

उपर्युक्त सूचना के आधार पर, निम्न प्रश्नों के उत्तर दीजिए :

(i) लागत फलन C(h) को h के पदों में लिखिए । *1*

(ii) क्रांतिक बिंदु ज्ञात कीजिए । *1*

(iii) (क) द्वितीय अवकलज परीक्षण द्वारा h का वह मान ज्ञात कीजिए, जिसके लिए पूल बनाने की लागत न्यूनतम हो । पूल बनाने की न्यूनतम लागत क्या है ? *2*

अथवा

Case Study – 2

37. A housing society wants to commission a swimming pool for its residents. For this, they have to purchase a square piece of land and dig this to such a depth that its capacity is 250 cubic metres. Cost of land is ₹ 500 per square metre. The cost of digging increases with the depth and cost for the whole pool is ₹ 4000 $(\text{depth})^2$.

Suppose the side of the square plot is x metres and depth is h metres.

On the basis of the above information, answer the following questions :

(i) Write cost C(h) as a function in terms of h. *1*

(ii) Find critical point. *1*

(iii) (a) Use second derivative test to find the value of h for which cost of constructing the pool is minimum. What is the minimum cost of construction of the pool ? *2*

OR

(iii) (ख) प्रथम अवकलज परीक्षण से पूल की ऐसी गहराई ज्ञात कीजिए कि पूल बनाने की लागत न्यूनतम हो । न्यूनतम लागत के लिए x और h के बीच का संबंध भी ज्ञात कीजिए । 2

प्रकरण अध्ययन – 3

38. एक कृषि संस्थान में, वैज्ञानिक बीजों की किस्मों को अलग-अलग वातावरणों में उगाने का प्रयोग करते हैं जिससे कि स्वस्थ पौधे उगें और अधिक उपज प्राप्त हो ।

एक वैज्ञानिक ने अवलोकन किया कि एक विशेष बीज अंकुरित होने के बाद बहुत तेज़ी से बढ़ रहा है । उसने बीज के अंकुरण के बाद से ही पौधे की वृद्धि को रिकॉर्ड किया था और उसने कहा कि इस वृद्धि को फलन $f(x) = \frac{1}{3}x^3 - 4x^2 + 15x + 2, \ 0 \leq x \leq 10$ से परिभाषित किया जा सकता है, जहाँ x दिनों की वह संख्या है जिनमें पौधा सूर्य के प्रकाश से उजागर था ।

उपर्युक्त सूचना के आधार पर, निम्न प्रश्नों के उत्तर दीजिए :

(i) इस फलन f(x) के क्रांतिक बिंदु कौन-से हैं ? 2

(ii) द्वितीय अवकलज परीक्षण का प्रयोग करके, फलन का न्यूनतम मान ज्ञात कीजिए । 2

(iii) (b) Use first derivative test to find the depth of the pool so that cost of construction is minimum. Also, find relation between x and h for minimum cost. *2*

Case Study – 3

38. In an agricultural institute, scientists do experiments with varieties of seeds to grow them in different environments to produce healthy plants and get more yield.

A scientist observed that a particular seed grew very fast after germination. He had recorded growth of plant since germination and he said that its growth can be defined by the function

$$f(x) = \frac{1}{3}x^3 - 4x^2 + 15x + 2, \quad 0 \le x \le 10$$

where x is the number of days the plant is exposed to sunlight.

On the basis of the above information, answer the following questions :

(i) What are the critical points of the function f(x) ? *2*

(ii) Using second derivative test, find the minimum value of the function. *2*

General Instructions :

Read the following instructions very carefully and strictly follow them :

(i) *This question paper contains **38** questions. **All** questions are **compulsory**.*

(ii) *This question paper is divided into **five** Sections – **A, B, C, D** and **E**.*

(iii) *In **Section A,** Questions no. **1** to **18** are Multiple Choice Questions (MCQs) and questions number **19** and **20** are Assertion-Reason based questions of **1** mark each.*

(iv) *In **Section B,** Questions no. **21** to **25** are Very Short Answer (VSA) type questions, carrying **2** marks each.*

) *In **Section C,** Questions no. **26** to **31** are Short Answer (SA) type questions, carrying **3** marks each.*

i) *In **Section D,** Questions no. **32** to **35** are Long Answer (LA) type questions carrying **5** marks each.*

ii) *In **Section E,** Questions no. **36** to **38** are case study based questions carrying **4** marks each.*

iii) *There is no overall choice. However, an internal choice has been provided in 2 questions in Section B, 3 questions in Section C, 2 questions in Section D and 2 questions in Section E.*

x) *Use of calculator is **not** allowed.*

SECTION A

*his section comprises 20 Multiple Choice Questions (MCQs) carrying **1** mark each. 20×1=20*

. If $A = \begin{bmatrix} 0 & 1 & -1 \\ 1 & 2 & 1 \\ 0 & 3 & -2 \end{bmatrix}$, then the value of $|A \text{ adj } (A)|$ is :

(A) -1 (B) 1

(C) 2 (D) 3

2. For two matrices A and B, given that $A^{-1} = \frac{1}{4}B$, then inverse of $(4A)$ is :

(A) $4B$ (B) B

(C) $\frac{1}{4}B$ (D) $\frac{1}{16}B$

3. If X, Y and XY are matrices of order 2 × 3, m × n and 2 × 5 respectively, then number of elements in matrix Y is :

(A) 6 (B) 10

(C) 15 (D) 35

4. The number of discontinuities of the function f given by

$$f(x) = \begin{cases} x+2, & \text{if} \quad x < 0 \\ e^{x}, & \text{if} \quad 0 \leq x \leq 1 \\ 2-x, & \text{if} \quad x > 1 \end{cases}$$

is :

(A) 0 (B) 1

(C) 2 (D) 3

. Let $y = f\left(\frac{1}{x}\right)$ and $f'(x) = x^3$. What is the value of $\frac{dy}{dx}$ at $x = \frac{1}{2}$?

(A) $-\frac{1}{64}$ (B) $-\frac{1}{32}$

(C) -32 (D) -64

. If $y = \log\sqrt{\sec\sqrt{x}}$, then the value of $\frac{dy}{dx}$ at $x = \frac{\pi^2}{16}$ is :

(A) $\frac{1}{\pi}$ (B) π

(C) $\frac{1}{2}$ (D) $\frac{1}{4}$

7. If $x = 3\cos\theta$ and $y = 5\sin\theta$, then $\frac{dy}{dx}$ is equal to :

(A) $-\frac{3}{5}\tan\theta$ (B) $-\frac{5}{3}\cot\theta$

(C) $-\frac{5}{3}\tan\theta$ (D) $-\frac{3}{5}\cot\theta$

8. For the function $f(x) = x^3$, $x = 0$ is a point of :

(A) local maxima (B) local minima

(C) non-differentiability (D) inflexion

9. The greatest integer function defined by $f(x) = [x]$, $1 < x < 3$ is not differentiable at $x =$

(A) 0 (B) 1

(C) 2 (D) $\frac{3}{2}$

0. If the radius of a circle is increasing at the rate of 0.5 cm/s, then the rate of increase of its circumference is :

(A) $\frac{2\pi}{3}$ cm/s (B) π cm/s

(C) $\frac{4\pi}{3}$ cm/s (D) 2π cm/s

1. $\int_{-\pi/4}^{\pi/4} x^3 \cos^2 x \, dx$ is equal to :

(A) 0 (B) -1

(C) 1 (D) 2

2. $\int \frac{x-3}{(x-1)^3} e^x \, dx$ is equal to :

(A) $\frac{2e^x}{(x-1)^3} + C$ (B) $\frac{-2e^x}{(x-1)^2} + C$

(C) $\frac{e^x}{(x-1)} + C$ (D) $\frac{e^x}{(x-1)^2} + C$

13. The area (in sq. units) of the region bounded by the curve $y = x$, x-axis, $x = 0$ and $x = 2$ is :

(A) $\frac{3}{2}$ (B) $\frac{1}{2}\log 2$

(C) 2 (D) 4

14. The number of arbitrary constants in the general solution of the differential equation $\frac{dy}{dx} + y = 0$ is :

(A) 0 (B) 1

(C) 2 (D) 3

5. What is the value of $\frac{\text{projection of } \vec{a} \text{ on } \vec{b}}{\text{projection of } \vec{b} \text{ on } \vec{a}}$

for vectors $\vec{a} = 2\hat{i} - 3\hat{j} - 6\hat{k}$ and $\vec{b} = 2\hat{i} - 2\hat{j} + \hat{k}$?

(A) $\frac{3}{7}$ (B) $\frac{7}{3}$

(C) $\frac{4}{3}$ (D) $\frac{4}{7}$

6. The direction ratios of the line $\frac{x-1}{3} = \frac{2-y}{1} = \frac{3z}{2}$ are :

(A) 3, 1, 2 (B) 4, 3, 2

(C) 9, – 3, 2 (D) 9, 3, 2

7. The Cartesian equation of the line passing through the point (1, – 3, 2) and parallel to the line $\vec{r} = 2\hat{i} - \hat{k} + \lambda(\hat{i} + \hat{j} + 2\hat{k})$ is :

(A) $\frac{x-1}{2} = \frac{y+3}{0} = \frac{z-2}{-1}$ (B) $\frac{x+1}{1} = \frac{y-3}{1} = \frac{z+2}{2}$

(C) $\frac{x+1}{2} = \frac{y-3}{0} = \frac{z+2}{-1}$ (D) $\frac{x-1}{1} = \frac{y+3}{1} = \frac{2-z}{-2}$

18. If $\vec{a}$ and $\vec{b}$ are two vectors such that $\vec{a}\,.\,\vec{b} > 0$ and $|\vec{a}\,.\,\vec{b}| = |\vec{a} \times \vec{b}|$, then the angle between $\vec{a}$ and $\vec{b}$ is :

(A) $\frac{\pi}{4}$ (B) $\frac{\pi}{3}$

(C) $\frac{2\pi}{3}$ (D) $\frac{3\pi}{4}$

*Questions number **19** and **20** are Assertion and Reason based questions. Two statements are given, one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer from the codes (A), (B), (C) and (D) as given below.*

(A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).

(B) Both Assertion (A) and Reason (R) are true, but Reason (R) is ***not*** the correct explanation of the Assertion (A).

(C) Assertion (A) is true, but Reason (R) is false.

(D) Assertion (A) is false, but Reason (R) is true.

9. *Assertion (A) :* $\cos^{-1}\left(\cos\dfrac{13\pi}{6}\right)$ is equal to $\dfrac{\pi}{6}$.

Reason (R) : The range of the principal value branch of the function $y = \cos^{-1} x$ is $[0, \pi]$.

0. *Assertion (A) :* If R and S are two events such that $P(R \mid S) = 1$ and $P(S) > 0$, then $S \subset R$.

Reason (R) : If two events A and B are such that $P(A \cap B) = P(B)$, then $A \subset B$.

SECTION B

his section comprises Very Short Answer (VSA) type questions of 2 marks each.

1. Find the value of $\cos^{-1}\left(\dfrac{1}{2}\right) - \tan^{-1}\left(-\dfrac{1}{\sqrt{3}}\right) + \text{cosec}^{-1}(-2)$.

2. (a) If $y = (\sin^{-1} x)^2$, then find $(1 - x^2)\dfrac{d^2y}{dx^2} - x\dfrac{dy}{dx}$.

OR

(b) If $y^x = x^y$, then find $\dfrac{dy}{dx}$.

23. Given that $f(x) = \dfrac{\log x}{x}$, find the point of local maximum of f(x).

24. (a) Find :

$$\int \frac{x^3 - 1}{x^3 - x}\, dx$$

OR

(b) Evaluate :

$$\int_{-4}^{0} |x + 2|\, dx$$

5. Find the angle between the lines

$$\frac{5 - x}{-7} = \frac{y + 2}{-5} = \frac{z}{1} \text{ and } \frac{x}{1} = \frac{y}{2} = \frac{z}{3}.$$

SECTION C

his section comprises Short Answer (SA) type questions of 3 marks each.

6. (a) Find a matrix A such that

$$A \begin{bmatrix} 4 & 0 \\ -1 & -2 \end{bmatrix} = \begin{bmatrix} 17 & 10 \\ 0 & -16 \end{bmatrix}.$$

Also, find A^{-1}.

OR

(b) Given a square matrix A of order 3 such that $A^2 = \begin{bmatrix} 3 & -4 & 4 \\ 0 & -1 & 0 \\ -2 & 2 & -3 \end{bmatrix}$,

show that $A^3 = A^{-1}$.

27. (a) If $x \sin (a + y) - \sin y = 0$,

prove that $\dfrac{dy}{dx} = \dfrac{\sin^2 (a + y)}{\sin a}$

OR

(b) Find $\dfrac{dy}{dx}$, if $y = (\cos x)^x + \cos^{-1} \sqrt{x}$.

8. (a) Find :

$$\int \frac{dx}{\cos x \sqrt{\cos 2x}}$$

OR

(b) Find :

$$\int \frac{5x - 3}{\sqrt{1 + 4x - 2x^2}} dx$$

9. Find the general solution of the differential equation

$$y\, dx - x\, dy + (x \log x)\, dx = 0.$$

0. If the vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$ represent the three sides of a triangle, then show that $\vec{a} \times \vec{b} = \vec{b} \times \vec{c} = \vec{c} \times \vec{a}$.

1. It is known that 20% of the students in a school have above 90% attendance and 80% of the students are irregular. Past year results show that 80% of students who have above 90% attendance and 20% of irregular students get ‘A’ grade in their annual examination. At the end of a year, a student is chosen at random from the school and is found to have an ‘A’ grade. What is the probability that the student is irregular ?

SECTION D

This section comprises Long Answer (LA) type questions of 5 marks each.

32. Check whether the relation S in the set of all real numbers ($\mathbb{R}$) defined by $S = \{(a, b) : a \leq b^3\}$ is reflexive, symmetric or transitive.

33. (a) Using integration, find the area of the region bounded by the curve $y = \sqrt{4 - x^2}$, the lines $x = -\sqrt{2}$ and $x = \sqrt{3}$ and the x-axis.

OR

(b) Using integration, evaluate the area of the region bounded by the curve $y = x^2$, the lines $y = 1$ and $y = 3$ and the y-axis.

4. (a) Find the shortest distance between the lines $\frac{x-8}{3} = \frac{y+9}{-16} = \frac{z-10}{7}$ and $\frac{x-15}{3} = \frac{y-29}{8} = \frac{z-5}{-5}$.

OR

(b) Find the point of intersection of the lines

$$\vec{r} = \hat{i} - \hat{j} + 6\hat{k} + \lambda(3\hat{i} - \hat{k}), \text{ and}$$

$$\vec{r} = -3\hat{j} + 3\hat{k} + \mu(\hat{i} + 2\hat{j} - \hat{k}).$$

Also, find the vector equation of the line passing through the point of intersection of the given lines and perpendicular to both the lines.

35. Solve the following linear programming problem graphically :

Minimise $Z = 6x + 7y$

subject to constraints

$x + 2y \geq 240$

$3x + 4y \leq 620$

$2x + y \geq 180$

$x, y \geq 0.$

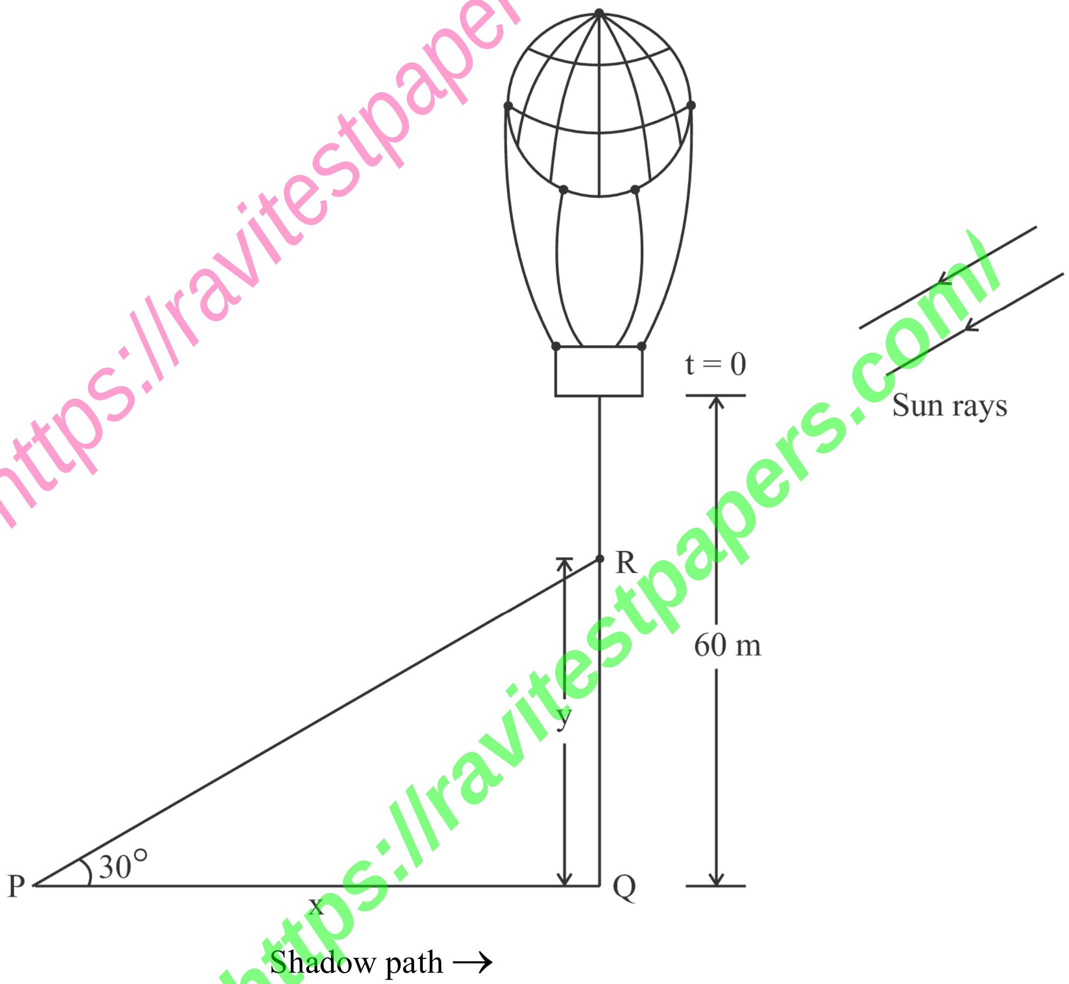
~~~

SECTION E

This section comprises 3 case study based questions of 4 marks each.

Case Study – 1

36. A sandbag is dropped from a balloon at a height of 60 metres.



When the angle of elevation of the sun is 30°, the position of the sandbag is given by the equation $y = 60 - 4.9\ t^2$, where y is the height of the sandbag above the ground and t is the time in seconds.
~~~

On the basis of the above information, answer the following questions :

(i) Find the relation between x and y, where x is the distance of the shadow at P from the point Q and y is the height of the sandbag above the ground.

(ii) After how much time will the sandbag be 35 metres above the ground ?

(iii) (a) Find the rate at which the shadow of the sandbag is travelling along the ground when the sandbag is at a height of 35 metres.

OR

(iii) (b) How fast is the height of the sandbag decreasing when 2 seconds have elapsed ?

Case Study – 2

7. A salesman receives a commission for each sale he makes together with a fixed daily income. The number of sales he makes in a day along with their probabilities are given in the table below :

X :	0	1	2	3	4	5
P(X) :	0.42	3k	0.3	0.05	2k	0.03

His daily income Y (in ₹) is given by :

$$Y = 800X + 50$$

On the basis of the above information, answer the following questions :

(i) Find the value of k.

(ii) Evaluate $P(X \geq 3)$.

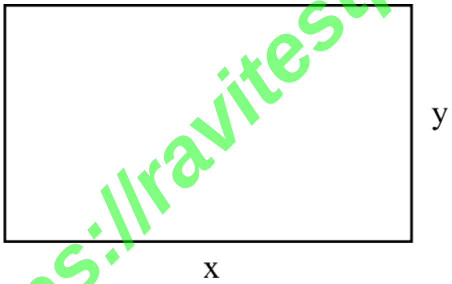
(iii) (a) Calculate the expected weekly income of the salesman assuming he works five days per week.

OR

(iii) (b) Calculate the expected weekly income of the salesman assuming he works only for three days of the week.

Case Study – 3

8. An architect is developing a plot of land for a commercial complex. When asked about the dimensions of the plot, he said that if the length is decreased by 25 m and the breadth is increased by 25 m, then its area increases by 625 m^2. If the length is decreased by 20 m and the breadth is increased by 10 m, then its area decreases by 200 m^2.



On the basis of the above information, answer the following questions :

(i) Formulate the linear equations in x and y to represent the given information.

(ii) Find the dimensions of the plot of land by matrix method.

All India 2018

CBSE Board Solved Paper

Time Allowed : 3 Hours | *Maximum Marks : 100*

General Instructions:

- (i) **All** questions are compulsory.
- (ii) The question paper consists of **29** questions divided into four sections **A**, **B**, **C** and **D**. Section **A** comprises of **4** questions of **one mark** each, Section **B** comprises of **8** questions of **two marks** each, Section **C** comprises of **11** questions of **four marks** each and Section **D** comprises of **6** questions of **six marks** each.
- (iii) All questions in Section **A** are to be answered in one word, one sentence or as per the exact requirement of the question.
- (iv) There is no overall choice. However, internal choice has been provided in **3** questions of four marks each and **3** questions of six marks each. You have to attempt only one of the alternatives in all such questions.
- (v) Use of calculators is not permitted. You may ask for logarithmic tables, if required.

SECTION - A

1. If $a * b$ denotes the larger of 'a' and 'b' and if $a \text{ o } b = (a * b) + 3$, then write the value of (5) o (10), where * and o are binary operations.

2. Find the magnitude of each of the two vectors $\vec{a}$ and $\vec{b}$, having the same magnitude such that the angle between them is 60° and their scalar product is $\frac{9}{2}$.

3. If the matrix $A = \begin{bmatrix} 0 & a & -3 \\ 2 & 0 & -1 \\ b & 1 & 0 \end{bmatrix}$ is skew symmetric, find the values of 'a' and 'b'.

4. Find the value of $\tan^{-1}\sqrt{3} - \cot^{-1}(-\sqrt{3})$.

SECTION - B

5. The total cost $C(x)$ associated with the production of x units of an item is given by $C(x) = 0.005x^3 - 0.02x^2 + 30x + 5000$. Find the marginal cost when 3 units are produced, where by marginal cost we mean the instantaneous rate of change of total cost at any level of output.

6. Differentiate $\tan^{-1}\left(\frac{1+\cos x}{\sin x}\right)$ with respect to x.

7. Given $A = \begin{bmatrix} 2 & -3 \\ -4 & 7 \end{bmatrix}$, compute A^{-1} and show that $2A^{-1} = 9I - A$.

8. Prove that:

$$3\sin^{-1} x = \sin^{-1}(3x - 4x^3),\ x \in \left[-\frac{1}{2}, \frac{1}{2}\right]$$

9. A black and a red die are rolled together. Find the conditional probability of obtaining the sum 8, given that the red die resulted in a number less than 4.

10. If θ is the angle between two vectors $\hat{i} - 2\hat{j} + 3\hat{k}$ and $3\hat{i} - 2\hat{j} + \hat{k}$, find sin θ.

11. Find the differential equation representing the family of curves $y = a\, e^{bx+5}$ where a and b are arbitrary constants.

12. Evaluate:

$$\int \frac{\cos 2x + 2\sin^2 x}{\cos^2 x} dx$$

SECTION - C

13. If $y = \sin(\sin x)$, prove that

$$\frac{d^2y}{dx^2} + \tan x \frac{dy}{dx} + y\cos^2 x = 0.$$

14. Find the particular solution of the differential equation

$e^x \tan y\, dx + (2 - e^x)\sec^2 y\, dy = 0$, given that $y = \frac{\pi}{4}$ when $x = 0$.

OR

Find the particular solution of the differential equation

$\frac{dy}{dx} + 2y\tan x = \sin x$, given that $y = 0$ when $x = \frac{\pi}{3}$.

15. Find the shortest distance between the lines

$\vec{r} = (4\hat{i} - \hat{j}) + \lambda(\hat{i} + 2\hat{j} - 3\hat{k})$ and

$\vec{r} = (\hat{i} - \hat{j} + 2\hat{k}) + \mu(2\hat{i} + 4\hat{j} - 5\hat{k})$.

