

MINIMUM LEARNING MATERIALS

11th Standard

Maths

65 x 2 = 130

- 1) Write the following in roster form.
 $\{x \in \mathbb{N} : x^2 < 121 \text{ and } x \text{ is a prime}\}$
- 2) Write the following in roster form.
The set of all positive roots of the equation $(x-1)(x+1)(x^2-1) = 0$.
- 3) Write the following in roster form $\{x \in \mathbb{N} : 4x + 9 < 52\}$
- 4) Simplify and hence the value of n: $3^{2n}9^{23^{-n}}/3^{3n} = 27$
- 5) Find the value of $\sin 105^\circ$
- 6) Solve for x $|4x - 5| \geq -2$
- 7) Solve for x $|x| - 10 < -3$
- 8) Solve $\frac{1}{|2x-1|} < 6$ and express the solution using the interval notation.
- 9) Solve $2|x+1| - 6 \leq 7$ and graph the solution set in a number line.
- 10) If $n(p(A)) = 1024$, $n(A \cup B) = 15$ and $n(p(B)) = 32$, then find $n(A \cap B)$.
- 11) Show that $\tan(45^\circ + A) = \frac{1 + \tan A}{1 - \tan A}$
- 12) Discuss the following relations for reflexivity, symmetricity and transitivity :
The relation R defined on the set of all positive integers by "mRn if m divided n".
- 13) Discuss the following relations for reflexivity, symmetricity and transitivity:
Let A be the set consisting of all the members of a family. The relation R defined by "aRb if a is not a sister of b".
- 14) Discuss the following relations for reflexivity, symmetricity and transitivity :
On the set of natural numbers, the relation R is defined by "xRy if $x + 2y = 1$ ".
- 15) Find the value $\sin(-1110^\circ)$
- 16) Find the value of n if $\frac{1}{8!} + \frac{1}{9!} = \frac{n}{10!}$
- 17) If ${}^{(n-1)}P_3 : {}^n P_4 = 1 : 10$, find n
- 18) Find the equation of the lines passing through the point (1, 1)
(i) with y-intercept (-4)

(ii) with slope 3

(iii) and $(-2, 3)$

(iv) and the perpendicular from the origin makes an angle 60° with x- axis.

19) Write the equation of the lines through the point $(1, -1)$

(i) parallel to $x + 3y - 4 = 0$

(ii) perpendicular to $3x + 4y = 6$

20) Find the distance between the parallel lines

$$12x + 5y = 7 \text{ and } 12x + 5y + 7 = 0.$$

21) Write the first 6 terms of the sequences whose n^{th} term a_n given below

$$a_n = \begin{cases} n+1 & \text{if } n \text{ is odd} \\ n & \text{if } n \text{ is even} \end{cases}$$

22) Write the first 6 terms of the sequences whose n^{th} term a_n given below

$$z_n = \begin{cases} n \\ a_{n-1} + a_{n-2} + a_{n-3} \end{cases}$$

23) If ${}^{15}C_{2r-1} = {}^{15}C_{2r+4}$, find r .

24) Find the number of subsets of A if $A = \{x : x = 4n + 1, 2 \leq n \leq 5, n \in N\}$.

25) Solve $3|x-2| + 7 = 19$ for x .

26) If $A = \{1, 2, 3, 4\}$ and $B = \{3, 4, 5, 6\}$, find $n((A \cup B) \times (A \cap B) \times (A \Delta B))$

27) If $p(A)$ denotes the power set of A, then find $n(P(P(P(\phi))))$.

28) Solve $3x - 5 \leq x + 1$ for x .

29) If ${}^{10}P_r = {}^7P_{r+2}$ find r .

30) A Mathematics club has 15 members. In that 8 are girls. 6 of the members are to be selected for a competition and half of them should be girls. How many ways of these selections are possible?

31) If $A + B = 45^\circ$, find $\tan 22 \frac{1^\circ}{2}$

32) Find the angle between the lines $3x^2 + 10xy + 8y^2 + 14x + 22y + 15 = 0$.

33) If $A = \begin{bmatrix} 1 & -1 & 2 \\ -2 & 1 & 3 \\ 0 & -3 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & -3 \\ -1 & 1 \\ 1 & 2 \end{bmatrix}$ find AB and BA if they exist.

34)

Find the value of x if
$$\begin{vmatrix} x-1 & x & x-2 \\ 0 & x-2 & x-3 \\ 0 & 0 & x-3 \end{vmatrix} = 0$$

35)

Prove that
$$\begin{bmatrix} \sec^2\theta & \tan^2\theta & 1 \\ \tan^2\theta & \sec^2\theta & -1 \\ 38 & 36 & 2 \end{bmatrix} = 0$$

36) Without expanding, evaluate the following determinants:

$$\begin{vmatrix} x+y & y+z & z+x \\ z & x & y \\ 1 & 1 & 1 \end{vmatrix}$$

37) Find the area of the triangle whose vertices are (0, 0), (1, 2) and (4, 3).

38)

Identify the singular and non-singular matrices:
$$\begin{bmatrix} 0 & a-b & k \\ b-a & 0 & 5 \\ -k & -5 & 0 \end{bmatrix}$$

39) Find $\vec{a} \cdot \vec{b}$ when \vec{a} and \vec{b} represent the points (2, 3, - 1) and (- 1, 2, 3).

40) Find $(\vec{a} + 3\vec{b}) \cdot (2\hat{a} - \hat{b})$ if $\vec{a} = \hat{i} + \hat{j} + 2\hat{k}$ and $\vec{b} = 3\hat{i} + 2\hat{j} - \hat{k}$

41) Find the angle between the vectors $5\hat{i} + 3\hat{j} + 4\hat{k}$ and $6\hat{i} - 8\hat{j} - \hat{k}$.

42) Find the angle between the vectors $2\hat{i} + 3\hat{j} - 6\hat{k}$ and $6\hat{i} - 3\hat{j} + 2\hat{k}$

43) Find the projection of the vector $\hat{i} + 3\hat{j} + 7\hat{k}$ on the vector $2\hat{i} + 6\hat{j} + 3\hat{k}$.

44) Find λ , when the projection of $\vec{a} = \lambda\hat{i} + \hat{j} + 4\hat{k}$ on $\vec{b} = 2\hat{i} + 6\hat{j} + 3\hat{k}$ is 4 units.

45) Find $|\vec{a} \times \vec{b}|$, where $\vec{a} = 3\hat{i} + 4\hat{j}$ and $\vec{b} = \hat{i} + \hat{j} + \hat{k}$.

46) Find the magnitude of $\vec{a} \times \vec{b}$ if $\vec{a} = 2\hat{i} + \hat{j} + 3\hat{k}$ and $\vec{b} = 3\hat{i} + 5\hat{j} - 2\hat{k}$.

