Alternating Current LONG ANSWERS QUESTIONS

12th Standard

Physics

5 Marks 39 x 5 = 195

1) A sinusoidal voltage of peak value 283 V and frequency 50 Hz is applied to a series LCR circuit in which R = 3 Ω, L = 25.48 mH, and C = 796 μF. Find (a) the impedance of the circuit; (b) the phase difference between the voltage across the source and the current; (c) the power dissipated in the circuit; and (d) the power factor.

Answer : (a) To find the impedance of the circuit, we first calculate X_{L} and X_{C}.

$X_L = 2\pi vL$

$= 2 \times 3.14 \times 50 \times 25.48 \times 10^{-3}\Omega = 8\Omega$

$X_c = \frac{1}{2\pi vC} = \frac{1}{2\times3.14\times50\times796\times10^{-6}} = 4\Omega$

Therefore.

$Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{3^2 + (8-4)^2} = 5\Omega$

$Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{3^2 + (8-4)^2} = 5\Omega$

(b) Phase difference, $\phi = \tan^{-1}\frac{X_C - X_L}{R}$

$= \tan^{-1}\left(\frac{4-8}{3}\right) = -53.1^{\circ}$

Since ϕ is negative, the current in the circuit lags the voltage across the source.

(c) The power dissipated in the circuit is

$P = I^2 R$

Now, $I = \frac{i_m}{\sqrt{2}} = \frac{1}{\sqrt{2}}\left(\frac{283}{5}\right) = 40\ \mathrm{A}$

Therefore, $P = (40\ \mathrm{A})^2 \times 3\Omega = 4800\ \mathrm{W}$

(d) Power factor $= \cos\phi = \cos(-53.1^{\circ}) = 0.6$

2) A resistor of 200 Ω and a capacitor of 15.0 μF are connected in series to a 220 V, 50 Hz ac source. (a) Calculate the current in the circuit; (b) Calculate the voltage (rms) across the resistor and the capacitor. Is the algebraic sum of these voltages more than the source voltage? If yes, resolve the paradox.

Answer : Given

$R = 200\Omega$, $C = 15.0\mu F = 15.0 \times 10^{-6} F$

$V = 220$ V, $v = 50$ Hz

(a) In order to calculate the current, we need the impedance of the circuit. It is

$$Z = \sqrt{R^2 + X_C^2} = \sqrt{R^2 + (2\pi v C)^{-2}}$$

$$= \sqrt{(200\Omega)^2 + \left(2 \times 3.14 \times 50 \times 15.0 \times 10^{-6}\ \mathrm{F}\right)^{-2}}$$

$$= \sqrt{(200\Omega)^2 + (212.3\Omega)^2}$$

= 291.67Ω

Therefore, the current in the circuit is

$$I = \frac{V}{Z} = \frac{220\ \mathrm{V}}{291.5\Omega} = 0.755\ \mathrm{A}$$

(b) Since the current is the same throughout the circuit, we have

$$V_R = IR = (0.755\ \mathrm{A})(200\Omega) = 151\ \mathrm{V}$$

$$V_C = IX_C = (0.755\ \mathrm{A})(212.3\Omega) = 160.3\ \mathrm{V}$$

The algebraic sum of the two voltages, V_R and V_C is 311.3 V which is more than the source voltage of 220 V. How to resolve this paradox? As you have learnt in the text, the two voltages are not in the same phase. Therefore, they cannot be added like ordinary numbers. The two voltages are out of phase by ninety degrees. Therefore, the total of these voltages must be obtained using the Pythagorean theorem:

$$V_{R+C} = \sqrt{V_R^2 + V_C^2}$$

= 220 V

Thus, if the phase difference between two voltages is properly taken into account, the total voltage across the resistor and the capacitor is equal to the voltage of the source.

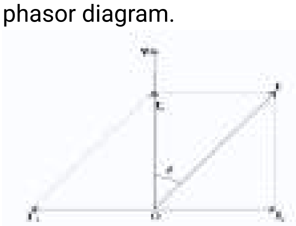
3) (a) For circuits used for transporting electric power, a low power factor implies large power loss in transmission. Explain.

(b) Power factor can often be improved by the use of a capacitor of appropriate capacitance in the circuit. Explain

Answer : (a) We know that P = I V cosϕ where cosϕ is the power factor. To supply a given power at a given voltage, if cosϕ is small, we have to increase current accordingly. But this will lead to large power loss (I^2R) in transmission.

(b) Suppose in a circuit, current I lags the voltage by an angle ϕ. Then power factor ϕ = R/Z.

We can improve the power factor (tending to 1) by making Z tend to R. Let us understand, with the help of a phasor diagram.



how this can be achieved. Let us resolve I into two components. I_p along the applied voltage V and I_q perpendicular to the applied voltage. I_q as you have learnt in Section 7.7, is called the wattless component since corresponding to this component of current, there is no power loss. I_P is known as the power component because it is in phase with the voltage and corresponds to power loss in the circuit.

It's clear from this analysis that if we want to improve power factor, we must completely neutralize the lagging wattless current I_q by an equal leading wattless current I'_q. This can be done by connecting a capacitor of appropriate value in parallel so that I_q and I'_q cancel each other and P is effectively I_p V.

4) A 100 Ω resistor is connected to a 220 V, 50 Hz ac supply

(a) What is the rms value of current in the circuit?

(b) What is the net power consumed over a full cycle?

Answer : Given: The values of resistor is 100 Ω and the supply voltage is 100 V. (a) The RMS current is given as,
I= V R
Where, the supply voltage is V and the value of resistor is R.
By substituting the given values in the above equation, we get,
I= 220 100 =2.2 A
Thus, the value of RMS current in the conductor is 2.2 A.
(b) Power consumed over a full cycle is given as,
P=VI
Where, the supply voltage is V and the RMS current is I.
By substituting the given values in the above equation, we get
P=220×2.2 =484 W
Thus, power consumed over a full cycle is 484 W.

5) (a) The peak voltage of an AC supply is 300 V. What is its rms voltage?
(b) The rms value of current in an AC circuit is 10 A. What is the peak current?

Answer : a) Given: The peak voltage of supply is 100 V.
The rms voltage is give as,
v m = 2 ×V
Where, the peak value of supply voltage is v m and its rms value is V.
By substituting the given values in the above equation, we get
300= 2 ×V V= 300 2 =212.1 V
Thus, the value of rms voltage is 212.1V.
b) Given: The rms current in an ac circuit is 10 A.
The peak current in the circuit is given as,i m = 2 ×I
Where, the peak current in an ac circuit is i m and its rms value is I.
By substituting the given values in the above equation, we get
i m = 2 ×10 =14.1 A
Thus, the value of peak current in the given ac circuit is 14.1 A.

6) A series LCR circuit with R = 20 Ω, L = 1.5 H and C = 35 μF is connected to a variable-frequency 200 V ac supply. When the frequency of the supply equals the natural frequency of the circuit, what is the average power transferred to the circuit in one complete cycle?

