RAVI MATHS TUITION CENTER, NEAR VILLIVAKKAM RLY STATION, CHENNAI – 82.
WHATSAPP - 8056206308

Trigonometry 10

11th Standard

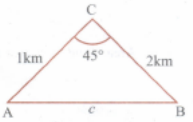
Maths

Time : 01:30:00 Hrs

Total Marks : 75

15 x 5 = 75

1) A plane is 1 km from one landmark and 2 km from another. From the planes point of view the land between them subtends an angle of 45^0. How far apart are the land marks?
Answer : Let A, B be the land marks and C be the position of the plane,
Given ∠ACB 45°



Using cosine formula,
$c^2 = a^2 + b^2 - 2ab \cos C$
$c^2 = 2^2 + 1^2 - 2(2)(1) \cos 45^0$
$= 4 + 1 - 4\left(\frac{1}{\sqrt{3}}\right)$
$= 5 - 2 \times \cancel{2}^{\sqrt{2}}\left(\frac{1}{\cancel{\sqrt{2}}}\right)$
$c^2 = 5 - 2\sqrt{2}$
$c^2 = \sqrt{5 - 2\sqrt{2}} km$

2) If $\frac{cos^4\alpha}{cos^2\beta} + \frac{sin^4\alpha}{sin^2\beta} = 1$ prove that $\frac{cos^4\beta}{cos^2\alpha} + \frac{sin^4\beta}{sin^2\alpha} = 1$
Answer : Given $\frac{cos^4\alpha}{cos^2\beta} + \frac{sin^4\alpha}{sin^2\beta}$
⇒ $\cos^4\alpha \sin^2\beta + \sin^4\alpha \cos^2\beta = \cos^2\beta \sin^2\beta$
⇒ $\cos^4\alpha(1-\cos^2\beta) + \cos^2\beta(1-\cos^2\alpha)2 = \cos^2\beta(1-\cos^2\beta)$
$\cos^4\alpha - \cancel{\cos^4\alpha \cos^2\beta} + \cos^2\beta - 2\cos^2\alpha\cos^2\beta + \cancel{\cos^4\alpha\cos^2\beta} = \cos^2\beta - \cos^4\beta$
⇒ $\cos^4\alpha - 2\cos^2\alpha\cos^2\beta + \cos^4\beta = 0$
⇒ $(\cos^2\alpha - \cos^2\beta)^2 = 0$
⇒ $\cos^2\alpha - \cos^2\beta = 0$
⇒ $\cos^2\alpha = \cos^2\beta$...(1)
⇒ $1 - \sin^2\alpha = 1 - \sin^2\beta$
⇒ $\sin^2\alpha = \sin^{-2}\beta$
$\frac{cos^4\alpha}{cos^2\beta} + \frac{sin^4\alpha}{sin^2\beta} = 1$
LHS= $\frac{cos^4\beta}{cos^2\alpha} + \frac{sin^4\beta}{sin^2\alpha}$
= $\frac{cos^2\beta cos^2\beta}{cos^2\alpha} + \frac{sin^2\beta sin^2\beta}{sin^2\alpha}$
$= cos^2\beta + sin^2\beta = 1$ =RHS

3) Eliminate θ from the equation a sec θ - c tan θ = b and b sec θ + d tan θ = C
Answer : a secθ-c tanθ=b...(1)
b secθ+d tanθ=c...(2)
(1) x b → ab secθ - bc tan θ = b^2
(1) x a → ⁻ab secθ + ⁻ad tan θ = ⁻ac

-tan θ (bc + ad) = b^2-ac
tanθ= $\frac{ac-b^2}{bc+ad}$

(1) x d → ad secθ - cd tan θ = bd
(1) x c → bc secθ + cd tan θ =c^2

(ad+bc) secθ = bd + c^2
secθ= $\frac{bd+c^2}{ad+bc}$
We know $\sec^2\theta - \tan^2\theta = 1$

⇒ $(\frac{bd+c^2}{ad+bc})^2 - (\frac{ac-b^2}{bc+ad})^2 = 1$
⇒ $\frac{(bd+c^2)^2}{(ad+bc)^2} - \frac{(ac-b^2)^2}{(bc+ad)^2} = 1$
⇒ $\frac{(bd+c^2)^2-(ac-b^2)^2}{(ad+bc)^2} = 1$
⇒ $(bd+c^2)^2-(ac-b^2)^2=(ad+bc)^2$
⇒ $(c^2+bd)^2=(ad+bc)^2+(ac-b^2)^2$
Thus θ is eliminated.