16. Two numbers are selected at random (without replacement) from the first five positive integers. Let X denote the larger of the two numbers obtained. Find the mean and variance of X.

17. Using properties of determinants, prove that

$$\begin{vmatrix} 1 & 1 & 1+3x \\ 1+3y & 1 & 1 \\ 1 & 1+3z & 1 \end{vmatrix} = 9(3xyz + xy + yz + zx)$$

18. Find the equations of the tangent and the normal, to the curve $16x^2 + 9y^2 = 145$ at the point (x_1, y_1), where $x_1 = 2$ and $y_1 > 0$.

OR

Find the intervals in which the function

$$f(x) = \frac{x^4}{4} - x^3 - 5x^2 + 24x + 12 \text{ is}$$

(a) strictly increasing, (b) strictly decreasing.

19. Find:

$$\int \frac{2\cos x}{(1 - \sin x)(1 + \sin^2 x)}dx$$

20. Suppose a girl throws a die. If she gets 1 or 2, she tosses a coin three times and notes the number of tails. If she gets 3, 4, 5 or 6, she tosses a coin once and notes whether a 'head' or 'tail' is obtained. If she obtained exactly one 'tail', what is the probability that she threw 3, 4, 5 or 6 with the die?

21. Let $\vec{a} = 4\hat{i} + 5\hat{j} - \hat{k}, \vec{b} = \hat{i} - 4\hat{j} + 5\hat{k}$ and $\vec{c} = 3\hat{i} + \hat{j} - \hat{k}$. Find a vector $\vec{d}$ which is perpendicular to both $\vec{c}$ and $\vec{b}$ and $\vec{d} \cdot \vec{a} = 21$.

22. An open tank with a square base and vertical sides is to be constructed from a metal sheet so as to hold a given quantity of water. Show that the cost of material will be least when depth of the tank is half of its width. If the cost is to be borne by nearby settled lower income families, for whom water will be provided, what kind of value is hidden in this question?

23. If $(x^2 + y^2)^2 = xy$, find $\frac{dy}{dx}$.

OR

If $x = a(2\theta - \sin 2\theta)$ and $y = a(1 - \cos 2\theta)$, find $\frac{dy}{dx}$ when $\theta = \frac{\pi}{3}$.

SECTION - D

24. Evaluate :

$$\int_0^{\pi/4} \frac{\sin x + \cos x}{16 + 9\sin 2x}dx$$

OR

Evaluate :

$$\int_1^3 (x^2 + 3x + e^x)dx$$

as the limit of the sum.

25. A factory manufactures two types of screws A and B, each type requiring the use of two machines, an automatic and a hand-operated. It takes 4 minutes on the automatic and 6 minutes on the hand-operated machines to manufacture a packet of screws 'A' while it takes 6 minutes on the automatic and 3 minutes on the hand-operated machine to manufacture a packet of screws 'B'. Each machine is available for at most 4 hours on any day. The manufacture can sell a packet of screws 'A' at a profit of 70 paise and screws 'B' at a profit of ₹ 1. Assuming that he can sell all the screws he manufactures, how many packets of each type should the factory owner produce in a day in order to maximize his profit? Formulate the above LPP and solve it graphically and find the maximum profit?

26. Let $A = \{x \in Z : 0 \le x \le 12\}$. Show that $R = \{(a, b) : a, b \in A, |a - b| \text{ is divisible by } 4\}$ is an equivalence relation. Find the set of all elements related to 1. Also write the equivalence class [2].

OR

Show that the function $f : \mathbf{R} \to \mathbf{R}$ defined by

$f(x) = \dfrac{x}{x^2+1}, \forall\, x \in \mathbf{R}$ is neither one-one nor onto. Also, if

$g : \mathbf{R} \to \mathbf{R}$ is defined as $g(x) = 2x - 1$, find $fog\,(x)$.

27. Using integration, find the area of the region in the first quadrant enclosed by the x-axis, the line $y = x$ and the circle $x^2 + y^2 = 32$.

28. If $A = \begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}$, find A^{-1}.

Use it to solve the system of equations

$2x - 3y + 5z = 11$

$3x + 2y - 4z = -5$

$x + y - 2z = -3.$

OR

Using elementary row transformations, find the inverse of

the matrix $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 7 \\ -2 & -4 & -5 \end{bmatrix}$.

29. Find the distance of the point $(-1, -5, -10)$ from the point of intersection of the line $\vec{r} = 2\hat{i} - \hat{j} + 2\hat{k} + \lambda(3\hat{i} + 4\hat{j} + 2\hat{k})$ and the plane $\vec{r} \cdot (\hat{i} - \hat{j} + \hat{k}) = 5$.

All India 2017

CBSE Board Solved Paper

Time Allowed : 3 Hours *Maximum Marks : 100*

General Instructions:

- (i) All questions are compulsory.
- (ii) This question paper contains **29** questions.
- (iii) Question **1-4** in **Section A** are very short-answer type questions carrying **1** mark each.
- (iv) Questions **5-12** in **Section B** are short-answer type questions carrying **2** marks each.
- (v) Question **13-23** in **Section C** are long-answer-**I** type questions carrying **4** marks each.
- (vi) Question **24-29** in **Section D** are long answer-**II** Type Questions carrying **6** marks each.
- (vii) Please write down the serial number of the Question before attempting it.

SECTION - A

1. If for any 2 × 2 square matrix A, A (adj A) = $\begin{bmatrix} 8 & 0 \\ 0 & 8 \end{bmatrix}$, then write the value of |A|.

2. Determine the value of '*k*' for which the following function is continuous at $x = 3$:

$$f(x) = \begin{cases} \dfrac{(x+3)^2 - 36}{x-3}, & x \neq 3 \\ k \quad , & x = 3 \end{cases}$$

3. Find : $\displaystyle\int \frac{\sin^2 x - \cos^2 x}{\sin x \cos x} \text{dx}$

4. Find the distance between the planes $2x - y + 2z = 5$ and $5x - 2.5y + 5z = 20$.

SECTION - B

5. If A is a skew-symmetric matrix of order 3, then prove that det A = 0.

6. Find the value of c in Rolle's theorem for the function $f(x) = x^3 - 3x$ in $\left[-\sqrt{3}, 0\right]$.

7. The volume of a cube is increasing at the rate of 9cm^3/s. How fast is its surface area increasing when the length of an edge is 10 cm?

8. Show that the function $f(x) = x^3 - 3x^2 + 6x - 100$ is increasing on R.

9. The *x*–coordinate of a point on the line joining the points P (2, 2, 1) and Q(5, 1, –2) is 4. Find its z-coordinate.

10. A die, whose faces are marked 1, 2, 3 in red and 4, 5, 6 in green, is tossed. Let A be the event "number obtained is even" and B be the event "number obtained is red." Find if A and B are independent events.

11. Two tailors, A and B earn ₹ 300 and ₹ 400 per day respectively. A can stitch 6 shirts and 4 pairs of trousers while B can stitch 10 shirts and 4 pairs of trousers per day. To find how many days should each of them work and if it is desired to produce at least 60 shirts and 32 pairs of trousers at a minimum labour cost, formulate this as an LPP.

12. Find : $\displaystyle\int \frac{dx}{5 - 8x - x^2}$

SECTION - C

13. If $\tan^{-1}\dfrac{x-3}{x-4} + \tan^{-1}\dfrac{x+3}{x+4} = \dfrac{\pi}{4}$, then find the value of x.

14. Using properties of determinants, prove that

$$\begin{vmatrix} a^2 + 2a & 2a+1 & 1 \\ 2a+1 & a+2 & 1 \\ 3 & 3 & 1 \end{vmatrix} = (a-1)^3$$

OR

Find matrix A such that

$$\begin{pmatrix} 2 & -1 \\ 1 & 0 \\ -3 & 4 \end{pmatrix} A = \begin{pmatrix} -1 & -8 \\ 1 & -2 \\ 9 & 22 \end{pmatrix}$$

15. If $x^y + y^x = a^b$, then find $\frac{dy}{dx}$.

OR

If $e^y(x+1) = 1$, then show that $\frac{d^2y}{dx^2} = \left(\frac{dy}{dx}\right)^2$.

16. Find : $\int \frac{\cos\theta}{(4+\sin^2\theta)(5-4\cos^2\theta)}\,d\theta$

17. Evaluate : $\int_0^{\pi} \frac{x\tan x}{\sec x + \tan x}\,dx$

OR

Evaluate : $\int_1^4 \{|x-1| + |x-2| + |x-4|\}\,dx$

18. Solve the differential equation $(\tan^{-1}x - y)\,dx = (1+x^2)\,dy$.

19. Show that the points A, B, C with position vectors $2\hat{i}-\hat{j}+\hat{k}, \hat{i}-3\hat{j}-5\hat{k}$ and $3\hat{i}-4\hat{j}-4\hat{k}$ respectively, are the vertices of a right-angled triangle. Hence find the area of the triangle.

20. Find the value of λ, if four points with position vectors $3\hat{i}+6\hat{j}+9\hat{k}, \hat{i}+2\hat{j}+3\hat{k}, 2\hat{i}+3\hat{j}+\hat{k}$ and $4\hat{i}+6\hat{j}+\lambda\hat{k}$ are coplanar.

21. There are 4 cards numbered 1, 3, 5 and 7, one number on one card. Two cards are drawn at random without replacement. Let X denote the sum of the numbers on the two drawn cards. Find the mean and variance of X.

22. Of the students in a school, it is known that 30% have 100% attendance and 70% students are irregular. Previous year results report that 70% of all students who have 100% attendance attain A grade and 10% irregular students attain A grade in their annual examination. At the end of the year, one students is chosen at random from the school and he was found to have an A grade. What is the probability that the student has 100% attendance? Is regularity required only in school? Justify your answer.

23. Maximise $Z = x + 2y$

subject to the constraints

$x + 2y \geq 100$

$2x - y \leq 0$

$2x + y \leq 200$

$x, y \geq 0$

Solve the above LPP graphically.

SECTION - D

24. Determine the product $\begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix}\begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix}$ and use it to solve the system of equations $x - y + z = 4$, $x - 2y - 2z = 9$, $2x + y + 3z = 1$.

25. Consider f : $R - \left\{-\frac{4}{3}\right\} \rightarrow R - \left\{\frac{4}{3}\right\}$ given by $f(x) = \frac{4x+3}{3x+4}$. Show that f is bijective. Find the inverse of f and hence find $f^{-1}(0)$ and x such that $f^{-1}(x) = 2$.

OR

Let A = $Q \times Q$ and let * be a binary operation on A defined by (a, b) * (c, d) = (ac, b + ad) for (a, b), (c, d) ∈A. Determine, whether * is commutative and associative. Then, with respect to *on A

(i) find the identity element in A

(ii) find the invertible elements of A

26. Show that the surface area of a closed cuboid with square base and given volume is minimum, when it is a cube.

27. Using the method of integration, find the area of the triangle ABC, coordinates of whose vertices are A(4, 1), B(6, 6) and C(8, 4).

OR

Find the area enclosed between the parabola $4y = 3x^2$ and the straight line $3x - 2y + 12 = 0$.

28. Find the particular solution of the differential equation $(x-y)\frac{dy}{dx} = (x+2y)$, given that $y = 0$ when $x = 1$.

29. Find the coordinates of the point where the line through the points (3, –4, –5) and (2, –3, 1) crosses the plane determined by the points (1, 2, 3), (4, 2, –3) and (0, 4, 3).

OR

A variable plane which remains at a constant distance 3p from the origin cuts the coordinate axes at A, B, C. Show that the locus of the centroid of triangle ABC is $\frac{1}{x^2} + \frac{1}{y^2} + \frac{1}{z^2} = \frac{1}{p^2}$.

All India 2016

CBSE Board Solved Paper

Time Allowed : 3 Hours *Maximum Marks : 100*

General Instructions:

(i) All questions are compulsory.
(ii) Please check that this Question Paper contains 26 questions.
(iii) Marks for each question are indicated against it.
(iv) Question **1** to **6** in **Section-A** are Very Short Answer Type Questions carrying **one** mark each.
(v) Question **7** to **19** in **Section-B** are Long Answer **I** Type Questions carrying **4** marks each.
(vi) Question **20** to **26** in **Section-C** are Long Answer **II** Type Questions carrying **6** marks each.
(vii) Please write down the serial number of the Question before attempting it.

SECTION - A

1. The two vectors $\hat{j}+\hat{k}$ and $3\hat{i}-\hat{j}+4\hat{k}$ represent the two sides AB and AC, respectively of a ΔABC. Find the length of the median through A.

2. Find the vector equation of a plane which is at a distance of 5 units from the origin and its normal vector is $2\hat{i}-3\hat{j}+6\hat{k}$.

3. Find the maximum value of $\begin{vmatrix} 1 & 1 & 1 \\ 1 & 1+\sin\theta & 1 \\ 1 & 1 & 1+\cos\theta \end{vmatrix}$

4. If A is a square matrix such that $A^2 = I$, then find the simplified value of $(A-I)^3+(A+I)^3-7A$.

5. Matrix $A=\begin{bmatrix} 0 & 2b & -2 \\ 3 & 1 & 3 \\ 3a & 3 & -1 \end{bmatrix}$ is given to be symmetric, find values of a and b.

6. Find the position vector of a point which divides the join of points with position vectors $\vec{a}-2\vec{b}$ and $2\vec{a}+\vec{b}$ externally in the ratio 2 : 1.

SECTION - B

7. Find the general solution of the following differential equation :

$$\left(1+y^2\right)+\left(x-e^{\tan^{-1}y}\right)\frac{dy}{dx}=0$$

8. Show that the vectors $\vec{a}, \vec{b}$ and $\vec{c}$ are coplanar if $\vec{a}+\vec{b}, \vec{b}+\vec{c}$ and $\vec{c}+\vec{a}$ are coplanar.

9. Find the vector and Cartesian equations of the line through the point (1, 2, –4) and perpendicular to the two lines.

$$\vec{r}=\left(8\hat{i}-19\hat{j}+10\hat{k}\right)+\lambda\left(3\hat{i}-16\hat{j}+7\hat{k}\right) \text{ and}$$

$$\vec{r}=\left(15\hat{i}+29\hat{j}+5\hat{k}\right)+\mu\left(3\hat{i}+8\hat{j}-5\hat{k}\right)$$

10. Three persons A, B and C apply for a job of Manager in a Private Company. Chances of their selection (A, B and C) are in the ratio 1 : 2 : 4. The probabilities that A, B and C can introduce changes to improve profits of the company are 0.8, 0.5 and 0.3 respectively. If the change does not take place, find the probability that it is due to the appointment of C.

OR

A and B throw a pair of dice alternately. A wins the game if he gets a total of 7 and B wins the game if he gets a total of 10. If A starts the game, then find the probability that B wins.

11. Prove that:

$$\tan^{-1}\frac{1}{5}+\tan^{-1}\frac{1}{7}+\tan^{-1}\frac{1}{3}+\tan^{-1}\frac{1}{8}=\frac{\pi}{4}$$

OR

Solve for x :

$2\tan^{-1}(\cos x) = \tan^{-1}(2 \operatorname{cosec} x)$

12. The monthly incomes of Aryan and Babban are in the ratio 3 : 4 and their monthly expenditures are in the ratio 5 : 7. If each saves ₹15,000 per month, find their monthly incomes using matrix method. This problem reflects which value?

13. If $x = a \sin 2t\,(1 + \cos 2t)$ and $y = b \cos 2t\,(1 - \cos 2t)$, find the values of $\frac{dy}{dx}$ at $t = \frac{\pi}{4}$ and $t = \frac{\pi}{3}$.

OR

If $y = x^x$, prove that $\frac{d^2y}{dx^2} - \frac{1}{y}\left(\frac{dy}{dx}\right)^2 - \frac{y}{x} = 0.$

14. Find the values of p and q for which

$$f(x) = \begin{cases} \dfrac{1 - \sin^3 x}{3\cos^2 x} & , \text{ if } x < \dfrac{\pi}{2} \\ p & , \text{ if } x = \pi/2 \\ \dfrac{q(1 - \sin x)}{(\pi - 2x)^2} & , \text{ if } x > \pi/2 \end{cases}$$

is continuous at $x = \pi/2$.

15. Show that the equation of normal at any point t on the curve $x = 3 \cos t - \cos^3 t$ and $y = 3 \sin t - \sin^3 t$ is $4\,(y \cos^3 t - x \sin^3 t) = 3 \sin 4t$.

16. Find $\int \frac{(3\sin\theta - 2)\cos\theta}{5 - \cos^2\theta - 4\sin\theta}\,d\theta$

OR

Evaluate $\int_0^{\pi} e^{2x} \cdot \sin\left(\frac{\pi}{4} + x\right) dx$

17. Find $\int \frac{\sqrt{x}}{\sqrt{a^3 - x^3}}\,dx.$

18. Evaluate $\int_{-1}^{2} \left|x^3 - x\right| dx.$

19. Find the particular solution of the differential equation $(1 - y^2)\,(1 + \log x)\,dx + 2xy\,dy = 0$, given that $y = 0$ when $x = 1$.

SECTION - C

20. Find the coordinate of the point P where the line through A(3, – 4, – 5) and B(2, – 3, 1) crosses the plane passing through three points L(2, 2, 1), M(3, 0, 1) and N(4, – 1, 0). Also, find the ratio in which P divides the line segment AB.

21. An urn contains 3 white and 6 red balls. Four balls are drawn one by one with replacement from the urn. Find the probability distribution of the number of red balls drawn. Also find mean and variance of the distribution.

22. A manufacturer produces two products A and B. Both the products are processed on two different machines. The available capacity of first machine is 12 hours and that of second machine is 9 hours per day. Each unit of product A requires 3 hours on both machines and each unit of product B requires 2 hours on first machine and 1 hour on second machine. Each unit of product A is sold at ₹7 profit and B at a profit of ₹4. Find the production level per day for maximum profit graphically.

23. Let $f : \mathrm{N} \to \mathrm{N}$ be a function defined as $f(x) = 9x^2 + 6x - 5$. Show that $f : \mathrm{N} \to \mathrm{S}$, where S is the range of f, is invertible. Find the inverse of f and hence find $f^{-1}(43)$ and $f^{-1}(163)$.

24. Prove that $\begin{vmatrix} yz - x^2 & zx - y^2 & xy - z^2 \\ zx - y^2 & xy - z^2 & yz - x^2 \\ xy - z^2 & yz - x^2 & zx - y^2 \end{vmatrix}$ is divisible by $(x + y + z)$ and hence find the quotient.

OR

Using elementary transformations, find the inverse of the matrix $A = \begin{pmatrix} 8 & 4 & 3 \\ 2 & 1 & 1 \\ 1 & 2 & 2 \end{pmatrix}$ and use it to solve the following system of linear equations :

$8x + 4y + 3z = 19$

$2x + y + z = 5$

$x + 2y + 2z = 7$

25. Show that the altitude of the right circular cone of maximum volume that can be inscribed in a sphere of radius r is $\frac{4r}{3}$. Also find maximum volume in terms of volume of the sphere.

OR

Find the intervals in which $f(x) = \sin 3x - \cos 3x$, $0 < x < \pi$, is strictly increasing or strictly decreasing.

26. Using integration find the area of the region $\left\{(x, y) : x^2 + y^2 \le 2ax,\ y^2 \ge ax,\ x, y \ge 0\right\}$.

All India 2015

CBSE Board Solved Paper

Time Allowed : 3 Hours *Maximum Marks : 100*

General Instructions:

(i) **All** questions are compulsory.

(ii) Please check that this Question Paper contains **26** questions.

(iii) Marks for each question are indicated against it.

(iv) Question **1** to **6** in **Section-A** are Very Short Answer Type Questions carrying **one** mark each.

(v) Question **7** to **19** in **Section-B** are Long Answer **I** Type Questions carrying **4** marks each.

(vi) Question **20** to **26** in **Section-C** are Long Answer **II** Type Questions carrying **6** marks each.

(vii) Please write down the serial number of the Question before attempting it.

SECTION - A

1. Write the value of $\vec{a}\cdot\left(\vec{b}\times\vec{a}\right)$.

2. If $\vec{a} = \hat{i} + 2\hat{j} - \hat{k}, \vec{b} = 2\hat{i} + \hat{j} + \hat{k}$ and $\vec{c} = 5\hat{i} - 4\hat{j} + 3\hat{k}$, then find the value of $\left(\vec{a} + \vec{b}\right)\cdot\vec{c}$.

3. Write the direction ratios of the following line:

$$x = -3, \frac{y-4}{3} = \frac{2-z}{1}$$

4. If $A = \begin{bmatrix} 2 & 3 \\ 5 & -2 \end{bmatrix}$, then write A^{-1}.

5. Find the differential equation representing the curve $y = cx + c^2$.

6. Write the integrating factor of the following differential equation:

$(1 + y^2)dx - (\tan^{-1} y - x)\, dy = 0$

SECTION - B

7. Using the properties of determinants, prove the following:

$$\begin{vmatrix} 1 & x & x+1 \\ 2x & x(x-1) & x(x+1) \\ 3x(1-x) & x(x-1)(x-2) & x(x+1)(x-1) \end{vmatrix} = 6x^2\left(1-x^2\right)$$

8. If $x = \alpha \sin 2t\,(1 + \cot 2t)$ and $y = \beta\cos 2t\,(1 - \cos 2t)$. show that $\frac{dy}{dx} = \frac{\beta}{\alpha}\tan t$.

9. Find: $\frac{d}{dz}\cos^{-1}\left(\frac{z - z^{-1}}{z + z^{-1}}\right)$

10. Find the derivative of the following function

$f(x) = \cos^{-1}\left[\sin\sqrt{\frac{1+z}{2}}\right] + x^x$ w.r.t. x, at $x = 1$

11. Evaluate:

$$\int_0^{\frac{\pi}{2}} \frac{2^{\sin x}}{2^{\sin x} + 2^{\cos x}}dx$$

OR

Evaluate : $\int_0^{\frac{3}{2}} |\, x\cdot\cos(\pi x)\,|\, dx$

12. To raise money for an orphanage, students of three schools A, B and C organised an exhibition in their locality, where they sold paper bags, scrap-books and pastel sheets made by them using recycled paper, at the rate of ₹ 20, ₹ 15 and ₹ 5 per unit respectively. School A sold 25 paper bags, 12 scrap-books and 34 pastel sheets. School B sold 22 paper bags, 15 scrap-books and 28 pastel sheets while school C sold 26 paper bags, 18 scrap-books and 36 pastel sheets. Using matrix, find the total amount raised by each school.

By such exhibition, which values are generated in the students?

13. Prove that:

$$2\tan^{-1}\left(\sqrt{\frac{a-b}{a+b}}\tan\frac{x}{2}\right)=\cos^{-1}\left(\frac{a\cos x+b}{a+b\cos x}\right)$$

OR

Solve the following for x :

$$\tan^{-1}\left(\frac{x-2}{x-3}\right)+\tan^{-1}\left(\frac{x+2}{x+3}\right)=\frac{\pi}{4},|x|<1.$$

14. If $A=\begin{pmatrix}2 & 0 & 1\\ 2 & 1 & 3\\ 1 & -1 & 0\end{pmatrix}$, find $A^2 - 5A + 16I$.

15. Show that four points A, B, C and D whose position vectors are $4\hat{i}+5\hat{j}+\hat{k},-\hat{j}-\hat{k},3\hat{i}+9\hat{j}+4\hat{k}$ and $4\left(-\hat{i}+\hat{j}+\hat{k}\right)$ respectively are coplanar.

16. Show that the following two lines are coplanar:

$$\frac{x-a+d}{a-d}=\frac{y-a}{a}=\frac{z-a-d}{a+d} \text{ and}$$

$$\frac{x-b+c}{\beta-\tau}=\frac{y-b}{\beta}=\frac{z-b-c}{\beta+\tau}$$

OR

Find the acute angle between the plane $5x - 4y + 7z - 13 = 0$ and the y-axis.

17. A and B throw a die alternatively till one of them gets a number greater than four and wins the game. If A starts the game, what is the probability of B winning?

OR

A die is thrown three times. Events A and B are defined as below:

A: 5 on the first and 6 on the second throw.

B: 3 or 4 on the third throw.

Find the probability of B, given that A has already occurred.

18. Evaluate: $\int\left(\sqrt{\cot x}+\sqrt{\tan x}\right)dx$

19. Find: $\int\frac{x^3-1}{x^3+x}dx$

SECTION - C

20. Using integration, find the area of the region bounded by the lines $y = 2 + x$, $y = 2 - x$, $x = 2$.

21. Find the differential equation for all the straight lines, which are at a unit distance from the origin.

OR

Show that the differential equation : $2xy\frac{dy}{dx}=x^2+3y^2$ is homogeneous and solve it.