47) Evaluate the following limits :

$$\lim_{x \rightarrow 2} \frac{x^4 - 16}{x - 2}$$

48)

Evaluate the following limits :
$$\lim_{x \rightarrow 0} \frac{\sqrt{1+x^2} - 1}{x}$$

49) Differentiate the following with respect to x : $y = x^3 + 5x^2 + 3x + 7$

50) Differentiate the following with respect to x : $y = e^x + \sin x + 2$

51) Differentiate the following with respect to x : $y = (x - \frac{1}{x})^2$

52) Differentiate the following with respect to x : $y = xe^x \log x$

53) Differentiate the following with respect to x : $y = \frac{\cos x}{x^3}$

54) Differentiate : $y = e^{\sin x}$.

55) Integrate the following with respect to x : $\frac{e^{2x}-1}{e^x}$

56) Integrate the following with respect to x : $e^{ax} \cos bx$

57) When a pair of fair dice is rolled, what are the probabilities of getting the sum (i) 7 (ii) 7 or 9 (iii) 7 or 12?

58) If A and B are mutually exclusive events $P(A) = \frac{3}{8}$ and $P(B) = \frac{1}{8}$, then find (i) $P(\bar{A})$ (ii) $P(A \cup B)$ (iii) $P(\bar{A} \cap B)$ (iv) $P(\bar{A} \cup \bar{B})$

59) If A and B are two events associated with a random experiment for which $P(A) = 0.35$, $P(A \text{ or } B) = 0.85$, and $P(A \text{ and } B) = 0.15$. Find (i) $P(\text{only } B)$ (ii) $P(\bar{B})$ (iii) $P(\text{only } A)$

60) Given that $P(A) = 0.52$, $P(B) = 0.43$, and $P(A \cap B) = 0.24$, find (i) $P(\bar{A} \cap \bar{B})$ (ii) $P(A \cup B)$ (iii) $P(\bar{A} \cap B)$ (iv) $P(\bar{A} \cup \bar{B})$

61) If $P(A) = 0.6$, $P(B) = 0.5$ and $P(A \cap B) = 0.2$ Find $P(A/\bar{B})$

62) Given that $P(A) = 0.52$, $P(B) = 0.43$, and $P(A \cap B) = 0.24$, find

$$P(\bar{A} \cap \bar{B})$$

63) Given that $P(A) = 0.52$, $P(B) = 0.43$, and $P(A \cap B) = 0.24$, find

$$P(A \cup B)$$

64) Find the position vector of a point R which divides the line joining points P and Q whose position vectors are $\hat{i} + 2\hat{j} - \hat{k}$ and $-\hat{i} + \hat{j} + \hat{k}$ respectively in the ratio 2:1 externally.

65) Find the distance between the line $4x + 3y + 4 = 0$ and a point (7, -3)

66) State whether the following sets are finite or infinite.

$$\{x \in \mathbb{N} : x \text{ is an even prime number}\}$$

67) If $f: R \rightarrow R$ is defined as $f(x) = 2x^2 - 1$, find the pre-image of 17, 4 and -2.

68) Let $A = \{a, b, c\}$, and $R = \{(a, a) (b, b) (a, c)\}$. Write down the minimum number of ordered pairs to be included to R to make it

Equivalence.

69)

$$\text{Evaluate } \left(\left[(256)^{\frac{-1}{4}} \right]^{\frac{-1}{4}} \right)^3$$

70) Discuss the nature of roots of $4x^2 - x - 2 = 0$

71) Write $f(x) = x^2 + 5x + 4$ in completed square form.

72) Rationalize the denominator of $\frac{\sqrt{5}}{(\sqrt{6} + \sqrt{2})}$

73) Solve $23x < 100$ when
(i) x is a natural number
(ii) x is an integer.

74) Prove that $\sin 4\alpha = 4 \tan \alpha \cdot \frac{1 - \tan^2 \alpha}{(1 + \tan^2 \alpha)^2}$

75) Express each of the following product as a sum or difference. $\sin \frac{x}{2} \cos \frac{3x}{2}$.

76) Express each of the following as a sum or difference. $\sin 35^\circ \cos 28^\circ$

77) Express each of the following as a product.
 $\sin 75^\circ - \sin 35^\circ$

78) Prove that $\cos(30 + x) = \frac{\sqrt{3} \cos x - \sin x}{2}$

79) Find the value $\tan (1050^\circ)$

80) Find $\sin 15^\circ$, $\cos 15^\circ$ and $\tan 15^\circ$. Hence evaluate $\cot 75^\circ + \tan 75^\circ$.

81) Solve: $\sin 2x + \sin 6x + \sin 4x = 0$

82) Simplify: $\cos A + \cos (120^\circ + A) + \cos (120^\circ - A)$

83) Show that $\cos 10^\circ \cos 30^\circ \cos 50^\circ \cos 70^\circ = \frac{3}{16}$.

84) Find the values of $\cos 2A$, A lies in the first quadrant, when $\cos A = \frac{15}{17}$

85) How many two-digit numbers can be formed using 1, 2, 3, 4, 5 without repetition of digits?

86) In how many ways 5 persons can be seated in a row?

- 87) Evaluate $\frac{n!}{r!(n-r)!}$ when $n = 50$, $r = 47$.
- 88) If $n! + (n - 1)! = 30$, then find the value of n .
- 89) How many chord can be drawn through 21 points on a circle?
- 90) In how many ways can 5 girls can 3 boys be seated in a row so that no two boys are together?
- 91) Show that the sum of $(m + n)^{\text{th}}$ and $(m - n)^{\text{th}}$ term of an A.P is equal to twice the m^{th} term.
- 92) Find the sum up to n terms of the series : $1 + \frac{6}{7} + \frac{11}{49} + \frac{16}{343} + \dots$
- 93) Write the n^{th} term of the following sequences
 $\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \frac{5}{6}$
- 94) In the binomial expansion of $(1+a)^{m+n}$, Prove that the coefficients of a^m and a^n are equal.
- 95) Find the general term in the expansion of $\left(\frac{4x}{5} - \frac{5}{2x}\right)^9$
- 96) Determine x so that the line passing through $(3, 4)$ and $(x, 5)$ makes 135° with the positive direction of x -axis.
- 97) Find the distance between the parallel lines.
 $3x - 4y + 5 = 0$ and $6x - 8y - 15 = 0$.
- 98) Find the value of a and p if the equation $x \cos a + y \sin a = p$ is the normal form of the line $\sqrt{3}x + y + 2 = 0$
- 99) If p is the length of the perpendicular from the origin to the line $\frac{x}{a} + \frac{y}{b} = 1$, then prove that $\frac{1}{p^2} = \frac{1}{a^2} + \frac{1}{b^2}$
- 100) Find the value of p for which the lines $8px + (2 - 3p)y + 1 = 0$ and $px + 8y + 7 = 0$ are perpendicular.
- 101) Find the equation of the straight line through the intersection of $5x - 6y = 1$ and $3x + 2y + 5 = 0$ and perpendicular to the straight line $3x - 5y + 11 = 0$.
- 102) Show that the lines are $3x + 2y + 9 = 0$ and $12x + 8y - 15 = 0$ are parallel lines.
- 103) Find the acute angle between the pair of lines given by $2x^2 - 5xy - 7y^2 = 0$.
- 104) If the area of the triangle with vertices $(-3, 0)$, $(3, 0)$ and $(0, k)$ is 9 square units, find the values of k .

