RAVI MATHS TUITION CENTER, NEAR VILLIVAKKAM RLY STATION, CHENNAI – 82.
WHATSAPP - 8056206308

Differential Calculus - Limits and Continuity 3

11th Standard

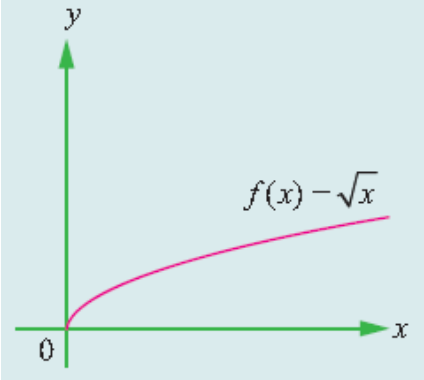
Maths

Time : 01:00:00 Hrs

Total Marks : 40

20 x 2 = 40

1) Consider the function f(x) = $\sqrt{x}, x \geq 0$.
Does$lim_{x\to 0} f(x)$ exist?
Answer : No. f(x) = $\sqrt{x}$ is not even defined for x < 0.



Therefore as x $\to 0^-$, $lim_{x\to 0^-}\sqrt{x}$ does not exist.
However, $lim_{x\to 0^+}\sqrt{x} = 0$. Therefore $lim_{x\to 0}\sqrt{x}$ does not exist.

2) Complete the table using calculator and use the result to estimate the limit.
$lim_{x\to 2}\frac{x-2}{x^2-x-2}$

x	1.9	1.99	1.999	2.001	2.01	2.1
f(x)						

Answer : $lim_{x\to 2}\frac{x-2}{x^2-x-2} = lim_{x\to 2}\frac{x-2}{(x-2)(x+1)} = lim_{x\to 2}\frac{1}{(x+1)}$

x	1.9	1.99	1.999	2.001	2.01	2.1
f(x)	$\frac{1}{1.9+1} = \frac{1}{2.9}$ =0.34	$\frac{1}{1.99+1} = \frac{1}{2.99}$ =0.33	$\frac{1}{1.999+1} = \frac{1}{2.999}$ =0.33	$\frac{1}{2.001+1} = \frac{1}{3.001}$ =0.33	$\frac{1}{2.01+1} = \frac{1}{3.01}$ =0.33	$\frac{1}{2.1+1} = \frac{1}{3.1}$ =0.32

$lim_{x\to 2}\frac{x-2}{x^2-x-2} = 0.\overline{3}$

3) Complete the table using calculator and use the result to estimate the limit.
$lim_{x\to 0}\frac{\sqrt{x+3}-\sqrt{3}}{x}$

x	-0.1	-0.01	-0.001	0.001	0.01	0.1
f(x)						

Answer : Let f(x)=$lim_{x\to 0}\frac{\sqrt{x+3}-\sqrt{3}}{x}$

x	-0.1	-0.01	-0.001	0.001	0.01	0.1

f(x)	$\frac{\sqrt{3-0.1}-\sqrt{3}}{-0.1}$ $=\frac{1.703-1.732}{-0.1}$ =0.29	$\frac{\sqrt{3-0.01}-\sqrt{3}}{-0.01}$ $=\frac{1.703-1.732}{-0.01}$ =0.288	$\frac{\sqrt{3-0.001}-\sqrt{3}}{-0.001}$ $=\frac{0.000287}{-0.001}$ =0.288	$\frac{\sqrt{3+0.001}-\sqrt{3}}{0.001}$ $=\frac{0.000287}{0.001}$ =0.289	$\frac{\sqrt{3+0.01}-\sqrt{3}}{0.01}$ $=\frac{0.00287}{0.01}$ =0.288	$\frac{\sqrt{3+0.1}-\sqrt{3}}{0.1}$ $=\frac{0.0287}{0.1}$ =0.286

$\therefore lim_{x\to 0}\frac{\sqrt{x+3}-\sqrt{3}}{x} = 0.288$

4) Complete the table using calculator and use the result to estimate the limit.