Answer : The supply frequency and the natural frequency are equal at resonance condition in the circuit.
Given Resistance of the resistor, R = 20 Ω
Given Inductance of the inductor, L = 1.5 H
Given Capacitance of the capacitor , C = 35 μF = 30 x 10^{-6} F
An AC source with a voltage of V = 200 V is connected to the LCR circuit,
We know that the Impedance of the above combination can be calculated by the following relation,

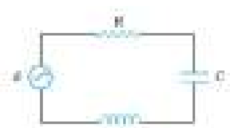
$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

At resonant condition in the circuit , $X_L = X_C$
Therefore , Z = R = 20 Ω
We know that Current in the network is given by the relation :

$$I = \frac{V}{Z} = \frac{200}{20} = 10A$$

Therefore, the average power that is being transferred to the circuit in one full cycle :
V I = 200 x 10 = 2000 W

7) Given bellow shows a series LCR circuit connected to a variable frequency 230 V source. L = 5.0 H, C = 80μF, R = 40 Ω.



(a) Determine the source frequency which drives the circuit in resonance.
(b) Obtain the impedance of the circuit and the amplitude of current at the resonating frequency.
(c) Determine the rms potential drops across the three elements of the circuit. Show that the potential drop across the LC combination is zero at the resonating frequency.

Answer : Given that the Inductance of the inductor in the circuit is, L = 5.0 H
Given that the Capacitance of the capacitor in the circuit is , C = 80 μH = 80 x 10^{-6} F
Given that Resistance of the resistor in the circuit, R = 40 Ω
Value of Potential of the variable voltage supply, V = 230 V
(a) We know that the Resonance angular frequency can be obtained by the following relation :
$\omega_r = \frac{1}{\sqrt{LC}} \omega_r = \frac{1}{\sqrt{5x80x10-6}} \omega_r = \frac{10^3}{20} = 50 \mathrm{rad/sec}$
Thus, the circuit encounters resonance at a frequency of 50 rad/s.
(b) We know that the Impedance of the circuit can be calculated by the following relation :
$Z = \sqrt{R^2 + (X_L - X_C)^2}$
At resonant condition,
$X_L = X_C$
Z = R = 40 Ω
At resonating frequency amplitude of the current can be given by the following relation :
$I_0 = \frac{V_0}{Z}$
where,
V_0 = peak voltage = $\sqrt{2}V$
Therefore,
$I_0 = \frac{\sqrt{2}V}{Z} = \frac{\sqrt{2}\times 230}{40} = 8.13 \text{ A}$
Thus, at resonant condition, the impedance of the circuit is calculated to be 40 Ω and the amplitude of the current is found to be 8.13 A
c) rms potential drop across the inductor in the circuit,
$(V_L)_{rms}$ = I x ω_r L
Where,
$I_{rms} = \frac{I_0}{\sqrt{2}} = \frac{\sqrt{2}V}{\sqrt{2}Z} = \frac{230}{40} = \frac{23}{4} A$
Therefore, $(V_L)_{rms}$
$\frac{23}{4} \times 50 \times 5 = 1437.5 \text{ V}$
We know that the Potential drop across the capacitor can be calculated with the following relation :
$(V_c)_{rms} = I \times \frac{1}{\omega_r C} = \frac{23}{4} \times \frac{1}{50\times 80 \times 10^{-6}} = 1437.5V$
We know that the Potential drop across the resistor can be calculated with the following relation :
$(V_R)_{rms} = IR = \frac{23}{4} \times 40 = 230 \text{ V}$
Now, Potential drop across the LC connection can be obtained by the following relation :
$V_{LC} = I(X_L - X_C)$
At resonant condition,
$X_L = X_C$
$V_{LC} = 0$
Therefore, it has been proved from the above equation that the potential drop across the LC connection is equal to zero at a frequency at which resonance occurs.

8) A capacitor of unknown capacitance, a resistance of 100 ohm and an inductor of self inductance L = $4/\pi^2$ henry are connected in series across an a.c. source of 200 V and 50 hz. Calculate the value of capacitance and the current that flows in the circuit, when the current is in phase with the voltage.

Answer : $Here,\ R = 100\Omega,\ L = \frac{4}{\pi^2}H,\ E_v = 200V,\ v = 50Hz,\ C = ?,\ I_v = ?$
$As\ current\ is\ in\ phase\ with\ voltage,\ therefore,$
$X_C = X_L\ ;\ \frac{1}{\omega C} = \omega L,$
$C = \frac{1}{\omega^2 L} = \frac{1}{(2\pi v)^2 L}$
$C = \frac{1}{4\pi^2 v^2 L} = \frac{1\times\pi^2}{4\times\pi^2\times 50^2\times 4} = 25\times 10^{-6} = 25\mu F$
$As\ X_C = X_L,\ \therefore\ Z = R = 100\Omega$
$I_v = \frac{E_v}{Z} = \frac{200}{100} = 2A$

9) A resistor of 12Ω, a capacitor of reactance 14 ohm and a pure inductor of inductance 0.1H are joined in series and placed across 200V, 50Hz a.c. supply. Calculate

(i) current in the circuit
(ii) phase angle between current and voltage. Take π = 3

Answer : $Here,\ R = 12\Omega,\ X_C = 14ohm,\ L = 0.1H$
$E_v = 20V,\ v = 50hz,\ I_v = ?,\ \phi = ?$
$X_L = \omega L = 2\pi v L = 2 \times 3 \times 50 \times 0.1 = 30\ ohm$
$Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{12^2 + (30 - 14)^2} = 20ohm$
$I_v = \frac{E_v}{Z} = \frac{200}{20} = 10A$
$tan\phi = \frac{X_L - X_C}{R} = \frac{30-14}{12} = 1.33$
$\phi = tan^{-1}(1.33) = 53.13^\circ$

10) An armature coil consists of 20 turns of wire each of area A=0.09 m^2 and total resistance 15.0 ohm. It rotates in a magnetic field of 0.5 T at a constant frequency of $\frac{150}{\pi}$ Hz. Calculate the value of
(i) maximum and
(ii) average induced e.m.f. produced in the coil.

Answer : $Here,\ N = 20,\ A = 0.09\ m^2\ R = 15.0ohm,\ B = 0.5T$
$v = \frac{150}{\pi}Hz,\ e_0 = ?$
$e_0 = NAB\omega - NAB(2\pi v) = 20 \times 0.09 \times 0.5\left(2\pi \times \frac{150}{\pi}\right) = 270V$
As e.m.f. produced is alternating, average induced e.m.f. produced in the coil is zero.

11) (i) A voltage V = $V_0\ sin\ \omega t$ applied to a series L-C-R circuit derives a current I = $I_0\ sin\ \omega t$ in the circuit. Deduce the expression for the average power dissipated in the circuit.
(ii) For circuits used for transporting electric power, a low power factor implies large power loss in transmission. Explain.
(iii) Define the term wattless current.