105) Construct an $m \times n$ matrix $A = [a_{ij}]$, where a_{ij} is given by

$$a_{ij} = \frac{(i-2j)^2}{2} \text{ with } m = 2, n = 3$$

106) Construct an $m \times n$ matrix $A = [a_{ij}]$, where a_{ij} is given by

$$a_{ij} = \frac{|3i-4j|}{4} \text{ with } m = 3, n = 4$$

107)

If $A = \begin{bmatrix} 1 & a \\ 0 & 1 \end{bmatrix}$, then compute A^4 .

108)

Evaluate : $\begin{vmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{vmatrix}$

109)

Without expanding the determinant, prove that $\begin{vmatrix} s & a^2 & b^2 + c^2 \\ s & b^2 & c^2 + a^2 \\ s & c^2 & a^2 + b^2 \end{vmatrix} = 0$

110)

Show that $\begin{vmatrix} b+c & bc & b^2c^2 \\ c+a & ca & c^2a^2 \\ a+b & ab & a^2b^2 \end{vmatrix} = 0$

111) Find the vectors of magnitude 6 which are perpendicular to both vectors

$$\vec{a} = 4\hat{i} - \hat{j} + 3\hat{k} \text{ and } \vec{b} = -2\hat{i} + \hat{j} - 2\hat{k}$$

112)

For any two vectors \vec{a} and \vec{b} , prove that $|\vec{a} \times \vec{b}|^2 + (\vec{a} \cdot \vec{b})^2 = |\vec{a}|^2 |\vec{b}|^2$

113) Find the direction cosines of the line joining (2, 3, 1) and (3, -1, 2).

114) Show that the vectors $2\hat{i} - 3\hat{j} + 4\hat{k}$ are $-4\hat{i} + 6\hat{j} - 8\hat{k}$ are collinear.

115) In problem, using the table estimate the value of the limit

$$\lim_{x \rightarrow 0} \frac{\sqrt{x+3} - \sqrt{3}}{x}$$

x	-0.1	-0.01	-0.001	0.001	0.01	0.1
f(x)	0.2911	0.2891	0.2886	0.2886	0.2885	0.28631

116) Evaluate the following limits :

$$\lim_{x \rightarrow 1} \frac{x^m - 1}{x^n - 1}, m \text{ and } n \text{ are integers.}$$

117)

Evaluate the following limits : $\lim_{x \rightarrow 0} \frac{\tan 2x}{\sin 5x}$

- 118) Examine the continuity of the following: $e^x \tan x$
- 119) Differentiate the following: $y = \cos (\tan x)$
- 120) Differentiate the following: $y = (1 + \cos^2 x)^6$
- 121) Find f' if $f(x) = x \cos x$.
- 122) Find the derivatives of the following : $y = x^{\log x} + (\log x)^x$
- 123) Find the derivatives of the following : $\sqrt{xy} = e^{(x-y)}$
- 124) Find the derivatives of the following : $x^y = y^x$
- 125) Integrate the function with respect to x : $e^{-x} \cos 2x$
- 126) Integrate the following with respect to x : $2 \cos x - 4 \sin x + 5 \sec^2 x + \operatorname{cosec}^2 x$
- 127) Evaluate : $\int \sqrt{1 + \sin 2x} dx$
- 128) Integrate the following with respect to x : $\frac{\sin \sqrt{x}}{\sqrt{x}}$
- 129) Integrate the following with respect to x : $\frac{\cot x}{\log (\sin x)}$
- 130) Integrate the following with respect to x : $\frac{\sin^{-1}}{\sqrt{1-x^2}}$
- 131) Evaluate the following integrals : $\int e^{3x} \cos 2x dx$
- 132) Evaluate the following integrals : $\int e^{-5x} \sin 3x dx$
- 133) If two coins are tossed simultaneously, then find the probability of getting
(i) one head and one tail (ii) at most two tails
- 134) A single card is drawn from a pack of 52 cards. What is the probability that
The card is either a queen or 9?
- 135) If $P(A) = 0.6$, $P(B) = 0.5$, and $P(A \cap B) = 0.2$ Find $P(\bar{A}/B)$
- 136) If $P(A) = 0.6$, $P(B) = 0.5$, and $P(A \cap B) = 0.2$. Find (i) $P(A/B)$
- 137) If A and B are two independent events such that, $P(A) = 0.4$ and $P(A \cup B) = 0.9$. Find $P(B)$.

$$65 \times 3 = 195$$

138) Show that $\sin^2 \frac{\pi}{18} + \sin^2 \frac{2\pi}{9} + \sin^2 \frac{7\pi}{18} + \sin^2 \frac{4\pi}{9} = 2$

139) Show that $\frac{\sin 8x \cos x - \sin 6x \cos 3x}{\cos 2x \cos x - \sin 3x \sin 4x} = \tan 2x$

140) Show that $\frac{(\cos \theta - \cos 3\theta)(\sin 8\theta + \sin 2\theta)}{(\sin 5\theta - \sin \theta)(\cos 4\theta - \cos 6\theta)} = 1$

- 141) If $\sin x = \frac{15}{17}$ and $\cos y = \frac{12}{13}$, $0 < x < \frac{\pi}{2}$, $0 < y < \frac{\pi}{2}$, find the value of $\sin (x + y)$
- 142) If $\sin x = \frac{15}{17}$ and $\cos y = \frac{12}{13}$, $0 < x < \frac{\pi}{2}$, $0 < y < \frac{\pi}{2}$, find the value of $\cos (x - y)$
- 143) If $\sin x = \frac{15}{17}$ and $\cos y = \frac{12}{13}$, $0 < x < \frac{\pi}{2}$, $0 < y < \frac{\pi}{2}$, find the value of $\tan (x + y)$
- 144) The formula for converting from Fahrenheit to Celsius temperatures is $y = \frac{5x}{9} - \frac{160}{9}$. Find the inverse of this function and determine whether the inverse is also a function.
- 145) Solve for x $\left| 3 - \frac{3}{4}x \right| \leq \frac{1}{4}$
- 146) Prove that $\frac{\sin x + \sin 3x + \sin 5x + \sin 7x}{\cos x + \cos 3x + \cos 5x + \cos 7x} = \tan 4x$
- 147) By taking suitable sets A, B, C, verify the following results:
 $(B - A) \cup C = (B \cup C) - (A - C)$
- 148) Compute $\log_9^{27} - \log_{27}^9$
- 149) Solve $\log_8 x + \log_4 x + \log_2 x = 11$
- 150) If $n(A \cap B) = 3$ and $n(A \cup B) = 10$ then find $n(P(A \Delta B))$
- 151) Solve $\log_4 2^{8x} = 2 \log_2^8$
- 152) If $a^2 + b^2 = 7ab$. Show that $\log \frac{a+b}{3} = \frac{1}{2} (\log a + \log b)$
- 153) Prove $\log \frac{a^2}{bc} + \log \frac{b^2}{ca} + \log \frac{c^2}{ab} = 0$
- 154) Prove that $\sin 105^\circ + \cos 105^\circ = \cos 45^\circ$
- 155) If $\frac{\log x}{y-z} = \frac{\log y}{z-x} = \frac{\log z}{x-y}$, then prove that $xyz = 1$
- 156) How many three-digit numbers are there with 3 in the unit place?
 (i) with repetition
 (ii) without repetition.
- 157) Find the domain of $\frac{1}{1 - 2\sin x}$
- 158) Find the distinct permutations of the letters of the word MISSISSIPPI?
- 159) If ${}^nP_r = 720$. If ${}^nC_r = 120$, find n, r = ?
- 160) Find the equation of the lines passing through the point of intersection lines $4x - y + 3 = 0$ and $5x + 2y + 7 = 0$
 (i) through the point $(-1, 2)$