4) If $A+B+C=180^0$,prove that $\sin^2A+\sin^2B-\sin^2C=2\sin A\sin B\cos C$
Answer : LHS=$\sin^2A+\sin^2B-\sin^2C$
=$\frac{1}{2}\left[2sin^2A+sin^2B-2sin^2C\right]$
=$\frac{1}{2}[(1-cos2A)+(1-cos2B)-(1-cos2C)]$
=$\frac{1}{2}[1-(cos2A+cos2B+cos2C)]$
=$\frac{1}{2}-\frac{1}{2}\left(cos2A+cos2B+\left(2cos^2C-1\right)\right)$
=$\frac{1}{2}-\frac{1}{2}\left(2cos\left(\frac{2A+2B}{2}\right)cos\left(\frac{2A-2B}{2}\right)+2cos^2C-1\right)$
=$\frac{1}{2}-\left(cos\,(A+B)\,cos\,(A+B)-cos^2C-\frac{1}{2}\right)$
=-(-cosCcos(A-B)$\cos^2$C)
=cosCcos(A-B)+$\cos^2$C
=cosC[cos(A-B)+cosC]
=cosC[cos(A-B)-cos(A+B)]
=cosC2sinAsinB
=2sinAsinBcosC=RHS
Hence proved.

5) If A+B+C=2s, prove that sin(s-A)sin(s-B)+sin s.sin(s-C)=sinA sinB.
Answer : $Given \quad A+B+C=2s$
$LHS=sin(s-A)sin(s-B)+sins.\,sin(s-C)$
$=\frac{1}{2}[cos(s-A-s+B)-cos(s-A+s-B)]+\frac{1}{2}[cos(s-s+C-cos(s+s-C)] \quad [\because sinAsinB=\frac{1}{2}[cos$
$=\frac{1}{2}[cos(B-A)-cos(2s-A-B)+cosC-cos(2s-C)$
$=\frac{1}{2}[cos(B-A)-cos(2s-A-B)+cosC-cos(2s-C)$
$=\frac{1}{2}$[cos(A-B)-cosC+~~cos~~C-~~cos~~(A+B)]
=sinA sin B =RHS
Hence proved

6) Prove that $\cos 5\theta = 16\cos^2\theta - 20\cos^3\theta + 5\cos\theta$.
Answer : LHS = $\cos 5\theta = \cos(3\theta + 2\theta)$
= $\cos 3\theta\cos 2\theta - \sin 3\theta\sin 2\theta$
= $(4\cos^3\theta - 3\cos\theta)(2\cos^2\theta - 1) - [3\sin\theta - 4\sin^3\theta]\,2\sin\theta\cos\theta$
= $8\cos^5\theta - 10\cos^3\theta + 3\cos\theta - 6\cos\theta + 6\cos^3\theta + 8\cos\theta(1 + \cos^4\theta - 2$
$\cos^2\theta)$
= $16\cos^5\theta - 20\cos^3\theta + 5\cos\theta$
= RHS
Hence proved.