Answer : (i) Average power delivered by an AC circuit is
$P_{av} = V_{rms}I_{rms}cos\phi$
where is minimum, the power delivered is minimum and hence, power dissipated will be maximum for the circuit.

12) Anand on entering his apartment, switched ON the tube light, but it did not work. So he called the electrician. The electrician inspected the tube light and suggested a replacement of the choke. On replacing the choke, Anand found the tube light working.
(i) What is the function of choke?
(ii) What type of choke coil is used to reduce high frequency of alternating current?
(iii) Identify the values exhibited here.

Answer : (i) To reduce the current in the circuit without any heat loss.
(ii) Choke coils with air core are used. These are called rf choke coils.
(iii) Concern for conserving energy and to avoid short circuits.

13) Subhash wanted to see the work of a transformer. He bought a transformer from a shop. He connected the primary to an a.c. supply. At that time an aluminium ring in his hand falls into the core of the transformer. Without noticing that he switched on the power supply. The aluminium ring flew yp into the air. He became panic. His father, an electrical engineer in Electricity Board explained the reason.
(a) What value does he exhibit ?
(b) Bring ou the reason for the above activity.

Answer : (a) Curiosity
(b) Induced current in the aluminium ring acts in the opposite direction to than in the coil and so magnetic field.

14) A series LCR circuit is connected to an ac source having voltage V = V_m sin cot. Derive the expression for the instantaneous current I and its phase relationship to the applied voltage.
Obtain the condition for resonance to occur. Define 'power factor'. State the conditions under which it is (i) maximum and (ii) minimum.

Answer : Phase difference between voltage and current,

$tan\ \phi = \frac{X_L - X_C}{R}$

and $I_0 = \frac{V_0}{Z} = \frac{V_0}{\sqrt{(X_L - X_C)^2 + R^2}}$

$\therefore$ Expression of AC,

$I = I_0 \sin\left(\omega t - \phi\right)$

Conditions for resonanceInductive reactance must be equal to capacitive reactance

i.e. $X_L = X_C$

As, $X_L = X_C$

$\Rightarrow\ \omega_0 L = \frac{1}{\omega_0 C}$

$\Rightarrow\ \omega_0^2 = \frac{1}{LC} \Rightarrow \omega_0 = \frac{1}{\sqrt{LC}}$

where ,ω_0 = resonant angular frequency Impedance becomes minimum and equal to ohmic resistance

i.e. Z = Z. minimum = R

AC becomes maximum

$\therefore\ I_{max} = \frac{V_{max}}{Z_{min}} = \frac{V_{max}}{R}$

Voltage and current arrives in same phase.

Power factor

i.e. $\cos\phi = \frac{P_{av}}{V_{rms} I_{rms}} = \frac{True\ Power}{Apparent\ Power}$

Also, $cos\ \phi = \frac{R}{Z} = \frac{R}{\sqrt{R^2 + (X_L - X_C)^2}}$

The power factor is maximum

i.e. cos ϕ.= + 1, in L-C-R series AC circuit when circuit is in resonance. The power factor is minimum when phase angle between V and I is 90°, i.e. either pure inductive circuit or pure capacitive AC circuit.

15) (a) Draw a labelled diagram of an a.c. generator and state its working principle.

(b) How is magnetic flux linked with the armature coil changed in a generator ?

(c) Derive the expression for maximum value of the induced emf and state the rule that gives the direction of the induced emf.

(d) Show the variation of the emf generated versus time as the armature is rotated with respect to the direction of the magnetic field.

Answer : 

(a) It works on the principle of electromagnetic induction, i.e. when a coil continuously rotates in a magnetic field, the magnetic flux associated with it keeps on changing; thus an emf is induced in it.

(b) When the coil rotates in a magnetic field, its effective area i.e. A cos S, (i.e. area normal to the magnetic field) keeps on changing. Hence magnetic flux NBA cos S, keeps on changing.

(c) Let the coil be rotating with angular velocity 'm', at any instant 't' when the normal to the plane of the coil makes an angle t with the magnetic field. Hence magnetic flux,

$\emptyset = NBA\cos\omega$,therefore induced emf (ϵ)

$\epsilon = -\frac{d\epsilon}{dt}$

$\Rightarrow \epsilon = NBA\omega\sin\omega t$

Induced emf will be maximum when $\omega t = 90^\circ$

Hence,$\epsilon_{max} = NBA\omega$

The direction of induced emf can be determined using Fleming's right hand rule. Alternatively, Statement of the above rule.

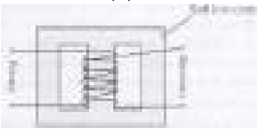
(d) 

16) (a) Draw a schematic arrangement for winding of primary and secondary coil in a transformer when the two coils are wound on top of each other.

(b) State the underlying principle of a transformer and obtain the expression for the ratio of secondary to primary voltage in terms of the

(i) number of secondary and primary windings and
(ii) primary and secondary currents.
(c) Write the main assumption involved in deriving the above relations.
(d) Write any two reasons due to which energy losses may occur in actual transformers.

Answer : **(a)**



(b) Principle of a transformer: When alternating current flows through the primary coil, an emf is induced in the neighbouring (secondary) coil

Let $\frac{d\phi}{dt}$ be the rate of change of flux through each turn of the primary and the secondary coil

$\frac{\varepsilon_1}{\varepsilon_2} = -N_1\frac{d\phi}{dt} / -N_2\frac{d\phi}{dt} = \frac{N_1}{N_2}$

$\frac{V_1}{V_2} = \frac{N_1}{N_2}$

But for an ideal transformer,

$\frac{V_1}{V_2} = \frac{I_2}{I_1}$

From equation (1) and (2)

$\frac{V_1}{V_2} = \frac{N_1}{N_2} = \frac{I_2}{I_1}$

(c) Main assumptions

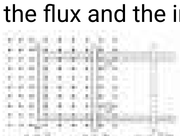
(i) The primary resistance and current are small

(ii) The flux linked with the primary and secondary coils is same / there is no leakage of flux from the core.

(iii) Secondary current is small.

(d) Reason due to which energy loses may occur Flux leakage/Resistance of the coils/Eddy currents/Hysteresis.

17) State Faraday's law of electromagnetic induction. Figure shows rectangular conductor PQRS in which the conductor PQ is free to move in a uniform magnetic field B perpendicular to the plane of the paper. The field extends from x = 0 to x = b and is zero for x > b. Assume that only the arm PQ possesses resistance r. When the arm PQ is pulled outward from x = 0 to x =2b and is then moved backward to x = 0 with constant speed v. Obtain the expressions for the flux and the induced emf. Sketch the variations of these quantities with distance 0 < x < 2b.