(ii) Parallel to $x - y + 5 = 0$

(iii) Perpendicular to $x - 2y + 1 = 0$.

161) Find the equation of the line passing through the point (1, 5) and also co-ordinate axes in the ratio 3: 10.

162) A polygon has 90 diagonals. Find the number of its sides?

163) There are 5 teachers and 20 students. Out of them a committee of 2 teachers and 3 students is to be formed. Find the number of ways in which this can be done. Further find in how many of these committees

(i) a particular teacher is included?

(ii) a particular student is excluded?

164) Find the value of λ for which the equation $12x^2 - 10xy + 2y^2 + 11x - 5y + \lambda = 0$ represents a pair of straight lines.

165) If $X = \{1, 2, 3, \dots, 10\}$ and $A = \{1, 2, 3, 4, 5\}$, find the number of sets $B \subseteq X$ such that $A - B = \{4\}$.

166) If A and B are two sets so that $n(B - A) = 2n(A - B) = 4n(A \cap B)$ and if $n(A \cup B) = 14$, then find $n(P(A))$.

167) Solve $|2x - 3| = |x - 5|$.

168) If $n(A) = 10$ and $n(A \cap B) = 3$, find $n((A \cap B') \cap A)$.

169) If ${}^{(n+2)}P_4 = 42 \times {}^nP_2$, find n.

170) In how many ways 5 boys and 4 girls can be seated in a row so that no two girls are together.

171) If ${}^nP_r = 11880$ and ${}^nC_r = 495$, Find n and r.

172) Prove $\log \frac{75}{16} - 2\log \frac{5}{9} + \log \frac{32}{243} = \log 2$

173) If $\log_2 x + \log_4 x + \log_{16} x = \frac{7}{2}$, find the value of x.

174) If the product of the 4th, 5th and 6th terms of a geometric progression is 4096 and if the product of the 5th, 6th and 7th terms of it is 32768, find the sum of first 8 terms of the geometric progression.

175) **Solve:**

(i) $\frac{3(x-2)}{5} \leq \frac{5(2-x)}{3}$

(ii) $\frac{5-x}{3} < \frac{x}{2} - 4$.

176) If $A = \begin{bmatrix} 4 & 2 \\ -1 & x \end{bmatrix}$ and such that $(A - 2I)(A - 3I) = O$, find the value of x.

177)

Prove that
$$\begin{vmatrix} 1 & 1 & 1 \\ x & y & z \\ x^2 & y^2 & z^2 \end{vmatrix} = (x - y)(y - z)(z - x).$$

178)

Show that
$$\begin{vmatrix} 0 & c & b \\ c & 0 & a \\ b & a & 0 \end{vmatrix} = \begin{vmatrix} b^2 + c^2 & ab & ac \\ ab & c^2 + a^2 & bc \\ ab & bc & a^2 + b^2 \end{vmatrix}$$

179)

Let A and B be two points with position vectors $2\vec{a} + 4\vec{b}$ and $2\vec{a} - 8\vec{b}$. Find the position vectors of the points which divide the line segment joining A and B in the ratio 1:3 internally and externally.

180) If D and E are the midpoints of the sides AB and AC of a triangle ABC,

prove that
$$\vec{BE} + \vec{DC} = \frac{3}{2}\vec{BC}$$

181)

If D is the midpoint of the side BC of a triangle ABC, prove that
$$\vec{AB} + \vec{AC} = 2\vec{AD}$$

182) Show that the points whose position vectors are $2\hat{i} + 3\hat{j} - 5\hat{k}$, $3\hat{i} + \hat{j} - 2\hat{k}$ and, $6\hat{i} - 5\hat{j} + 7\hat{k}$ are collinear

183) Find a point whose position vector has magnitude 5 and parallel to the vector $4\hat{i} - 3\hat{j} + 10\hat{k}$.

184) Show that the points (4, - 3, 1), (2, - 4, 5) and (1, - 1, 0) form a right angled triangle.

185) Find the unit vectors perpendicular to each of the vectors $\vec{a} + \vec{b}$ and $\vec{a} - \vec{b}$, where $\vec{a} = \hat{i} + \hat{j} + \hat{k}$ and $\vec{b} = \hat{i} + 2\hat{j} + 3\hat{k}$.

186) Find the derivatives of the following functions with respect to corresponding independent variables : $y = x \sin x \cos x$

187) Find the derivatives of the following functions with respect to corresponding independent variables: $y = e^{-x} \cdot \log x$

188) Differentiate the following: $y = \frac{\sin^2 x}{\cos x}$

189) Find $\frac{dy}{dx}$ if $x = at^2$; $y = 2at$, $t \neq 0$.

190) Find y' , y'' and y''' if $y = x^3 - 6x^2 - 5x + 3$.

191) Find the derivative of $\sin x^2$ with respect to x^2 .

192) Evaluate the following integrals : $\frac{12}{(4x-5)^3} + \frac{6}{3x+2} + 16e^{4x+3}$

193) Evaluate the following integrals : $\frac{15}{\sqrt{5x-4}} - 8\cot(4x+2)\operatorname{cosec}(4x+2)$

194) Evaluate : $\int \frac{1}{\sin^2 x \cos^2 x} dx$

195) Evaluate : $\int (\tan x + \cot x)^2 dx$

196) What is the chance that (i) non-leap year (ii) leap year should have fifty three Sundays?

197) A bag contains 7 red and 4 black balls, 3 balls are drawn at random. Find the probability that (i) all are red (ii) one red and 2 black.

198) A cricket club has 16 members, of whom only 5 can bowl. What is the probability that in a team of 11 members at least 3 bowlers are selected?

199) Show that each of the given three vectors is a unit vector.

$\frac{1}{7}(2\hat{i} + 3\hat{j} + 6\hat{k})$; $\frac{1}{7}(3\hat{i} - 6\hat{j} + 2\hat{k})$; $\frac{1}{7}(6\hat{i} + 2\hat{j} - 3\hat{k})$ Also, show that they are mutually perpendicular to each other.