22. Find the direction ratios of the normal to the plane, which passes through the points (1,0,0) and (0,1,0) and makes angle $\frac{\pi}{4}$ with the plane x + y = 3. Also find the equation of the plane.

23. If the function $f : R \to R$ be defined by $f(x) = 2x - 3$ and $g : R \to R$ by $g(x) = x^3 + 5$, then find the value of $(fog)^{-1}(x)$.

OR

Let A = Q × Q, where Q is the set of all rational numbers, and x be a binary opertation defined on A by $(a, b)*(c,d) = (ac, b + ad)$, for all $(a, b)\ (c,d) \in A$.
Find

(i) the identity element in A

(ii) the invertible element of A.

24. If the function $f(x) = 2x^3 - 9mx^2 + 12m^2x + 1$. where $m > 0$ attains its maximum and minimum at p and q respectively such that $p^2 = q$, then find the value of m.

25. The postmaster of a local post office wishes to hire extra helpers during the Deepawali season because of large increase in the volume of mail handling and delivery. Because of the limited office space and the budgetary conditions, the number of temporary helpers must not exceed 10. According to past experience, a man can handle 300 letters and 80 packages per day, on the average, and a woman can handle 400 letters and 50 packets per day. The postmaster believes that the daily volume of extra mail and packages will be no less than 3400 and 680 respectively. A man receives ₹ 225 a day and a woman receives ₹ 200 a day. How many men and women helpers should be hired to keep the pay-roll at a minimum? Formulate an LPP and solve it graphically.

26. 40% students of a college reside in hostel and the remaining reside outside. At the end of the year, 50% of the hostelers got A grade while from outside students, only 30% got A grade in the examination. At the end of the year, a student of the college was chosen at random and was found to have gotten A grade. What is the probability that the selected student was a hosteler?

All India 2014

CBSE Board Solved Paper

Time Allowed : 3 Hours *Maximum Marks : 100*

General Instructions:

- (i) **All** questions are compulsory.
- (ii) The question paper consists of **29** questions divided into three sections A, B and C. Section A comprises of **10** questions of **one** mark each, section B comprises of **12** questions of **four** marks each and section C comprises of **07** questions of **six** marks each.
- (iii) All questions in Section A are to be answered in one word, one sentence or as per the exact requirement of the question.
- (iv) There is no overall choice. However, internal choice has been provided in **04** questions of **four** marks each and **02** questions of **six** marks each. You have to attempt only one of the alternatives in all such questions.
- (v) Use of calculator is not permitted. You may ask for logarithmic tables, if required.

SECTION - A

1. If R = $\{(x, y) : x + 2y = 8\}$ is a relation on N, write the range of R.

2. If $\tan^{-1}x + \tan^{-1}y = \frac{\pi}{4}$, $xy < 1$, then write the value of $x + y + xy$.

3. If A is a square matrix such that $A^2 = A$, then write the value of $7A - (I + A)^3$, where I is an identity matrix.

4. If $\begin{bmatrix} x-y & z \\ 2x-y & w \end{bmatrix} = \begin{bmatrix} -1 & 4 \\ 0 & 5 \end{bmatrix}$, find the value of $x + y$.

5. If $\begin{vmatrix} 3x & 7 \\ -2 & 4 \end{vmatrix} = \begin{vmatrix} 8 & 7 \\ 6 & 4 \end{vmatrix}$, find the value of x.

6. If $f(x) = \int_0^x t \sin t \, dt$, then write the value of $f'(x)$.

7. Evaluate: $\int_2^4 \frac{x}{x^2+1} dx$

8. Find the value of 'p' for which the vectors $3\hat{i} + 2\hat{j} + 9\hat{k}$ and $\hat{i} - 2p\hat{j} + 3\hat{k}$ are parallel.

9. Find $\vec{a}.(\vec{b} \times \vec{c})$, if $\vec{a} = 2\hat{i} + \hat{j} + 3\hat{k}$, $\vec{b} = -\hat{i} + 2\hat{j} + \hat{k}$ and $\vec{c} = 3\hat{i} + \hat{j} + 2\hat{k}$.

10. If the cartesian equations of a line are $\frac{3-x}{5} = \frac{y+4}{7} = \frac{2z-6}{4}$, write the vector equation for the line.

SECTION - B

11. If the function $f : R \to R$ be given by $f(x) = x^2 + 2$ and $g : R \to R$ be given by $g(x) = \frac{x}{x-1}$, $x \neq 1$, find fog and gof and hence find fog (2) and gof (–3).

12. Prove that

$$\tan^{-1}\left[\frac{\sqrt{1+x} - \sqrt{1-x}}{\sqrt{1+x} + \sqrt{1-x}}\right] = \frac{\pi}{4} - \frac{1}{2}\cos^{-1}x, \frac{-1}{\sqrt{2}} \leq x \leq 1$$

OR

If $\tan^{-1}\left(\frac{x-2}{x-4}\right) + \tan^{-1}\left(\frac{x+2}{x+4}\right) = \frac{\pi}{4}$, find the value of x.

13. Using properties of determinants, prove that

$$\begin{vmatrix} x+y & x & x \\ 5x+4y & 4x & 2x \\ 10x+8y & 8x & 3x \end{vmatrix} = x^3$$

14. Find the value of $\frac{dy}{dx}$ at $\theta = \frac{\pi}{4}$, if $x = ae^{\theta}(\sin\theta - \cos\theta)$ and $y = ae^{\theta}(\sin\theta + \cos\theta)$

15. If $y = Pe^{ax} + Qe^{bx}$, show that $\frac{d^2y}{dx^2} - (a+b)\frac{dy}{dx} + aby = 0$.

16. Find the value(s) of x for which $y = [x(x-2)]^2$ is an increasing function.

OR

Find the equations of the tangent and normal to the curve $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ at the point $(\sqrt{2}\,a, b)$.

17. Evaluate: $\int_0^{\pi} \frac{4x \sin x}{1+\cos^2 x} dx$

OR

Evaluate: $\int \frac{x+2}{\sqrt{x^2+5x+6}} dx$

18. Find the particular solution of the differential equation $\frac{dy}{dx} = 1 + x + y + xy$, given that $y = 0$ when $x = 1$.

19. Solve the differential equation

$$(1+x^2)\frac{dy}{dx} + y = e^{\tan^{-1} x}$$

20. Show that the four points A, B, C and D with position vectors $4\hat{i}+5\hat{j}+\hat{k}$, $-\hat{j}-\hat{k}$, $3\hat{i}+9\hat{j}+4\hat{k}$ and $4(-\hat{i}+\hat{j}+\hat{k})$ respectively are coplanar.

OR

The scalar product of the vector $\vec{a} = \hat{i}+\hat{j}+\hat{k}$ with a unit vector along the sum of vectors $\vec{b} = 2\hat{i}+4\hat{j}-5\hat{k}$ and $\vec{c} = \lambda\hat{i}+2\hat{j}+3\hat{k}$ is equal to one. Find the value of λ and hence find the unit vector along $\vec{b}+\vec{c}$.

21. A line passes through (2, –1, 3) and is perpendicular to the lines

$\vec{r} = (\hat{i}+\hat{j}-\hat{k}) + \lambda(2\hat{i}-2\hat{j}+\hat{k})$ and

$\vec{r} = (2\hat{i}-\hat{j}-3\hat{k}) + \mu(\hat{i}+2\hat{j}+2\hat{k})$.

Obtain its equation in vector and cartesian form.

22. An experiment succeeds thrice as often as it fails. Find the probability that in the next five trials, there will be at least 3 successes.

SECTION - C

23. Two schools A and B want to award their selected students on the values of sincerity, truthfulness and helpfulness. The school A wants to award ₹ x each, ₹ y each and ₹ z each for the three respective values to 3, 2 and 1 students respectively with a total award money of ₹ 1,600. School B wants to spend ₹ 2,300 to award its 4, 1 and 3 students on the respective values (by giving the same award money to the three values as before). If the total amount of award for one prize on each value is ₹ 900, using matrices, find the award money for each value. Apart from these three values, suggest one more value which should be considered for award.

24. Show that the altitude of the right circular cone of maximum volume that can be inscribed in a sphere of radius r is $\frac{4r}{3}$.

Also show that the maximum volume of the cone is $\frac{8}{27}$ of the volume of the sphere.

25. Evaluate: $\int \frac{1}{\cos^4 x + \sin^4 x} dx$

26. Using integration, find the area of the region bounded by the triangle whose vertices are (–1, 2), (1, 5) and (3, 4).

27. Find the equation of the plane through the line of intersection of the planes $x + y + z = 1$ and $2x + 3y + 4z = 5$ which is perpendicular to the plane $x - y + z = 0$. Also find the distance of the plane obtained above, from the origin.

OR

Find the distance of the point (2, 12, 5) from the point of intersection of the line $\vec{r} = 2\hat{i}-4\hat{j}+2\hat{k}+\lambda(3\hat{i}+4\hat{j}+2\hat{k})$ and the plane $\vec{r}.(\hat{i}-2\hat{j}+\hat{k}) = 0$

28. A manufacturing company makes two types of teaching aids A and B of Mathematics for class XII. Each type of A requires 9 labour hours for fabricating and 1 labour hour for finishing. Each type of B requires 12 labour hours for fabricating and 3 labour hours for finishing. For fabricating and finishing, the maximum labour hours available per week are 180 and 30 respectively. The company makes a profit of ₹ 80 on each piece of type A and ₹ 120 on each piece of type B. How many pieces of type A and type B should be manufactured per week to get a maximum profit? Make it as an LPP and solve graphically. What is the maximum profit per week?

29. There are three coins. One is a two-headed coin (having head on both faces), another is a biased coin that comes up heads 75% of the times and third is also a biased coin that comes up tails 40% of the times. One of the three coins is chosen at random and tossed, and it shows heads. What is the probability that it was the two-headed coin?

OR

Two numbers are selected at random (without replacement) from the first six positive integers. Let X denote the larger of the two numbers obtained. Find the probability distribution of the random variable X, and hence find the mean of the distribution.

All India 2013

CBSE Board Solved Paper

Time Allowed : 3 Hours ***Maximum Marks : 100***

General Instructions:

(i) **All** questions are compulsory.

(ii) The question paper contains of **29** questions divided into three sections A, B and C. Section A comprises of **10** questions of **one** mark each, section B comprises of **12** questions of **four** marks each and section C comprises of **07** questions of **six** marks each.

(iii) All questions in Section A are to be answered in one word, one sentence or as per the exact requirement of the question.

(iv) There is no overall choice. However, internal choice has been provided in **04** questions of **four** marks each and **02** questions of **six** marks each. You have to attempt only one of the alternatives in all such questions.

(v) Use of calculator is not permitted. You may ask for logarithmic tables, if required.

SECTION - A

1. Write the principal value of $\tan^{-1}\left(\sqrt{3}\right)-\cot^{-1}\left(-\sqrt{3}\right)$

2. Write the value of $\tan^{-1}\left[2\sin\left(2\cos^{-1}\frac{\sqrt{3}}{2}\right)\right]$

3. For what value of x, is the matrix $A=\begin{bmatrix}0 & 1 & -2\\ -1 & 0 & 3\\ x & -3 & 0\end{bmatrix}$ a skew-symmetric matrix?

4. If matrix $A=\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix}$ and $A^2 = kA$, then write the value of k.

5. Write the differential equation representing the family of curves $y = mx$, where m is an arbitrary constant.

6. If A_{ij} is the cofactor of the element a_{ij} of the determinant $\begin{vmatrix}2 & -3 & 5\\ 6 & 0 & 4\\ 1 & 5 & -7\end{vmatrix}$, then write the value of $a_{32}.A_{32}$.

7. P and Q are two points with position vectors $3\vec{a}-2\vec{b}$ and $\vec{a}+\vec{b}$ respectively. Write the position vector of a point R which divides the line segment PQ in the ratio 2 : 1 externally.

8. Find $\left|\vec{x}\right|$, if for a unit vector $\vec{a}$, $\left(\vec{x}-\vec{a}\right).\left(\vec{x}+\vec{a}\right)=15$

9. Find the length of the perpendicular drawn from the origin to the plane $2x-3y+6z+21=0$

10. The money to be spent for the welfare of the employees of a firm is proportional to the rate of change of its total revenue (marginal revenue). If the total revenue (in rupees) received from the sale of x units of a product is given by $R(x) = 3x^2 + 36x + 5$, find the marginal revenue, when x = 5, and write which value does the question indicate.

SECTION - B

11. Consider $f: R^+ \to [4,\infty)$ given by $f(x) = x^2 + 4$. Show that f is invertible with the inverse f^{-1} of f given by $f^{-1}(y) = \sqrt{y-4}$, where R^+ is the set of all non-negative real numbers.

12. Show that $\tan\left(\frac{1}{2}\sin^{-1}\frac{3}{4}\right)=\frac{4-\sqrt{7}}{3}$

OR

Solve the following equation: $\cos(\tan^{-1}x)=\sin\left(\cot^{-1}\frac{3}{4}\right)$

13. Using properties of determinants, prove the following:

$$\begin{vmatrix}x & x+y & x+2y\\ x+2y & x & x+y\\ x+y & x+2y & x\end{vmatrix}=9y^2(x+y)$$

14. If $y^x = e^{y-x}$, prove that $\frac{dy}{dx}=\frac{(1+\log y)^2}{\log y}$

15. Differentiate the following with respect to x:

$$\sin^{-1}\left(\frac{2^{x+1}.3^x}{1+(36)^x}\right)$$

16. Find the value of k, for which

$$f(x)=\begin{cases}\dfrac{\sqrt{1+kx}-\sqrt{1-kx}}{x}, & \text{if } -1\le x<0\\[2mm] \dfrac{2x+1}{x-1}, & \text{if } 0\le x<1\end{cases}$$

is continuous at x = 0

OR

If x = a $\cos^3\theta$ and y = a $\sin^3\theta$, then find the value of $\dfrac{d^2y}{dx^2}$ at $\theta=\dfrac{\pi}{6}$.

17. Evaluate : $\displaystyle\int\frac{\cos 2x-\cos 2\alpha}{\cos x-\cos\alpha}dx$

OR

Evaluate: $\displaystyle\int\frac{x+2}{\sqrt{x^2+2x+3}}dx$

18. Evaluate : $\displaystyle\int\frac{dx}{x(x^5+3)}$

19. Evaluate: $\displaystyle\int_0^{2\pi}\frac{1}{1+e^{\sin x}}dx$

20. If $\vec{a}=\hat{i}-\hat{j}+7\hat{k}$ and $\vec{b}=5\hat{i}-\hat{j}+\lambda\hat{k}$, then find the value of λ, so that $\vec{a}+\vec{b}$ and $\vec{a}-\vec{b}$ are perpendicular vectors.

21. Show that the lines $\vec{r}=3\hat{i}+2j-4\hat{k}+\lambda\left(\hat{i}+2\hat{j}+2\hat{k}\right)$; $\vec{r}=5\hat{i}-2j+\mu\left(3\hat{i}+2\hat{j}+6\hat{k}\right)$ are intersecting. Hence, find their point of intersection

OR

Find the vector equation of the plane through the points (2, 1, – 1) and (– 1, 3, 4) and perpendicular to the plane $x-2y+4z=10$.

22. The probabilities of two students A and B coming to the school in time are $\frac{3}{7}$ and $\frac{5}{7}$ respectively. Assuming that the events, 'A coming in time' and 'B coming in time' are independent, find the probability of only one of them coming to the school in time. Write at least one advantage of coming to school in time.

SECTION - C

23. Find the area of greatest rectangle that can be inscribed in an ellipse $\dfrac{x^2}{a^2}+\dfrac{y^2}{b^2}=1$.

OR

Find the equation of tangent to the curve $3x^2-y^2=8$, which pass through the point $\left(\dfrac{4}{3},0\right)$.

24. Find the area of the region bounded by the parabola $y=x^2$ and $y=|x|$.

25. Find the particular solution of the differential equation $(\tan^{-1}y-x)\,dy=(1+y^2)dx$, given that when x = 0, y = 0.

26. Find the equation of the plane passing through the line of intersection of the planes $\vec{r}.\left(\hat{i}+3\hat{j}\right)-6=0$ and $\vec{r}.\left(3\hat{i}-\hat{j}-4\hat{k}\right)=0$, whose perpendicular distance from origin is unity.

OR

Find the vector equation of the line passing through the point (1, 2, 3) and parallel to the plane $\vec{r}.\left(\hat{i}-\hat{j}+2\hat{k}\right)=5$ and $\vec{r}.\left(3\hat{i}+\hat{j}+\hat{k}\right)=6$

27. In a hockey match, both teams A and B scored same number of goals up to the end of the game, so to decide the winner, the referee asked both the captains to throw a die alternatively and decided that the team, whose captain gets a six first, will be declared the winner. If the captain of team A was asked to start, find their respective probabilities of winning the match and state whether the decision of the referee was fair or not.

28. A manufacturer considers that men and women workers are equally efficient and so he pays them at the same rate. He has 30 and 17 units of workers (male and female) and capital respectively, which he uses to produce two types of goods A and B. To produce one unit of A, 2 workers and 3 units of capital are required while 3 workers and 1 unit of capital is required to produce one unit of B. If A and B are prices at ₹ 100 and ₹ 120 per unit respectively, how should he use his resources to maximise the total revenue? Form the above as a LPP and solve graphically.

Do you agree with this view of the manufacturer that men and women workers are equally efficient and so should be paid at the same rate?

29. The management committee of a residential colony decided to award some of its members (say x) for honesty, some (say y) for helping others and some others (say z) for supervising the workers to keep the colony neat and clean. The sum of all the awardees is 12. Three times the sum of awardees for cooperation and supervision added to two times the number of awardees for honesty is 33. If the sum of the number of awardees for honesty and supervision is twice the number of awardees for helping others, using matrix method, find the number of awardees of each category. Apart from these values, namely, honesty, cooperation and supervision, suggest one more value which the management of the colony must include for awards.

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CBSE Board Solved Paper Term-II

Time Allowed : 2 Hours ***Maximum Marks : 40***

General Instructions:

(i) This question paper contains **three** Sections – Section **A**, **B** and **C**.

(ii) Each Section is compulsory.

(iii) Section-**A** has **6** short answer type-I questions of **2** marks each.

(iv) Section-**B** has **4** short answer type-II questions of **3** marks each.

(v) Section-**C** has **3** long answer type questions of **4** marks each.

(vi) There is an internal choice in some questions.

(vii) Question **14** is a Case Study based problem with **2** sub-parts of **2** marks each.

SECTION - A

Question Nos. 1 to 6 carry 2 marks each.

1. Find : $\int \frac{dx}{x^2 - 6x + 13}$

2. Find the general solution of the differential equation : $e^{dy/dx} = x^2$.

3. Write the projection of the vector $(\vec{b} + \vec{c})$ on the vector $\vec{a}$, where $\vec{a} = 2\hat{i} - 2\hat{j} + \hat{k}$, $\vec{b} = \hat{i} + 2\hat{j} - 2\hat{k}$ and $\vec{c} = 2\hat{i} - \hat{j} + 4\hat{k}$.

4. If the distance of the point (1, 1, 1) from the plane $x - y + z + \lambda = 0$ is $\frac{5}{\sqrt{3}}$, find the value (s) of λ.

5. Two cards are drawn successively with replacement from a well shuffled pack of 52 cards. Find the probability distribution of the number of spade cards.

6. A pair of dice is thrown and the sum of the numbers appearing on the dice is observed to be 7. Find the probability that the number 5 has appeared on atleast one die.

OR

The probability that A hits the target is $\frac{1}{3}$ and the probability that B hits it, is $\frac{2}{5}$. If both try to hit the target independently, find the probability that the target is hit.

SECTION - B

Question Nos. 7 to 10 carry 3 marks each.

7. Evaluate : $\int_0^{2\pi} \frac{dx}{1 + e^{\sin x}}$

8. Find the particular solution of the differential equation $x\frac{dy}{dx} - y = x^2 \cdot e^x$, given $y(1) = 0$.

OR

Find the general solution of the differential equation $x\frac{dy}{dx} = y(\log y - \log x + 1)$.

9. The two adjacent sides of a parallelogram are represented by vectors $2\hat{i} - 4\hat{j} + 5\hat{k}$ and $\hat{i} - 2\hat{j} - 3\hat{k}$. Find the unit vector parallel to one of its diagonals. Also, find the area of the parallelogram.

OR

If $\vec{a} = 2\hat{i} + 2\hat{j} + 3\hat{k}$, $\vec{b} = -\hat{i} + 2\hat{j} + \hat{k}$ and $\vec{c} = 3\hat{i} + \hat{j}$ are such that the vector $(\vec{a} + \lambda\vec{b})$ is perpendicular to vector $\vec{c}$, then find the value of λ.

10. Show that the lines :

$\frac{1-x}{2} = \frac{y-3}{4} = \frac{z}{-1}$ and $\frac{x-4}{3} = \frac{2y-2}{-4} = z - 1$ are coplanar.

SECTION - C

Question Nos. 11 to 14 carry 4 marks each.

11. Find the area of the region bounded by curve $4x^2 = y$ and the line $y = 8x + 12$, using integration.

12. Find : $\int \frac{x^2}{(x^2+1)(3x^2+4)} dx$

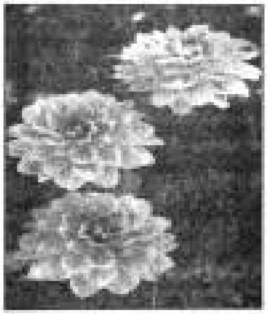
OR

Evaluate : $\int_{-2}^{1} \sqrt{5-4x-x^2}\, dx$

13. Find the distance of the point (1, –2, 9) from the point of intersection of the line $\vec{r} = 4\hat{i} + 2\hat{j} + 7\hat{k} + \lambda(3\hat{i} + 4\hat{j} + 2\hat{k})$ and the plane $\vec{r} \cdot (\hat{i} - \hat{j} + \hat{k}) = 10$.

Case Study Problem:

14. A shopkeeper sells three types of flower seeds A1, A2, A3. They are sold in the form of a mixture, where the proportions of these seeds are 4 : 4 : 2, respectively. The germination rates of the three types of seeds are 45% , 60% and 35% respectively.



Based on the above information:

(a) Calculate the probability that a randomly chosen seed will germinate;

(b) Calculate the probability that the seed is of type A2, given that a randomly chosed seed germinates.

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CBSE Board Solved Paper Term-I

Time Allowed : 90 Minutes **Maximum Marks : 40**

General Instructions:

(i) This question paper comprises **50** questions out of which **40** questions are to be attempted as per instructions. All questions carry equal marks.

(ii) The question paper consists of **three** Sections – Section **A**, **B** and **C**.

(iii) Section **A** contains **20** questions Attempt any **16** questions from Q. No. **1** to **20.**

(iv) Section **B** also contains **20** questions. Attempt any **16** questions from Q. No. **21** to **40**.

(v) Section **C** contains **10** questions including one Case Study. Attempt any **8** from Q. No. **41** to **50.**

(vi) There is only one correct option for every Multiple Choice Question (MCQ). Marks will not be awarded for answering more than one option.

(vii) There is no negative marking.

SECTION - A

In this section, attempt any 16 questions out of questions 1–20. Each question is of one mark.