Answer : The magnitude of the induced emf in a circuit is equal to the time rate of change of magnetic flux through the circuit.

Mathematically, the induced emf is given by

$\varepsilon = \frac{-d\phi}{dt}$

First consider the forward motion from x = 0 to x =2b

The flux ϕ_B linked with the section SPQR is

$\phi_B = Blx, o \leq x \leq b$

= Blb, b $\leq$ x $\leq$ 2b

The Induced emf is,

$\varepsilon = \frac{d\phi_B}{dt}$

= Blv 0 $\leq$ x $\leq$ b

= 0 b $\leq$ x $\leq$ 2b

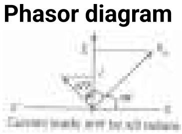
18) State the working of a.c generator with the help of a labelled diagram The coil of an a.c.generator having N turns each of area A, is rotated with a constant angular velocity ω Deduce the expression for the alternating e.m.f generated in the coil What the source of energy generation this device?

Answer : It works on the principle of electromagnetic induction, i.e. when a coil continuously rotates in a magnetic field, the magnetic flux associated with it keeps on changing; thus an emf is induced in it.
(b) When the coil rotates in a magnetic field, its effective area i.e. A cos θ , (i.e. area normal to the magnetic field) keeps on changing. Hence magnetic flux ϕ = NBAcos S keeps on changing.
(c) Let the coil be rotating with angular velocity 'm',
at any instant 't' when the normal to the plane of the coil makes an angle θ with the magnetic field. Hence magnetic flux, 12 ϕ = NBAcos mt, Therefore induced emf (e)
$\epsilon = -\frac{d\phi}{dt}$
$\Rightarrow \epsilon = NBA\omega sin\ \omega t$
Induced emf will be maximum when mt = 90° 12 Hence, £max = NBAω
Direction of induced emf can be determined using Flemming's Right hand rule. Alternatively: Statement of the above rule.

19) A device X is connected to an AC source, V = V_0 sin rot. The variation of voltage, current and power in one cycle is shown in the following graph.

Identify the device X.
(i) Which of the curves A, B, and C represents the voltage, current and the power consumed in the circuit? Justify the answer.
(ii) How does its impedance vary with frequency of the AC source? Show graphically.
(iii) Obtain an expression for the current in the circuit and its phase relation with AC voltage.

Answer : Device X is a capacitor.
As, the current is leading voltage by $\frac{\pi}{2}$ radius.
Curve A represents power,
Curve B represents voltage and
Curve C represents current
As, E(t) = E_0 sin ωt
Current, I(t) = I_0 cosωt
As, in the case of capacitor,
I = I_0sin$\left(\omega t + \frac{\pi}{2}\right)$ [current is leading voltage]
Average power, P= E (t)I(t) = $E_0 I_0$cosϕ/2
where, ϕ= phase difference
As, Xc= capacitive reactance = $\frac{1}{C\omega}$
where, ω is angular frequency
So, reactance or impedance decreases with increase in frequency
Graph of Xc versus co is shown below,
Phasor diagram



For a capacitor fed with an AC supply
V = $\frac{q}{C}$ or q = CV
$\therefore$ I = $\frac{dq}{dt} = \frac{X_0}{X_C} sin\left(\omega t + \frac{\pi}{2}\right)$

20) A $2\mu F$ capacitor, $100\ \Omega$ resistor and 8H inductor are connected in series with an AC source.
What should be the frequency of the source such that current drawn in the circuit is maximum? What is this frequency called?
If the peak value of emf of the source is 200 V, find the maximum current.
Draw a graph showing variation of amplitude of circuit current with changing frequency of applied voltage in a series L-C-R circuit for two different values of resistance R_1 and R_2(R_1 > R_2).
Define the term 'Sharpness of" Resonance'. Under what condition, does a circuit become more selective?

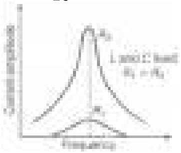
Answer : To draw maximum current from a series L-C-R circuit, the circuit at particular frequency

$X_L = X_C$.

V = $\frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\times314\sqrt{8\times2\times10^{-6}}} = 39.80$ Hz

This frequency is known as the series resonance frequency.

$I_0 = \frac{E_0}{R} = \frac{200}{100} = 2A$



Sharpness of resonance It is defined as the ratio of the voltage developed across the inductance (L) or capacitance (C) at resonance to the voltage developed across the resistance (R).

Q = $\frac{1}{R} = \sqrt{\frac{L}{C}}$

It may also be defined as the ratio of resonance angular frequency to the bandwidth of the circuit

Q = $\frac{\omega_r}{2\triangle\omega}$

Circuit become more selective if the resonance is more sharp, maximum current is more, the circuit is close to resonance for smaller range of $(2\triangle\omega)$ of frequencies. Thus, the tuning of the circuit will be good.

21) An AC source of emf, E = E_0 sin ωt is connected across a series combination of an inductor L, a capacitor C and a resistor, R. Obtain an expression for the equivalent impedance Z of the circuit and hence, find the value of (t) for the AC source for which Z = R.

Show that the phase angle ϕ (between the current flowing in this circuit and the voltage applied to it) can be obtained through the relation.

$\phi = tan^{-1}\left(\frac{Reactive\ impedence}{Resistance}\right)$

Answer : Also, from phasor diagram

$tan\ \phi = \frac{X_L - X_C}{R} = \frac{Reactive\ impedence}{Resistance}$

$\Rightarrow\ \phi = tan^{-1}\left[\frac{Reactive\ impedence}{Resistance}\right]$

Impedance, $Z = \sqrt{R^2 + (X_L - X_C)^2}$

where, Z = R

$\Rightarrow R = \sqrt{R^2 + (X_L - X_C)^2} \Rightarrow (X_L - X_C)^2 = 0$

$X_L = X_C$

$\Rightarrow\ \omega_0 L = \frac{1}{\omega_0 C} \Rightarrow \omega_0 = \frac{1}{\sqrt{LC}}$

where, ω_0 is resonant L-C-R series AC circuit.

22) Use Huygens' geometrical construction to show how a plane wavefront at t = 0 propagates and produces a wavefront at a later time?

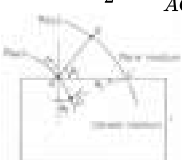
Verify, using Huygens' principle, Snell's law of refraction of a plane wave propagating from a denser to a rarer medium. When monochromatic light is incident on a surface separation two media, the reflected and refracted light both have the same frequency. Explain why?

Answer : Consider a plane wave moving through free space as shown in the figure. At t = 0, the wavefront is indicated by the plane labeled AA'. According to Huygens' principle, Each point on this wavefront is considered a point source. For clarity, only three-point sources on AA' are as shown in the figure below.