200) Let \vec{a} , \vec{b} and \vec{c} be non-coplanar vectors. Let A, B and C be the points whose position vectors with respect to the origin O are $\vec{a} + 2\vec{b} + 3\vec{c}$, $-2\vec{a} + 3\vec{b} + 5\vec{c}$ and $7\vec{a} - \vec{c}$ respectively. Then prove that A, B and C are collinear.

201) Find the separate equation of the following pair of straight lines.

$$6(x-1)^2 + 5(x-1)(y-2) - 4(y-2)^2 = 0$$

202) Find the separate equation of the following pair of straight lines.

$$2x^2 - xy - 3y^2 - 6x + 19y - 20 = 0$$

203) Consider the functions:

i) $f(x) = |x|$,

ii) $f(x) = |x-1|$

iii) $f(x) = |x+1|$

204) Let $f = \{(1, 2), (3, 4), (2, 2)\}$ and $s = \{(2, 1), (3, 1), (4, 2)\}$. Find $g \circ f$ and $f \circ g$.

205) Let $f = \{(1, 4), (2, 5), (3, 5)\}$ and $g = \{(4, 1), (5, 2), (6, 4)\}$. Find $g \circ f$. Can you find $f \circ g$?

206) Solve $-x^2 + 3x - 2 \geq 0$

- 207) Given that $\log_{10}2 = 0.30103$, $\log_{10}3 = 0.47712$ (approximately), find the number of digits in 28.3^{12}
- 208) In $\triangle ABC$, $A = 30^\circ$, $B = 60^\circ$ and $c = 10$, Find a and b .
- 209) Show that $\tan 75^\circ + \cot 75^\circ = 4$
- 210) Prove that $(1 + \tan 1^\circ)(1 + \tan 2^\circ)(1 + \tan 3^\circ) \dots (1 + \tan 44^\circ)$ is a multiple of 4.
- 211) Prove that $\frac{\sin 4x + \sin 2x}{\cos 4x + \cos 2x} = \tan 3x$
- 212) Solve the following equations $\sin \theta + \cos \theta = \sqrt{2}$
- 213) Prove that $\frac{\sin(4A-2B) + \sin(4B-2A)}{\cos(4A-2B) + \cos(4B-2A)} = \tan(A+B)$
- 214) Prove that $\tan 315^\circ \cot(-405^\circ) + \cot 495^\circ \tan(-585^\circ) = 2$.
- 215) How many different selections of 5 books can be made from 12 different books if,
- (i) Two particular books are always selected?
 - (ii) Two particular books are never selected?
- 216) A test consists of 10 multiple choice questions. In how many ways can the test be answered if
- (i) Each question has four choices?
 - (ii) The first four questions have three choices and the remaining have five choices?
 - (iii) Question number n has $n + 1$ choices?
- 217) In how many ways can the letters of the word SUCCESS be arranged so that all S's are together?
- 218) Out of 7 consonants and 4 vowels, how many strings of 3 consonants and 2 vowels can be formed?
- 219) Prove that if a, b, c are in HP, if and only if $\frac{a}{c} = \frac{a-b}{b-c}$.
- 220) If the 5th and 9th terms of a harmonic progression are $\frac{1}{19}$ and $\frac{1}{35}$, find the 12th term of the sequence.
- 221) Find the sum : $1 + \frac{4}{5} + \frac{7}{25} + \frac{10}{125} + \dots$
- 222) Find the coefficient of x^6 in the expansion of $(3 + 2x)^{10}$.

223) Without expanding, evaluate the following determinants:

$$(i) \begin{vmatrix} 2 & 3 & 4 \\ 5 & 6 & 8 \\ 6x & 9x & 12x \end{vmatrix} \quad (ii) \begin{vmatrix} x+y & y+z & z+x \\ z & x & y \\ 1 & 1 & 1 \end{vmatrix}$$

224)

For what value of x, the matrix $A = \begin{bmatrix} 0 & 1 & -2 \\ -1 & 0 & x^3 \\ 2 & -3 & 0 \end{bmatrix}$ is skew-symmetric.

225)

If a, b, c are p^{th} , q^{th} and r^{th} terms of an A.P, find the value of $\begin{vmatrix} a & b & c \\ p & q & r \\ 1 & 1 & 1 \end{vmatrix}$

226) The Farenheit temperature F and absolute temperature K satisfy a linear equation. Given that $K = 273$ when $F = 32$ and $K = 373$ when $F = 212$. Express K in terms of F and find the value of F when $K = 0$.

227) Evaluate the following integrals : $\int \frac{1}{x^2 - 2x + 5} dx$

228) Evaluate the following integrals : $\int \frac{1}{\sqrt{x^2 + 12x + 11}} dx$

229) Integrate the following functions with respect to x : $\frac{\cos 2x}{\sin^2 x \cos^2 x}$

230) Find the derivatives of the following : $(\cos x)^{\log x}$

231) Find the derivative of the $\tan (x + y) + \tan (x - y) = x$

232) A die is rolled. If it shows an odd number, then find the probability of getting 5.

233) Suppose ten coins are tossed. Find the probability to get (i) exactly two heads (ii) at most two heads (iii) at least two heads.

234) Show that the vectors $\vec{a} = 2\hat{i} + 3\hat{j} + 6\hat{k}$, $\vec{b} = 6\hat{i} + 2\hat{j} - 3\hat{k}$, and $\vec{c} = 3\hat{i} - 6\hat{j} + 2\hat{k}$ are mutually orthogonal.

235) If $|\vec{a}| = 5$, $|\vec{b}| = 6$, $|\vec{c}| = 7$ and $\vec{a} + \vec{b} + \vec{c} = \vec{0}$, find $\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}$.

236) If \vec{a} , \vec{b} are unit vectors and θ is the angle between them, show that

$$\tan \frac{\theta}{2} = \frac{|\vec{a} - \vec{b}|}{|\vec{a} + \vec{b}|}$$

237) If $\vec{a} = 2\hat{i} + 3\hat{j} - 4\hat{k}$, $\vec{b} = 3\hat{i} - 4\hat{j} - 5\hat{k}$, and $\vec{c} = -3\hat{i} + 2\hat{j} + 3\hat{k}$, find the magnitude and direction cosines of $3\vec{a} - 2\vec{b} + 5\vec{c}$

77 x 5 = 385

238) Simplify $\frac{1}{3-\sqrt{8}} - \frac{1}{\sqrt{8}-\sqrt{7}} + \frac{1}{\sqrt{7}-\sqrt{6}} - \frac{1}{\sqrt{6}-\sqrt{5}} + \frac{1}{\sqrt{5}-2}$

239) If $x = \sqrt{2} + \sqrt{3}$ find $\frac{x^2+1}{x^2-1}$

240) If $\operatorname{cosec} \theta - \sin \theta = a^3$ and $\sec \theta - \cos \theta = b^3$, then prove that $a^2b^2(a^2 + b^2) = 1$

241) Show that $\cot(A + 15^\circ) - \tan(A - 15^\circ) = \frac{4\cos 2A}{1 + 2\sin 2A}$

242) Prove that $\log 2 + 16\log \frac{16}{15} + 12\log \frac{25}{24} + 7\log \frac{81}{80} = 1$

243) Solve : $\log_2 x - 3\log_{\frac{1}{2}} x = 6$

244) How many numbers are there between 100 and 500 with the digits 0, 1, 2, 3, 4, 5 ? if

- (i) repetition of digits allowed
- (ii) the repetition of digits is not allowed.

