11TH MATHS SETS RELATIONS AND FUNCTIONS

MOST IMPORTANT QUESTION ANSWERS

288) Find the largest possible domain of the real valued function f(x) = $\frac{\sqrt{4-x^2}}{\sqrt{x^2-9}}$

Answer : Given f(x) = $\frac{\sqrt{4-x^2}}{\sqrt{x^2-9}}$

When x = 2, f(x) = 0

When x = -2, f(x) = 0

For all the other values, we get negative value in the square root which is not possible.

$\therefore$ Domain = {2, -2}

287) Write the values of f at -3, 5, 2, -1, 0 if

$$f(x) = \begin{cases} x^2+x-5 & if\ x \in (-\infty, 0) \\ x^2+3x-2 & if\ x \in (3, \infty) \\ x^2 & if\ x \in (0, 2) \\ x^2-3 & otherwise \end{cases}$$

Answer : f(-3) = $(-3)^2$ - 3 $\left[\therefore\ f(x) = x^2-3 \quad when\ x = -3\right]$

= 9 - 3 = 6

f(5) = 5^2 + 3(5)-2 $\left[\therefore f(x) = x^2+3x-2 \quad when \quad x = 5\right]$

= 25 + 15 - 2

= 38

f(2) = 2^2 - 3

= 4 - 3 = 1 $\left[\therefore\ f(x) = x^2-3\ when\ x = 2\right]$

f(-1) = $(-1)^2$ + (-1) -5 $\left[\therefore\ f(x) = x^2+x-5\ when\ x = -1\right]$

= 1-1-5 = -5

f(0) = 0^2-3 = -3 $\left[\therefore\ f(x) = x^2-3\ when\ x = 0\right]$

$\therefore$ f(-3) = 6, f(5) = 38, f(2) = 1, f(-1) = -5, f(0) = -3

278) On the set of natural number let R be the relation defined by aRb if 2a + 3b = 30. Write down the relation by listing all the pairs. Check whether it is reflexive

Answer : 2a + 3b = 30

R = {(3.8), (6,6), (9,4). (12,2)}

Not reflexive, Not Symmetric, transitive, hence not an equivalence relation.

279) On the set of natural number let R be the relation defined by aRb if 2a + 3b = 30. Write down the relation by listing all the pairs. Check whether it is symmetric

Answer : Given relation is 2a + 3b = 30 for all a, b $\in$ N.

2a + 3b = 30 $\Rightarrow$ 2a =30 - 3b

$\Rightarrow a = \frac{30-3b}{2}$

a	12	9	6	3
b	2	4	6	8

$\therefore$ The list of ordered pairs are (12, 2) (9, 4) (6, 6) (3, 8)

Symmetricity : (9, 4) $\in$ R $\Rightarrow$ (4, 9) $\notin$ R

$\therefore$ R is not symmetric

280) On the set of natural number let R be the relation defined by aRb if 2a + 3b = 30. Write down the relation by listing all the pairs. Check whether it is transitive

Answer : Given relation is 2a + 3b = 30 for all a, b ∈ N.

2a + 3b = 30 ⇒ 2a =30 - 3b

$\Rightarrow a=\frac{30-3b}{2}$

a	12	9	6	3
b	2	4	6	8

∴ The list of ordered pairs are (12, 2) (9, 4) (6, 6) (3, 8)

Transitivity: Clearly R is not transitive.

276) Let X = {a, b, c, d}, and R = {(a, a) (b, b) (a, c)}. Write down the minimum number of ordered pairs to be included to R to make it

(i) reflexive

(ii) symmetric

(iii) transitive

(iv) equivalence

Answer : X = {a, b, c, d}

R = {(a, a), (b, b), (a, c)}

(i) To make R reflexive we need to include (c, c) and (d, d)

(ii) To make R symmetric we need to include (c, a)

(iii) R is transitive

(iv) To make R reflexive we need to include (c, c)

To make R symmetric we need to include (c, c) and (c, a) for transitive

∴ The relation now becomes

R = {(a, a), (b, b), (a, c), (c, c), (c, a)}

∴ R is equivalence relation.