7) Solve the following equations $\cos\theta+\cos 3\theta=2\cos2\theta$
Answer : $2cos\left(\frac{\theta+3\theta}{2}\right).\,cos\left(\frac{3\theta-\theta}{2}\right)-2cos2\theta=0$
2cos2θ.cosθ-2cos2θ=0
cos2θ(cosθ-1)=0
cos2θ=0
or cosθ=1
cos2θ=0 or cosθ=1
cos2θ=0
2θ=(2n+1)$\frac{\pi}{2}$, n∊z
θ=(2n+1)$\frac{\pi}{4}$, n∊z
θ=(2n+1)$\frac{\pi}{4}$
cosθ=1
cosθ=cos0
θ=2nπ$\pm$0, n∊z
θ=2nπ, n∊z
∴ The solutions are θ=(2n+1)$\frac{\pi}{4}$or 2nπ, n∊z

8) In △ABC, Prove the following
$\frac{asin(B-C)}{b^2-c^2}=\frac{bsin(C-A)}{c^2-a^2}=\frac{csin(A-B)}{a^2-b^2}$

Answer : Let $\frac{a}{sinA}=\frac{b}{sinB}=\frac{c}{sinC}=k$
a=k sinA, b=k sinB, c=k sinC...(1)
Consider $\frac{asin(B-C)}{b^2-c^2}=\frac{ksinAsin(B-C)}{k^2sin^2B-k^2sin^2C}$
$\frac{ksin(B+C)sin(B-C)}{k^2\left(sin^2B-sin^2C\right)}=\frac{k\left(sin^2B-sin^2C\right)}{k^2\left(sin^2B-sin^2C\right)}=\frac{1}{k}$ ...(2)
$\frac{bsin(C-A)}{c^2-a^2}=\frac{ksinB \quad sin(C-A)}{k^2sin^2C-k^2sin^2A}=\frac{sin(C+A) \quad sin(C-A)}{ksin(C+A) \quad sin(C-A)}=\frac{1}{k}$ ...(3)
Similarly, $\frac{Csin(A-B)}{a^2-b^2}=\frac{1}{k}$ ...(4)
From (2), (3) and (4)
$\frac{asin(B-C)}{b^2-c^2}=\frac{bsin(C-A)}{c^2-a^2}=\frac{csin(A-B)}{a^2-b^2}$

9) Solve the following equation $tan\theta+tan\left(\theta+\frac{\pi}{3}\right)+tan\left(\theta+\frac{2\pi}{3}\right)=\sqrt{3}$

Answer : $\Rightarrow tan\theta+\frac{tan\theta+tan\frac{\pi}{3}}{1-tan\theta.tan\frac{\pi}{3}}+\frac{tan\theta+tan\frac{2\pi}{3}}{1-tan\theta.tan\frac{2\pi}{3}}=3$
$\Rightarrow tan\theta+\frac{tan\theta+\sqrt{3}}{1-\sqrt{3}tan\theta}+\frac{tan\theta-\sqrt{3}}{1+\sqrt{3}tan\theta}=\sqrt{3}$
$\Rightarrow tan\theta+\frac{(1+\sqrt{3}tan\theta)(tan\theta+\sqrt{3})+(tan\theta-\sqrt{3})(1-\sqrt{3}tan\theta)}{1-3tan^2\theta}=\sqrt{3}$
$\Rightarrow tan\theta+\frac{8tan\theta}{1-3tan^2\theta}=\sqrt{3}$
$\Rightarrow tan\theta+\frac{8tan\theta}{1-3tan^2\theta}=\sqrt{3}3.9$
$\Rightarrow \frac{tan\theta-3tan^2\theta+8tan\theta}{1-3tan^2\theta}=\sqrt{3}$
$\Rightarrow \frac{3\left(3tan\theta-tan^2\theta\right)}{1-3tan^2\theta}=\sqrt{3}$
$\Rightarrow 3\left(tan3\theta\right)=\sqrt{3}$
$\Rightarrow tan3\theta=\frac{\sqrt{3}}{3}=\frac{1}{\sqrt{3}}$
$\Rightarrow tan3\theta=tan\left(\frac{\pi}{6}\right)$
$\Rightarrow 3\theta=n\pi+\frac{\pi}{6},n\epsilon z$
$\Rightarrow \theta=\frac{n\pi}{3}+\frac{\pi}{18},n\epsilon z$
Solutions are $\theta=\frac{n\pi}{3}+\frac{\pi}{18},n\epsilon z$