1. Differential of log [log(log x^5)] w.r.t. x is

(a) $\dfrac{5}{x\log\left(x^5\right)\log\left(\log x^5\right)}$ (b) $\dfrac{5}{x\log\left(\log x^5\right)}$

(c) $\dfrac{5x^4}{\log\left(x^5\right)\log\left(\log x^5\right)}$ (d) $\dfrac{5x^4}{\log x^5\log\left(\log x^5\right)}$

2. The number of all possible matrices of order 2 × 3 with each entry 1 or 2 is

(a) 16 (b) 6 (c) 64 (d) 24

3. A function f : R → R is defined as $f(x) = x^3 + 1$. Then the function has

(a) on minimum value

(b) no maximum value

(c) both maximum and minimum values

(d) neither maximum value nor minimum value

4. If $\sin y = x \cos (a + y)$, then $\dfrac{dx}{dy}$ is

(a) $\dfrac{\cos a}{\cos^2(a+y)}$ (b) $\dfrac{-\cos a}{\cos^2(a+y)}$

(c) $\dfrac{\cos a}{\sin^2 y}$ (d) $\dfrac{-\cos a}{\sin^2 y}$

5. The points on the curve $\dfrac{x^2}{9}+\dfrac{y^2}{25}=1$, where tangent is parallel to x-axis are

(a) $(\pm 5, 0)$ (b) $(0, \pm 5)$ (c) $(0, \pm 3)$ (d) $(\pm 3, 0)$

6. Three points $P(2x, x + 3)$, $Q(0, x)$ and $R(x + 3, x + 6)$ are collinear, then x is equal to

(a) 0 (b) 2 (c) 3 (d) 1

7. The principal value of $\cos^{-1}\left(\dfrac{1}{2}\right)+\sin^{-1}\left(-\dfrac{1}{\sqrt{2}}\right)$ is

(a) $\dfrac{\pi}{12}$ (b) π (c) $\dfrac{\pi}{3}$ (d) $\dfrac{\pi}{6}$

8. If $(x^2 + y^2)^2 = xy$, then $\dfrac{dy}{dx}$ is

(a) $\dfrac{y+4x\left(x^2+y^2\right)}{4y\left(x^2+y^2\right)-x}$ (b) $\dfrac{y-4x\left(x^2+y^2\right)}{x+4\left(x^2+y^2\right)}$

(c) $\dfrac{y-4x\left(x^2+y^2\right)}{4y\left(x^2+y^2\right)-x}$ (d) $\dfrac{4y\left(x^2+y^2\right)-x}{y-4x\left(x^2+y^2\right)}$

9. If a matrix A is both symmetric and skew symmetric, then A is necessarily a

(a) Diagonal matrix (b) Zero square matrix

(c) Square matrix (d) Identity matrix

10. Let set X = {1, 2, 3} and a relation R is defined in X as : R = {(1, 3), (2, 2), (3, 2)}, then minimum ordered pairs which should be added in relation R to make it reflexive and symmetric are
(a) {(1,1), (2, 3), (1, 2)}
(b) {(3, 3), (3, 1), (1, 2)}
(c) {(1, 1), (3, 3), (3, 1), (2, 3)}
(d) {(1, 1), (3, 3), (3, 1), (1, 2)}

11. A linear programming problem is as follows:
Minimise Z = 2x + y
Subject to the constraints $x \geq 3, x \leq 9, y \geq 0$
$x - y \geq 0, x + y \leq 14$
The feasible region has
(a) 5 corner points including (0, 0) and (9, 5)
(b) 5 corner points including (7, 7) and (3, 3)
(c) 5 corner points including (14, 0) and (9, 0)
(d) 5 corner points including (3, 6) and (9, 5)

12. The function $f(x) = \begin{cases} \dfrac{e^{3x} - e^{-5x}}{x}, & \text{if } x \neq 0 \\ k, & \text{if } x = 0 \end{cases}$

is continuous at $x = 0$ for the value of k, is
(a) 3 (b) 5 (c) 2 (d) 8

13. If C_{ij} denotes the cofactor of element p_{ij} of the matrix
$P = \begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & 2 & 4 \end{bmatrix}$, then the value of $C_{31}.C_{23}$ is
(a) 5 (b) 24 (c) – 24 (d) – 5

14. The function $y = x^2e^{-x}$ is decreasing in the interval
(a) $(0, 2)$ (b) $(2, \infty)$
(c) $(-\infty, 0)$ (d) $(-\infty, 0) \cup (2, \infty)$

15. If $R = \{(x, y): x, y \in Z, x^2 + y^2 \leq 4\}$ is a relation in set Z, then domain of R is
(a) {0, 1, 2} (b) {–2, –1, 0, 1, 2}
(c) {0, –1, –2} (d) {–1, 0, 1}

16. The system of linear equations
$5x + ky = 5, 3x + 3y = 5$
will be consistant
(a) $k \neq -3$ (b) $k = -5$
(c) $k = 5$ (d) $k \neq 5$

17. The equation of the tangent to the curve $y(1 + x^2) = 2 - x$, where it crosses the x-axis is
(a) $x - 5y = 2$ (b) $5x - y = 2$
(c) $x + 5y = 2$ (d) $5x + y = 2$

18. If $\begin{bmatrix} 3c+6 & a-d \\ a+d & 2-3b \end{bmatrix} = \begin{bmatrix} 12 & 2 \\ -8 & -4 \end{bmatrix}$ are equal, then value of $ab - cd$ is
(a) 4 (b) 16
(c) – 4 (d) – 16

19. The principal value of $\tan^{-1}\left(\tan\dfrac{9\pi}{8}\right)$ is
(a) $\dfrac{\pi}{8}$ (b) $\dfrac{3\pi}{8}$ (c) $-\dfrac{\pi}{8}$ (d) $-\dfrac{3\pi}{8}$

20. For two matrices $P = \begin{bmatrix} 3 & 4 \\ -1 & 2 \\ 0 & 1 \end{bmatrix}$ and $Q^T = \begin{bmatrix} -1 & 2 & 1 \\ 1 & 2 & 3 \end{bmatrix}$

$P - Q$ is
(a) $\begin{bmatrix} 2 & 3 \\ -3 & 0 \\ 0 & -3 \end{bmatrix}$ (b) $\begin{bmatrix} 4 & 3 \\ -3 & 0 \\ -1 & -2 \end{bmatrix}$
(c) $\begin{bmatrix} 4 & 3 \\ 0 & -3 \\ -1 & -2 \end{bmatrix}$ (d) $\begin{bmatrix} 2 & 3 \\ 0 & -3 \\ 0 & -3 \end{bmatrix}$

SECTION - B

In this section, attempt any 16 questions out of questions 21–40. Each question is of one mark.

21. The function $f(x) = 2x^3 - 15x^2 + 36x + 6$ is increasing in the interval
(a) $(-\infty, 2) \cup (3, \infty)$ (b) $(-\infty, 2)$
(c) $(-\infty, 2] \cup [3, \infty)$ (d) $[3, \infty)$

22. If $x = 2\cos\theta - \cos 2\theta$ and $y = 2\sin\theta - \sin 2\theta$, then $\dfrac{dy}{dx}$ is
(a) $\dfrac{\cos\theta + \cos 2\theta}{\sin\theta - \sin 2\theta}$ (b) $\dfrac{\cos\theta - \cos 2\theta}{\sin 2\theta - \sin\theta}$
(c) $\dfrac{\cos\theta - \cos 2\theta}{\sin\theta - \sin 2\theta}$ (d) $\dfrac{\cos 2\theta - \cos\theta}{\sin 2\theta + \sin\theta}$

23. What is the domain of the function $\cos^{-1}(2x - 3)$?
(a) [–1, 1] (b) (1, 2) (c) (–1, 1) (d) [1, 2]

24. A matrix $A = [a_{ij}]_{3 \times 3}$ is defined by

$$a_{ij} = \begin{cases} 2i + 3j & , i < j \\ 5 & , i = j \\ 3i - 2j & , i > j \end{cases}$$

The number of elements in A which are more than 5, is
(a) 3 (b) 4 (c) 5 (d) 6

25. If a function f defined by

$$f(x) = \begin{cases} \dfrac{k\cos x}{\pi - 2x} & , \text{if } x \neq \dfrac{\pi}{2} \\ 3 & , \text{if } x = \dfrac{\pi}{2} \end{cases}$$

is continuous at $x = \dfrac{\pi}{2}$, then the value of k, is
(a) 2 (b) 3 (c) 6 (d) –6

26. For the matrix $X = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}, (X^2 - X)$ is

(a) $2I$ (b) $3I$ (c) I (d) $5I$

27. Let $X = \{x^2 : x \in N\}$ and the function $f : N \to X$ is defined by $f(x) = x^2, x \in N$. Then this function is

(a) injective only (b) not bijective

(c) surjective only (d) bijective

28. The corner points of the feasible region for a linear programming problem are $P(0, 5)$, $Q(1, 5)$, $R(4, 2)$ and $S(12, 0)$. The minimum value of the objective function $Z = 2x + 5y$ is at the point

(a) P (b) Q

(c) R (d) S

29. The equation of the normal to the curve $ay^2 = x^3$ at the point (am^2, am^3) is

(a) $2y - 3mx + am^3 = 0$

(b) $2x + 3my - 3am^4 - am^2 = 0$

(c) $2x + 3my + 3am^4 - 2am^2 = 0$

(d) $2x + 3my - 3am^4 - 2am^2 = 0$

30. If A is a square matrix of order 3 and $|A| = -5$, then $|\text{adj } A|$ is

(a) 125 (b) –25 (c) 25 (d) ±25

31. The simplest form of $\tan^{-1}\left[\dfrac{\sqrt{1+x} - \sqrt{1-x}}{\sqrt{1+x} + \sqrt{1-x}}\right]$ is

(a) $\dfrac{\pi}{4} - \dfrac{x}{2}$ (b) $\dfrac{\pi}{4} + \dfrac{x}{2}$

(c) $\dfrac{\pi}{4} - \dfrac{1}{2}\cos^{-1} x$ (d) $\dfrac{\pi}{4} + \dfrac{1}{2}\cos^{-1} x$

32. If for the matrix $A = \begin{bmatrix} \alpha & -2 \\ -2 & \alpha \end{bmatrix}$, $|A^3| = 125$, then the value of α is

(a) ±3 (b) – 3 (c) ±1 (d) 1

33. If $y = \sin(m \sin^{-1} x)$, then which one of the following equations is true?

(a) $(1 - x^2)\dfrac{d^2y}{dx^2} + x\dfrac{dy}{dx} + m^2 y = 0$

(b) $(1 - x^2)\dfrac{d^2y}{dx^2} - x\dfrac{dy}{dx} + m^2 y = 0$

(c) $(1 + x^2)\dfrac{d^2y}{dx^2} - x\dfrac{dy}{dx} - m^2 y = 0$

(d) $(1 + x^2)\dfrac{d^2y}{dx^2} + x\dfrac{dy}{dx} - m^2 x = 0$

34. The principal value of $[\tan^{-1}\sqrt{3} - \cot^{-1}(-\sqrt{3})]$ is

(a) π (b) $-\dfrac{\pi}{2}$ (c) 0 (d) $2\sqrt{3}$

35. The maximum value of $\left(\dfrac{1}{x}\right)^x$ is

(a) $e^{1/e}$ (b) e (c) $\left(\dfrac{1}{e}\right)^{1/e}$ (d) e^e

36. Let matrix $X = [x_{ij}]$ is given by $X = \begin{bmatrix} 1 & -1 & 2 \\ 3 & 4 & -5 \\ 2 & -1 & 3 \end{bmatrix}$. Then the matrix $Y = [m_{ij}]$, where m_{ij} = Minor of x_{ij}, is

(a) $\begin{bmatrix} 7 & -5 & -3 \\ 19 & 1 & -11 \\ -11 & 1 & 7 \end{bmatrix}$ (b) $\begin{bmatrix} 7 & -19 & -11 \\ 5 & -1 & -1 \\ 3 & 11 & 7 \end{bmatrix}$

(c) $\begin{bmatrix} 7 & 19 & -11 \\ -3 & 11 & 7 \\ -5 & -1 & -1 \end{bmatrix}$ (d) $\begin{bmatrix} 7 & 19 & -11 \\ -1 & -1 & 1 \\ -3 & -11 & 7 \end{bmatrix}$

37. A function $f : R \to R$ defined by $f(x) = 2 + x^2$ is

(a) not one-one

(b) one-one

(c) not onto

(d) neither one-one nor onto

38. A linear programming problem is as follow:

maximise / minimise objective function $Z = 2x - y + 5$

Subject to the constraints

$3x + 4y \le 60$

$x + 3y \le 30$

$x \ge 0, y \ge 0$

If the corner points of the feasible region are A(0, 10), B(12, 6), C(20, 0) and O(0, 0), then which of the following is true?

(a) Maximum value of Z is 40

(b) Minimum value of Z is – 5

(c) Difference of maximum and minimum values of Z is 35

(d) At two corner points, value of Z are equal

39. If x = –4 is a root of $\begin{vmatrix} x & 2 & 3 \\ 1 & x & 1 \\ 3 & 2 & x \end{vmatrix} = 0$, then the sum of the other two roots is

(a) 4 (b) – 3

(c) 2 (d) 5

40. The absolute maximum value of the function $f(x) = 4x - \dfrac{1}{2}x^2$ in the interval $\left[-2, \dfrac{9}{2}\right]$ is

(a) 8 (b) 9

(c) 6 (d) 10

SECTION - C

Attempt any 8 questions out of the questions 41-50. Each question is of one mark.

41. In a sphere of radius r, a right circular cone of height h, having maximum curved surface area is inscribed. The expression for the square of curved surface of cone is

(a) $2\pi^2 rh\,(2rh + h^2)$ (b) $\pi^2 hr\,(2rh + h^2)$

(c) $2\pi^2 r\,(2rh^2 - h^3)$ (d) $2\pi^2 r^2\,(2rh - h^2)$

42. The corner points of the feasible region determined by a set of constraints (linear inequalities) are P(0, 5), Q(3, 5), R(5, 0) and S(4, 1) and the objective function is $Z = ax + 2by$ where $a, b > 0$. The condition on a and b such that the maximum Z occurs at Q and S is

(a) $a - 5b = 0$ (b) $a - 3b = 0$

(c) $a - 2b = 0$ (d) $a - 8b = 0$

43. If curves $y^2 = 4x$ and $xy = c$ cut at right angles, then the value of c is

(a) $4\sqrt{2}$ (b) 8

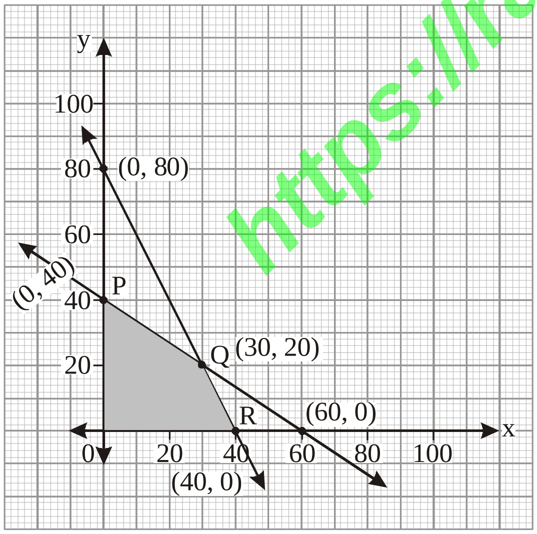
(c) $2\sqrt{2}$ (d) $-4\sqrt{2}$

44. The inverse of the matrix $X = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 4 \end{bmatrix}$ is

(a) $24\begin{bmatrix} 1/2 & 0 & 0 \\ 0 & 1/3 & 0 \\ 0 & 0 & 1/4 \end{bmatrix}$ (b) $\dfrac{1}{24}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

(c) $\dfrac{1}{24}\begin{bmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 4 \end{bmatrix}$ (d) $\begin{bmatrix} 1/2 & 0 & 0 \\ 0 & 1/3 & 0 \\ 0 & 0 & 1/4 \end{bmatrix}$

45. For an L.P.P. the objective function is $Z = 4x + 3y$ and the feasible region determined by a set of constraints (linear inequations) is shown in the graph.



Which one of the following statements is true?

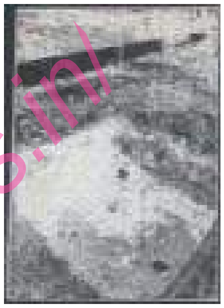
(a) Maximum value of Z is at R.

(b) Maximum value of Z is at Q.

(c) Value of Z at R is less than the value at P.

(d) Value of Z at Q is less than the value at R.

Case Study



In a residential society comprising of 100 houses, there were 60 children between the ages of 10-15 years. They were inspired by their teachers to start composting to ensure that biodegradable waste is recycled. For this purpose, instead of each child doing it for only his/her house, children convinced the Residents Welfare Association to do it as a society initiative. For this they identified a square area in the local park. Local authorities charged amount of ₹ 50 per square metre for space so that there is no misuse of the space and Resident Welfare Association takes it seriously. Association hired a labourer for digging out 250 m^3 and he charged ₹ 400 ×(depth)^2. Association will like to have minimum cost.

Based on this information, answer the any 4 of the following questions.

46. Let side of square plot is x m and its depth is h metres, then cost c for the pit is

(a) $\dfrac{50}{h} + 400\,h^2$ (b) $\dfrac{12500}{h} + 400\,h^2$

(c) $\dfrac{250}{h} + h^2$ (d) $\dfrac{250}{h} + 400\,h^2$

47. Value of h (in m) for which $\dfrac{dc}{dh} = 0$ is

(a) 1.5 (b) 2

(c) 2.5 (d) 3

48. $\dfrac{d^2c}{dh^2}$ is given by

(a) $\dfrac{25000}{h^3} + 800$ (b) $\dfrac{500}{h^3} + 800$

(c) $\dfrac{100}{h^3} + 800$ (d) $\dfrac{500}{h^3} + 2$

49. Value of x (in m) for minimum cost is

(a) 5 (b) $10\sqrt{\dfrac{5}{3}}$

(c) $5\sqrt{5}$ (d) 10

50. Total minimum cost of digging the pit (in ₹) is

(a) 4100 (b) 7500

(c) 7850 (d) 3220

All India 2021-22

CBSE Board Sample Paper Term-II

Time Allowed : 2 Hours *Maximum Marks : 40*

General Instructions:

(i) This question paper contains **three** sections – **A**, **B** and **C**. Each part is compulsory.

(ii) Section-**A** has **6** short answer type (SA1) questions of **2** marks each.

(iii) Section-**B** has **4** short answer type (SA2) questions of **3** marks each.

(iv) Section-**C** has **4** long answer type questions (LA) of **4** marks each.

(v) There is an internal choice in some of the questions.

(vi) Question **14** is a case-based problem having **2** sub parts of **2** marks each.

SECTION - A

Question Nos. 1 to 6 carry 2 marks each.

1. Find : $\int \frac{\log x}{(1+\log x)^2} dx$

OR

Find : $\int \frac{\sin 2x}{\sqrt{9-\cos^4 x}} dx$

2. Write the sum of the order and the degree of the following differential equation:

$$\frac{d}{dx}\left(\frac{dy}{dx}\right) = 5$$

3. If $\hat{a}$ and $\hat{b}$ are unit vectors, then prove that

$|\hat{a}+\hat{b}| = 2\cos\frac{\theta}{2}$, where θ is the angle between them.

4. Find the direction cosines of the following line:

$$\frac{3-x}{-1} = \frac{2y-1}{2} = \frac{z}{4}$$

5. A bag contains 1 red and 3 white balls. Find the probability distribution of the number of red balls if 2 balls are drawn at random from the bag one-by-one without replacement.

6. Two cards are drawn at random from a pack of 52 cards one-by-one without replacement. What is the probability of getting first card red and second card jack?

SECTION - B

Question Nos. 7 to 10 carry 3 marks each.

7. Find : $\int \frac{x+1}{(x^2+1)x} dx$

8. Find the general solution of the following differential equation:

$$x\frac{dy}{dx} = y - x\sin\left(\frac{y}{x}\right)$$

OR

Find the particular solution of the following differential equation, given that $y = 0$ when $x = \frac{\pi}{4}$:

$$\frac{dy}{dx} + y\cot x = \frac{2}{1+\sin x}$$

9. If $\vec{a} \neq \vec{0}, \vec{a}.\vec{b} = \vec{a}.\vec{c}, \vec{a}\times\vec{b} = \vec{a}\times\vec{c}$, then show that $\vec{b} = \vec{c}$.

10. Find the shortest distance between the following lines:

$$\vec{r} = (\hat{i} + \hat{j} - \hat{k}) + s(2\hat{i} + \hat{j} + \hat{k})$$

$$\vec{r} = (\hat{i} + \hat{j} + 2\hat{k}) + t(4\hat{i} + 2\hat{j} + 2\hat{k})$$

OR

Find the vector and the cartesian equations of the plane containing the point $\hat{i} + 2\hat{j} - \hat{k}$ and parallel to the lines

$\vec{r} = (\hat{i} + 2\hat{j} + 2\hat{k}) + s(2\hat{i} - 3\hat{j} + 2\hat{k})$ and

$\vec{r} = (3\hat{i} + \hat{j} - 2\hat{k}) + t(\hat{i} - 3\hat{j} + \hat{k})$

SECTION - C

Question Nos. 11 to 14 carry 4 marks each.

11. Evaluate : $\int_{-1}^{2} |x^3 - 3x^2 + 2x|\, dx$

12. Using integration, find the area of the region in the first quadrant enclosed by the line $x + y = 2$, the parabola $y^2 = x$ and the x-axis.

OR

Using integration, find the area of the region

$\{(x, y) : 0 \le y \le \sqrt{3}x, x^2 + y^2 \le 4\}$

13. Find the foot of the perpendicular from the point (1, 2, 0) upon the plane $x - 3y + 2z = 9$. Hence, find the distance of the point (1, 2, 0) from the given plane.

Case-Based/Data-Based:

14.

An insurance company believes that people can be divided into two classes: those who are accident prone and those who are not. The company's statistics show that an accident-prone person will have an accident at sometime within a fixed one-year period with probability 0.6, whereas this probability is 0.2 for a person who is not accident prone. The company knows that 20 percent of the population is accident prone.

Based on the above information, answer the following questions

(a) What is the probability that a new policyholder will have an accident within a year of purchasing a policy?

(b) Suppose that a new policyholder has an accident within a year of purchasing a policy. What is the probability that he or she is accident prone?

All India 2021-22

CBSE Board Sample Paper Term-I

Time Allowed : 90 Minutes **_Maximum Marks : 40_**

General Instructions:

(i) This question paper contains three sections – **A**, **B** and **C**. Each part is compulsory.

(ii) Section-**A** has **20** MCQs, attempt any **16** out of **20**.

(iii) Section-**B** has **20** MCQs, attempt any **16** out of **20**.

(iv) Section-**C** has **10** MCQs, attempt any **8** out of **10**.

(v) All questions carry equal marks.

(vi) There is no negative marking.

SECTION - A

*In this section, attempt **any 16** questions out of questions 1-20. Each question is of 1 mark weightage.*

1. $\sin\left[\frac{\pi}{3}-\sin^{-1}\left(-\frac{1}{2}\right)\right]$ is equal to

(a) $\frac{1}{2}$ (b) $\frac{1}{3}$ (c) –1 (d) 1

2. The value of k (k < 0) for which the function *f* defined as

$$f(x)=\begin{cases}\frac{1-\cos kx}{x\sin x}, & x\neq 0\\ \frac{1}{2}, & x=0\end{cases}$$ is continuous at $x = 0$ is:

(a) ±1 (b) –1 (c) $\pm\frac{1}{2}$ (d) $\frac{1}{2}$

3. If A = [a_{ij}] is a square matrix of order 2 such that $a_{ij} = \begin{cases}1, & \text{when } i\neq j\\ 0, & \text{when } i=j\end{cases}$, then A^2 is:

(a) $\begin{bmatrix}1 & 0\\ 1 & 0\end{bmatrix}$ (b) $\begin{bmatrix}1 & 1\\ 0 & 0\end{bmatrix}$ (c) $\begin{bmatrix}1 & 1\\ 1 & 0\end{bmatrix}$ (d) $\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$

4. Value of *k*, for which $A=\begin{bmatrix}k & 8\\ 4 & 2k\end{bmatrix}$ is a singular matrix is:

(a) 4 (b) –4 (c) ±4 (d) 0

5. Find the intervals in which the function f given by $f(x) = x^2 - 4x + 6$ is strictly increasing:

(a) $(-\infty, 2)\cup(2,\infty)$ (b) $(2,\infty)$

(c) $(-\infty, 2)$ (d) $(-\infty, 2]\cup(2,\infty)$

6. Given that A is a square matrix of order 3 and | A | = – 4, then | adj A | is equal to:

(a) –4 (b) 4 (c) –16 (d) 16

7. A relation R in set A = {1,2,3} is defined as R = {(1, 1), (1, 2), (2, 2), (3, 3)}. Which of the following ordered pair in R shall be removed to make it an equivalence relation in A?