With these sources for the wavelets, we draw circular arcs, each of radius c$\triangle$t. where c is the speed of light in vacuum and $\triangle$t is sometime interval during which the wave propagates. The surface drawn target to these wavelets is the plane BB', which is the wavefront at a later time and is parallel to AA'.
Consider ray 1 strikes the surface and the subsequent time interval ray 2 strikes the surface as shown in the given figure. During the time interval. the wave at A sends out a Huygens' wavelet (the light brown are passing through D) and the light refracts into the material. making an angle a2 with the normal to the surface. In the same time interval, the wave at B sends out a Huygens' wavelet (the light brown are passing through C) and the light continues to propagate in the same direction. The radius of the wavelet from A is AD = $v_2\triangle$t, where v_2 is the wave speed in the second medium. The radius of the wavelet from B is BC = $v_1\triangle$t. where VI is the wave speed in the original medium.
From $\triangle$S. ABC and ADC. we find that

$sin\theta_1 = \frac{BC}{AC} = \frac{v_1\triangle t}{AC}$ ...(i)

and $sin\theta_2 = \frac{AD}{AC} = \frac{v_2\triangle t}{AC}$ ..(ii)



On dividing Eq.(i) by the Eq.(ii), we get

$\frac{sin\theta_1}{sin\theta_2} = \frac{v_1}{v_2}$

We know that, $v_1 = \frac{c}{n_1}$ and $v_2 = \frac{c}{n_2}$

Therefore, $\frac{sin\theta_1}{sin\theta_2} = \frac{c/n_1}{c/n_2} = \frac{n_2}{n_1} = 1\mu_2$

Which is Snell's law of refraction. The reflection and refraction phenomenon occur due to interaction of corpuscles of incident light and the atoms of matter on receiving light energy, the atoms are forced to oscillate about their mean positions with the same frequency as incident light. According to Maxwell's classical theory, the frequency of light emitted by a charged oscillator is same as its frequency of oscillation. Thus. the frequency of reflected and refracted light is same as the incident frequency.

23) A plane wavefront approaches a plane surface separating two media. If medium 1 is optically denser and medium 2 is optically rarer, using Huygens' principle, explain and show how a refracted wavefront is constructed?
Verify Snell's law. When a light wave travels from a rarer to a denser medium, the speed decreases. Does it imply a reduction in its energy? Explain.

Answer : Let a plane wavefront AB is incident at the interface XY separating two media such that medium 1 is optically denser than medium 2. Let time t is taken by the wave to reach from B to C,

then, BC = V_1 t ...(i)
where, VI is the velocity of light in medium 1. In the duration of time t, the secondary wavelets emitted from point A gets spread over a hemisphere of radius,
AE = V_2t ...(ii)
in the medium 2 and $v_2 > v_1$.
The tangent plane CE from C over this hemisphere of radius v_2t will be the new refracted wavefront of AB.
It is the evidence that angle of refraction r is greater than angle of incidence i.
By geometry
$\angle N_2AE = \angle EGA = r$ (angle of refraction)
Also, $\angle PAN_1 = \angle BAC = i$ (angle of incidence)
Now, in $\triangle$ABC,
sin i = $\frac{BC}{AC} = \frac{v_1t}{AC}$ [from Eq.(i)]......(iii)
In $\triangle$AEC, sin r = $\frac{AE}{AC} = \frac{v_2t}{AC}$ from Eq.(ii)]..(iv)
Now, $\frac{sin\ i}{sin\ r} = \frac{\frac{v_1t}{AC}}{\frac{v_2t}{AC}}$
$\frac{sin\ i}{sin\ r} = \frac{v_1}{v_2}$
= constant
= $1\mu_2$
Where $1\mu_2$ = refractive index of second medium w.r.t. first medium.
Hence, Snell's law of refraction is verified.
No, energy carried by the wave does not depend on its speed instead, it depends on the amplitude of a wave.

24) Raj who is in class XII was demonstrated with an experiment of Faraday's laws of electromagnetic induction by his Physics teacher. While his teacher was explaining the experiment, her lecture and raised a question that "Is there any possibility of induced emf due to earth's magnetism"? After listening his question, the Physics teacher was stunned for a moment and without giving answer, discusses this question in group discussion. In the group discussion, students came out with correct answer.
(i) Write the values that you learnt from this incident.
(ii) What can be the reason for Rays questions?

Answer : (i) Team spirit and curiosity.
(ii) When the wire in N-S direction is dropped feely, none of the components of earth's magnetic field is intercepted. So, no induced emf is produced. When the wire is dropped freely in E-W direction horizontal component of earth's magnetic field is intercepted. So emf is induced in the coil.

25) Show that in the free oscillations of an LC circuit, the sum of energies stored in the capacitor and the inductor is constant in time.

Answer : Let q_0 be the initial charge on a capacitor. Let the charged capacitor be connected to an inductor of inductance L. As you have studied in this LC circuit will sustain an oscillation with frquency

$\omega\left(=2\pi v=\frac{1}{\sqrt{LC}}\right)$

At an instant t, charge q on the capacitor and the current i are given by:

q (t) = q_o cos ωt

i (t) = - q_o ω sin ωt

Energy stored in the capacitor at time t is

$U_E=\frac{1}{2}CV^2=\frac{1}{2}\frac{q^2}{C}=\frac{q_0^2}{2C}\cos^2(\omega t)$

Energy stored in the inductor at time t is

$U_M=\frac{1}{2}Li^2$

$=\frac{1}{2}Lq_0^2\omega^2\sin^2(\omega t)$

$=\frac{q_0^2}{2C}\sin^2(\omega t)\quad(\because\omega^2=1/\sqrt{LC})$

Sum of energies

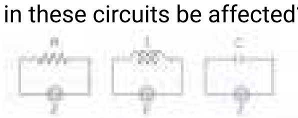
$U_E+U_M=\frac{q_0^2}{2C}\left[\cos^2\omega t+\sin^2\omega t\right]$

$=\frac{q_0^2}{2C}$

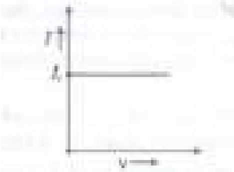
This sum is constant in time as q_o and C, both are time-independent. Note that it is equal to the initial energy of the capacitor.

26) (i) What do you understand by sharpness of resonance in a series L -C-R circuit? Derive an expression for Q-factor of the circuit.

(ii) Three electrical circuits having AC sources of variable frequency are shown in the figures. Initially, the current flowing in each of these is same. If the frequency of the applied AC source is increased, how will the current flowing in these circuits be affected? Give the reason for your answer.



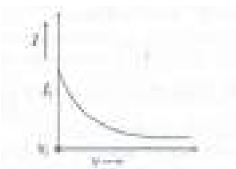
Answer : (ii) Let initially, I_r be current flowing in all the three circuits. If frequency of applied AC source is increased, then the change in current will occur in following manner.
(a) AC circuit containing resistance only where,
V_i = initial frequency of AC source.



