RAVI MATHS TUITION CENTER, CHENNAI-82. WHATSAPP - 8056206308

Differential Equations

12th Standard

Maths

5 x 2 = 10

1) For each of the differential equations in Exercises, find the general solution:
$\frac{dy}{dx} = \frac{1-\cos x}{1+\cos x}$

2) For each of the differential equations in Exercises, find the general solution:
$\sec^2 x \tan y dx + \sec^2 y \tan x dy = 0$

3) Find the differential equation of the family of lines passing through the origin.

4) Find the sum of the order and degree of the following differential equations :
$\frac{d^2y}{dx^2} + \sqrt[3]{\frac{dy}{dx}} + (1+x) = 0$

5) Find the general solution of differential equation $\log\left(\frac{dy}{dx}\right) = x + 1$

5 x 3 = 15

6) Form the differential equation representing the family of curves y = e^{2x} (A + Bx), where A and B are constants.

7) Show that the general solution of the differential equation $\frac{dy}{dx} + \frac{y^2+y+1}{x^2+x+1} = 0$ is given by (x + y + 1) = A (1 - x - y - 2xy), where A is parameter.

8) Solve: $(x^3 + y^3)$ dy - x^2y dx = 0.

9) Solve the differential equation : $\frac{dy}{dx}$ = e^{x-y} + x^3 e^{-y}.

10) Find the general solution of the differential equations: $ydx - \left(x + 2y^2\right)dy = 0$.

5 x 5 = 25

11) Find the particular solution satisfying the given condition : $x^2dy + \left(xy + y^2\right)dx = 0$; y = 1, when x = 1.

12) Find the particular solution of the differential equation $\frac{dy}{dx} = \frac{xy}{x^2+y^2}$ given that y = 1, when x = 0.

13) Find the particular solution of the differential equation : x^2dy = y(x + y)dx = 0, when x = 1, y = 1.

14) Find the particular solution of the differential equation
$(3xy+y^2)$dx+(x^2+xy)dy = 0 for x = 1, y = 1

15) Solve the following differential equation:
$x\cos\left(\frac{y}{x}\right)(ydx + xdy) = y\sin\left(\frac{y}{x}\right)(xdy - ydx)$

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1) The given differential equation is:
$\frac{dy}{dx} = \frac{1-\cos x}{1+\cos x}$
$\Rightarrow \frac{dy}{dx} = \frac{2\sin^2\frac{x}{2}}{2\cos^2\frac{x}{2}} = \tan^2\frac{x}{2}$
$\Rightarrow \frac{dy}{dx} = \left(\sec^2\frac{x}{2} - 1\right)$
Separating the variables, we get:
$dy = \left(\sec^2\frac{x}{2} - 1\right)dx$
Now, integrating both sides of this equation, we get:
$\int dy = \int\left(\sec^2\frac{x}{2} - 1\right)dx = \int \sec^2\frac{x}{2}dx - \int dx$
$\Rightarrow y = 2\tan\frac{x}{2} - x + \mathrm{C}$
This is the required general solution of the given differential equation.

2) The given differential equation is:
$\sec^2 x \tan y dx + \sec^2 y \tan x dy = 0$
$\Rightarrow \frac{\sec^2 x \tan y dx + \sec^2 y \tan x dy}{\tan x \tan y} = 0$
$\Rightarrow \frac{\sec^2 x}{\tan x}dx + \frac{\sec^2 y}{\tan y}dy = 0$
$\Rightarrow \frac{\sec^2 x}{\tan x}dx = -\frac{\sec^2 y}{\tan y}dy$
Integrating both sides of this equation, we get:
$\int \frac{\sec^2 x}{\tan x}dx = -\int \frac{\sec^2 y}{\tan y}dy$
$\therefore \frac{d}{dx}(\tan x) = \frac{dt}{dx}$
$\Rightarrow \sec^2 x = \frac{dt}{dx}$
Now, $\int \frac{\sec^2 x}{\tan x}dx = \int_t^1 dt.$
$= \log t$
$= \log(\tan x)$
Let $\sec^2 x dx = dt$
Similarly, $\int \frac{\sec^2 x}{\tan x}dy = \log(\tan y)$
Substituting these values in equation (1), we get:
$\log(\tan x) = -\log(\tan y) + \log \mathrm{C}$
$\Rightarrow \log(\tan x) = \log\left(\frac{\mathrm{C}}{\tan y}\right)$
$\Rightarrow \tan x = \frac{\mathrm{C}}{\tan y}$
$\Rightarrow \tan x \tan y = \mathrm{C}$
This is the required general solution of the given differential equation.