(a) (1, 1) (b) (1, 2) (c) (2, 2) (d) (3, 3)

8. If $\begin{bmatrix}2a+b & a-2b\\ 5c-d & 4c+3d\end{bmatrix}=\begin{bmatrix}4 & -3\\ 11 & 24\end{bmatrix}$, then value of a + b – c + 2d is:

(a) 8 (b) 10 (c) 4 (d) –8

9. The point at which the normal to the curve $y = x + \frac{1}{x}, x > 0$ is perpendicular to the line $3x - 4y - 7 = 0$ is:

(a) (2, 5/2) (b) (±2, 5/2)

(c) (–1/2, 5/2) (d) (1/2, 5/2)

10. sin ($\tan^{-1}x$), where |x| < 1, is equal to:

(a) $\frac{x}{\sqrt{1-x^2}}$ (b) $\frac{1}{\sqrt{1-x^2}}$

(c) $\frac{1}{\sqrt{1+x^2}}$ (d) $\frac{x}{\sqrt{1+x^2}}$

11. Let the relation R in the set A = {x ∈ Z : 0 ≤ x ≤ 12}, given by R = {(a, b) : |a – b| is a multiple of 4}. Then [1], the equivalence class containing 1, is:

(a) {1, 5, 9} (b) {0, 1, 2, 5}

(c) ϕ (d) A

12. If $e^x + e^y = e^{x+y}$, then $\frac{dy}{dx}$ is:

(a) e^{y-x} (b) e^{x+y} (c) $-e^{y-x}$ (d) $2e^{x-y}$

13. Given that matrices A and B are of order $3 \times n$ and $m \times 5$ respectively, then the order of matrix C = 5A +3B is:

(a) 3×5 and m = n (b) 3×5

(c) 3×3 (d) 5×5

14. If y = 5 cos x – 3 sin x, then $\frac{d^2y}{dx^2}$ is equal to:

(a) –y (b) y (c) 25 y (d) 9 y

15. For matrix $A = \begin{bmatrix} 2 & 5 \\ -11 & 7 \end{bmatrix}$, (*adj A*)' is equal to :

(a) $\begin{bmatrix} -2 & -5 \\ 11 & -7 \end{bmatrix}$ (b) $\begin{bmatrix} 7 & 5 \\ 11 & 2 \end{bmatrix}$

(c) $\begin{bmatrix} 7 & 11 \\ -5 & 2 \end{bmatrix}$ (d) $\begin{bmatrix} 7 & -5 \\ 11 & 2 \end{bmatrix}$

16. The points on the curve $\frac{x^2}{9}+\frac{y^2}{16}=1$ at which the tangents are parallel to y-axis are:

(a) (0, ±4) (b) (±4, 0) (c) (±3, 0) (d) (0, ±3)

17. Given that $A = [a_{ij}]$ is a square matrix of order 3 × 3 and $|A| = -7$, then the value of $\sum_{i=1}^{3} a_{i2}A_{i2}$, where A_{ij} denotes the cofactor of element a_{ij} is:

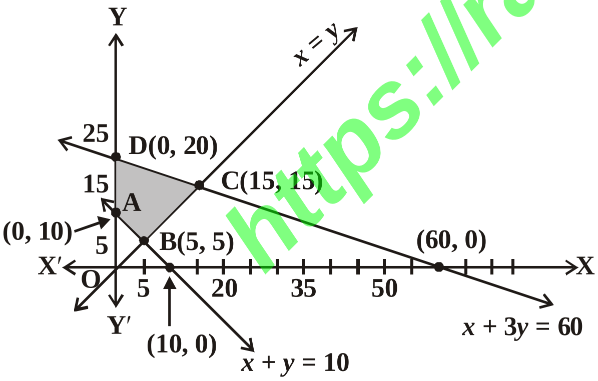
(a) 7 (b) –7 (c) 0 (d) 49

18. If $y = \log(\cos e^x)$, then $\frac{dy}{dx}$ is :

(a) $\cos e^{x-1}$ (b) $e^{-x}\cos e^x$

(c) $e^x \sin e^x$ (d) $-e^x \tan e^x$

19. Based on the given shaded region as the feasible region in the graph, at which point(s) is the objective function Z = 3x + 9y maximum?



(a) Point B

(b) Point C

(c) Point D

(d) every point on the line segment CD

20. The least value of the function $f(x) = 2\cos x + x$ in the closed interval $\left[0, \frac{\pi}{2}\right]$ is :

(a) 2 (b) $\frac{\pi}{6}+\sqrt{3}$

(c) $\frac{\pi}{2}$ (d) The least value does not exist.

SECTION - B

*In this section, attempt **any 16** questions out of the questions 21-40. Each question is of 1 mark weightage.*

21. The function $f: R \to R$ defined as $f(x) = x^3$ is:

(a) One-one but not onto

(b) Not one-one but onto

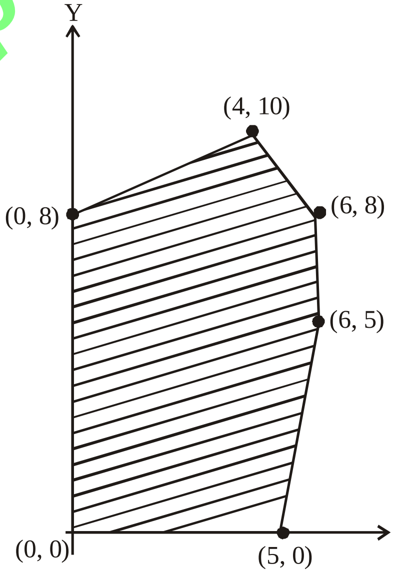
(c) Neither one-one nor onto

(d) One-one and onto

22. If x = a sec θ, y = b tan θ, then $\frac{d^2y}{dx^2}$ at $\theta = \frac{\pi}{6}$ is :

(a) $\frac{-3\sqrt{3}b}{a^2}$ (b) $\frac{-2\sqrt{3}b}{a}$ (c) $\frac{-3\sqrt{3}b}{a}$ (d) $\frac{-b}{3\sqrt{3}a^2}$

23. In the given graph, the feasible region for a LPP is shaded. The objective function Z = 2x – 3y, will be minimum at:



(a) (4, 10) (b) (6, 8)

(c) (0, 8) (d) (6, 5)

24. The derivative of $\sin^{-1}(2x\sqrt{1-x^2})$ w.r.t $\sin^{-1}x$, $\frac{1}{\sqrt{2}} < x < 1$, is

(a) 2 (b) $\frac{\pi}{2}-2$ (c) $\frac{\pi}{2}$ (d) –2

25. If $A = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}$, then :

(a) $A^{-1} = B$ (b) $A^{-1} = 6B$

(c) $B^{-1} = B$ (d) $B^{-1} = \frac{1}{6}A$

26. The real function $f(x) = 2x^3 - 3x^2 - 36x + 7$ is:

(a) Strictly increasing in $(-\infty, -2)$ and strictly decreasing in $(-2, \infty)$

(b) Strictly decreasing in $(-2, 3)$

(c) Strictly decreasing in $(-\infty, 3)$ and strictly increasing in $(3, \infty)$

(d) Strictly decreasing in $(-\infty, -2) \cup (3, \infty)$

27. Simplest form of $\tan^{-1}\left(\frac{\sqrt{1+\cos x} + \sqrt{1-\cos x}}{\sqrt{1+\cos x} - \sqrt{1-\cos x}}\right)$, $\pi < x < \frac{3\pi}{2}$ is

(a) $\frac{\pi}{4} - \frac{x}{2}$ (b) $\frac{3\pi}{2} - \frac{x}{2}$

(c) $-\frac{x}{2}$ (d) $\pi - \frac{x}{2}$

28. Given that A is a non-singular matrix of order 3 such that $A^2 = 2A$, then value of $|2A|$ is:

(a) 4 (b) 8 (c) 64 (d) 16

29. The value of b for which the function $f(x) = x + \cos x + b$ is strictly decreasing over R is:

(a) $b < 1$ (b) No value of b exists

(c) $b \le 1$ (d) $b \ge 1$

30. Let R be the relation in the set N given by $R = \{(a, b) : a = b - 2, b > 6\}$, then

(a) $(2, 4) \in R$ (b) $(3, 8) \in R$

(c) $(6, 8) \in R$ (d) $(8, 7) \in R$

31. The point(s), at which the function f given by

$$f(x) = \begin{cases} \frac{x}{|x|}, & x < 0 \\ -1, & x \ge 0 \end{cases}$$ is continuous, is/are :

(a) $x \in R$ (b) $x = 0$

(c) $x \in R - \{0\}$ (d) $x = -1$ and 1

32. If $A = \begin{bmatrix} 0 & 2 \\ 3 & -4 \end{bmatrix}$ and $kA = \begin{bmatrix} 0 & 3a \\ 2b & 24 \end{bmatrix}$, then the values of k, a and b respectively are:

(a) $-6, -12, -18$ (b) $-6, -4, -9$

(c) $-6, 4, 9$ (d) $-6, 12, 18$

33. A linear programming problem is as follows:

Minimize $Z = 30x + 50y$ subject to the constraints,

$3x + 5y \ge 15$
$2x + 3y \le 18$
$x \ge 0, y \ge 0$

In the feasible region, the minimum value of Z occurs at

(a) a unique point (b) no point

(c) infinitely many points (d) two points only

34. The area of a trapezium is defined by function f and given by $f(x) = (10 + x)\sqrt{100 - x^2}$, then the area when it is maximised is :

(a) 75 cm^2 (b) $7\sqrt{3}\text{ cm}^2$

(c) $75\sqrt{3}\text{ cm}^2$ (d) 5 cm^2

35. If A is square matrix such that $A^2 = A$, then $(I + A)^3 - 7A$ is equal to

(a) A (b) I + A (c) I – A (d) I

36. If $\tan^{-1} x = y$, then :

(a) $-1 < y < 1$ (b) $\frac{-\pi}{2} \le y \le \frac{\pi}{2}$

(c) $\frac{-\pi}{2} < y < \frac{\pi}{2}$ (d) $y \in \left\{\frac{-\pi}{2}, \frac{\pi}{2}\right\}$

37. Let $A = \{1, 2, 3\}$, $B = \{4, 5, 6, 7\}$ and let $f = \{(1, 4), (2, 5), (3, 6)\}$ be a function from A to B. Based on the given information, f is best defined as:

(a) Surjective function (b) Injective function

(c) Bijective function (d) Function

38. For $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$, then $14A^{-1}$ is given by :

(a) $14\begin{bmatrix} 2 & -1 \\ 1 & 3 \end{bmatrix}$ (b) $\begin{bmatrix} 4 & -2 \\ 2 & 6 \end{bmatrix}$

(c) $2\begin{bmatrix} 2 & -1 \\ 1 & -3 \end{bmatrix}$ (d) $2\begin{bmatrix} -3 & -1 \\ 1 & -2 \end{bmatrix}$

39. The point(s) on the curve $y = x^3 - 11x + 5$ at which the tangent is $y = x - 11$ is/are:

(a) $(-2, 19)$ (b) $(2, -9)$

(c) $(\pm 2, 19)$ (d) $(-2, 19)$ and $(2, -9)$

40. Given that $A = \begin{bmatrix} \alpha & \beta \\ \gamma & -\alpha \end{bmatrix}$ and $A^2 = 3I$, then

(a) $1 + \alpha^2 + \beta\gamma = 0$ (b) $1 - \alpha^2 - \beta\gamma = 0$

(c) $3 - \alpha^2 - \beta\gamma = 0$ (d) $3 + \alpha^2 + \beta\gamma = 0$

SECTION - C

In this section, attempt ***any 8*** *questions. Each question is of 1 mark weightage. Questions 46-50 are based on a case-study.*

41. For an objective function $Z = ax + by$, where $a, b > 0$; the corner points of the feasible region determined by a set of constraints (linear inequalities) are (0, 20), (10, 10), (30, 30) and (0, 40). The condition on a and b such that the maximum Z occurs at both the points (30, 30) and (0, 40) is:

(a) $b - 3a = 0$ (b) $a = 3b$

(c) $a + 2b = 0$ (d) $2a - b = 0$

42. For which value of m is the line y = mx + 1 a tangent to the curve $y^2 = 4x$?

(a) $\frac{1}{2}$ (b) 1 (c) 2 (d) 3

43. The maximum value of $[x(x-1)+1]^{1/3}$, $0 \le x \le 1$ is

(a) 0 (b) $\frac{1}{2}$ (c) 1 (d) $\sqrt[3]{\frac{1}{3}}$

44. In a linear programming problem, the constraints on the decision variables x and y are $x - 3y \ge 0$, $y \ge 0$, $0 \le x \le 3$. The feasible region

(a) is not in the first quadrant

(b) is bounded in the first quadrant

(c) is unbounded in the first quadrant

(d) does not exist

45. Let $A = \begin{bmatrix} 1 & \sin\alpha & 1 \\ -\sin\alpha & 1 & \sin\alpha \\ -1 & -\sin\alpha & 1 \end{bmatrix}$, where $0 \le \alpha \le 2\pi$, then

(a) $|A| = 0$ (b) $|A| \in (2, \infty)$

(c) $|A| \in (2, 4)$ (d) $|A| \in [2, 4]$

Case Study

The fuel cost per hour for running a train is proportional to the square of the speed it generates in km per hour. If the fuel costs ₹ 48 per hour at speed 16 km per hour and the fixed charges to run the train amount to ₹ 1200 per hour.

Assume the speed of the train as υ km/h.

Based on the given information, answer the following questions.

46. Given that the fuel cost per hour is k times the square of the speed the train generates in km/h, the value of k is:

(a) $\frac{16}{3}$ (b) $\frac{1}{3}$ (c) 3 (d) $\frac{3}{16}$

47. If the train has travelled a distance of 500 km, then the total cost of running the train is given by function:

(a) $\frac{15}{16}\upsilon + \frac{600000}{\upsilon}$ (b) $\frac{375}{4}\upsilon + \frac{600000}{\upsilon}$

(c) $\frac{5}{16}\upsilon^2 + \frac{150000}{\upsilon}$ (d) $\frac{3}{16}\upsilon + \frac{6000}{\upsilon}$

48. The most economical speed to run the train is:

(a) 18 km/h (b) 5 km/h (c) 80 km/h (d) 40 km/h

49. The fuel cost for the train to travel 500 km at the most economical speed is:

(a) ₹ 3750 (b) ₹ 750

(c) ₹ 7500 (d) ₹ 75000

50. The total cost of the train to travel 500 km at the most economical speed is:

(a) ₹ 3750 (b) ₹ 75000

(c) ₹ 7500 (d) ₹ 15000

All India 2020

CBSE Board Solved Paper

Time Allowed : 3 Hours *Maximum Marks : 80*

General Instructions:

Read the following instructions very carefully and strictly follow them:

(i) This question paper comprises **four** Sections A, B, C and D. This question paper carries **36** questions. **All** questions are compulsory.

(ii) **Section A:** Questions no. **1** to **20** comprises of **20** questions of **1** mark each.

(iii) **Section B:** Questions no. **21** to **26** comprises of **6** questions of **2** marks each.

(iv) **Section C:** Questions no. **27** to **32** comprises of **6** questions of **4** marks each.

(v) **Section D:** Questions no. **33** to **36** comprises of **4** questions of **6** marks each.

(vi) There is no overall choice in the question paper. However, an internal choice has been provided in **3** questions of **one** mark, **2** questions of **two** marks, **2** questions of **four** marks and **2** questions of **six** marks. Only one of the choices in such questions have to be attempted.

(vii) In addition to this, separate instructions are given with each section and question, wherever necessary.

(viii) Use of calculators is not permitted.

SECTION - A

Question numbers 1 to 20 carry 1 mark each.

Question numbers 1 to 10 are multiple choice type questions. Select the correct option.

1. If A is a square matrix of order 3 and |A| = 5, then the value of |2A′| is

(a) –10 (b) 10

(c) –40 (d) 40

2. If A is a square matrix such that $A^2 = A$, then $(I - A)^3 + A$ is equal to

(a) I (b) 0

(c) I – A (d) I + A

3. The principal value of $\tan^{-1}\left(\tan \frac{3\pi}{5}\right)$ is

(a) $\frac{2\pi}{5}$ (b) $\frac{-2\pi}{5}$

(c) $\frac{3\pi}{5}$ (d) $\frac{-3\pi}{5}$

4. If the projection of $\vec{a} = \hat{i} - 2\hat{j} + 3\hat{k}$ on $\vec{b} = 2\hat{i} + \lambda\hat{k}$ is zero, then the value of λ is

(a) 0 (b) 1

(c) $\frac{-2}{3}$ (d) $\frac{-3}{2}$

5. The vector equation of the line passing through the point (–1, 5, 4) and perpendicular to the plane $z = 0$ is

(a) $\vec{r} = -\hat{i} + 5\hat{j} + 4\hat{k} + \lambda(\hat{i} + \hat{j})$

(b) $\vec{r} = -\hat{i} + 5\hat{j} + (4 + \lambda)\hat{k}$

(c) $\vec{r} = \hat{i} - 5\hat{j} - 4\hat{k} + \lambda\hat{k}$

(d) $\vec{r} = \lambda\hat{k}$

6. The number of arbitrary constants in the particular solution of a differential equation of second order is (are)

(a) 0 (b) 1

(c) 2 (d) 3

7. $\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \sec^2 x\,dx$ is equal to

(a) -1 (b) 0

(c) 1 (d) 2

8. The length of the perpendicular drawn from the point (4, –7, 3) on the *y*-axis is

(a) 3 units (b) 4 units

(c) 5 units (d) 7 units

9. If A and B are two independent events with $P(A) = \frac{1}{3}$ and $P(B) = \frac{1}{4}$, then $P(B'|A)$ is equal to

(a) $\frac{1}{4}$ (b) $\frac{1}{3}$

(c) $\frac{3}{4}$ (d) 1

10. The corner points of the feasible region determined by the system of linear inequalities are (0, 0), (4, 0), (2, 4) and (0, 5). If the maximum value of $z = ax + by$, where $a, b > 0$ occurs at both (2, 4) and (4, 0), then

(a) $a = 2b$ (b) $2a = b$

(c) $a = b$ (d) $3a = b$

Fill in the blanks in question numbers 11 to 15.

11. A relation R in a set A is called ________, if $(a_1, a_2) \in R$ implies $(a_2, a_1) \in R$, for all $a_1, a_2 \in A$.

12. The greatest integer function defined by

$f(x) = [x]$, $0 < x < 2$ is not differentiable at $x =$ ________

13. If A is a matrix of order 3 × 2, then the order of the matrix A′ is _______.

OR

A square matrix A is said to be skew-symmetric, if ________.

14. The equation of the normal to the curve $y^2 = 8x$ at the origin is ________.

OR

The radius of a circle is increasing at the uniform rate of 3 cm/sec. At the instant when the radius of the circle is 2 cm, its area increases at the rate of _________ cm²/s.

15. The position vectors of two points A and B are $\overrightarrow{OA} = 2\hat{i} - \hat{j} - \hat{k}$ and $\overrightarrow{OB} = 2\hat{i} - \hat{j} + 2\hat{k}$, respectively. The position vector of a point P which divides the line segment joining A and B in the ratio 2 : 1 is _________.

Question numbers 16 to 20 are very short answer type questions.

16. If $A = \begin{bmatrix} 2 & 0 & 0 \\ -1 & 2 & 3 \\ 3 & 3 & 5 \end{bmatrix}$, then find A (adj A).

17. Find : $\int x^4 \log x\,dx$

OR

Find: $\int \frac{2x}{\sqrt[3]{x^2+1}}\,dx$

18. Evaluate: $\int_1^3 |2x-1|\,dx$

19. Two cards are drawn at random and one-by-one without replacement from a well-shuffled pack of 52 playing cards. Find the probability that one card is red and the other is black.

20. Find: $\int \frac{dx}{\sqrt{9-4x^2}}$

SECTION - B

Question numbers 21 to 26 carry 2 marks each.

21. Prove that $\sin^{-1}(2x\sqrt{1-x^2}) = 2\cos^{-1}x, \frac{1}{\sqrt{2}} \le x \le 1$.

OR

Consider a bijective function $f = R_+ \to (7, \infty)$ given by $f(x) = 16x^2 + 24x + 7$, where R_+ is the set of all positive real numbers. Find the inverse function of *f*.

22. If $x = at^2$, $y = 2at$, then find $\frac{d^2y}{dx^2}$.

23. Find the points on the curve $y = x^3 - 3x^2 - 4x$ at which the tangent lines are parallel to the line $4x + y - 3 = 0$.

24. Find a unit vector perpendicular to each of the vectors $\vec{a}$ and $\vec{b}$ where $\vec{a} = 5\hat{i} + 6\hat{j} - 2\hat{k}$ and $\vec{b} = 7\hat{i} + 6\hat{j} + 2\hat{k}$.

OR

Find the volume of the parallelopiped whose adjacent edges are represented by $2\vec{a}$, $-\vec{b}$ and $3\vec{c}$, where

$\vec{a} = \hat{i} - \hat{j} + 2\hat{k}$, $\vec{b} = 3\hat{i} + 4\hat{j} - 5\hat{k}$, and $\vec{c} = 2\hat{i} - \hat{j} + 3\hat{k}$.

25. Find the value of *k* so that the lines $x = -y = kz$ and $x - 2 = 2y + 1 = -z + 1$ are perpendicular to each other.

26. The probability of finding a green signal on a busy crossing X is 30%. What is the probability of finding a green signal on X on two consecutive days out of three?

SECTION - C

Question number 27 to 32 carry 4 marks each.

27. Let N be the set of natural numbers and R be the relation on N × N defined by (a, b) R (c, d) if ad = bc for all $a, b, c, d \in$ N. Show that R is an equivalence relation.

28. If $y = e^{x^2 \cos x} + (\cos x)^x$, then find $\frac{dy}{dx}$.

29. Find: $\int \sec^3 x\, dx$

30. Find the general solution of the differential equation

$y\, e^y\, dx = (y^3 + 2x\, e^y)dy$.

OR

Find the particular solution of the differential equation

$x\frac{dy}{dx} = y - x\tan\left(\frac{y}{x}\right)$, given that $y = \frac{\pi}{4}$ at $x = 1$.

31. A furniture trader deals in only two items – chairs and tables. He has ₹ 50,000 to invest and a space to store at most 35 items. A chair costs him ₹ 1000 and a table costs him ₹ 2000. The trader earns a profit of ₹ 150 and ₹ 250 on a chair and table, respectively. Formulate the above problem as an LPP to maximise the profit and solve it graphically.

32. There are two bags, I and II. Bag I contains 3 red and 5 black balls and Bag II contains 4 red and 3 black balls. One ball is transferred randomly from Bag I to Bag II and then a ball is drawn randomly from Bag II. If the ball so drawn is found to be black in colour, then find the probability that the transferred ball is also black.

OR

An urn contains 5 red, 2 white and 3 black balls. Three balls are drawn, one-by-one, at random without replacement. Find the probability distribution of the number of white balls. Also, find the mean and the variance of the number of white balls drawn.

SECTION - D

Question numbers 33 to 36 carry 6 marks each.

33. If A = $\begin{bmatrix} 1 & 2 & -3 \\ 3 & 2 & -2 \\ 2 & -1 & 1 \end{bmatrix}$, then find A^{-1} and use it to solve the following system of the equations:

$x + 2y - 3z = 6$

$3x + 2y - 2z = 3$

$2x - y + z = 2$

OR

Using properties of determinants, prove that

$$\begin{vmatrix} (b+c)^2 & a^2 & bc \\ (c+a)^2 & b^2 & ca \\ (a+b)^2 & c^2 & ab \end{vmatrix}$$

$= (a - b)(b - c)(c - a)(a + b + c)(a^2 + b^2 + c^2)$.

34. Using integration, find the area of the region bounded by the triangle whose vertices are (2, –2), (4, 5) and (6, 2).

35. Show that the height of the right circular cylinder of greatest volume which can be inscribed in a right circular cone of height h and radius r is one-third of the height of the cone, and the greatest volume of the cylinder is $\frac{4}{9}$ times the volume of the cone.

36. Find the equation of the plane that contains the point A(2, 1, –1) and is perpendicular to the line of intersection of the planes $2x + y - z = 3$ and $x + 2y + z = 2$. Also find the angle between the plane thus obtained and the y-axis.

OR

Find the distance of the point P(–2, –4, 7) from the point of intersection Q of the line

$\vec{r} = (3\hat{i} - 2\hat{j} + 6\hat{k}) + \lambda(2\hat{i} - \hat{j} + 2\hat{k})$ and the plane

$\vec{r}.(\hat{i} - \hat{j} + \hat{k}) = 6$. Also write the vector equation of the line PQ.

All India 2019

CBSE Board Solved Paper

Time Allowed : 3 Hours ***Maximum Marks : 100***

General Instructions:

(i) **All** questions are compulsory.

(ii) The question paper consists of **29** questions divided into four sections **A**, **B**, **C** and **D**. Section **A** comprises of **4** questions of **one mark** each, Section **B** comprises of **8** questions of **two marks** each, Section **C** comprises of **11** questions of **four marks** each and Section **D** comprises of **6** questions of **six marks** each.