10) In a $\triangle$ABC, if $\frac{sin \quad A}{sin \quad C}=\frac{sin(A-B)}{sin(B-c)}$,prove that a^2,b^2,$c^2$are in arithmetic progression

Answer : Using sine formula,
Let $\frac{sinA}{a}=\frac{sinB}{b}=\frac{sinC}{c}=k$
sin A=ak, sin B=bk, sin C=ck
Given $\frac{sin \quad A}{sin \quad C}=\frac{sin(A-B)}{sin(B-C)}$
$\Rightarrow \frac{sin(B+C)}{sin(A+B)}=\frac{sin(A-B)}{sin(B-C)}$
⇒ Sin(B+C)sin(B-C)=sin(A+B)sin(A-B)
⇒ $sin^2B-sin^2C=sin^2A-sin^2B$
⇒ $k^2b^2-k^2c^2=k^2a^2-k^2b^2$
⇒ $b^2-c^2=a^2-b^2$
⇒ $2b^2=a^2+c^2$
⇒ a^2,b^2,c^2 are in A.P.

11) If A + B + C = π, prove the following
i. cos A + cos B + cos C = 1 + 4 sin $\left(\frac{A}{2}\right)$ sin $\left(\frac{B}{2}\right)$ sin $\left(\frac{C}{2}\right)$
ii. sin $\left(\frac{A}{2}\right)sin\left(\frac{B}{2}\right)sin\left(\frac{C}{2}\right)\leq\frac{1}{8}$
iii. 1 < cos A + cos B + cos C $\leq\frac{3}{2}$

Answer : i. cos A + cos B + cos C = 2 cos $\left(\frac{A+B}{2}\right)$ cos $\left(\frac{A-B}{2}\right)$ + cos C
= 2cos $\left(\frac{\pi}{2}-\frac{c}{2}\right)cos\left(\frac{A}{2}-\frac{B}{2}\right)+cosC$
= 2sin $\left(\frac{c}{2}\right)cos\left(\frac{A}{2}-\frac{B}{2}\right)+1-2sin^2\left(\frac{C}{2}\right)$
= 1+2sin $\left(\frac{c}{2}\right)\left[cos\left(\frac{A}{2}-\frac{B}{2}\right)-sin\left(\frac{C}{2}\right)\right]$
= 1+2sin $\left(\frac{c}{2}\right)\left[cos\left(\frac{A}{2}-\frac{B}{2}\right)-cos\left(\frac{\pi}{2}-\frac{C}{2}\right)\right]$
= 1+2sin $\left(\frac{c}{2}\right)\left[cos\left(\frac{A}{2}-\frac{B}{2}\right)-cos\left(\frac{A}{2}+\frac{B}{2}\right)\right]$
= 1+ 4sin $\left(\frac{A}{2}\right)sin\left(\frac{B}{2}\right)sin\left(\frac{C}{2}\right)$
ii. Let u= sin $\left(\frac{A}{2}\right)sin\left(\frac{B}{2}\right)sin\left(\frac{C}{2}\right)$
$=-\frac{1}{2}\left[cos\frac{A+B}{2}-cos\frac{A-B}{2}\right]sin\frac{c}{2}$

$= -\frac{1}{2}[cos\frac{A+B}{2} - cos\frac{A-B}{2}]cos\frac{A+B}{2}$
$= cos^2\frac{A+B}{2} - cos\frac{A-B}{2}cos\frac{A+B}{2} + 2u = 0,$ which is quadratic in cos $\frac{A+B}{2}$
Since cos $\frac{A+B}{2}$ is real number, the above equation has a solution.
Thus, the discriminant b^2 - 4ac $\geq$ 0, which gives
$= cos^2\frac{A-B}{2} - 8u \geq 0 \Rightarrow u \leq \frac{1}{8}cos^2\frac{A-B}{2} \leq \frac{1}{8}$
Hence, sin $(\frac{A}{2})sin(\frac{B}{2})sin(\frac{C}{2}) \leq \frac{1}{8}$
iii. From (i) and (ii), we have cos A + cos B + cos C > 1 and cos A + cos B + cos C ≤ 1 + 4 × $\frac{1}{8}$
Thus, we get 1 < cos A + cos B + cos C $\leq \frac{3}{2}$