3) Generate equation of family of lines passing through origin y = mx $m = \frac{y}{x}$ $\frac{dy}{dx} = m$ $\frac{dy}{dx} = \frac{y}{x}$
$x\frac{dy}{dx} - y = 0$

4) Here, $\left\{\frac{d^2y}{dx^2} + (1+x)\right\}^3 = -\frac{dy}{dx}$
Thus, order is 2 and degree is 3. So, the sum is 5.

5) $\log\left(\frac{dy}{dx}\right) = x + 1$
$\Rightarrow \frac{dy}{dx} = e^{x+1}$

$\Rightarrow dy = e^{x+1}dx$
Integrating both the sides,
$\int dy = \int e^{x+1}dx$
$\Rightarrow y = e^{x+1} + C$

5 x 3 = 15

6) $y = e^{2x}(a + bx)$
Differentiating both sides with respect to x , we get:
$y' = 2e^{2x}(a + bx) + e^{2x} \cdot b$
$\Rightarrow y' = e^{2x}(2a + 2bx + b)$
Multiplying equation (1) with equation (2) and then subtracting it from equation (2), we get:
$y' - 2y = e^{2x}(2a + 2bx + b) - e^{2x}(2a + 2bx)$
$\Rightarrow y' - 2 = be^{2x}$
Differentiating both sides with respect to x , we get:
$y''k - 2y' = 2be^{2x}$
Dividing equation (4) by equation (3), we get:
$\frac{y''-2y'}{y'-2y} = 2$
$\Rightarrow y'' - 2y' = 2y' - 4y$
$\Rightarrow y'' - 4y' + 4y = 0$
This is the required differential equation of the given curve.

7) We have: $\frac{dy}{dx} + \frac{y^2+y+1}{x^2+x+1} = 0$
$\Rightarrow \frac{dy}{y^2+y+1} + \frac{dx}{x^2+x+1} = 0$ |**Variables Separable**
Integrating, $\int \frac{dy}{y^2+y+1} + \int \frac{dx}{x^2+x+1} =$ Constant
$\Rightarrow \int \frac{dy}{\left(y^2+y+\frac{1}{4}\right)+\frac{3}{4}} + \int \frac{dx}{\left(x^2+x+\frac{1}{4}\right)+\frac{3}{4}} =$ Constant
$\Rightarrow \int \frac{dy}{\left(\frac{\sqrt{3}}{2}\right)^2+\left(y+\frac{1}{2}\right)^2} + \int \frac{dx}{\left(\frac{\sqrt{3}}{2}\right)^2+\left(x+\frac{1}{2}\right)^2} =$ Constant
$\Rightarrow \frac{1}{\frac{\sqrt{3}}{2}} tan^{-1} \frac{y+\frac{1}{2}}{\frac{\sqrt{3}}{2}} + \frac{1}{\frac{\sqrt{3}}{2}} tan^{-1} \frac{x+\frac{1}{2}}{\frac{\sqrt{3}}{2}} = \frac{1}{\frac{\sqrt{3}}{2}} tan^{-1} A$
$\Rightarrow tan^{-1} \frac{2y+1}{\sqrt{3}} + tan^{-1} \frac{(2x+1)}{\sqrt{3}} = tan^{-1} A$
$tan^{-1} \frac{\frac{2y+1}{\sqrt{3}} + \frac{2x+1}{\sqrt{3}}}{1-\left(\frac{2y+1}{\sqrt{3}}\right)\left(\frac{2x+1}{\sqrt{3}}\right)} tan^{-1} A'$
$\Rightarrow \frac{(2y+1)+(2x+1)}{[3-(2x+2y+4xy+1)]} = \sqrt{3}A'$
$= 2(x + y + 1) = A(2 - 2x - 2y - 4xy)$,
where $\sqrt{3}A' = A$
$\Rightarrow x + y + 1 = A(1 - x - y - 2xy)$,
which is true.