(iii) All questions in Section **A** are to be answered in one word, one sentence or as per the exact requirement of the question.

(iv) There is no overall choice. However, internal choice has been provided in **1** question of Section A, **3** questions of Section B, **3** questions of Section C and **3** questions of Section D. You have to attempt only one of the alternatives in all such questions.

(v) Use of calculators is not permitted. You may ask for logarithmic tables, if required.

SECTION - A

1. Form the differential equation representing the family of curves $y = \frac{A}{x} + 5$, by eliminating the arbitrary constant A.

2. If A is a square matrix of order 3, with $|A| = 9$, then write the value of $|2.\text{adj}A|$·

3. Find the acute angle between the planes $\vec{r}.(\hat{i}-2\hat{j}-2\hat{k})=1$ and $\vec{r}.(3\hat{i}-6\hat{j}+2\hat{k})=0$.

OR

Find the length of the intercept, cut off by the plane $2x + y - z = 5$ on the x-axis.

4. If $y = \log(\cos e^x)$, then find $\frac{dy}{dx}$.

SECTION - B

5. Find: $\int_{-\frac{\pi}{4}}^{0} \frac{1+\tan x}{1-\tan x}dx$

6. Let * be an operation defined as $* : R \times R \to R$ such that $a * b = 2a + b$, $a, b \in R$. Check if * is a binary operation. If yes, find if it is associative too.

7. X and Y are two points with position vectors $3\vec{a}+\vec{b}$ and $\vec{a}-3\vec{b}$ respectively. Write the position vector of a point Z which divides the line segment XY in the ratio 2 : 1 externally.

OR

Let $\vec{a} = \hat{i} + 2\hat{j} - 3\hat{k}$ and $\vec{b} = 3\hat{i} - \hat{j} + 2\hat{k}$ be two vectors. Show that the vectors $(\vec{a}+\vec{b})$ and $(\vec{a}-\vec{b})$ are perpendicular to each other.

8. If A and B are symmetric matrices, such that AB and BA are both defined, then prove that AB – BA is a skew symmetric matrix.

9. 12 cards numbered 1 to 12 (one number on one card), are placed in a box and mixed up thoroughly. Then a card is drawn at random from the box. If it is known that the number on the drawn card is greater than 5, find the probability that the card bears an odd number.

10. Out of 8 outstanding students of a school, in which there are 3 boys and 5 girls, a team of 4 students is to be selected for a quiz competition. Find the probability that 2 boys and 2 girls are selected.

OR

In a multiple choice examination with three possible answers for each of the five questions, what is the probability that a candidate would get four or more correct answers just by guessing?

11. Solve the following differential equation:

$$\frac{dy}{dx} + y = \cos x - \sin x$$

12. Find: $\int x.\tan^{-1} x dx$

OR

Find: $\int \frac{dx}{\sqrt{5-4x-2x^2}}$

SECTION - C

13. Using properties of determinants, find the value of x for which

$$\begin{vmatrix} 4-x & 4+x & 4+x \\ 4+x & 4-x & 4+x \\ 4+x & 4+x & 4-x \end{vmatrix} = 0$$

14. Solve the differential equation $\frac{dy}{dx} = 1 + x^2 + y^2 + x^2y^2$, given that $y = 1$ when $x = 0$.

OR

Find the particular solution of the differential equation $\frac{dy}{dx} = \frac{xy}{x^2+y^2}$, given that $y = 1$ when $x = 0$.

15. Let A = R – {2} and B = R – {1}. If $f : A \to B$ is a function defined by $f(x) = \frac{x-1}{x-2}$, show that f is one-one and onto. Hence, find f^{-1}.

OR

Show that the relation S on the set $A = \{x \in Z : 0 \le x \le 12\}$ given by $S = \{(a, b) : a, b \in Z, |a - b| \text{ is divisible by } 3\}$ is an equivalence relation.

16. Integrate the function $\frac{\cos(x+a)}{\sin(x+b)}$ w.r.t.x.

17. If $x = \sin t$, $y = \sin pt$, prove that $(1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} + p^2y = 0$.

OR

Differentiate $\tan^{-1}\left[\frac{\sqrt{1+x^2}-\sqrt{1-x^2}}{\sqrt{1+x^2}+\sqrt{1-x^2}}\right]$ with respect to $\cos^{-1}x^2$.

18. Prove that: $\cos^{-1}\left(\frac{12}{13}\right) + \sin^{-1}\left(\frac{3}{5}\right) = \sin^{-1}\left(\frac{56}{65}\right)$

19. If $y = (x)^{\cos x} + (\cos x)^{\sin x}$, find $\frac{dy}{dx}$.

20. Prove that $\int_0^a f(x)dx = \int_0^a f(a-x)dx$, and hence evaluate $\int_0^1 x^2(1-x)^n dx$.

21. Find the value of x, for which the four points A(x, –1, –1), B(4, 5, 1), C(3, 9, 4) and D(– 4, 4, 4) are coplanar.

22. A ladder 13 m long is leaning against a vertical wall. The bottom of the ladder is dragged away from the wall along the ground at the rate of 2 cm/sec. How fast is the height on the wall decreasing when the foot of the ladder is 5 m away from the wall ?

23. Find the vector equation of the plane determined by the points A(3, –1, 2), B(5, 2, 4) and C(–1, –1, 6). Hence, find the distance of the plane, thus obtained, from the origin.

SECTION - D

24. Using integration, find the area of the greatest rectangle that can be inscribed in an ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.

25. An insurance company insured 3000 cyclists, 6000 scooter drivers and 9000 car drivers. The probability of an accident involving a cyclist, a scooter driver and a car driver are 0.3, 0.05 and 0.02 respectively. One of the insured persons meets with an accident. What is the probability that he is a cyclist?

26. Using elementary row transformations, find the inverse of the matrix $\begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}$.

OR

Using matrices, solve the following system of linear equations:

$x + 2y - 3z = -4$
$2x + 3y + 2z = 2$
$3x - 3y - 4z = 11$

27. Using integration, find the area of the region bounded by the parabola $y^2 = 4x$ and the circle $4x^2 + 4y^2 = 9$.

OR

Using the method of integration, find the area of the region bounded by the lines $3x - 2y + 1 = 0$, $2x + 3y - 21 = 0$ and $x - 5y + 9 = 0$.

28. A dietician wishes to mix two types of food in such a way that the vitamin contents of the mixture contains at least 8 units of vitamin A and 10 units of vitamin C. Food I contains 2 units/kg of vitamin A and 1 unit/kg of vitamin C. It costs ₹ 50 per kg to produce food I. Food II contains 1 unit/kg of vitamin A and 2 units/kg of vitamin C and it costs ₹ 70 per kg to produce food II. Formulate this problem as a LPP to minimise the cost of a mixture that will produce the required diet. Also find the minimum cost.

29. Find the vector equation of a line passing through the point (2, 3, 2) and parallel to the line $\vec{r} = (-2\hat{i} + 3\hat{j}) + \lambda(2\hat{i} - 3\hat{j} + 6\hat{k})$. Also, find the distance between these two lines.

OR

Find the coordinates of the foot of the perpendicular Q drawn from P(3, 2, 1) to the plane $2x - y + z + 1 = 0$. Also, find the distance PQ and the image of the point P treating this plane as a mirror.

Ravi home tutions

MODEL PAPER 5

12th Standard CBSE

Maths

1) Set A has 3 elements and the set B has 4 elements. Then the number of injective functions that can be defined from set A to set B is 1

(a) 144 (b) 12 (c) 24 (d) 64

2) If A = diag(3, -1), then matrix A is 1

(a) $\begin{bmatrix} 0 & 3 \\ 0 & -1 \end{bmatrix}$ (b) $\begin{bmatrix} -1 & 0 \\ 3 & 0 \end{bmatrix}$ (c) $\begin{bmatrix} 3 & 0 \\ 0 & -1 \end{bmatrix}$ (d) $\begin{bmatrix} 3 & -1 \\ 0 & 0 \end{bmatrix}$

3) Derivative of cot x° with respect to x is 1

(a) cosec x° (b) cosec x° cot x° (c) -1° cosec2 x° (d) -1° cosec x° cot x°

4) The position vector of a point which divides the join of points with position vectors $\vec{a}+\vec{b}$ and $2\vec{a}-\vec{b}$ in the ratio 1:2 internally is 1

(a) $\frac{3\vec{a}+a\vec{b}}{3}$ (b) $\vec{a}$ (c) $\frac{5\vec{a}-\vec{b}}{3}$ (d) $\frac{4\vec{a}+\vec{b}}{3}$

5) Let A = {1, 2, 3}. Then number of relations containing (1, 2) and (1, 3) which are reflexive and symmetric but not transitive is 1

(a) 1 (b) 2 (c) 3 (d) 4

6) $\int \frac{dx}{sin^2xcos^2x}$ equals 1

(a) tan x + cot x + C (b) tan x + cot x + C (c) tan x cot x + C

(d) tan x – cot 2x + C

7) $\int_1^{\sqrt{3}} \frac{dx}{1+x^2}$ equals 1

(a) $\frac{\pi}{3}$ (b) $\frac{2\pi}{3}$ (c) $\frac{\pi}{6}$ (d) $\frac{\pi}{12}$

8) A homogeneous differential equation of the from $\frac{dx}{dy}=h\left(\frac{x}{y}\right)$ can be solved by making the substitution. 1

(a) y = vx (b) v = yx (c) x = vy (d) x = v

9) Let A = {1, 2, 3, ... , n} and B = {a, b}. Then the number of surjections from A into B is 1

(a) $^nP_2$ (b) 2^n-2 (c) 2^n -1 (d) None of these

ed by $f(x)=x^2+5, \forall x \in Z$ is one-one or not. 1

11) The vector in the direction of vector $\hat{i}-2\hat{j}+2\hat{k}$ that has magnitude 12 is 1

12) Find the vector equation for the line which passes through the point (1,2, 3) and is parallel to the line $\frac{x-1}{-2}=\frac{1-y}{3}=\frac{3-z}{-4}$. 1

13) a) If ɵ is the angle between two vectors $\vec{a}$and $\vec{b}, \vec{a}.\vec{b} \geq 0$ then only when 1

(a) $0<\theta<\frac{\pi}{2}$ (b) $0\leq\theta\leq\frac{\pi}{2}$ (c) $0<\theta<\pi$ (d) $0\leq\theta\leq\pi$

(OR)

b) Suppose that two cards are drawn at random from a deck of cards. Let X be the number of aces obtained. Then the value of E(X) is 1

(a) $\frac{37}{221}$ (b) $\frac{5}{13}$ (c) $\frac{1}{13}$ (d) $\frac{2}{13}$

14) a) If A is an invertible matrix of order 2, then det (A^{-1}) is equal to 1

(a) det (A) (b) $\frac{1}{det(A)}$ (c) 1 (d) 0

(OR)

b) The line y = mx + 1 is a tangent to the curve y^2 = 4x if the value of m is 1

(a) 1 (b) 2 (c) 3 (d) $\frac{1}{2}$

15) a) If $y=\tan(x+y),$ then find $\frac{ay}{dx}$ 1

(OR)

b) Evaluate $\int_2^3 3^x dx$ 1

16) a) The area of the region bounded by the curve y = sinx between the ordinates x = 0 $x=\frac{\pi}{2}$ and the X-axis is. 1

(a) 2 sq units (b) 4 sq units (c) 3 sq units (d) 1 sq unit

(OR)

b) Find the minor of the element of second row and third column in the determinant $\begin{vmatrix} 3 & -2 & 4 \\ 5 & 2 & 1 \\ 1 & 6 & -5 \end{vmatrix}$ 1

17) a) Let A be the set of all human beings in
whether the relation R = {(x, y) : x is wife of y ; x, Y∈A} is reflexive, symmetric and transitive.

(OR)

b) Which of the following binary operations are commutative? 2

(i) * On Z defined by a*b=a^2+b^2

(ii) * On Q defined by a*b=a^2+2b

18) If P(E) = $\frac{7}{13}$,P(F)=$\frac{9}{13}$ and P(E∩F)=$\frac{4}{13}$,then evaluate : 2

(a) $P(\overline{E}/F)$ (b) $P(\overline{E}/F)$

19) If $A=\begin{bmatrix} x & 0 \\ 1 & 1 \end{bmatrix}$ and $B=\begin{bmatrix} 1 & 0 \\ 5 & 1 \end{bmatrix}$ then find the value of x for which A^2 = R. 2

20) Differentiate each of the following $\log\tan\frac{x}{2}$ 2

21) If $\vec{a}=2\hat{i}+\hat{j}+3\hat{k}$ and $\vec{b}=3\hat{i}+5\hat{j}-2\hat{k}$ then find $|\vec{a}\times\overrightarrow{b}|$ 2

22) Find the direction cosines of the normal to the plane $2x+3y-z=5$ and also find the distance from origin to the given plane. 2

23) Three events A, Band Chave probabilities $\frac{2}{5}$, $\frac{1}{3}$ and $\frac{1}{2}$,respectively,If $P(A\cap C)=\frac{1}{5}$ and $P(B\cap C)=\frac{1}{4}$ then find the values of P(C / B) and $P\left(A'\cap C'\right)$. 2

24) a) Examine the differentiability of the function 2

$$f(x)=\begin{cases} x[x], & \text{if } 0\le x<2 \\ (x-1)x, & \text{if } 2\le x<3 \end{cases} \quad \text{at } x=2$$

(OR)

b) If $y=\tan^{-1}x,$ then find $\frac{d^2y}{dx^2}$ in terms of y 2

25) a) $f(x)=x^2, x\in R$ Find $\frac{f(1.1)-f(1)}{1.1-1}$ 2

(OR)

b) Find the slope and tangent and normal to the curve 2

$x^2+2y+y^2=0\ at\ (-1,2)$.

$\int\frac{dx}{x^2+4x+8}$ 2

(OR)

b) Find the general solution of the following differential equation. 2

$\frac{dy}{dx}=\frac{xy+y}{xy+x}$

27) a) Verify that the function y = a cos x + b sin x, where a, b ϵ R is a solution of differential equation $\frac{d^2y}{dx^2}+y=0$. 3

(OR)

b) Find the area between the curve $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ and the x - axis between x = 0 and x = a. Draw a rough sketch of the curve also. 3

28) Let A be the set of all students of a Boys'school. Show that the relation R in A given by: 3
R={(a,b) : a sister of b}
is empty relation and the relation R' given by:
R'={(a,b): the difference between heights of a and b is less than 3 metres} is the universal relation.

29) Find the coordinates of the foot of perpendicular and the length of the perpendicular: drawn from the point P (5, 4, 2) to the line : 3
$\vec{r}=\left(-\hat{i}+3\hat{j}-5+\hat{k}\right)+\lambda(2\hat{i}+3\hat{j}-\hat{k})$
Also find the image of P in this line

30) Verify that the following problem has no feasible solution: 3
Maximize Z=4x$_1$+4x$_2$, subject to the constraints:
$2x_1 3x_2\le 18, x_1+x_2\ge 12, x_1, x_2\ge 0.$

31) a) Find the value of 'a' for which the function 'f' defined as: 3

$$f\left(x\right)=\begin{cases} a\quad sin\frac{\pi}{2}(x+), & x\le 0 \\ \frac{tan\quad x-sin\quad x}{x^3}, & x>0 \end{cases}$$

is continuous at x=0.

(OR)

b) For what values of 'a' and 'b', the function 'f' is defined as: 3

$$f\left(x\right)=\begin{cases} 3ax+b & if\quad x<1 \\ 11\quad if & x=1 \\ 5ax-2b & if\quad x>1 \end{cases}$$ is continuous at x=1.

32) a) If xsin(a+y)+sin a cos(a+y)=0, then prov

$\frac{dy}{dx} = \frac{sin^2(a+y)}{sin \quad a}$

(OR)

b) If $x = a \quad cos\theta + b \quad sin\theta, \quad y = a \quad sin\theta - b \quad cos\theta$, show that $y^2\frac{d^2y}{dx^2} - x\frac{dy}{dx} - y = 0$ 3

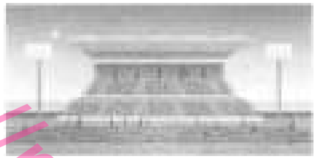
33) a) Find the equation of the tangent to the curve $x^2 + 3y = 3$ which is parallel to the line

$y - 4x + 5 = 0$ 3

(OR)

b) Show that the function $f(x) = \cos^2 x$ is strictly decreasing on $\left(0, \frac{\pi}{2}\right)$. 3

dents of class XII of two schools, say school A and school B. For which a team from each school is chosen. Remaining students of class XII of school A and B are respectively sitting on the plane represented by the equation $\vec{r} \cdot (\hat{i} + \hat{j} + 2\hat{k}) = 5$ and $\vec{r} \cdot (\hat{i} - \hat{j} + \hat{k}) = 6$,to cheer up the team of their respective schools.



Based on the above information, answer the following questions.

(i) The cartesian equation of the plane on which students of school A are seated is

(a) 2x - y +z = 8 **(b) 2x + y + z = 8** **(c) x + y + 2z = 5** **(d) x + y + z = 5**

(ii) The magnitude of the normal to the plane on which students of school B are seated, is

(a) $\sqrt{5}$ **(b)** $\sqrt{6}$ **(c)** $\sqrt{3}$ **(d)** $\sqrt{2}$

(iii) The intercept form of the equation of the plane on which students of school B are seated, is

(a) $\frac{x}{6} + \frac{y}{6} + \frac{z}{6} = 1$ **(b)** $\frac{x}{3} + \frac{y}{(-6)} + \frac{z}{6} = 1$ **(c)** $\frac{x}{3} + \frac{y}{6} + \frac{z}{6} = 1$ **(d)** $\frac{x}{3} + \frac{y}{6} + \frac{z}{3} = 1$

(iv) Which of the following is a student of school B?

(a) Mohit sitting at (1, 2, 1) **(b) Ravi sitting at (0,1,2)** **(c) Khushi sitting at (3, 1, 1)** **(d) Shewta sitting at (2, -1, 2)**

(v) The distance of the plane, on which students of school B are seated, from the origin is

(a) 6 units **(b)** $\frac{1}{\sqrt{6}}$ **units** **(c)** $\frac{5}{\sqrt{6}}$ **units** **(d)** $\sqrt{6}$ **units**

35) A differential equation is said to be in the v expressible in the form j(x) dx = g(y) dy. The solution of this equation is given by $\int f(x)dx = \int g(y)dy + c$ where c is the constant of integration.

Based on the above information, answer the following questions.

(i) If the solunon of the differential equation $\frac{dy}{dx} = \frac{ax+3}{2y+f}$ represents a circle, then the value of' a 'is

(a) 2 **(b) - 2** **(c) 3** **(d) - 4**

(ii) The diiftfterential equation $\frac{dy}{dx} = \frac{\sqrt{1-y^2}}{y}$ deterrnines a family of circle with

(a) variable radii and fixed centre (0,1)
(b) variable radii and fixed centre (0,-1)
(c) fixed radius 1 and variable centre on x-axis
(d) fixed radius 1 and variable centre on y-axis

(iii) If y' = y + 1,y (0) = 1, theny (In 2) =

(a) 1 **(b) 2** **(c) 3** **(d) 4**

(iv) The solution of the differential equation $\frac{dy}{dx} = e^{x-y} + x^2e^{-y}$ is

(a) $e^x = \frac{y^3}{3} + e^y + c$ **(b)** $e^y = \frac{x^2}{3} + e^x + c$ **(c)** $e^y = \frac{x^3}{3} + e^x + c$ **(d) none of these**

(v) If $\frac{dy}{dx} = y\sin 2x, y(0) = 1$ then its solution is

(a) y =esin^{2}x **(b) y=sin^2** **(c) y=cos^{2}x** **(d) y=cos^{2}x**

36) a) using integration, find the area of the region bounded by the curves y = x^2 and y = x. 5

(OR)

b) Evaluate : $\int \frac{x^2+x+1}{(x+2)(x^2+1)}dx$ 5

37) a) Show that the altitude of the right circular cone of maximum volume that can be inscribed in a sphere of radius r is $\frac{4r}{3}$ Also show that the maximum volume of the cone is $\frac{8}{27}$ of the volume of the sphere. 5

(OR)

b) If $A = \begin{bmatrix} 2 & 3 & 1 \\ 1 & 2 & 2 \\ -3 & 1 & -1 \end{bmatrix}$,find $A^{-1}$and hence solve the system of equations 2x + y- 3z =13, 3x+2y+z=4 and x+2y-z=8. 5

another bag contains 2 red and 6 black balls. One of the two bags is selected at random and two balls are drawn at random (without replacement) from the bag which are both found to be red. Find the probability that the balls are drawn from the first bag. 5

(OR)

b) Find the particular solution of the following differential equation given that : y = 0, when x = 1 $(x^2 + xy)\,dy = (x^2 + y^2)\,dx$. 5

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MODEL PAPER 4

12th Standard CBSE

Maths

1) The diagonal elements of a skew symmetric matrix are 1
(a) all zeroes (b) are all equal to some scalar k(≠ 0) (c) can be any number
(d) none of these

2) Write the number of points where f(x) = |x + 2| + |x – 3| is not differentiable 1
(a) 2 (b) 3 (c) 0 (d) 1

3) An operation * on Z⁺ is defined as a*b=a-b. Is the operation * a binary operation? Justify your answer. 1

4) Differentiate sin {log (x^3-1)}, with respect to x. 1

5) For what value of λ are the vectors $\vec{a} = 2\vec{i} + \lambda\vec{j} + \vec{k}$ and $\vec{b} = \vec{i} - 2\vec{j} + 3\vec{k}$ perpendicular to each other? 1

6) If a line has direction ratios 2,-1,-2, then what are its direction cosines? 1

7) Find the point on the curve $y=x^2-4x+5$,where tangent to the curve is parallel to the x-axis. 1

8) Given P(A)=0.2, P(B)=0.3 and $P(A \cap B) = 0.3$ Find P(A/B) 1

9) If $\begin{bmatrix} x-y & z \\ 2x-y & w \end{bmatrix} = \begin{bmatrix} -1 & 4 \\ 0 & 5 \end{bmatrix}$, find the value of x + y. 1

10) State the reason why the Relation R=[(a,b) : $a \le b^2$ on the set R of the real numbers is not reflexive 1

11) Let A = [a_{ij}]be a matric of order 2 x 3 and a_{ij} = $\frac{i-j}{i+j}$, write the value of a_{23} 1

12) Find the derivative of sin $(cos^2\,(\sqrt{x}))$. 1

13) a) Find λ, if the vectors 1
$\vec{a} = \hat{i} + 3\hat{j} + \hat{k}, \vec{b} = 2\hat{i} - \hat{j} - \hat{k}$ and $\vec{c} = \lambda\hat{j} + 3\hat{k}$ are coplanar.