$lim_{x\to 0}\frac{sinx}{x}$

x	-0.1	-0.01	-0.001	0.001	0.01	0.1
f(x)						

Answer : Let f(x)= $\frac{sinx}{x}$

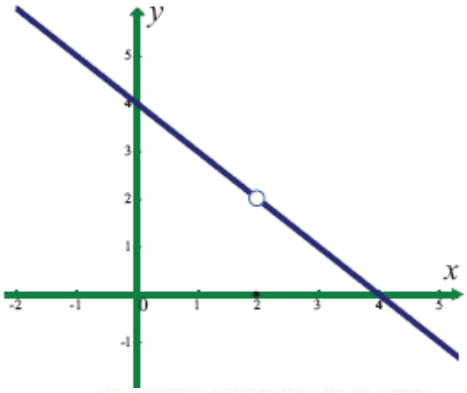
x	-0.1	-0.01	-0.001	0.001	0.01	0.1
f(x)	$\frac{sin(-0.1)}{-0.1}$ $\frac{-sin(0.1)}{-0.1}$ =0.998	$\frac{sin(-0.01)}{-0.01}$ $\frac{-sin(0.01)}{-0.01}$ =0.999	$\frac{sin(-0.001)}{-0.001}$ $\frac{-sin(0.001)}{-0.001}$ =0.9999	$\frac{sin(0.001)}{0.001}$ =0.9999	$\frac{sin(0.01)}{0.01}$ =0.999	$\frac{sin(0.1)}{0.1}$ =0.998

$lim_{x\to 0}\frac{sinx}{x} = 1$

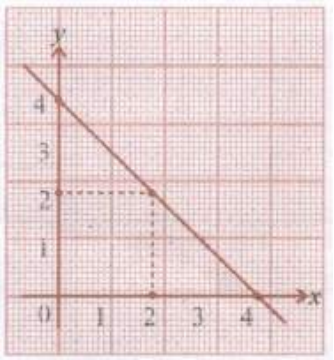
5) Use the graph to find the limits (if it exists). If the limit does not exist, explain why?

$lim_{x\to 2}f(x)$

where $f(x) = \begin{cases} 4-x, & x\neq 2 \\ 0, & x=2 \end{cases}$.



Answer :

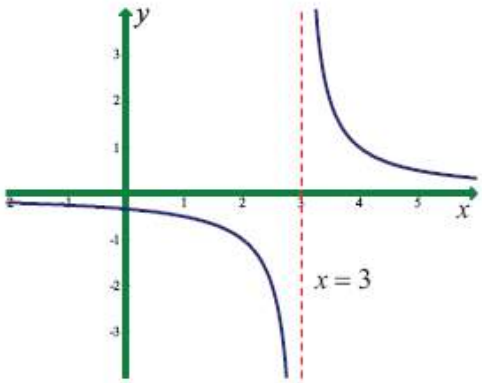


At x = 2, the value of the curve on y-axis is 2.

$\therefore lim_{x\to 2}f(x) = 2$

6) Use the graph to find the limits (if it exists). If the limit does not exist, explain why?

$lim_{x\to 3}\frac{1}{x-3}$



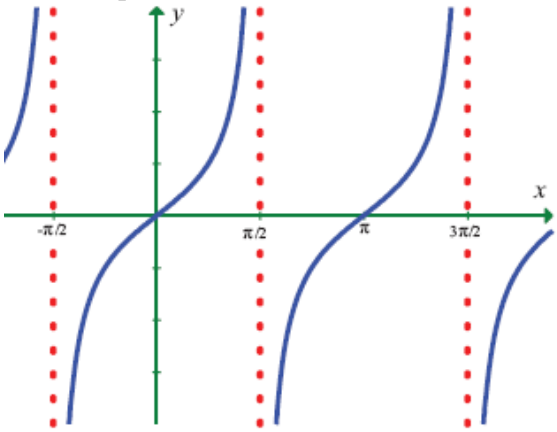
Answer : $lim_{x\to 3}\frac{1}{x-3}$

At x = 3, the curve does not meet the y-axis.