262) If the function $f: R \rightarrow R$ be given by $f(x)=x^2+2$ and g : R → **R** be given by $g(x)=\frac{\dot{x}}{x-1}$ find f o g and g o f

Answer : Clearly

Range f = domain g and

runge g = domain f

fog and gof both exist.

$$\text{Now, } (\text{ fog })(x)=f(g(x))$$

$$=f\left(\frac{x}{x-1}\right)=\left(\frac{x}{x-1}\right)^2+2$$

$$(\text{ fog })(x)=\frac{x^2}{(x-1)^2}+2$$

$$\text{And } (\text{ gof })(x)=\mathrm{g}[\mathrm{f}(x)]$$

$$=g\left(x^2+2\right)=\frac{x^2+2}{\left(x^2+2\right)-1}$$

$$(\text{gof})(x)=\frac{x^2+2}{x^2+1}$$

275) Write the values of f at -4, 1, -2, 7, 0 if

$$f(x)=\begin{cases}-x+4 & if -\infty < x \leq -3\\ x+4 & if -3 < x < -2\\ x^2-x & if -2 \leq x < 1\\ x-x^2 & if 1 \leq x < 7\\ 0 & otherwise\end{cases}$$

Answer : f(-4) = +4 + 4 [∴ f(x) = -x + 4 when x = -4]

=8

f(1) = $1-1^2$ [∴ f(x) = $x-x^2$ when x = 1]

f(1) = 0

f(-2) = $(-2)^2-(-2)$ [∴ f(x) = x^2-x when x = -2]

= 4+2 = 6

f(7) = 0 [∴ f(x) = 0 when x = 7]

f(0 = 0^2-0 [∴ f(x) = x^2 - x when x = 0]

=0

∴ f(-4) = 8, f(1) = 0, f(-2) = 6, f(7) = 0 and f(0) = 1

261) Let f : {2, 3, 4, 5} {3, 4, 5, 9} and g : { 3, 4, 5, 9} → {7, 11, 15} be functions defined as f(2) = 3, f(3) = 4, f(4) = f(5) = 5 and g(3) = g(a) = 7 and g(5) = g(9) = 11 find g o f.

Answer : Range f = {3, 4, 5}

it is a subset of domain of g.

So, g o f exists

And, g o f : {2, 3, 4, 5} + {7, 11, 15} such that

$(\text{gof})(2)=g(f(2))=g(3)=7$

$(\text{gof})(3)=g(f(3))=g(4)=7$

$(\text{gof})(4)=g(f(4))=g(5)=11$

$(\text{gof})(5)=g(f(5))=g(5)=11$

Hence gof : {2, 3, 4, 5} + {7,11, 15} and such that

gof = {(2, 7), (3 ,7), (4, 11), (5, 11)}.

254) Find the range of the following functions given by $f(x)=\frac{1}{2-sin\ 3x}$.

Answer : We have $f(x)=\frac{1}{2-sin\ 3x}$

-1 ≤ sin 3x ≤ 1 for all x ∈ R

⇒ -1 ≤ - sin 3x ≤ 1 for all x ∈ R

⇒ 1 ≤ 2 - sin 3x ≤ 3 for all x ∈ R

⇒ 2 - sin 3x ≠ 0 for all x ∈ R

⇒ f(x) = $\frac{1}{2-sin\ 3x}$ is defined for all x ∈ R

Hence, domain (f) =R

Range of f: As discused above

1 ≤ 2 - sin 3x ≤ 3 for all x ∈ R

⇒ $\frac{1}{3} \leq \frac{1}{2-sin\ 3x}$ -sin3x ≠ 0 for all x ∈ R

⇒ $\frac{1}{3}$ ≤ f(x) ≤1 for all x ∈ R.

⇒ f(x) ∈ R [1/3, 1]

Hence, range (f) = [1/3, 1]

253) Find the domain and range of the function f(x) = $\frac{1}{\sqrt{x-5}}$.