8) $-\frac{x^3}{3y^3} + log|y| = c$ is required solution.

9) $\int e^y dy = \int \left(e^x + x^3\right) dx$
$\Rightarrow e^y = e^x + \frac{x^4}{4} + c$

10) The given equation is $ydx - \left(x + 2y^2\right) dy = 0$
$\Rightarrow \frac{dx}{dy} - \frac{x}{y} = 2y$
|**Linear Equation in x**
Computing with $\frac{dx}{dy} + Px = Q$,
we have: $'P' = -\frac{1}{y} and \quad 'Q' = 2y.$
$\therefore \; I.F. = e^{\int Pdy} = e^{\int -\frac{1}{y}dy} = e^{-log|y|}$
$= e^{log^{-1}} = y^{-1} = \frac{1}{y}.$
Multiplying (1) by, $\frac{1}{y}$ we get

$\frac{1}{y}\frac{dx}{dy}-\frac{x}{y^2}=2\Rightarrow\frac{d}{dy}\left(x.\frac{1}{y}\right)=2.$

Integrating,$x.\frac{1}{y}=\int 2.dy+c\Rightarrow\frac{x}{y}=2y+c$

$\Rightarrow x=2y^2+cy,$

Which is the reqd. general solution.

5 x 5 = 25

11) Given, $x^2dy+\left(xy+y^2\right)dx=0$

$\therefore\ \frac{dy}{dx}=\frac{-\left(xy+y^2\right)}{x^2}$

Put y = vx

$\Rightarrow\frac{dy}{dx}=v+x\frac{dv}{dx}$

$\therefore$ The differential equation becomes

$v+x\frac{dv}{dx}=-\left(v+v^2\right)$

$\Rightarrow\frac{dv}{v^2+2v}=\frac{dx}{x}$

$\Rightarrow\int\frac{dv}{(v+1)^2-1^2}=-\int\frac{dx}{x}$

$\Rightarrow\frac{1}{2}\log\frac{v}{v+2}=-\log x+\log C$

$\Rightarrow\frac{C}{x}=\sqrt{\frac{y}{y+2x}}$

If x = 1, y = 1 then $c=\frac{1}{\sqrt{3}}$

$\Rightarrow\frac{1}{\sqrt{3}x}=\sqrt{\frac{y}{y+2x}}$

12) Given differential equation is

$\frac{dy}{dx}=\frac{y/x}{1+(y/x)^2}=f\left(\frac{y}{x}\right)$

Hence, homogeneous.

Put y = vx

$\Rightarrow\frac{dy}{dx}=v+x\frac{dv}{dx}$

$\therefore$ The differential equation becomes

$v+x\frac{dv}{dx}=\frac{v}{1+v^2}$

$x\frac{dv}{dx}=\frac{v}{1+v^2}-v$

$=-\frac{v^3}{1+v^2}$

$\Rightarrow\log|v|-\frac{1}{2v^2}=-\log|x|+C$

$\Rightarrow\log v+\log x-\frac{1}{2v^2}=C$

$\Rightarrow\log(vx)-\frac{1}{2v^2}=C$

$\therefore\log y-\frac{x^2}{2y^2}=C$

For particular solution

x = 0, y = 1 $\Rightarrow$ c = 0

$\therefore\log y-\frac{x^2}{2y^2}=0$

13) The given D.E. is $x^2dy + y(x + y)dx = 0$... (i)

when x = 1, y = 1

From (i), $\frac{dy}{dx}=-\frac{xy+y^2}{x^2}$ ... (ii)