Hence, the limit does not exist.

7) Use the graph to find the limits (if it exists). If the limit does not exist, explain why?

$lim_{x\to \frac{\pi}{2}} tanx$



Answer : $lim_{x\to \frac{\pi}{2}} tanx$

At x = $\frac{\pi}{2}$, the curve does not meets the y-axis.

$\therefore lim_{x\to \frac{\pi}{2}} tanx$ does not exist.

8) If f(2) = 4, can you conclude anything about the limit of f(x) as x approaches 2?

Answer : Given f(2) = 4

Since only the value of the function is given, we cannot conclude anything about the limit of f(x) as x approaches 2.

9) Calculate $lim_{x\to 3}(x^3 - 2x + 6)$.

Answer : P (x)= x^3-2x+6 is a polynomial.

Hence, $lim_{x\to 3}p(x) = p(3) = 3^3 - 2\times 3 + 6 = 27$.

10) Compute $lim_{x\to 0}[\frac{x^2+x}{x} + 4x^3 + 3]$.

Answer : $lim_{x\to 0}[\frac{x^2+x}{x} + 4x^3 + 3] = lim_{x\to 0}(\frac{x^2+x}{x}) + lim_{x\to 0}(4x^3 + 3)$

$lim_{x\to 0}(x + 1) + lim_{x\to 0}(4x^3 + 3)$

=(0 + 1) + (0 + 3)

=4.

11) Compute $lim_{x\to 1}\frac{\sqrt{x}-1}{x-1}$.

Answer : Here $lim_{x\to 1}(x - 1) = 0$. In such cases, rationalise the numerator

$lim_{x\to 1}\frac{\sqrt{x}-1}{x-1} = lim_{x\to 1}\frac{(\sqrt{x}-1)}{(\sqrt{x}-1)(\sqrt{x}+1)} = lim_{x\to 1}\frac{1}{\sqrt{x}+1} = \frac{lim_{x\to 1}(1)}{lim_{x\to 1}(\sqrt{x}+1)} = \frac{1}{2}$.

12) Calculate $lim_{t\to 1}\frac{\sqrt{t}-1}{t-1}$

Answer : $lim_{t\to 1}\frac{\sqrt{t}-1}{t-1} = lim_{x\to 1}\frac{t^{\frac{1}{2}}-1^{\frac{1}{2}}}{t-1} = \frac{1}{2}(1)^{\frac{1}{2}-1} = \frac{1}{2}.$

13) Evaluate the following limits :

$lim_{x\to 2}\frac{x^4-16}{x-2}$

Answer : $lim_{x\to 2}\frac{x^4-16}{x-2} = lim_{x\to 2}\frac{x^4-2^4}{x-2}$ $[\because lim_{x\to a}\frac{x^n-a^n}{x-a} = n.\,a^{n-1}]$

$=4(2)^{4-1} = 4(2)^3 = 4(8) = 32$

14) Evaluate the following limits :

$lim_{x\to 5}\frac{\sqrt{x+4}-3}{x-5}$

Answer : $lim_{x\to 5}\frac{\sqrt{x+4}-3}{x-5}$

Multiplying and dividing by $\sqrt{x+4}+3$ we get,

$lim_{x\to 5}\frac{\sqrt{x+4}-3}{x-5}\frac{\sqrt{x+4}+3}{\sqrt{x+4}+3} = lim_{x\to 5}\frac{(x+4)-3^2}{x-5[\sqrt{x+4}+3]} = lim_{x\to 5}\frac{(x+4)-9}{(x-5)[\sqrt{x+4}+3]}$

$= \lim_{x\to 5}\frac{x-5}{(x-5)\left[\sqrt{x+4}+3\right]} = \lim_{x\to 5}\frac{1}{\sqrt{x+4}+3}$