Answer : Given that : f(x) = $\frac{1}{\sqrt{x-5}}$

Here, it is clear that/ex) is real when x - 5 > 0 ⇒ x > 5

Hence, the domain = $(5, \infty)$

Now to find the range put

$f(x) = y = \frac{1}{\sqrt{x-5}}$

$\Rightarrow \sqrt{x-5} = \frac{1}{y} \Rightarrow x - 5 = \frac{1}{y^2}$

$\Rightarrow x = \frac{1}{y^2} + 5$

For x ∈ $(5, \infty)$, y ∈ R^+.

Hence, the range of f = R^+

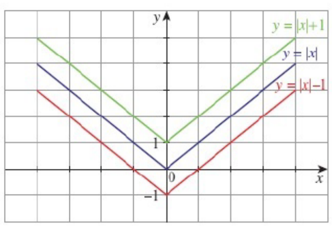
232) Consider the functions:

(i) f(x) = |x|

(ii) f(x) = |x| − 1

(iii) f(x) = |x| + 1

Answer :



f(x) = |x| − 1 causes the graph of the functionf(x) = |x| shifts to the downward for one unit. f(x) = |x| + 1 causes the graph of the function f(x) = |x| shifts to the upward for one unit.

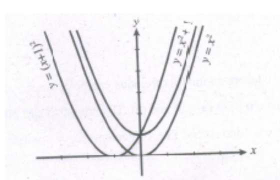
228) Consider the functions:

i) $f(x) = x^2$,

ii) $f(x) = x^2 + 1$,

iii) $f(x) = (x+1)^2$

Answer :



f(x) = x^2 + 1causes the graph of the function f(x) = x^2 shifts to the upward for one unit.

f(x) = $(x + 1)^2$ causes the graph of the function f(x) = x^2 shifts to the left for one unit.

218) Find the largest possible domain for the real valued function given by $f(x) = \frac{\sqrt{9-x^2}}{x^2-1}$.

Answer : If x < -3 or x > 3, then x^2 will be greater than 9 and hence 9 - x^2 will become negative which has no square root in R.

So x must lie on the interval [- 3, 3].

Also if $x \geq -1$ or $x \leq 1$, then x^2-1 will become negative or zero. If it is negative, x^2 - 1 has no square root in R. If it is zero, f is not defined. So, x must lie outside [- 1, 1].

That is x must lie on $(-\infty, -1] \cup [1, \infty)$, Combining these two conditions, the largest possible domain for f is $[-3, 3] \cap ((-\infty, -1) \cup (1, \infty))$. That is $[-3, -1) \cup (1, 3]$.

217) Find the range of the function $f(x) = \frac{1}{1-3\cos x}$.

Answer : Clearly

$-1 \leq \cos x \leq 1$

$\Rightarrow -3 \geq -3\cos x \geq -3$

$\Rightarrow -3 \leq -3\cos x \leq 3$

$\Rightarrow 1 - 3 \leq 1 - 3\ \cos x \leq 1 + 3$

Thus we get $-2 \leq 1 - 3\ \cos\ x$ and $1 - 3\cos\ x \leq 4$.

By taking reciprocals, we get $\frac{1}{1-3\cos x} \leq -\frac{1}{2}$ and $\frac{1}{1-3\cos\ x} \geq \frac{1}{4}$.

Hence the range of f is $\left(-\infty, \frac{1}{2}\right] \cup \left[\frac{1}{4}, \infty\right)$.

216) Find the domain of $f(x) = \frac{1}{1-2\cos\ x}$.

Answer : The function is defined for all x ∈ R except 1 - 2 cos x = 0. That is, except $\cos x = \frac{1}{2}$. That is except $x = 2n\pi \pm \frac{\pi}{3}, n \in Z$. Hence the domain is $R - \{2n\pi \pm \frac{\pi}{2}\}, n \in Z$.