245) Resolve the following rational expressions into partial fractions.

$$\frac{x+12}{(x+1)^2(x-2)}$$

246) Resolve the following rational expressions into partial fractions.

$$\frac{2x^2+5x-11}{x^2+2x-3}$$

247) Resolve the following rational expressions into partial fractions.

$$\frac{7+x}{(1+x)(1+x^2)}$$

248) How many three-digit numbers, which are divisible by 5, can be formed using the digits 0, 1, 2, 3, 4, 5 if

- (i) repetition of digits are not allowed?
- (ii) repetition of digits are allowed?

249) If a, b, c are respectively the p^{th} , q^{th} and r^{th} terms of a GP. show that $(q - r) \log a + (r - p) \log b + (p - q) \log c = 0$.

250) By the principle of mathematical induction, prove that for $n \geq 1$

$$1^3 + 2^3 + 3^3 + \dots + n^3 = \left[\frac{n(n+1)}{2} \right]^2$$

251) By the principle of mathematical induction, prove that for $n \geq 1$,

$$1^2 + 3^2 + 5^2 + \dots + (2n-1)^2 = \frac{n(2n-1)(2n+1)}{3}$$

252) By the principle of mathematical induction, prove that for $n \geq 1$

$$1 \cdot 2 + 2 \cdot 3 + \dots + n(n+1) = \frac{n(n+1)(n+2)}{3}$$

253) 8 women and 6 men are standing in a line.

(i) How many arrangements are possible if any individual can stand in any position?

(ii) In how many arrangements will all 6 men be standing next to one another?

(iii) In how many arrangements will no two men be standing next to one another?

254) If $(n+1)C_8 : (n-3)P_4 = 57:16$, find n .

255) Prove that $\sqrt[3]{x^3+6} - \sqrt[3]{x^3+3}$ is approximately equal to $\frac{1}{x^2}$ when x is sufficiently large.

256) Prove that $\sqrt{\frac{1-x}{1+x}}$ is approximately equal to $1 - x + \frac{x^2}{2}$ when x is very small.

257) How many strings are there using the letters of the word INTERMEDIATE, if

(i) The vowels and consonants are alternative

(ii) All the vowels are together

(iii) Vowels are never together

(iv) No two vowels are together.

258) if the binomial co-efficients of three consecutive terms in the expansion of $(a+x^n)$ are in the ratio 1:7:42 then find n

259) In the binomial coefficient of $(1+x)^n$ the Coefficients of the 5th, 6th and 7th terms are in A.P find all values of n

260) If the letters of the word GARDEN are permuted in all possible ways and the strings thus formed are arranged in the dictionary order, then find the ranks of the words

(i) GARDEN

(ii) DANGER

261) Prove that ${}^{2n}C_n = \frac{2^n \times 1 \times 3 \times \dots (2n-1)}{n!}$

262) Show that the equation $2x^2 - xy - 3y^2 - 6x + 19y - 20 = 0$ represents a pair of intersecting lines. Show further that the angle between them is $\tan^{-1}(5)$

263) A committee of 7 peoples has to be formed from 8 men and 4 women. In how many ways can this be done when the committee consists of

- (i) exactly 3 women?
- (ii) at least 3 women?
- (iii) at most 3 women?

- 264) Population of a city in the years 2005 and 2010 are 1,35,000 and 1,45,000 respectively. Find the approximate population in the year 2015. (assuming that the growth of population is constant)
- 265) Find p and q , if the following equation represents a pair of perpendicular lines $6x^2 + 5xy - py^2 + 7x + qy - 5 = 0$.
- 266) Find the value of k if the following equation represents a pair of straight lines. Further, find whether these lines are parallel or intersecting $12x^2 + 7xy - 12y^2 - x + 7y + k = 0$.
- 267) For what value of k does the equation $12x^2 + 2kxy + 2y^2 + 11x - 5y + 2 = 0$ represent two straight lines.
- 268) Prove that one of the straight line given by $ax^2 + 2hxy + by^2 = 0$ will bisect the angle between the co-ordinate axes if $(a + b)^2 = 4h^2$
- 269) Show that the equation $9x^2 - 24xy + 16y^2 + 12x + 16y - 12 = 0$ represents a pair parallel lines. find the distance between them
- 270) Show that the equation $4x^2 + 4xy + y^2 - 6x - 3y - 4 = 0$ represents a pair of parallel lines. Find the distance between them
- 271) Let $S = \{1, 2, 3\}$ and $\rho = \{(1, 1), (1, 2), (2, 2), (1, 3), (3, 1)\}$.
- (i) Is ρ reflexive? If not, state the reason and write the minimum set of ordered pairs to be included to ρ so as to make it reflexive.
 - (ii) Is ρ symmetric? If not, state the reason, write minimum number of ordered pairs to be included to ρ so as to make it symmetric and write minimum number of ordered pairs to be deleted from ρ so as to make it symmetric,
 - (iii) Is ρ transitive? If not, state the reason, write minimum number of ordered pairs to be included to ρ so as to make it transitive and write minimum number of ordered pairs to be deleted from ρ so as to make it transitive.
 - (iv) Is ρ an equivalence relation? If not, write the minimum ordered pairs to be included to ρ so as to make it an equivalence relation.
- 272) Let $A = \{0, 1, 2, 3\}$. Construct relations on A of the following types:
- (i) not reflexive, not symmetric, not transitive.
 - (ii) not reflexive, not symmetric, transitive.
- 273) Find the largest possible domain for the real valued function f defined by $f(x) = \sqrt{x^2 - 5x + 6}$.
- 274) Find the range of the function $f(x) = \frac{1}{1 - 3\cos x}$.

275) Find the largest possible domain for the real valued function given by

$$f(x) = \frac{\sqrt{9-x^2}}{x^2-1}.$$

276) An exam paper contains 8 questions, 4 in Part A and 4 in Part B.

Examiners are required to answer 5 questions. In how many ways can this be done if

- (i) There are no restrictions of choosing a number of questions in either parts.
- (ii) At least two questions from Part A must be answered.

277) By the principle of mathematical induction, prove that, for all integers $n \geq 1$,

$$1^2 + 2^2 + 3^2 + \dots + n^2 = \frac{n(n+1)(2n+1)}{6}.$$

278) Prove that $\sqrt[3]{x^3+7} - \sqrt[3]{x^3+4}$ is approximately equal to $\frac{1}{x^2}$ when x is large.

279) Prove that $\sqrt[3]{x^3+7} - \sqrt[3]{x^3+4}$ is approximately equal to $\frac{1}{x^2}$ when x is large.