(OR)

b) Let X = {1,2,3,4}. A function is defined from X to N as R = {x, f(x)): x ϵ X, f(x) = $^{x}p_{x-1}$} . Then find the range of f 1

14) a) The curves y = ae^{-x} and y = be^{x} are orthogonal if 1
(a) a = b (b) a = -b (c) ab = -1 (d) ab = 1

(OR)

b) A line makes angle α, β, γ with x-axis, y-axis and z-axis respectively then cos 2α + cos 2β + cos 2γ is equal to 1
(a) 2 (b) 1 (c) -2 (d) -1

1

(OR)

b) $\int tan^{-1}(cot\,x)dx.$ 1

16) a) If $A^T = \begin{bmatrix} 3 & 4 \\ -1 & 2 \\ 0 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} -1 & 2 & 1 \\ 1 & 2 & 3 \end{bmatrix}$, then find $A^T - B^T$. 1

(OR)

b) Find the Value of determinant $\triangle = \begin{vmatrix} 2 & 2 & 2 \\ x & y & z \\ y+z & z+x & z+y \end{vmatrix}$ 1

17) a) $\int sin^2x \quad cos^2xdx$ 2

(OR)

b) $\int \frac{cosx}{cos(x+\alpha)}dx$ 2

18) Let * be a binary operation On the set of all non-zero real numbers given by a*b = $\frac{ab}{5}$ 2
For all a,b $\in R - [0]$
Find the value of x, given that (i) 2 * (x*5) =6 (ii) 3 (x*3) = 9

19) If E and F be two events such that P(E) = $\frac{1}{3}, P(F) = \frac{1}{4}$,find $P(E \cup F)$ if E and F are independent events. 2

20) If P(A)=$\frac{2}{5}, P(B) = \frac{1}{3}, P(A \cap B) = \frac{1}{5}$, then find $P(\bar{A}/\bar{B})$. 2

21) Let f: R →R be defined by f(x) = x^2 +1 Then, find pre-images of 17 and - 3. 2

22) If matrix $\begin{bmatrix} 0 & a & 3 \\ 2 & b & -1 \\ c & 1 & 0 \end{bmatrix}$ is a skew-symmetric matrix, then find the values of a, b and c 2

23) Find the derivative of xe^xsin x w.r.t. x 2

24) a) Solve the differential equation $\frac{dy}{dx} = xy + x + y + 1$ 2

(OR)

b) Find the general solution of differential equation $\frac{dy}{dx} = e^{3x-4y}$ 2

25) a) Find the symmetric and skew-symmetric matrices of matrix matrices of matrix $A = \begin{bmatrix} 0 & -2 & 4 \\ 2 & 0 & -1 \\ -4 & 1 & 0 \end{bmatrix}$ 2

(OR)

b) Find A^{-1},if $A = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$ and show that $A^{-1} = \frac{A^2-3I}{2}$ 2

26) a) Discuss the continuity of the function $f(x)=\begin{cases}\frac{\sin 5x}{x}, \\ 5,\ \text{if } x=0\end{cases}$ 5

(OR)

b) If $f(x)=\sin 2x-\cos 2x,\ \text{then find } f'\left(\frac{\pi}{6}\right)$ 2

27) a) Show that the relation R in the set {1,2,3} given by:
R={(1,1), (2,2), (3,3), (1,2), (2,3)} is reflexive but neither symmetric nor transitive. 3

(OR)

b) For the matrix $A=\begin{bmatrix}1 & 5\\ 6 & 7\end{bmatrix}$, verify that: 3
(i) $A+A\prime$is a symmetric matrix
(ii) $A-A\prime$is a skew-symmetric matrix.

28) Find the co - factors of the elements of the determinant $\begin{vmatrix}2 & -3 & 5\\ 6 & 0 & 4\\ 1 & 5 & -7\end{vmatrix}$ and verify that $a_{11}A_{31}+a_{12}A_{32}+a_{13}A_{33}=0$. 3

29) Construct a 3×2 matrix whose elements are given by:$a_{ij}=\frac{1}{2}|[i-3j]|$ 3

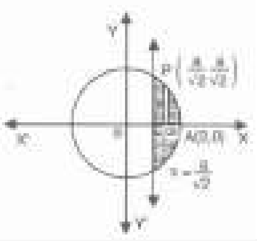
30) Verify that the function $y=e^{-3x}$ is a solution of the differential equation $\frac{d^2y}{dx^2}+\frac{dy}{dx}-6y=0$. 3

31) Find the area of the region bounded by the ellipse $\frac{x^2}{16}+\frac{y^2}{9}=1$ 3

32) a) $f(x)=\begin{cases}kx^2, & if \quad x\le 2\\ 3 & if \quad x>2\end{cases}\ at \quad x=2.$ 3

(OR)

b) Find the area of the smaller part of the circle x^2 + y^2 = a^2 cut off by the line $x=\frac{a}{\sqrt{2}}$ 3



33) a) $Find \quad \frac{dy}{dx}, \quad if \quad x^{2/3}+y^{2/3}=a^{2/3}$ 3

(OR)

b) $If \quad y=3e^{2x}+2e^{3x}, prove \quad that \quad \frac{d^2y}{dx^2}-5\frac{dy}{dx}+6y=0$ 3

34) a) $\int_2^5 f(x)dx$, when: $f(x)=|x-2|+|x-3|+|x-5|$. 5

(OR)

b) Solve the following differential equation $e^x\tan y dx+(1-e^x)\sec^2 y dy=0$ 5

$\frac{x-\cos x}{4x-\pi}, x\neq\frac{\pi}{4}$ is
$k, x=\frac{\pi}{4}$ 5
continuous at $x=\frac{\pi}{4}$.

(OR)

b) Find the intervals in which the following functions is strictly increasing or strictly decreasing. 5
$f(x)=2x^3-3x^2-36x+7$

36) a) If$A=\begin{bmatrix}1 & 3 & 2\\ 2 & 0 & -1\\ 1 & 2 & 3\end{bmatrix}$, then show that $A^3-4A^2-3A+11I=O$. 5

(OR)

b) Find the equation of the plane which is perpendicular to the plane 5x + 3Y + 6z + 8 = 0 and which contains the line of intersection of the planes x + 2y + 3z - 4 = 0 and 2x +y - z + 5 = 0. 5

37) Let $f:A\rightarrow B$ and $g:B\rightarrow C$ Cbe two functions defined on non-empty sets A, B, C, then gof $:A\rightarrow C$ be is called the composition off and g defined as, 4
$gof(x)=g\{f(x)\}\forall x\in A$.
Consider the functions $f(x)=\begin{cases}\sin x, & x\ge 0\\ 1-\cos x, & x\le 0\end{cases}, g(x)=e^x$ and then answer the following questions.

(i) The function gof(x) is defined as
(a) $gof(x)=\begin{cases}e^x & ,x\ge 0\\ 1-e^{\cos x} & ,x\le 0\end{cases}$ **(b)** gof(x) $=\begin{cases}e^{\sin x} & ,x\le 0\\ e^{1-\cos x} & ,x\ge 0\end{cases}$
(c) $gof(x)=\begin{cases}e^{\sin x} & ,x\le 0\\ 1-e^{\cos x} & ,x\ge 0\end{cases}$ **(d)** $gof(x)=\begin{cases}e^{\sin x} & ,x\ge 0\\ e^{1-\cos x} & ,x\le 0\end{cases}$

(ii) $\frac{d}{dx}\{\text{gof}(x)\}=$
(a) $[gof(x)]'=\begin{cases}\cos x\cdot e^{\sin x} & ,x\ge 0\\ e^{1-\cos x}\cdot\sin x & ,x\le 0\end{cases}$ **(b)** $[gof(x)]'=\begin{cases}\cos x\cdot e^{\sin x} & ,x\ge 0\\ -\sin x\cdot e^{1-\cos x} & ,x\le 0\end{cases}$
(c) $[gof(x)]'=\begin{cases}\cos x\cdot e^{\sin x} & ,x\ge 0\\ \sin x\cdot(1-\cos x) & ,x\le 0\end{cases}$ **(d)** $[gof(x)]'=\begin{cases}\cos x\cdot e^{\sin x} & ,x\ge 0\\ (1-\sin x)\cdot e^{1-\cos x} & ,x\le 0\end{cases}$

(iii) R.H.D. of gof(x) at x = 0 is
(a) 0 **(b) 1** **(c) -1** **(d) 2**

(iv) L.H.D. of gof(x) at x = 0 is
(a) 0 **(b) 1** **(c) -1** **(d) 2**

(v) The value of $f'(x)$ at $x=\frac{\pi}{4}$ is
(a) 1/9 **(b)** $1/\sqrt{2}$ **(c) 1/2** **(d) not defined**

38) Linear programming is a method for finding the optimal v minimum) of quantities subject to the constraints when relationship is expressed as linear equations or inequations.

Based on the above information, answer the following questions.

(i) The optimal value of the objective function is attained at the points

(a) on X-axis **(b) on Y-axis** **(c) which are corner points of the feasible region** **(d) none of these**

(ii) The graph of the inequality 3x + 4y < 12 is

(a) half plane that contains the origin **(b) half plane that neither contains the origin nor the points of the line 3x + 4y =12.** **(c) whole XOY-plane excluding the points on line 3x + 4y = 12** **(d) none of these**

(iii) The feasible region for an LPP is shown in the figure. Let Z = 2x + 5y be the objective function. Maximum of Z occurs at

(a) (7,0) **(b) (6,3)** **(c) (0,6)** **(d) (4,5)**

(iv) The corner points of the feasible region determined by the system of linear constraints are (0, 10), (5, 5), (15, 15), (0, 20). Let Z = px + qy, where p, q > 0. Condition on p and q so that the maximum of Z occurs at both the points (15, 15) and (0, 20) is

(a) p = q **(b) p = 2q** **(c) q=2p** **(d) q=3p**

(v) The corner points of the feasible region determined by the system of linear constraints are (0, 0), (0, 40), (20,40), (60,20), (60, 0). The objective function is Z = 4x + 3y. Compare the quantity in Column A and Column B

Column A	**Column B**
Maximum of Z	325

(a) The quantity in column A is greater **(b) The quantity in column B is greater** **(c) The two quantities are equal** **(d) The relationship cannot be determine basis of the information supplied**

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MODEL PAPER 3

12th Standard CBSE

Maths

1) A relation S in the set of real numbers is defined as xSy ⇒ x – y+ $\sqrt{3}$ is an irrational number, then relation S is 1

(a) reflexive (b) reflexive and symmetric (c) transitive

(d) symmetric and transitive

2) If $\Delta = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$ and A_{ij} is Cofactors of aij, then value of Δ is given by 1

(a) $a_{11} A_{31} + a_{12} A_{32} + a_{13} A_{33}$ (b) $a_{11} A_{11} + a_{12} A_{21} + a_{13} A_{31}$

(c) $a_{21} A_{11} + a_{22} A_{12} + a_{23} A_{13}$ (d) $a_{11} A_{11} + a_{21} A_{21} + a_{31} A_{31}$

3) The line y = x + 1 is a tangent to the curve $y^2 = 4x$ at the point 1

(a) (1, 2) (b) (2, 1) (c) (1, – 2) (d) (– 1, 2)

4) The probability of obtaining an even prime number on each die, when a pair of dice is rolled is 1

(a) 0 (b) $\frac{1}{3}$ (c) $\frac{1}{12}$ (d) $\frac{1}{36}$

5) Let f, g and h be functions from R to R. Then, 1

(a) $(f+g)oh = \text{fog+goh}$ (b) $(f+g)oh = foh + goh$

(c) $(f \cdot g)oh = (foh) + (goh)$ (d) $(f \cdot g)oh = (fog) \cdot (goh)$

6) If $\int_0^1 \frac{e^t}{1+t} dt = a,$ then $\int_0^1 \frac{e^t}{(1+t)^2} dt$ is equal to 1

(a) $a - 1 + \frac{e}{2}$ (b) $a + 1 - \frac{e}{2}$ (c) $a - 1 - \frac{e}{2}$ (d) $a + 1 + \frac{e}{2}$

7) $\int sin\ 3x\ sin\ 2x\ dx.$ 1

8) Let A = {1, 2, 3}. Then find the number of equivalence relations containing (1, 2) 1

A and B such that AB =0 but BA $\neq$ 0 1

10) Find the transpose of matrix $\begin{bmatrix} -1 & 5 & 6 \\ \sqrt{3} & 5 & 6 \\ 2 & 3 & -1 \end{bmatrix}$ 1

11) Examine the following function for continuity. 1

$f(x) = \frac{1}{x-5}, x \neq 5$

12) Find the order and degree, of each of the following differential equation, if defined 1

$\left(\frac{ds}{dt}\right)^4 + 3s\frac{d^2s}{dt^2} = 0$

13) a) If $f(x) = \int_0^x t\ sin\ dt$, then f'(x) is 1

(a) cosx + x sin x (b) x sinx (c) x cosx (d) sinx + x cosx

(OR)

b) The area of the circle $x^2 + y^2 = 16$ exterior to the parabola $y^2 = 6x$ is 1

(a) $\frac{4}{3}(4\pi - \sqrt{3})$ (b) $\frac{4}{3}(4\pi + \sqrt{3})$ (c) $\frac{4}{3}(8\pi - \sqrt{3})$ (d) $\frac{4}{3}(8\pi + \sqrt{3})$

14) a) Let A be a diagonal A = (d_1, d_2, ..., dn) write the value of | A | 1

(OR)

b) If $f(x) = \{4 - (x-7)^2$, then find $f^{-4}(x)$. 1

15) a) Let A = {1, 2, 3}. Then number of equivalence relations containing (1, 2) is 1

(a) 1 (b) 2 (c) 3 (d) 4

(OR)

b) If n = p, then the order of the matrix 7X – 5Z is: 1

(a) p × 2 (b) 2 × n (c) n × 3 (d) p × n

16) a) Which of the following differential equat solution?

(a) $\frac{d^2y}{dx^2}-x^2\frac{dy}{dx}+xy=x$ (b) $\frac{d^2y}{dx^2}+x\frac{dy}{dx}+xy=x$ (c) $\frac{d^2y}{dx^2}-x^2\frac{dy}{dx}+xy=0$

(d) $\frac{d^2y}{dx^2}+x\frac{dy}{dx}+xy=0$

(OR)

b) If $\vec{a}$ are $\vec{b}$ two collinear vectors, then which of the following are incorrect: 1

(a) $\vec{b}$ = $\lambda\vec{a}$for some scalar λ (b) $\vec{a}\pm\vec{b}$

(c) the respective components of $\vec{a}$and $\vec{b}$ are not proportional

(d) both the vectors $\vec{a}$and $\vec{b}$have same direction, but different magnitudes

17) a) If $A=[1 \quad 2 \quad 3]$ and $B=\begin{bmatrix}-2\\3\\1\end{bmatrix}$, find AB and BA. 2

(OR)

b) If is $A=\begin{bmatrix}0&b&-2\\3&1&3\\2a&3&-1\end{bmatrix}$ skew symmetric matrix, find the values of a and b. 2

18) A die is rolled. If E = (1,3,5), F(2,3) and G{2, 3,4,5}, find 2

(a) P(E∪F)/G

(b) [(E∩F)/G].

19) Find the angle between the lines 2

$\vec{r}=(2i-5j+k)+l(i+j+3k)$

and $\vec{r}=(i-j-k)+\mu(2i-3j+k)$

20) If $f:[1,\infty)\rightarrow[2,\infty)$ is defined by $f(x)=x+\frac{1}{x}$, then find $f^{-1}(x)$ 2

21) Find the values of x, y and z, if $\begin{bmatrix}-2x+y\\x+y+z\\x+y\end{bmatrix}=\begin{bmatrix}-3\\3\\3\end{bmatrix}$ 2

22) Find the inverse of the matrix $\begin{bmatrix}\cos\theta&\sin\theta\\-\sin\theta&\cos\theta\end{bmatrix}$ 2

23) Find the general solution of the following differential equation. 2

$\frac{dy}{dx}=\sin^{-1}x$

2

(OR)

b) Find the sum of the order and degree of the following differential equations : 2

$\frac{d^2y}{dx^2}+\sqrt[3]{\frac{dy}{dx}}+(1+x)=0$

25) a) If $\left(x^2+y^2\right)^2=xy,\ \text{find}\ \frac{dy}{dx}$ 2

(OR)

b) Find the relationship between a and b so that the function f defined by f 2

$f(x)=\begin{cases}ax+1, \text{ if } x\leq 3\\ bx+3, \text{ if } x>3\end{cases}$ is

continuous at x = 3.

26) a) Show that the function $f(x)=\tan x-x$ is always increasing in $x\in R$. 2

(OR)

b) Evaluate the following integral. 2

$\int\frac{(x^3+8)(x-1)}{x^2-2x+4}dx$

27) a) Given:3 $\begin{bmatrix}x&y\\z&w\end{bmatrix}=\begin{bmatrix}x&6\\-1&2w\end{bmatrix}+\begin{bmatrix}4&x+y\\z+w&3\end{bmatrix}$, find the values of x, y, z and w. 3

(OR)

b) Show that $*:R\times R\rightarrow R$ defined by a $a*b=a+2ab$ is not communicative. 3

28) Prove that the function given by: 3

$f(x)cosxis$:

(a) strictly decreasing in $(0,\pi)$

(b) strictly increasing in $(\pi,2\pi)$

(c) neither increasing nor decreasing in $(0,2\pi)$

29) A die is thrown three times.Events A and B are defined as below: 3

A: 4 on the third throw

B: 6 on the first and 5 on the second throw.

Find the probability of A,given that B has already occured.

30) Find the slope of the tangent to the curve $y=x^3-3x+2$ at the point, where x-coordinate is 3. 3

31) Find the value of 'p' so that the lines:

$\frac{1-x}{3} = \frac{7y-14}{2p} = \frac{z-3}{2} \quad and \quad \frac{7-7x}{3p} = \frac{y-5}{1} = \frac{6-z}{5}$

are at right angles. 3

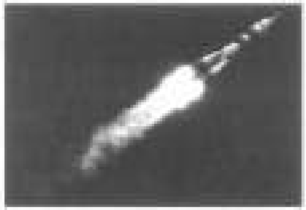
32) Find the area of the region bounded by x^2 - 4y, y = 2, y = 4 and the y - axis in the first quadrant. 3

33) a) Find $\frac{dy}{dx}$, if $x = a \quad cos\theta, \quad y = a \quad sin\theta$

(OR)

b) $Find \quad \frac{dy}{dx}, \quad if \quad x = at^2, \quad y = 2at.$ 3

34) The upward speed v(t) of a rocket at time t is approximated by $v(t) = at^2 + bt + c, 0 \le t \le 100$,where a, band c are constants. It has been found that the speed at times t = 3, t = 6 and t = 9 seconds are respectively 64, 133 and 208 miles per second. 4



If $\begin{pmatrix} 9 & 3 & 1 \\ 36 & 6 & 1 \\ 81 & 9 & 1 \end{pmatrix}^{-1} = \frac{1}{18}\begin{pmatrix} 1 & -2 & 1 \\ -15 & 24 & -9 \\ 54 & -54 & 18 \end{pmatrix}$,then answer the following questions

(i) The value of b + c is

(a) 20 **(b) 21** **(c) 3/4** **(d) 4/3**

(ii) The value of a + c is

(a) 1 **(b) 20** **(c) 4/3** **(d) none of these**

(iii) v(t) is given by

(a) t^2+20t+l **(b)** $\frac{1}{3}t^2 + 20t + 1$ **(c)** $t^2 + \frac{1}{3}t + 20$ **(d) P + t + 1**

(iv) The speed at time t = 15 seconds is

(a) 346 miles/see **(b) 356 miles/see** **(c) 366 miles/see** **(d) 376 miles/see**

(v) The time at which the speed of rocket is 784 miles/see is

(a) 20 seconds **(b) 30 seconds** **(c) 25 seconds** **(d) 27 seconds**

= a if f(x) has

(a) Discontinuity of first kind $\lim_{h\to 0} f(a-h)$ and $\lim_{h\to 0} f(a+h)$ both exist but are not equal. If is also known as irremovable discontinuity.

(b) Discontinuity of second kind: If none of the limits $\lim_{h\to 0} f(a-h)$ and $\lim_{h\to 0} f(a+h)$ exist.

(c) Removable discontinuity: $\lim_{h\to 0} f(a-h)$ and $\lim_{h\to 0} f(a+h)$ both exist and equal but not equal to f(a).

Based on the above information, answer the following questions.

(i) If $f(x) = \begin{cases} \frac{x^2-9}{x-3}, & \text{for } x \ne 3 \\ 4, & \text{for } x = 3 \end{cases}$,then at x= 3

(a) f has removable discontinuity **(b) f is continuous**

(c) f has irremovable discontinuity **(d) none of these**

(ii) Let $f(x) = \begin{cases} x+2, & \text{if } x \le 4 \\ x+4, & \text{if } x > 4 \end{cases}$,then at x = 4

(a) f is continuous **(b) f has removable discontinuity**

(c) f has irremovable discontinuity **(d) none of these**

(iii) Consider the function f(x) defined $f(x) = \begin{cases} \frac{x^2-4}{x-2} \\ 5 \end{cases}$,for $x \ne 2$

(a) f has removable discontinuity **(b) f has irremovable discontinuity**

(c) f is continuous **(d) f is continuous if f(2) = 3**

(iv) If $f(x) = \begin{cases} \frac{x-|x|}{x}, & if \ x \ne 0 \\ 2, & if \ x = 0 \end{cases}$,then x=0

a) f is continuous **(b) f has removable discontinuity**

(c) f has irremovable discontinuity **(d) none of these**

(v) If $f'(x) = \begin{cases} \frac{e^x-1}{\log(1+2x)}, & \text{if } x \ne 0 \\ 7, & \text{if } x = 0 \end{cases}$,then at x = 0

(a) f is continuous if f(0) = 2 **(b) f is continuous**

(c) f has irremovable discontinuity **(d) f has removable discontinuity**

36) a) If $A = \begin{pmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{pmatrix}$ and $A^3 - 6A^2 + 7A + kI^3 = 0$, find k. 5

(OR)

b) Find the points of local maxima, local minima and the points of inflexion of the function $f(x) = x^5 - 5x^4 + 5x^3 - 1$. Also, find the corresponding local maximum and local minimum values. 5

37) a) A figure consists of a semicircle with a rectangle on its diameter. Given the perimeter of the figure, find its dimensions in order that the area may be maximum. 5

(OR)

b) An insurance company insured 2,000 cyclists, 4,000 scooter drivers and 6,000 motorbike drivers. The probability of an accident involving a cyclist, scooter driver and a motorbike driver are 0.01, 0.03 and 0.15 respectively. One of the insured persons meets with an accident. What is the probability that he is a scooter driver?
Which mode of transport would you suggest to a student and why? 5

38) a) Find the vector and cartesian forms of the equation of the plane passing through the point (1, 2, - 4) and parallel to the lines
$\vec{r} = \left(\hat{i} + 2\hat{j} - 4\hat{k}\right) + \lambda\left(2\hat{i} + 3\hat{j} + 6\hat{k}\right)$
and $\vec{r} = \left(\hat{i} - 3\hat{j} + 5\hat{k}\right) + \mu\left(\hat{i} + \hat{j} - \hat{k}\right)$
Also, find the distance of the point (9, -8, -10) from the plane thus obtained. 5

(OR)

b) Evaluate : $\int \frac{8}{(x+2)(x^2+4)} dx$ 5

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Ravi home tuitions

MODEL PAPER 2

12th Standard CBSE

Maths

1) Let f : R → R be defined as f (x) = 3x. Choose the correct answer 1

(a) f is one-one onto (b) f is many-one onto (c) f is one-one but not onto

(d) f is neither one-one nor onto

2) Matrices A and B will be inverse of each other only if 1

(a) AB = BA (b) AB = BA = 0 (c) AB = 0, BA = I (d) AB = BA = I

3) Which of the following functions are decreasing on 0, $\frac{\pi}{2}$? 1

(a) cos x (b) cos 2x (c) cos 3x (d) tan x

4) If $\frac{d}{dx}f(x)=4x^3-\frac{3}{x^4}$ such that f (2) = 0. Then f (x) is 1

(a) $x^4+\frac{1}{x^3}-\frac{129}{8}$ (b) $x^3+\frac{1}{x^4}+\frac{129}{8}$ (c) $x^4+\frac{1}{x^3}+\frac{129}{8}$ (d) $x^3+\frac{1}{x^4}-\frac{129}{8}$

5) Smaller area enclosed by the circle $x^2 + y^2 = 4$ and the line x + y = 2 is 1

(a) 2 (π – 2) (b) π - 2 (c) 2π - 1 (d) 2(π+2)

6) The amount of pollution content added in air in a city due to X diesel vehicles is given by P(x)=0.005x^3+0.02x^2 +30x.Find the marginal increase in pollution content when 3 diesel vehicles are added and write which value is indicated in the above question? 1

7) Write the vector equation of the following line: $\frac{x-5}{3}=\frac{y+4}{7}=\frac{6-z}{2}$ 1

8) Write the element $a_{23} of a 3\times 3 matrix A=[a_{ij}]$ whose elements a_{ij} are given by $a_{ij}=\frac{|i-j|}{2}$ 1

9) If $A=\begin{bmatrix}1 & 2\\ 3 & -1\end{bmatrix}$ and $B=\begin{bmatrix}1 & 3\\ -1 & 1\end{bmatrix}$ Then write the value of |AB|. 1

10) Evaluate thefollowing integral. 1

$\int\frac{(1+\cos x)}{x+\sin x}dx$

11) Solve the following differential equation. 1

$\frac{dy}{dx}-\frac{x}{x^2+1}=0$

12) An urn contains 6 red and 3 black balls, Two balls are randomly drawn. Let X represents the number of black balls. What are the possible values of X? 1

3 is strictly increasing on R. 1

(OR)

b) If $P(1,5,4)$ and $Q(4,1,-2)$ then find the direction ratios of $\overrightarrow{PQ}$. 1

14) a) Find the cofactor of a$_{12}$ in the following. $\begin{vmatrix}2 & -3 & 5\\ 6 & 0 & 4\\ 1 & 5 & -7\end{vmatrix}$ 1

(OR)

b) If $\begin{bmatrix}x & +3y & y\\ 7 & -x & 4\end{bmatrix}=\begin{bmatrix}4 & -1\\ 0 & 4\end{bmatrix}$, find the values of x and y. 1

15) a) Evaluate the integral: $\int\frac{x^2+4x}{x^3+6x^2+5}dx$ 1

(OR)

b) Differentiate the following w.r.t. x, or find $\frac{dy}{dx}$. $y=\frac{sin(ax+b)}{cos(cx+d)}$. 1

16) a) If f(X)=X+7 and g(X)=X-7, $X\in R$, find fog(7). ? 1

(OR)

b) if $f:R\rightarrow R$ defined by $f(x)=\frac{2x-1}{5}$ is an invertible function, then find $f^{-1}(x)$. 1

17) Solve the matrix equation $\begin{bmatrix}x^2\\ y^2\end{bmatrix}-3\begin{bmatrix}x\\ 2y\end{bmatrix}=\begin{bmatrix}-2\\ -9\end{bmatrix}$ 2

18) Let A be the set of all human beings in a town at a particular time. Determine whether the relation R = {(x, y) : x is wife of y ; x, Y∈A} is reflexive, symmetric and transitive. 2

19) $\int sin^{-1}(cosx)dx$ 2

20) $\int logxdx$ 2

21) Let * be a binary operation On the set of all non-zero real numbers given by a*b = $\frac{ab}{5}$ 2

For all a,b $\in R-[0]$

Find the value of x, given that (i) 2 * (x*5) =6 (ii) 3 (x*3) = 9

22) If E and F are two events such that $P(E)=\frac{1}{4}$, $P(E)=\frac{1}{2}$ and $P(E\cap F)=\frac{1}{8}$, 2

find

(a) P(E or F)

(b) P(not E and not F).