212) If A and B are two sets so that n(B - A) = 2n(A - B) = $4n(A \cap B)$ and if $n(A \cup B) = 14$, then find n(P (A)).

Answer : To find n(P(A)), we need n(A)

Let $n(A \cap B) = k$. Then $n(A - B) = 2k$ and $n(B - A) = 4k$

Now $n(A \cup B) = n(A - B) + n(B - A) + n(A \cap B) = 7k$.

It is given that $n(A \cup B) = 14$. Thus 7k = 14 and hence k = 2

So n(A - B) = 4 and n(B - A) = 8. As $n(A) = n(A - B) + n(A \cap B)$, we get n(A) = 6 and hence n(P(A)) = 2^6 = 64

206) Find the range of the function $\frac{1}{2cosx-1}$

Answer : Range of cosine function is -1 $\leq$cos x $\leq$ 1

$\Rightarrow$ -2 $\leq$ 2 cos x $\leq$ 2 (Multiplied by 2)

$\Rightarrow$ -2 -1 $\leq$ 2 cos x -1 $\leq$ 2-1

$\Rightarrow$ -3 $\leq$ 2 cos x-1 $\leq$ 1

$\Rightarrow \frac{-1}{3} > \frac{1}{2cosx-1} > \frac{1}{1}$

$\Rightarrow \frac{-1}{3} f(x) > 1$

$\therefore\ Range = \left(-\infty, -\frac{1}{3}\right] \cup [1, \infty)$

205) Find the domain of $\frac{1}{1-2sinx}$

Answer : Let f(x) = $\frac{1}{1-2sinx}$

When the denominator is 0,

1-2 sin x = 0

$\Rightarrow$ 1 = 2 sin x

$\Rightarrow sin \quad x = \frac{1}{2}$

$\Rightarrow sin \quad x = sin\frac{\pi}{6}$

$\Rightarrow x = n\pi + (-1)^n \frac{\pi}{6} n \in Z \quad [\because sin \quad x = sin\alpha \Rightarrow x = n\pi + (-1)^n \alpha \quad n \in Z]$

Domain of f(x) is R - $\left(n\pi + (-1)^n \frac{\pi}{6}\right), n \in Z$

196) If n (p(A)) = 1024, n(A∪B) = 15 and n(p(B)) = 32, then find n(A∩B).

Answer : Given n(p(A)) = 1024 = 2^{10}

$\Rightarrow$ n(A) = 10 [∴ if n(A) = n, then n(p(A)) = 2^n]

n(p(B)) = 32 = 2^5

$\Rightarrow$ n(B) = 5.

We know that,

n(A∪ B) = n(A) + n(B) - n(A∩B)

$\Rightarrow$ 15 = 10+5 - (A∩B)

$\Rightarrow$ n(A∩B) = 0.

177) Find the domain and range of the function f(x) = $\frac{x^2-9}{x-3}$.

Answer : We have f(x) = $\frac{x^2-9}{x-3}$

Domain of f : Clearly f(x) is not defined for x - 3 = 0 i.e. x = 3. Therefore, Domain (f) = R- {3}

Range of f: Let f(x) = y. Then,

f(x) = y ⇒ $\frac{x^2-9}{x-3}$=y ⇒ x+3 = y

it follows from the above relation that y takes all real values except 6 when x takes values in the ser R - {3}. Therefore, Range (f) = R {6}.

176) Find the range of the following functions given by f(x) = 1 + 3 cos 2x.

Answer : Given that: f(x) = 1 + 3 cos 2x

We know that -1 ≤ cos 2x ≤ 1

⇒ -3 ≤ 3 cos 2x ≤ 3 ⇒ -3 + 1 ≤ 1 + 3 cos 2x ≤ 3 + 1

⇒ -2 ≤ 1 + 3 cos 2x ≤ 4 ⇒ -2 ≤ f(x) ≤ 4

Hence the range of f = [-2, 4]

175) Find the domain of each of the following functions given by:

f(x) = $\frac{x^3-x+3}{x^2-1}$.