280) Prove that for any natural number n , $a^n - b^n$ is divisible by $a-b$, where $a > b$.

281) Prove that $3^{2n+2} - 8n - 9$ is divisible by 8 for all $n \geq 1$.

282)

$$\text{If } A = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix} \text{ and } A^3 - 6A^2 + 7A + KI = O, \text{ find the value of } k.$$

283) Verify the property $A(B + C) = AB + AC$, when the matrices A , B , and C are given by

$$A = \begin{bmatrix} 2 & 0 & -3 \\ 1 & 4 & 5 \end{bmatrix}, B = \begin{bmatrix} 3 & 1 \\ -1 & 0 \\ 4 & 2 \end{bmatrix} \text{ and } C = \begin{bmatrix} 4 & 7 \\ 2 & 1 \\ 1 & -1 \end{bmatrix}.$$

284)

$$\text{Prove that } \begin{vmatrix} a^2 & bc & ac + c^2 \\ a^2 + ab & b^2 & ac \\ ab & b^2 + bc & c^2 \end{vmatrix} = 4a^2b^2c^2$$

285)

$$\text{Using Factor Theorem, prove that } \begin{vmatrix} x+1 & 3 & 5 \\ 2 & x+2 & 5 \\ 2 & 3 & x+4 \end{vmatrix} = (x-1)^2(x+9).$$

286)

Prove that
$$\begin{vmatrix} 1 & x^2 & x^3 \\ 1 & y^2 & y^3 \\ 1 & z^2 & z^3 \end{vmatrix} = (x - y)(y - z)(z - x)(xy + yz + zx).$$

287)

Prove that
$$|A| = \begin{vmatrix} (q+r)^2 & p^2 & p^2 \\ q^2 & (r+p)^2 & q^2 \\ r^2 & r^2 & (p+q)^2 \end{vmatrix} = 2pqr(p + q + r)^3.$$

288) Solve the following problems by using Factor Theorem :

Show that
$$\begin{vmatrix} x & a & a \\ a & x & a \\ a & a & x \end{vmatrix} = (x - a)^2(x + 2a)$$

289) Solve the following problems by using Factor Theorem :

Show that
$$\begin{vmatrix} b+c & a & a^2 \\ c+a & b & b^2 \\ a+b & c & c^2 \end{vmatrix} = (a + b + c)(a - b)(b - c)(c - a).$$

290)

Show that
$$\begin{vmatrix} 1 & 1 & 1 \\ x & y & z \\ x^2 & y^2 & z^2 \end{vmatrix} = (x - y)(y - z)(z - x).$$

291) If ABCD is a quadrilateral and E and F are the midpoints of AC and BD

respectively, then prove that $\overrightarrow{AB} + \overrightarrow{AD} + \overrightarrow{CB} + \overrightarrow{CD} = 4 \overrightarrow{EF}.$

292) Prove that the points whose position vectors $2\hat{i} + 4\hat{j} + 3\hat{k}$, $4\hat{i} + \hat{j} + 9\hat{k}$ and $10\hat{i} - \hat{j} + 6\hat{k}$ form a right angled triangle.

293) Show that the vectors $5\hat{i} + 6\hat{j} + 7\hat{k}$, $7\hat{i} - 8\hat{j} + 9\hat{k}$, $3\hat{i} + 20\hat{j} + 5\hat{k}$ are coplanar.

294) Show that the points whose position vectors $4\hat{i} + 5\hat{j} + \hat{k}$, $-\hat{j} - \hat{k}$, $3\hat{i} + 9\hat{j} + 4\hat{k}$ and $-4\hat{i} + 4\hat{j} + 4\hat{k}$ are coplanar.

295) Let $\vec{a}, \vec{b}, \vec{c}$ be three vectors such that $|\vec{a}| = 3$, $|\vec{b}| = 4$, $|\vec{c}| = 5$ and each one of them being perpendicular to the sum of the other two, find $|\vec{a} + \vec{b} + \vec{c}|$

|.

296) Three vectors \vec{a} , \vec{b} and \vec{c} are such that $|\vec{a}| = 2$, $|\vec{b}| = 3$, $|\vec{c}| = 4$, and $\vec{a} + \vec{b} + \vec{c} = \vec{0}$. Find $4\vec{a} \cdot \vec{b} + 3\vec{b} \cdot \vec{c} + 3\vec{c} \cdot \vec{a}$.

297) If $y = e^{\tan^{-1} x}$, Show that $(1 + x^2) y'' + (2x - 1) y' = 0$.

298) If $y = \frac{\sin^{-1} x}{\sqrt{1-x^2}}$, Show that $(1 - x^2) y_2 - 3x y_1 - y = 0$.

299) If $y = (\cos^{-1} x)^2$, prove that $(1 - x^2) \frac{d^2 y}{dx^2} - x \frac{dy}{dx} - 2 = 0$. Hence find y_2 when $x = 0$

300) Evaluate $\int \frac{x+3}{(x+2)^2(x+1)} dx$

301) Integrate the following functions with respect to x : $\frac{3x-9}{(x-1)(x+2)(x^2+1)}$

302) Evaluate the following integrals : $\int \frac{3x+5}{x^2+4x+7} dx$

303) Evaluate the following integrals : $\int \frac{x+1}{x^2-3x+1} dx$

304) Evaluate the following integrals : $\int \frac{2x+3}{\sqrt{x^2+x+1}} dx$

305) Evaluate the following integrals : $\int \frac{5x-7}{\sqrt{3x-x^2-2}} dx$

306) A problem in Mathematics is given to three students whose chances of solving $\frac{1}{3}$, $\frac{1}{4}$ and $\frac{1}{5}$ (i) What is the probability that the problem is solved? (ii) What is the probability that exactly one of them will solve it?

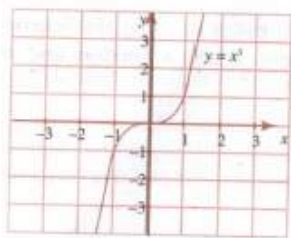
307) A factory has two Machines-I and II. Machine-I produces 60% of items and Machine-II produces 40% of the items of the total output. Further 2% of the items produced by Machine-I are defective whereas 4% produced by Machine-II are defective. If an item is drawn at random what is the probability that it is defective?

308) The chances of A, B, and C becoming manager of a certain company are 5 : 3 : 2. The probabilities that the office canteen will be improved if A, B, and C become managers are 0.4, 0.5 and 0.3 respectively. If the office canteen has been improved, what is the probability that B was appointed as the manager?

309) The chances of X, Y and Z becoming managers of a certain company are 4 : 2 : 3. The probabilities that bonus scheme will be introduced if X, Y and Z become managers are 0.3, 0.5 and 0.4 respectively. If the bonus scheme has

been introduced, what is the probability that Z was appointed as the manager?