23) Find the minors of the diagonal elements of the determinant $\begin{vmatrix} -i & 1 & i \\ 1 & -i & i \end{vmatrix}$ 2

24) a) Show that the function $f(x) = \begin{cases} \frac{\sin x}{x} + \cos x, & \text{if } x \neq 0 \\ 2, & \text{if } x = 0 \end{cases}$ is continuous at x = O. 2

(OR)

b) Discuss the continuity of the function $f(x) = \begin{cases} \frac{\sin 5x}{x}, \text{ if } x \neq 0 \text{ at } x = 0. \\ 5, \text{ if } x = 0 \end{cases}$ 2

25) a) Find X and Y, if $2X + 3Y = \begin{bmatrix} 2 & 3 \\ 4 & 0 \end{bmatrix} and 3X + 2Y = \begin{bmatrix} 2 & -2 \\ -1 & 5 \end{bmatrix}$ 2

(OR)

b) If $A = \begin{bmatrix} 2 & 3 & -5 \\ 0 & -1 & 4 \end{bmatrix}, then\ verify\ that (3A)' = 3A'$ 2

26) a) Show that each of three vectors is a unit vector:
$\frac{1}{7}(2\hat{i} + 3\hat{j} + 6\hat{k}), \frac{1}{7}(6\hat{i} - 3\hat{j} + 2\hat{k}), \frac{1}{7}(3\hat{i} - 6\hat{j} + 2\hat{k})$ 2

(OR)

b) Find the projection of $\vec{a} + \vec{b}$ on $\vec{a} - \vec{b}, \vec{a} = i + 2j + k, \vec{b} = 3\hat{i} + \hat{j} - \hat{k}$. 2

27) Find the equation of the curve passing through the point (-2,3) given that the slope of the tangent to the curve at point (x,y) is $\frac{2x}{y^2}$. 3

28) Show that the relation R in the set Z of integers given by:
R={(a,b):2divides a-b} is an equivalence relation 3

29) show that the points: A(a,b+c), B(b,c+a), C(c,a+b) are collinear. 3

30) $Differentiate \quad sin^2 x \quad w.r.t. \quad e^{cosx}$ 3

31) a) Let a*b=a+2b-5, find 3*5. 3

(OR)

b) $Find \quad \frac{d^2y}{dx^2}, \quad if \quad y = x^3 + tan \quad x.$ 3

Show that the function f (x) $\begin{cases} & f \quad x \neq 0 \\ 1 & , \quad if \quad x = 0 \end{cases}$ is not continuous at x=0. 3

(OR)

b) $f(x) = \begin{cases} kx + 1, & if \quad x \leq 5 \\ 3X - 5, & if \quad x > 5 \end{cases} \quad at \quad x = 5.$ 3

33) a) In the matrix, $A = \begin{bmatrix} 2 & 5 & 19 & -7 \\ 35 & -2 & 5/2 & 12 \\ \sqrt{3} & 1 & -5 & 17 \end{bmatrix}$ write: 3

(i) The order of the matrix.

(ii) The number of elements.

(iii) Write the elements $a_{13}, a_{21}, a_{33}, a_{24}, a_{23}$.

(OR)

b) If $x \begin{bmatrix} 2 \\ 3 \end{bmatrix} + y \begin{bmatrix} -1 \\ 1 \end{bmatrix} = \begin{bmatrix} 10 \\ 5 \end{bmatrix}$, find the values of x and y. 3

34) a) If A= $\begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}$ find A^{-1}. Using A^{-1} solve the following system of equations: 2x-3y+5z=16; 3x+2y-4z=-4; x+y-2z=-3 5

(OR)

b) Evaluate the integral $\int \frac{sin\ x}{(1 - cos\ x)(2 - cos\ x)} dx$ 5

35) a) Show that the diffrential equation (xe$^{x/y}$ +y)dx = xdy is homogeneous. Find the particular solution of this differential equation, given that x = 1 when y = 1. 5

(OR)

b) Using integration find the area of the region in the first quadrant enclosed by the X-axis, the line y=x and the circle x^2 + y^2 = 32. 5

36) a) Let $f(x) = \begin{cases} \frac{1 - sin^3 x}{3 cos^2 x} & , if \quad x < \frac{\pi}{2} \\ a & , if \quad x = \frac{\pi}{2} \\ \frac{b(1 - sin \quad x)}{(\pi - 2x)^2} & , if \quad x > \frac{\pi}{2} \end{cases}$ If f (x) be a continuous function at x=$\frac{\pi}{2}$, find a and b. 5

(OR)

b) Show that the right circular cone of least curved surface and volume has an altitude equal to $\sqrt{2}$ times the radius of the base. 5

37) Let $A=\begin{bmatrix}1 & 0\\ 2 & 1\end{bmatrix}$,and U$_1$ U$_2$ are first and second columns respectively of a 2 x 2 matrix U. Also, let the column matrices U$_l$ and U$_2$ satisfying $AU_1=\begin{bmatrix}1\\0\end{bmatrix}$ and $AU_2=\begin{bmatrix}2\\3\end{bmatrix}$

Based on the above information, answer the following questions

(i) The matrix U$_1$ + U$_2$ is equal to

(a) $\begin{bmatrix}1\\-1\end{bmatrix}$ **(b)** $\begin{bmatrix}2\\-2\end{bmatrix}$ **(c)** $\begin{bmatrix}3\\-3\end{bmatrix}$ **(d)** $\begin{bmatrix}4\\-4\end{bmatrix}$

(ii) The value of lUl is

(a) 2 **(b) -2** **(c) 3** **(d) -3**

(iii) If $X=[3 \quad 2]U\begin{bmatrix}3\\2\end{bmatrix}$,then the value of lXl =

(a) 3 **(b) -3** **(c) -5** **(d) 5**

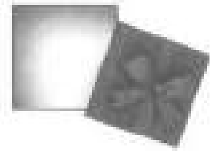
(iv) The minor of element at the position a$_{22}$ in U is

(a) 1 **(b) 2** **(c) -2** **(d) -1**

(v) If $U=[a_{ij}]_{2\times 2}$,then the value of a$_{11}$A$_{11}$ + a$_{12}$A$_{l2}$ where A$_{ij}$ denotes the cofactor of a$_{ij}$ is

(a) **1** **(b) 2** **(c) -3** **(d) 3**

38) Megha wants to prepare a handmade gift box for her friend's birthday at home. For making lower part of box, she takes a square piece of cardboard of side 20 cm.



Based on the above information, answer the following questions.

(i) If x cm be the length of each side of the square cardboard which is to be cut off from corners of the square piece of side 20 cm, then possible value of x will be given by the interval

(a) [0,20] **(b) (0,10)** **(c) (0,3)** **(d) None of these**

(ii) Volume of the open box formed by folding up the cutting corner can be expressed as

(a) V = x(20 - 2x)(20 - 2x) **(b)** $V=\frac{x}{2}(20+x)(20-x)$

(c) $V=\frac{x}{3}(20-2x)(20+2x)$ **(d) V = x(20 - 2x)(20 - x)**

(iii) The values of x for which $\frac{dV}{dx}=0$,are

(a) 3,4 **(b)** $0, \frac{10}{3}$ **(c) 0,10** **(d)** $10, \frac{10}{3}$

(iv) Megha is interested in maximising the volume of the box. So, what should be the side of the square to be cut off so that the volume of the box is maximum?

(a) 12 cm **(b) 8 cm** **(c)** $\frac{10}{3}$ cm **(d) 2cm**

(v) The maximum value of the volume is

(a) $\frac{17000}{27}$ cm^3 **(b)** $\frac{11000}{27}$ cm^3 **(c)** $\frac{8000}{27}$ cm^3 **(d)** $\frac{16000}{27}$ cm^3

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MODEL PAPER 1

12th Standard CBSE

Maths

General Instructions: 1. This question paper contains two parts A and B. Each part is compulsory. Part A carries 24 marks and Part B carries 56 marks 2. Part-A has Objective Type Questions and Part -B has Descriptive Type Questions 3. Both Part A and Part B have choices

Part – A: 1. It consists of two sections- I and II. 2. Section I comprises of 16 very short answer type questions. 3. Section II contains 2 case studies. Each case study comprises of 5 case-based MCQs. An examinee is to attempt any 4 out of 5 MCQs.

Part – B: 1. It consists of three sections- III, IV and V. 2. Section III comprises of 10 questions of 2 marks each. 3. Section IV comprises of 7 questions of 3 marks each. 4. Section V comprises of 3 questions of 5 marks each. 5. Internal choice is provided in 3 questions of Section –III, 2 questions of SectionIV and 3 questions of Section-V. You have to attempt only one of the alternatives in all such questions

) Let R be a relation on the set L of lines defined by l_1 R l_2 if l_1 is perpendicular to l_2, then relation R is 1

(a) reflexive and symmetric (b) symmetric and transitive (c) equivalence relation (d) symmetric

) If a matrix has 6 elements, then number of possible orders of the matrix can be 1

(a) 2 (b) 4 (c) 3 (d) 6

) Given ∫ 2^x dx = f(x) + C, then f(x) is 1

(a) 2^x (b) $2^x \log_e 2$ (c) $\frac{2^x}{log_e 2}$ (d) $\frac{2^x}{log_e 2}$

) $\begin{vmatrix} x & 2 \\ 18 & x \end{vmatrix} = \begin{vmatrix} 6 & 2 \\ 18 & 6 \end{vmatrix}$, then x is equal to 1

(a) 6 (b) ±6 (c) -6 (d) 0

) The rate of change of the area of a circle with respect to its radius r at r = 6 cm is 1

(a) 10π (b) 12π (c) 8π (d) 11π

6) The total revenue received from the sale of x units of a product is given by $R(x)=3x^2+36x+5$.Find the marginal revenue when x=5. 1

7) Evaluate $\begin{vmatrix} \cos 15^\circ & \sin 15^\circ \\ \sin 75^\circ & \cos 75^\circ \end{vmatrix}$ 1

8) Integrate $\left(\frac{2a}{\sqrt{x}} - \frac{b}{x^2} + 3c\sqrt[3]{x^2}\right)$ w.r.t. x 1

9) c 1

$\int \frac{x^3+5x^2-4}{x^2} dx$

10) Write a vector in the direction of the vector $\hat{i} - 2\hat{j} + 2\hat{k}$ that has magnitude 9 units. 1

11) a) Find the distance of a point (2, 3, 4) from X-axis. 1

(OR)

b) Events E and F are such that P (not E or not F) = 0.25. State whether E and F are mutually exclusive. 1

2) Given functions f(x) = $\frac{x^2-4}{x-2}$ and g(x) = x + 2, x < = R. Then which of the following is 1

(a) f is continuous at x = 2, g is continuous at x = 2

(b) f is continuous at x = 2, g is not continuous at x = 2

(c) f is not continuous at x = 2, g is continuous at x = 2

(d) f is not continuous at x = 2, g is not continuous at x = 2

3) The total revenue Rupees in received from the sale of x units of an article is given by R(x) = 3x² + 36x + 5. The marginal revenue when x = 15 is 1

(a) 126 (b) 116 (c) 96 (d) 90

4) a) Examine the continuity of the following functions at the given point. 1

f(x) = 5x - 3 at x = -3

(OR)

b) The radius of a circle is increasing at the rate of 0.7 crri/s. What is the rate of increase of its circumference? 1

5) a) Find the order and degree of the differential equation $\left[1+\left(\frac{dy}{dx}\right)^2\right]^{3/2} = \frac{d^2y}{dx^2}$ 1

(OR)

b) Solve the following differential equation 1

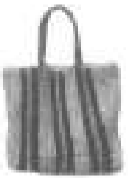
$\frac{dy}{dx} = x^3e^{-2y}$

16) a) Let R=[(a,a^2) : a is a prime number less than 5) be a relation Find the range of R 1

(OR)

b) Write the number of all possible matrices of order 2 x 2 with each entry 1,2 or 3. 1

17) Three shopkeepers Salim, Vijay and Venket are using polythene bags, handmade bags (prepared by prisoners) and newspaper's envelope as carry bags. It is found that the shopkeepers Salim, Vijay and Venket are using (20,30,40), (30,40,20) and (40, 20, 30) polythene bags, handmade bags and newspaper's envelopes respectively. The shopkeepers Salim, Vijay and Venket spent Rs.250, Rs.270 and Rs.200 on these carry bags respectively.



Using the concept of matrices and determinants, answer the following questions.

(i) What is the cost of one polythene bag?

(a) Rs.1 (b) Rs.2 (c) Rs.3 (d) Rs.5

(ii) What is the cost of one handmade bag?

(a) Rs.1 (b) Rs.2 (c) Rs.3 (d) Rs. 5

(iii) What is the cost of one newspaper envelope

(a) Rs.1 (b) Rs.2 (c) Rs.3 (d) Rs. 5

(iv) Keeping in mind the social conditions, which shopkeeper is better?

(a) Salim (b) Vijay (c) Venket (d) None of these

(v) Keeping in mind the environmental conditions, which shopkeeper is better?

(a) Salim (b) Vijay (c) Venket (d) None of these

18) Two farmers Shyam and Balwan Singh cultivate only three varieties of pulses namely Urad, Masoor and Mung. The sale (in Rs.) of these varieties of pulses by both the farmers in the month of September and October are given by the following matrices A and B. 4

September sales (in ₹)

$$A = \begin{array}{c} \begin{array}{ccc} \text{Urad} & \text{Masoor} & \text{Mung} \end{array} \\ \begin{bmatrix} 10000 & 20000 & 30000 \\ 50000 & 30000 & 10000 \end{bmatrix} \end{array} \begin{array}{l} \text{Shyam} \\ \text{Balwan Singh} \end{array}$$

October sales (in ₹)

$$B = \begin{array}{c} \begin{array}{ccc} \text{Urad} & \text{Masoor} & \text{Mung} \end{array} \\ \begin{bmatrix} 5000 & 10000 & 6000 \\ 20000 & 10000 & 10000 \end{bmatrix} \end{array} \begin{array}{l} \text{Shyam} \\ \text{Balwan Singh} \end{array}$$

Jsing algebra of matrices, answer the following questions.

(i) The combined sales of Masoor in September and October, for farmer Balwan Singh, is

(a) Rs. 30000	**(b) Rs. 90000**	**(c) Rs. 40000**	**(d) Rs. 135000**

(ii) The combined sales of Urad in September and October, for farmer Shyam is

(a) Rs. 20000	**(b) Rs. 30000**	**(c) Rs. 36000**	**(d) Rs. 15000**

(iii) Find the decrease in sales of Mung from September to October, for the farmer Shyam.

(a) Rs. 24000	**(b) Rs. 10000**	**(c) Rs. 30000**	**(d) No change**

(iv) If both farmers receive 2% profit on gross sales, compute the profit for each farmer and or each variety sold in October.

(a) $\begin{array}{c} \begin{array}{ccc} \text{Urad} & \text{Masoor} & \text{Mung} \end{array} \\ \begin{bmatrix} 100 & 200 & 220 \\ 400 & 300 & 200 \end{bmatrix} \end{array} \begin{array}{l} \text{Shyam} \\ \text{Balwan Singh} \end{array}$ (b) $\begin{array}{c} \begin{array}{ccc} \text{Urad} & \text{Masoor} & \text{Mung} \end{array} \\ \begin{bmatrix} 100 & 200 & 120 \\ 400 & 200 & 200 \end{bmatrix} \end{array} \begin{array}{l} \text{Shyam} \\ \text{Balwan Singh} \end{array}$ (c) $\begin{array}{c} \begin{array}{ccc} \text{Urad} & \text{Masoor} & \text{Mung} \end{array} \\ \begin{bmatrix} 130 & 200 & 220 \\ 400 & 250 & 200 \end{bmatrix} \end{array} \begin{array}{l} \text{Shyam} \\ \text{Balwan Singh} \end{array}$ (d) $\begin{array}{c} \begin{array}{ccc} \text{Urad} & \text{Masoor} & \text{Mung} \end{array} \\ \begin{bmatrix} 100 & 200 & 120 \\ 250 & 200 & 220 \end{bmatrix} \end{array} \begin{array}{l} \text{Shyam} \\ \text{Balwan Singh} \end{array}$

(v) Which variety of pulse has the highest selling value in the month of September for the armer Balwan Singh?

(a) Jrad	**(b) Masoor**	**(c) Mung**	**(d) All of these have the same price**

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9) a) 2

If $A = \begin{bmatrix} 1 & 4 \\ 3 & 2 \\ 2 & 1 \end{bmatrix}$ $B = \begin{bmatrix} 5 & 2 \\ -1 & 0 \\ 1 & 1 \end{bmatrix}$, then find the matrix X for which A + B - X = 0.

(OR)

b) Define Reflexive.Give one example. 2

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20) Define symmetric Relation.Give one example 2

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21) Find the unit vector in the direction of $\vec{a}+\vec{b}$ if $\vec{a}=2\hat{i}+\hat{j}+3\hat{k}$, and $\vec{b}=\hat{i}+2\hat{j}-\hat{k}$ 2

22) One card is drawn is drawn from a pack of 52 cards. Find the probability of getting : 2

(a) a red card

(b) a jack of hearts

(c) a black face card

(d) a king.

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23) Find the direction cosines of the vector joining the points A(I, 2, - 3) and B(- 1, - 2, 1) directed from B to A. 2

---

24) Show that the function f (x)= $\begin{cases} x^3+3 & , \quad if \quad x \neq 0 \\ 1 & , \quad if \quad x = 0 \end{cases}$ is not continuous at x=0. 2

---

5) a) Show that the solution of differential equation : $y = \left(2x^2 - 1\right) + ce^{-x^2}$ is $\frac{dy}{dx} + 2xy - 4x^3 = 0$ 2

(OR)

b) From the differential equation of equation y = a cos2x + b sin2x, where a and b are constant. 2

---

5) a) $\int sin2xcos3xdx$ 2

(OR)

b) $\int \frac{dx}{1+sinx}$ 2

---

7) a) If f(x) = x cos x + e^x, then find f'(0). 2

(OR)

b) Verify that the function y = a cos x + b sin x, where a, b ϵ R is a solution of differential equation $\frac{d^2y}{dx^2} + y = 0$. 2

---

3) a) Find the value of $\lim_{x \to 3} \frac{x^2+2x-15}{x-3}$ 2

(OR)

b) If a*b=3a+4b, then value 3*4 is........... 2

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29) If $x \begin{bmatrix} 2 \\ 3 \end{bmatrix} + y \begin{bmatrix} -1 \\ 1 \end{bmatrix} = \begin{bmatrix} 10 \\ 5 \end{bmatrix}$, find the values of x and y. 3

---

30) $Find \quad \frac{dy}{dx} \quad if \quad y + sin \quad y = cos \quad x$ 3

---

31) If P(A)=$\frac{7}{13}$,P(B)=$\frac{9}{13}$ and P$(A \cap B)$=$\frac{4}{13}$,evaluate P(A/B). 3

---

32) Find the area of the region bounded by the curve y^2 = x and the lines x = 1 , x = 4 and the x - axis. 3

33) Differentiate $\frac{\sqrt{a+x}+\sqrt{a-x}}{\sqrt{a+x}-\sqrt{a-x}}$ 3

34) A relation R on a set A is said to be an equivalence relation on A iff it is 3

(a) Reflexive i.e.., $(a, a) \in R \,\forall\, a \in A$

(b) Symmetric i.e., $(a, b) \in R \Rightarrow (b, a) \in R \,\forall\, a, b \in A$

(c) Transitive i.e., $(a, b) \in R$ and $(b, c) \in R \Rightarrow (a, c) \in R \,\forall\, a, b, c \in A$

Based on the above information, answer the following questions.

(i) If the relation R = {(1, 1), (1, 2), (1, 3), (2,2), (2, 3), (3,1), (3, 2), (3, 3)} defined on the set A = {1, 2, 3}, then R is

(a) reflexive (b) symmetric (c) transitive (d) equivalence

(ii) If the relation R = {(1, 2), (2,1), (1, 3), (3, I)} defined on the setA = {1, 2, 3}, then R is

(a) reflexive (b) symmetric (c) transitive (d) equivalence

(iii) If the relation R on the set N of all natural numbers defined as R = {(x, y) : y = x + 5 and x < 4}, then R is

(a) reflexive (b) symmetric (c) transitive (d) equivalence

(iv) If the relation R on the set A = {1, 2, 3, , 13, 14}defined as R = {(x, y) : 3x - y = 0}, then R s

(a) reflexive (b) symmetric (c) transitive (d) equivalence

35) a) Find dy/dx in the following: $ax + by^2 = cos \quad y$ 3

(OR)

b) Find dy/dx in the following: $x^3 + x^2y + xy^2 + y^3 = 81$ 3

36) a) Three friends A, B and C are playing a dice game. The numbers rolled up by them in their first three chances were noted and given by A = {l, 5},B = {2, 4, 5} and C = {l, 2, 5} as A reaches the cell 'SKIP YOUR NEXT TURN' in second throw. 5

Based on the above information, answer the following questions.

(i) P (A I B) =

$(a)\ \frac{1}{6}$ $(b)\ \frac{1}{3}$ $(c)\ \frac{1}{2}$ $(d)\ \frac{2}{3}$

(ii) P (B I C) =

$(a)\ \frac{2}{3}$ $(b)\ \frac{1}{12}$ $(c)\ \frac{1}{9}$ $(d)\ 0$

(iii) P (A ∩ B I C) =

$(a)\ \frac{1}{6}$ $(b)\ \frac{1}{2}$ $(c)\ \frac{1}{12}$ $(d)\ \frac{1}{3}$

(iv) P (A I C) =

$(a)\ \frac{1}{4}$ $(b)\ 1$ $(c)\ \frac{2}{3}$ **(d) None of these**

(v) P (A ∪ B I C) =

$(a)\ 0$ $(b)\ \frac{1}{2}$ $(c)\ \frac{2}{3}$ $(d)\ 1$

(OR)

b) If $A = \begin{pmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{pmatrix}$, prove that $A^3 - 6A^2 + 7A + 2I = 0$ 5

7) a) Using integration, find the area of the region bounded by the curve $x^2 = 4y$ and the line

x = 4y - 2. 5

(OR)

b) Find : $\int \frac{x^2}{(x^2+1)(x^2+4)}dx$ 5

3) a) Find the distance of the point$3\hat{i}-2\hat{j}+\hat{k}$from the plane 3x+y-z+2=0 measured parallel to the line $\frac{x-1}{2}=\frac{y+2}{-3}=\frac{z-1}{1}$. Also, find the foot of the . Also, find the foot of the perpendicular from the given point upon the given plane. 5

(OR)

b) Find graphically, the maximum value of Z = 2x + 5y, subject to constraints given below: $2x + 4y \leq 8 \Rightarrow x + 2y \leq 4$ 5

$3x + y \leq 6$

$x + y \leq 4$

$x \geq 0, y \geq 0$

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