Answer : Given that f(x) = $\frac{x^3-x+3}{x^2-1}$

Here, f(x) is not defined if $x^2 - 1 \neq 0$

$(x - 1)(x + 1) \neq 0$

$x \neq 1, x \neq -1$

Hence, the domain of f = R - {-1, 1}

174) If f(x) = $\frac{x-1}{x+1}$, then show that, $f\left(\frac{-1}{x}\right) = \frac{-1}{f(x)}$.

Answer : $f\left(\frac{-1}{x}\right) = \frac{\frac{1}{x}-1}{\frac{1}{x}+1} = \frac{-\left(\frac{1}{x}+1\right)}{-\left(\frac{1}{x}-1\right)} = \frac{1+x}{1-x} = \frac{1}{\frac{1-x}{1+x}} = \frac{1}{\left(\frac{x-1}{x+1}\right)} = \frac{-1}{f(x)}$

Hence, $f\left(\frac{-1}{x}\right) = \frac{-1}{f(x)}$.

167) Find the domain of $d(x) = \frac{1}{1-3cosx}$

Answer : The function is defined for all x ∈ R except 1-2cos x = 0. That is, except $\cos x = \frac{1}{2}$. That is except, $x = 2n\pi \pm \frac{\pi}{3}n \in Z$. Hence the domain is $R - \{2n\pi \pm \frac{\pi}{3}\}, n \in Z$

152) Consider f : {1, 2, 3} ⟶ {a, b, c} given by f(1) = a, f(2) = b and f(3) = c. Show that $(f^{-1})^{-1} = f$

Answer : Given f(1) = a, f(2) = b, f(3) = c

∴ f = {(1, a) (2, b) (3, c)}...(1)

Clearly f is a bijection and invertible

∴ f^{-1} = {(a, 1) (b, 2) (c, 3)}

⇒ $(f^{-1})^{-1}$ = {(1, a) (2, b) (3, c)}...(2)

From (1) and (2), it is clear that $f = (f^{-1})^{-1}$

141) Show that the relation R on the set A = {1, 2, 3} given by R = {(1, 1) (2, 2) (3, 3) (1, 2) (2, 3)} is reflexive but neither symmetric nor transitive.

Answer : Since A = {1, 2, 3}

(1,1) (2, 2) (3, 3) ∈ R ⇒ is reflexive.

Also (1, 2) ∈ R but (2, 1) ∉ R ⇒ R is not symmetric.

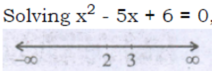
And (1, 2) ∈ R, (2, 3) ∈ R but (1, 3) ∉ R ⇒ R is not transitive.

R is reflexive but neither symmetric nor transitive.

134) Find the largest possible domain for the real valued function f defined by $f(x) = \sqrt{x^2 - 5x + 6}$.

Answer : As we are finding the square root of $x^2 - 5x + 6$, we must have $x^2 - 5x + 6 \geq 0$ for all x in the domain.

Solving $x^2 - 5x + 6 = 0$, we get x = 2 and 3. Now draw the number line as below:



Now we have three intervals. $(-\infty, 2)$, (2, 3) and $(3, \infty)$.

(i) Take any point in $(-\infty, 2)$, say x = 1. Clearly $x^2 - 5x + 6$ is positive.

(ii) Take any point in (2, 3), say x = 2.5. Clearly $x^2 - 5x + 6$ is negative.

(iii) Take anypoint in $(3, \infty)$ say x = 4. Clearly $x^2 - 5x + 6$ is positive.

For all x, in the intervals $(-\infty, 2)$ and $(3, \infty)$, $x^2 - 5x + 6$ is positive. At x = 2,3 the value of $x^2 - 5x + 6$ is zero. Thus, $\sqrt{x^2 - 5x + 6}$ is defined for all x in $(-\infty, 2] \cup [3, \infty)$. Hence the domain of $\sqrt{x^2 - 5x + 6}$ is $(-\infty, 2] \cup [3, \infty)$.