- 310) A consulting firm rents car from three agencies such that 50% from agency L, 30% from agency M and 20% from agency N. If 90% of the cars from L, 70% of cars from M and 60% of the cars from N are in good conditions
- (i) what is the probability that the firm will get a car in good condition?
 - (ii) if a car is in good condition, what is probability that it has come from agency N?
- 311) The probability that a girl, preparing for competitive examination will get a State Government service is 0.12, the probability that she will get a Central Government job is 0.25, and the probability that she will get both is 0.07. Find the probability that (i) she will get atleast one of the two jobs (ii) she will get only one of the two jobs.
- 312) Let $A = \{0, 1, 2, 3\}$. Construct relations on A of the following types:
- (i) not reflexive, symmetric, not transitive.
 - (ii) not reflexive, symmetric, transitive.
- 313) Let $A = \{0, 1, 2, 3\}$. Construct relations on A of the following types:
- (i) reflexive, not symmetric, not transitive.
 - (ii) reflexive, not symmetric, transitive.
- 314) Let $A = \{0, 1, 2, 3\}$. Construct relations on A of the following types:
- (i) reflexive, symmetric, not transitive.
 - (ii) reflexive, symmetric, transitive.
- 315) For the given curve $y = x^3$ given in figure draw, try to draw with the same scale
- (i) $y = -x^3$
 - (ii) $y = x^3 + 1$
 - (iii) $y = x^3 - 1$
 - (iv) $y = (x + 1)^3$



- 316) From the curve $y = |x|$, draw
- (i) $y = |x + 1| + 1$
 - (ii) $y = |x - 1| - 1$
 - (iii) $y = |x + 2| - 3$

317) Write the values of f at -3,5,2,-1,0 if

$$f(x) = \begin{cases} x^2 + x - 5 & \text{if } x \in (-\infty, 0) \\ x^2 + 3x - 2 & \text{if } x \in (3, \infty) \\ x^2 & \text{if } x \in (0, 2) \\ x^2 - 3 & \text{otherwise} \end{cases}$$

318) Resolve the following rational expressions into partial fractions.

$$\frac{x}{(x^2+1)(x-1)(x+2)}$$

319) Resolve into partial fractions: $\frac{2x}{(x^2+1)(x-1)}$

320) Solve the linear inequalities and exhibit the solution set graphically: $x + y \geq 3$, $2x - y \leq 5$, $-x + 2y \leq 3$.

321) Integrate the following with respect to x : $\frac{2x+1}{\sqrt{9+4x-x^2}}$

322) If $x = a(\theta + \sin \theta)$, $y = a(1 - \cos \theta)$ then prove that at $\theta = \frac{\pi}{2}$, $y'' = \frac{1}{a}$

323) If $\sin y = x \sin(a + y)$, then prove that $\frac{dy}{dx} = \frac{\sin^2(a+y)}{\sin a}$, $a \neq n\pi$.

324) A firm manufactures PVC pipes in three plants viz, X, Y, and Z. The daily production volumes from the three firms X, Y and Z are respectively 2000 units, 3000 units, and 5000 units. It is known from the past experience that 3% of the output from plant X, 4% from plant Y and 2% from plant Z are defective. A pipe is selected at random from a day's total production,

(i) find the probability that the selected pipe is a defective one.

(ii) if the selected pipe is a defective, then what is the probability that it was produced by plant Y?

325) X speaks truth in 70 percent of cases, and Y in 90 percent of cases. What is the probability that they likely to contradict each other in stating the same fact?

326) A factory has two machines I and II. Machine-I produces 40% of items of the output and Machine-II produces 60% of the items. Further 4% of items produced by Machine-I are defective and 5% produced by Machine-II are defective. If an item is drawn at random, find the probability that it is a defective item.

327) A factory has two machines I and II. Machine I produces 40% of items of the output and Machine II produces 60% of the items. Further 4% of items produced by Machine I are defective and 5% produced by Machine II are

defective. An item is drawn at random. If the drawn item is defective, find the probability that it was produced by Machine II. (See the previous example, compare the questions).

328) A year is selected at random. What is the probability that

(i) it contains 53 Sundays (ii) it is a leap year which contains 53 Sundays.

329) Suppose the chances of hitting a target by a person X is 3 times in 4 shots, by Y is 4 times in 5 shots, and by Z is 2 times in 3 shots. They fire simultaneously exactly one time. What is the probability that the target is damaged by exactly 2 hits?

330) Prove that $\cos A \cos 2A \cos 2^2 A \cos 2^3 A \dots \cos 2^{n-1} A = \frac{\sin 2^n A}{2^n \sin A}$.

331) If $A + B + C = 180^\circ$, prove that $\sin^2 A + \sin^2 B + \sin^2 C = 2 + 2 \cos A \cos B \cos C$

332) If $A + B + C = \pi$, prove that $\cos^2 A + \cos^2 B + \cos^2 C = 1 - 2 \cos A \cos B \cos C$.

333) If $A + B + C = 180^\circ$, prove that $\sin^2 A + \sin^2 B - \sin^2 C = 2 \sin A \sin B \cos C$

334) If $A + B + C = 180^\circ$, prove that $\tan \frac{A}{2} \tan \frac{B}{2} + \tan \frac{B}{2} \tan \frac{C}{2} + \tan \frac{C}{2} \tan \frac{A}{2} = 1$

335) Prove that $32(\sqrt{3}) \sin \frac{\pi}{48} \cos \frac{\pi}{48} \cos \frac{\pi}{24} \cos \frac{\pi}{12} \cos \frac{\pi}{6} = 3$.

336) If $A + B + C = \frac{\pi}{2}$, prove the following $\cos 2A + \cos 2B + \cos 2C = 1 + 4 \sin A \sin B \sin C$

337) Show that $\cos \frac{\pi}{15} \cos \frac{2\pi}{15} \cos \frac{3\pi}{15} \cos \frac{4\pi}{15} \cos \frac{5\pi}{15} \cos \frac{6\pi}{15} \cos \frac{7\pi}{15} = \frac{1}{128}$

338) If ${}^{(n+2)}C_7 : {}^{(n-1)}P_4 = 13 : 24$ find n.

339)

$$\text{Show that } \begin{vmatrix} 2bc - a^2 & c^2 & b^2 \\ c^2 & 2ca - b^2 & a^2 \\ b^2 & a^2 & 2ab - c^2 \end{vmatrix} = \begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}^2.$$

340) If a, b, c are all positive and are p^{th} , q^{th} and r^{th} terms of a G.P., show that

$$\begin{vmatrix} \log a & p & 1 \\ \log b & q & 1 \\ \log c & r & 1 \end{vmatrix} = 0.$$

341) Compute the sum of first n terms of the following series $6 + 66 + 666 + \dots$

342)

Find the Co-efficient of x^{15} in $\left(x^2 + \frac{1}{x^3}\right)^{10}$

343)

Find the Co-efficient of x^6 and the co -efficient of x^2 in $\left(x^2 - \frac{1}{x^3}\right)^6$

344) If n is a positive integer, show that $9^{n+1} - 8n - 9$ is always divisible by 64