Frequency of AC source
There is no effect on current with the increase in frequency.
(b) AC circuit containing inductance only with the increase of frequency of AC source, inductive reactance increases as,

$$I=\frac{V_{\text{rms}}}{X_L}=\frac{V_{\text{rms}}}{2\pi vL} \Rightarrow X_L=2\pi vL$$

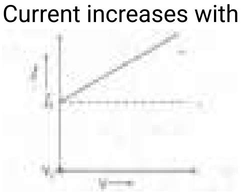
For given circuit $I \propto \frac{1}{v}$
Current decreases with the increase in frequency.



Frequency of AC source
(c) AC circuit containing capacitor only

$$X_C=\frac{1}{\omega C}=\frac{1}{2\pi vC}$$
$$\text{Current, } I=\frac{V_{\text{rms}}}{X_C}=\frac{V_{\text{rms}}}{\left(\frac{1}{2\pi \mathrm{VC}}\right)}$$

I = $2\pi vCV_{rms}$
For given circuit, I $\propto$ V
Current increases with the increase in frequency.



Frequency of AC source

27) A resistor of 400 0, an inductor of 5/ πH and a capacitor of $\frac{50}{\pi}\mu\mathrm{F}$ are connected in series across a source of alternating voltage of 140 sin 100 πtV Find the voltage (rms) across the resistor, the inductor and the capacitor. Is the algebraic sum of these voltage more than the source voltage? If yes, resolve the paradox.

Answer : Given, applied voltage, V = 140sin100πt V

$C=\frac{50}{\pi}\mu\mathrm{F}=\frac{50}{\pi}\times10^{-6}\ \mathrm{F}$

$L=\frac{5}{\pi}\mathrm{H}, R=400\Omega$

Comparing with V = $V_0 \sin \omega t$, we get

V_0 = 140 V and $\omega = 100\pi$

Inductive reactance, $X_L=\omega L=100\pi\times\frac{5}{\pi}=500\Omega$

Capacitive reactance, $X_C=\frac{1}{\omega C}=\frac{1}{100\pi\times\frac{50}{\pi}\times10^{-6}}$

= 200 Ω

Impedance of the circuit, $Z=\sqrt{R^2+(X_L-X_C)^2}$

$=\sqrt{(400)^2+(500-200)^2}$

$=\sqrt{160000+90000}=500\Omega$

Maximum current in the circuit,

$I_0=\frac{V_0}{Z}=\frac{140}{500}$

$I_{\text{rms}}=\frac{I_0}{\sqrt{2}}=\frac{140}{500\times\sqrt{2}}=0.2\ \mathrm{A}$

V_{rms} across resistor, $V_R = I_{rms}R$

= 0.2 x 400 = 80V

V_{rms} across inductor, $V_L = I_{rms} X_L$

= 0.2 x 500 = 100V

V_{ems} across capacitor, $V_c = I_{rms} X_C$

= 0.2 x 200 = 40 V

Now, $V \neq V_R + V_L + V_C$

Because V_C V_L and V_R are not in same phase, instead

$V=\sqrt{V_R^2+(V_L-V_C)^2}$

$=\sqrt{80^2+(100-40)^2}=100\ \mathrm{V}$

which is same as that of applied rms voltage.

28) A resistance of 20 Ω is connected to a source of alternating current rated 110 V, 50 Hz. Find the
(i) rms current.
(ii) maximum instantaneous current in the resistor.
(iii) time taken by the current to change from its maximum value to the rms value.

Answer : Given, resistance, R = 20 Ω

The rms value of voltage, E_{rms} = 110 V

Frequency, v = 50 Hz

(i) $I_{\text{rms}}=\frac{E_{\text{rms}}}{R}=\frac{110}{20}=5.5\ \mathrm{A}$

(ii) $I_0 = \sqrt{2}\ I_{rms}$ = 1.414 $\times$ 5.5 = 7.8 A

(iii) Let the AC be represented by $I = I_0 \cos\omega t$

At t = 0, $I = I_0 \cos 0^\circ = I_0$ (max) = 7.8 A

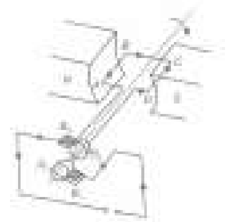
At t= t, let I = $I_V=\frac{I_0}{\sqrt{2}}=I_0\cos\omega t$

$$\because \cos\omega t=\frac{1}{\sqrt{2}}=\cos\frac{\pi}{4}\Rightarrow\omega t=\frac{\pi}{4}\Rightarrow 2\pi v t=\frac{\pi}{4}$$

$\therefore \quad t=\frac{1}{8\mathrm{v}}=\frac{1}{8\times50}=2.5\times10^{-3}\ \mathrm{s}$

29) (a) State the principle on which an ac generator works. Draw a labelled diagram and explain its working.
(b) A conducting rod held horizontally along East-West direction is dropped from rest from a certain height near the Earth's surface. Why should there be an induced emf across the ends of the rod? Draw a plot showing the instantaneous variation of emf as a function of time from the instant it begins to fall.

Answer : An alternating current generator, designed by Nikola Tesla, is based upon the principle of electromagnetic induction.



Working: On rotating the coil in the magnetic field, the magnetic flux linked with the coil changes continuously. This changing magnetic flux induces an emf in the circuit.

Mathematically,

$$\varepsilon=-\frac{d\phi_B}{dt}=-\frac{d}{dt}(NBA\cos\omega t)$$

$$=NBA\omega\sin\omega t=\varepsilon_0\sin\omega t$$

Here ω_0 = NBAω = peak/amplitude of induced emf

(b) When a rod falls, it cuts across the horizontal component of earth's magnetic field. As a result, the change in magnetic flux takes place and an emf is induced at the ends of the rod.

The induced emf is given by

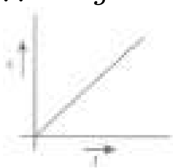
ε = B/v

The velocity of the rod at any instant of time is given by

$$v=u+at$$

$$[\because u=0, a=g]$$

$$\therefore\ v=gt\Rightarrow\varepsilon\propto t$$



30) (a) An a.c. source of voltage V = V_0 sin rot is connected to a series combination of L, C and R. Use the phasor diagram to obtain expressions for impedance of the circuit and phase angle between voltage and current. Find the condition when current will be in phase with the voltage. What is the circuit in this condition called?

(b) In a series LR circuit X_L = R and power factor of the circuit is P_1. When capacitor with capacitance C such that X_L= X_C is put in series the power factor becomes P_2 Calculate $\frac{P_1}{P_2}$.

Answer : (a) (i)

Take the voltage of source

V = V_m sin ωt ....(i)

To determine the phase relation between current and voltage at any instant of time, we use a phasor technique.

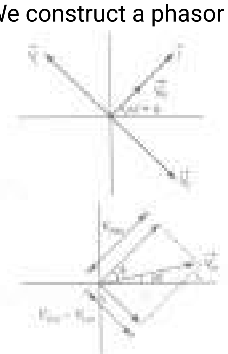
As all the three components are in series, the same amount of current flows through them at any instant of time.