$= \frac{1}{\sqrt{5+4}+3} = \frac{1}{\sqrt{9}+3} = \frac{1}{3+3} = \frac{1}{6}$

15) Evaluate the following limits :$lim_{x\to 0}\frac{\sqrt{1+x^2}-1}{x}$

Answer : $lim_{x\to 0}\frac{\sqrt{1+x^2}-1}{x}$

Multiplying and dividing by $(\sqrt{1+x^2}+1)$ we get,

$lim_{x\to 0}\frac{\sqrt{1+x^2}-1}{x}\times\frac{\sqrt{1+x^2}+1}{\sqrt{1+x^2}+1} = lim_{x\to 0}\frac{(1+x^2)-1}{x[\sqrt{1+x^2}+1]} = lim_{x\to 0}\frac{x^2}{x[\sqrt{1+x^2}+1]}$

$= lim_{x\to 0}\frac{x^2}{x\sqrt{1+x^2}+1} = \frac{0}{\sqrt{1}+1} = \frac{0}{2} = 0$

16) Evaluate the following limits :

$lim_{x\to 5}\frac{\sqrt{x-1}-2}{x-5}$

Answer : $lim_{x\to 5}\frac{\sqrt{x-1}-2}{x-5}$

Multiplying and dividing by$(\sqrt{x-1}+2)$ we get,

$lim_{x\to 5}\frac{\sqrt{x-1}-2}{x-5}\times\frac{\sqrt{x-1}+2}{\sqrt{x-1}+2} = lim_{x\to 5}\frac{(x-1)-4}{x-5[\sqrt{x-1}+2]}$

$= \lim_{x\to 5}\frac{(x-5)}{(x-5)\left[\sqrt{x-1}+2\right]}$

$= lim_{x\to 5}\frac{1}{\sqrt{x-1}+2} = \frac{1}{\sqrt{5-1}+2}$

$\frac{1}{\sqrt{4}+2} = \frac{1}{2+2} = \frac{1}{4}$

$lim_{x\to 5}\frac{\sqrt{x-1}-2}{x-5} = \frac{1}{4}$

17) Calculate $lim_{x\to\infty}\frac{1-x^3}{3x+2}$

Answer : Dividing by x, we get

$\frac{1-x^3}{3x+2} = \frac{\frac{1}{x}-x^2}{3+\frac{2}{x}} \to -\infty\ as x\to\infty$

Therefore the limit does not exist.

18) Evaluate the following limits :$lim_{x\to\infty}(1+\frac{1}{x})^{7x}$

Answer : $lim_{x\to\infty}(1+\frac{1}{x})^{7x} = lim_{x\to\infty}[(1+\frac{1}{x})^x]^7$

Put $\frac{1}{x} = t$,

when $x \to \infty$ means $\frac{1}{x} \to 0$

$\therefore \frac{1}{x} \to 0$ means $t \to 0$

$= [lim_{t\to 0}[(1+t)^{\frac{1}{t}}]^7 = e^7 \ [\because lim_{x\to 0}(1+x)^{\frac{1}{x}} = e]$

$\therefore lim_{x\to\infty}(1+\frac{1}{x})^{7x} = e^7$

19) Evaluate the following limits :$lim_{x\to\infty}(1+\frac{k}{x})^{\frac{m}{x}}$

Answer : $lim_{x\to\infty}(1+\frac{k}{x})^{\frac{m}{x}}$

put $\frac{1}{x} = t$

$\therefore lim_{x\to\infty}(1+\frac{k}{x})^{\frac{m}{x}} = lim_{\frac{1}{x}\to\infty}(1+kt)^{mt} = lim_{t\to 0}(1+0)^{m(0)} = lim_{t\to 0}(1)^0 = 1$

20) Evaluate the following limits :$lim_{x\to 0}\frac{2\ arc\ sinx}{3x}$

Answer : $lim_{x\to 0}\frac{2\ arc\ sinx}{3x} = \frac{2}{3}.lim_{x\to 0}\frac{arc(sin\ x)}{x} = \frac{2}{3}\times 1 = \frac{2}{3}$