Put, y = vx

$\Rightarrow\frac{dy}{dx}=v+x\frac{dv}{dx}$

From (ii), $v+x\frac{dv}{dx}=\frac{xvx+v^2x^2}{x^2}$

$\Rightarrow x\frac{dv}{dx}=\frac{-vx^2-v^2x^2-vx^2}{x^2}$

$\Rightarrow x\frac{dv}{dx}=-2v-v^2$

$\Rightarrow\frac{dv}{v(v+2)}=-\frac{dx}{x}$

$\Rightarrow\frac{1}{2}\left[\frac{1}{v}-\frac{1}{v+2}\right]dv+\frac{dx}{x}=0$

Integrating it we get

$\frac{1}{2}[\log v - \log|v+2|] + \log x = C_1$

$\Rightarrow \log\frac{v}{|v+2|} + \log x^2 = 2C_1$

$\Rightarrow \log\left[\frac{y}{y+2x}x^2\right] = C_1$

$\Rightarrow \log\left[\frac{yx^2}{y+2x}\right] = C$

$\Rightarrow \log\left[\frac{1}{1+2}\right] = C \Rightarrow C = \log\frac{1}{3}$

$\therefore \log\left[\frac{yx^2}{y+2x}\right] = \log\frac{1}{3}$

$\Rightarrow$ $3yx^2 = y + 2x$

$3yx^2 - y = 2x$

$y(3x^2 - 1) = 2x$

$y = \frac{2x}{3x^2-1}$

14) The given D.E is

(3xy+y2)dx+(x2+xy0dy = 0

where x = 1, y = 1

$\Rightarrow \frac{dy}{dx} = -\frac{3xy+y^2}{x^2+xy}$ ..(i)

which is a linear homogeneous equation put,

y = vx

$\frac{dy}{dx} = v + x\frac{dv}{dx}$

From (i), $v + x\frac{dv}{dx} = -\frac{3xvx+v^2x^2}{x^2+x.vx}$

$= -\frac{(3v+v^2)}{1+v}$

x dv/dx = $-\frac{(3v+v^2)}{1+v} - v$

= $\frac{-3v-v^2-v-v^2}{1+v}$

$\Rightarrow \quad x\frac{dv}{dx} = \frac{-4v-2v^2}{1+v}$

$\Rightarrow \quad \frac{1+v}{2v^2+4v}dv = \frac{2}{(x^2-1)^2}$

Integrating both sides, we get

1/4log $|2v^2 + 4v|$ + log $|x|$ = $\log C_1$

$\Rightarrow \left(2\frac{y^2}{x^2} + 4\frac{y}{x}\right)^{1/4} = C_1$

$\Rightarrow 2x^2y^2+4x^3y = C$

When x = 1, y = 1, C = 6

Hence $2x^2y2+4x^3y = 6$

$\Rightarrow x^2y^2+2x^2y = 3$

15) $xcos\left(\frac{y}{x}\right)(ydx + xdy)$

$= ysin\left(\frac{y}{x}\right)(xdy - ydx)$

$\Rightarrow \left(y + x\frac{dy}{dx}\right)xcos\left(\frac{y}{x}\right)$

$= \left(x\frac{dy}{dx} - y\right)ysin\left(\frac{y}{x}\right)$

$\Rightarrow \frac{dy}{dx} = \frac{y\left\{xcos\left(\frac{y}{x}\right)+ysin\left(\frac{y}{x}\right)\right\}}{x\left\{ysin\left(\frac{y}{x}\right)-xcos\left(\frac{y}{x}\right)\right\}}$

Eqn (i) is homogeneous So, putting y = vx and $\frac{dy}{dx} = v + x\frac{dv}{dx}$

$v + x\frac{dv}{dx} = \frac{vx[xcosv+vxsinv]}{x[vxsinv-xcosv]}$

$\Rightarrow v + x\frac{dv}{dx} = \frac{v[cosv+vsinv]}{vsinv-cosv}$

$= \frac{2vcosv}{vsinv-cosv}$

$\Rightarrow \frac{1-cosv-vsinv}{vcosv}dv = -\frac{2dx}{x}$

$\Rightarrow \int\frac{1.cosv-vsinv}{vcosv}dv = -2\int\frac{dx}{x}$

$\Rightarrow \log |v \cos v| = -2\log x + \log C$

$\Rightarrow x^2 v \cos v = C$

$\Rightarrow xy \cos(y/x) = C$ [as $y = vx$]