Let it be

$I = I_m \sin(\omega t + \phi)$ ....(ii)

where Φ is the phase difference between the voltage across the source and current.

We construct a phasor diagram.



On applying Pythagoras theorem, we get

$V_m^2 = V_{Rm}^2 + (V_{Cm} - V_{Lm})^2$

$\text{Here } V_{Rm} = I_m R, V_{Cm} = I_m X_C, V_{Lm} = I_m X_L$

$V_m = I_m\sqrt{R^2 + (X_C - X_L)^2}$

$= I_m Z$

$\text{where } Z = \sqrt{R^2 + (X_C - X_L)^2}$

Z is called the impedance of the circuit.

(ii) Phase angle between voltage and current is given by $\phi = \tan^{-1}\left(\frac{V_{Cm} - V_{Lm}}{V_{Rm}}\right)$

The voltage and current will be in phase,

$\text{if } \phi = 0 \text{ or } V_{Cm} = V_{Lm} \text{ or } X_L = X_C$

This condition is called resonance condition of the circuit and the nature of circuit is purely resistive.

(b) Power factor (P) = $\frac{R}{Z}$

In a RL circuit, $Z = \sqrt{R^2 + X_L^2}$

$P_1 = \frac{R}{\sqrt{R^2+R^2}} = \frac{1}{\sqrt{2}} [\because R = X_L(\text{ given })]$

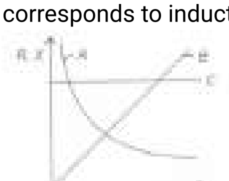
When a capacitor of capacitance C is connected, such that $X_L = X_C$ then

$I = \sqrt{R^2 + (X_L - X_C)^2} = R$

$\text{Power factor } = P_2 = \frac{R}{R} = 1$

$\therefore \frac{P_1}{P_2} = \frac{1}{\sqrt{2}}$

31) (a) Figure shows the variation of resistance and reactance versus angular frequency. Identify the curve which corresponds to inductive reactance and resistance.



(b) Show that a series LCR circuit at resonance behaves as a purely resistive circuit. Compare the phase relation between current and voltage in series LCR circuit for $(\text{i})\ X_L > X_C\ (\text{ii})\ X_L = X_C$ using phasor diagrams.

(c) What is an acceptor circuit and where it is used?

Answer : (a) Curve B: Inductive reactance; Curve C: Resistance

(b) At resonance, $X_L = X_C$

$\because$ Impedance $Z = \sqrt{R^2 + (X_L - X_C)^2} = R$

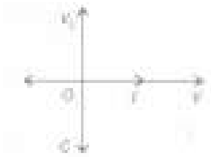
Phasor diagrams:

(i) For $X_L > X_C \Rightarrow V_L > V_C$

Phase difference = Φ

(ii) For $X_L = X_C \Rightarrow V_L = V_C$

Phase difference $= 0$



(c) An acceptor circuit is a series LCR circuit and it is used in tuning mechanism of radio or TV sets.

32) A resistor of 50 Ω, a capacitor of $\left(\frac{25}{\pi}\right)\mu\mathrm{F}$ and an inductor of $\left(\frac{4}{\pi}\right)$H are connected in series across. Calculate an AC sources whose voltage (in volt) is given by V =70 sin(100 πt). Calculate

i) the net reactance of the circuit

(ii) the impedance of the circuit

(iii) the effective value of current in the circuit

Answer : i)

$$R = 50\Omega$$
$$C = \frac{25}{\pi} \times 10^{-6}\ \mathrm{F}$$
$$L = \frac{4}{\pi}$$
$$\Rightarrow \quad V = 70\sin(100\pi t).$$
$$\Rightarrow \quad V_0 = 70\ \mathrm{V}$$
$$\therefore \quad \omega t = 100\pi r$$
$$\Rightarrow \quad \omega = 100\pi$$
$$\Rightarrow 2\pi f = 100\pi$$
$$\Rightarrow \quad f = 50\ \mathrm{Hz}$$

i) Net reactance of the circuit

$$X_L = \omega L = 2\pi f L = 2 \times \pi \times 50 \times \frac{4}{\pi} = 8 \times 50 = 200\Omega$$
$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C} = \frac{1}{2 \times \pi \times 50 \times \frac{25}{\pi} \times 10^{-6}}$$
$$= \frac{1 \times 10^6}{2 \times 50 \times 50} = \frac{10^6}{5000} = \frac{1000000}{5000} = 200\Omega$$

Net reactance of the circuit = $X_{L'}$ - Xc

=(200- 200) Ω=0 Ω

ii) Impedance, Z =
$$= \sqrt{R^2 + (X_L - X_C)^2}$$
$$= \sqrt{(50)^2 + (0)^2} = \sqrt{(50)^2} = 50\Omega$$

iii)
$$V_{rms} = \frac{70}{\sqrt{2}}$$
$$I_{rms} = \frac{V_{rms}}{\text{Impedance } (Z)} = \frac{70\sqrt{2}}{50} = \frac{49.5}{50} = 0.99\ \mathrm{A}$$

33) A series RL circuit with R =10 2 an L=$\left(\frac{100}{\pi}\right)$ mH is connected to an AC source of voltage V=141sin(100 π t), where V is in volts and t is in seconds. Calculate

(i) impedance of the circuit

(ii) phase angle, and

(ii) voltage drop across the inductor

Answer :

$$R = 10\Omega$$

$$L = \left(\frac{100}{\pi}\right)\mathrm{mH} = \frac{100}{\pi} \times 10^{-3}\mathrm{H}$$

$$V = 141 \sin(100\pi t)$$

$\therefore V_0 = 141$ V

$\therefore \omega t = 100\pi t$

$\Rightarrow 2\pi f = 100\pi$

$\Rightarrow f = 50$ Hz

(i) $Z = \sqrt{(X_L)^2 + R^2} = \sqrt{(10)^2 + (2\pi/L)^2}$

$= \sqrt{100 + \left(2 \times \pi \times 50 \times \frac{100}{\pi} \times 10^{-3}\right)^2}$

$= \sqrt{100 + \left(100 \times 100 \times 10^{-3}\right)^2} = \sqrt{100 + \left(10^4 \times 10^{-3}\right)^2}$

$= \sqrt{100 + (10)^2} \equiv \sqrt{200} = 10\sqrt{2}\Omega$

(ii) $\therefore \tan\phi = \frac{\omega L}{R} = \frac{2\times\pi\times50\times\frac{100}{\pi}\times10^{-3}}{10}$

$= \frac{100\times100\times10^{-3}}{10} = 1$

$\Rightarrow \phi = 45^\circ$

(iii) $V_L = IX_L = \frac{V_{\mathrm{ras}}}{Z} \cdot \omega L$

$= \frac{14\mathrm{I}}{\sqrt{2}\times10\sqrt{2}} \times 100\pi \times \frac{100}{\pi} \times 10^{-3} = 705\mathrm{volt}$

34) (a) Mention the factors on which the resonant frequency of a series LCR circuit depends. Plot a graph showing variation of impedance of a series LCR circuit with the frequency of the applied AC source.

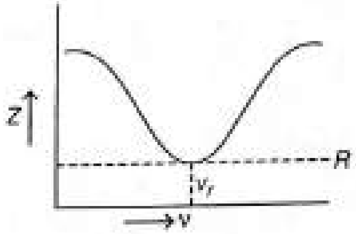
(b) With the help of a suitable diagram, explain the working of a step-up transformer.

(c) Write two causes of energy loss in a real transformer.

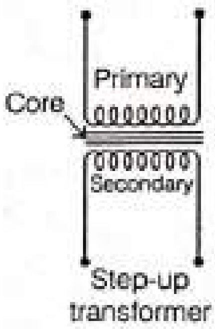
Answer : (a) In a series LCR circuit, the resonant frequency is given as $f_o = \frac{1}{2\pi\sqrt{LC}}$

From the above formula, ít is clear that resonant frequency depends on the value of inductance L and capacitance C present in the circuit.

The graph between impedance and frequency is given below



(b) The diagram of a step-up transformer is shown in the following figure.



Working

The given source of emf, say AC mains, is always connected to the primary coil. When alternating current flows through the primary coil, then in each cycle of current, the core is magnetised once in one direction and once in the opposite direction. Hence, an alternating magnetic flux is produced in the core. Since the secondary coil is also wound on the same core, the magnetic flux passing through it changes continuously due to the repeated magnetisation and demagnetisation of the core. Therefore, by mutual induction, an alternating emf of the same frequency is induced in the secondary coil.

(c) Energy Loss in Transformers

In real transformer, small energy losses occur due to the following reasons.

I. Flux leakage There is always some leakage of magnetic flux that is not all of the flux due to primary passes through the secondary. This is due to poor design of the core or the air gaps in the core. It can be reduced by winding the primary and secondary coils one over the other.

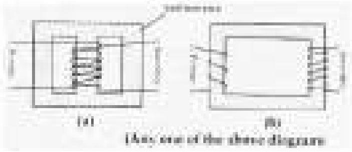
II. Resistance of the windings The wire used for the windings has some resistance and so, energy is also lost due to heat produced in the wire (I^2R). In high current, low voltage windings, energy losses are minimised by using thick wire.

35) Explain the principle, construction and working of a step-down transformer. Can it be used with a DC circuit?

Answer : Working of a transformer is based on the principle of mutual induction. Transformer cannot step up or step down a dc voltage because there is no change in magnetic flux. When dc voltage source is applied across a primary coil of a transformer, the current in primary coil remains same, so there is no change in magnetic flux and hence no voltage is induced across the secondary coil. Hence, a step up transformer cannot be used to step up DC voltage .

36) Write the function of a transformer. State its principle of working with the help of a Mention diagram. various energy losses in this device.

Answer : Conversion of ac of low voltage into ac of high voltage & vice versa. Mutual induction : When alternating voltage is applied to primary windings, emf is induced in the secondary windings.



Energy losses :

(a) Leakage of magnetic flux

(b) Eddy currents

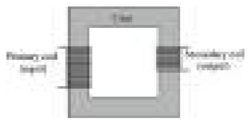
(c) Hysteresis loss

(d) Copper loss

37) (i) Draw a labelled diagram of a step-down transformer. State the principle of its working.

(ii) Express the turn ratio in terms of voltages.

(iii) Find the ratio ofprimary and secondary currentsin terms of turn ratio in an ideal transformer.

Answer : (i) 

Principle : A transformer works on the principle of mutual induction. Whenever the amount of magnetic flux linked with a coil changes, an emf is induced in the neighbouring coil.

Working : When an alternating current source is connected to the ends of primary coil, the current changes continuously in the primary coil, due to which magnetic flux linked with the secondary coil changes continuously. Therefore, the alternating emf of same frequency is developed across the secondary terminals. According to Faraday's laws, the e.m.f. induced in the primary coil,

EP = −NP △ ϕ/ △ t ....(i)

and emf induced in the secondary coil

ES=−NS △ ϕ/ △ t ....(ii)

From (i) and (ii)

ES/EP = NS/NP = K ....(ii)

For step-down transformer, K < 1.

∴ ES < EP

(ii) The induced emf in primary coil, EP=−NP(dϕ/dt)

The induced emf in secondary coil, ES=−NS(dϕ/dt)

ES/EP = NS/NP = K

Where K is the turns ratio or the transformation ratio.

(iii) If the transformer is ideal, then

Input electrical power = Output electrical power

$$E_pI_p = E_SI_S$$

$$\frac{E_S}{E_p} = \frac{I_p}{I_S}$$

$$\frac{I_p}{I_S} = \frac{E_S}{E_p} = \frac{N_S}{N_p} = K$$

38) What is the power dissipation in an AC circuit in which voltage and current are given by $V = 300sin(\omega t + -(\frac{\pi}{2})$ and I = 5 sin ωt?

Answer : Since phase difference between voltage and current is Φ = 90°

∴ Power factor = cos Φ = cos 90° = 0

Hence power dissipation is zero.

39) An inductor of 200 mH, capacitor of 400 μF and a resistance of 10 Ω are connected in series to an a.c. source of 50 V of varialbe frequency. Calculate (i) angular frequency at which maximum power dissipation occurs in the circuit and the corresponding value of effective current, and (ii) value of Q factor is the circuit.

Answer : Here, $L=200 \mathrm{~m} H=\frac{2}{10} H$

$C=400 \nu F=400 \times 10^{-6} \mathrm{~F}=4 \times 10^{-4} \mathrm{~F}$

$R=10 \mathrm{~ohm}, E_v=50 \mathrm{~V}$

(i) Maximum power dissipation occurs in the circuit at resonance, i.e., when

$\omega L=\frac{1}{\omega C}$

$\text{or } \omega=\frac{1}{\sqrt{L C}}=\frac{1}{\sqrt{\frac{2}{10} \times 4 \times 10^{-4}}}$

$\omega=\frac{1}{\sqrt{80 \times 10^{-6}}}=\frac{10^3}{8.944}$

$\omega=111.8 \mathrm{rad} / \mathrm{s}$

$I_v=\frac{E_v}{Z}=\frac{E_v}{R}=\frac{50}{10}=5 \mathrm{~A}$

(ii) $Q=\frac{1}{R} \sqrt{\frac{L}{C}}$

$Q=\frac{1}{10} \sqrt{\frac{2 / 10}{4 \times 10^{-4}}}=\frac{1}{10} \times \frac{100}{1.47}=2.237$