12) A ripple tank demonstrates the effect of two water waves being added together. The two waves are described by h = 8 cos t and h = 6 sin t, where t ∊ [0, 2π) is in seconds and h is the height in millimeters above still water. Find the maximum height of the resultant wave and the value of that which it occurs.
Answer : Let H be the height of the resultant wave at time t. Then H is given by
H = 8 cos t + 6 sin t
Let 8 cos t + 6 sin t = k cos(t - α)
= k(cos t cos α + sin t sin α)
Hence, k= 10 and $\tan\ \alpha = \frac{3}{4}$, so that
H = 10 cos(t - α)
Thus, the maximum of H = 10mm. The maximum occurs when t = α, where $\tan \alpha = \frac{3}{4}$.

13) Find the values of sin 18°.
Answer : Let θ = 18°, Then 5θ = 90°
3θ + 2θ = 90° ⇒2θ = 90° -3θ
sin 2θ = sin (90° - 3θ) = cos 3θ
2 sin θ cos θ = 4 $\cos^3$θ - 3 cos θ. Since cos θ = cos 18° ≠ 0, we have
2 sin θ = 4 $\cos^2$θ - 3 = 4 (1 - $\sin^2$θ) - 3
4 $\sin^2$θ + 2 sin θ - 1 = 0
$sin\ \theta = \frac{-2\pm\sqrt{4-4(4)(-1)}}{2(4)} = \frac{-1\pm\sqrt{5}}{4}$
Thus,$\sin\ 180^0 = \frac{\sqrt{5}-1}{4}$ (positive sign is taken sin 18o is in I quadrant. sinθ is positive).

14) Prove that for any a and b,-$\sqrt{a^2+b^2} \leq$ a sin θ+ b cos $\theta \leq \sqrt{a^2+b^2}$
Answer : Now, a sin θ+ b cos$\theta \leq \sqrt{a^2+b^2}[\frac{a}{\sqrt{a^2+b^2}}sin\theta + \frac{b}{\sqrt{a^2+b^2}}cos\theta]$
=$\sqrt{a^2+b^2}$ [cos θ sin θ + sin α cos θ]
$(where\ cos\alpha = \frac{a}{\sqrt{a^2+b^2}}, sin\alpha = \frac{b}{\sqrt{a^2+b^2}})$
$= \sqrt{a^2+b^2}sin(\alpha+\theta)$
Thus, a sin θ+ b cos $\theta \leq \sqrt{a^2+b^2}$
Hence, -$\sqrt{a^2+b^2} \leq$a sin θ+ b cos $\theta \leq \sqrt{a^2+b^2}$

15) In a triangle ABC, prove that $\frac{a^2+b^2}{a^2+c^2} = \frac{1+cos(A-B)cosC}{1+cos(A-C)cosB}$
Answer : The law of sine: $(\frac{a}{sinA}) = (\frac{b}{sinB}) = (\frac{c}{sinC}) = 2R$
LHS $= \frac{a^2+b^2}{a^2+c^2} = \frac{(2R\sin A)^2+(2R\sin B)^2}{(2R\sin A)^2+(2R\sin C)^2}$
$= \frac{\sin^2 A+\sin^2 B}{\sin^2 A+\sin^2 C} = \frac{1-\cos^2 A+\sin^2 B}{1-\cos^2 A+\sin^2 C}$
$= \frac{1-(\cos^2 A-\sin^2 B)}{1-(\cos^2 A-\sin^2 C)} = \frac{1-\cos(A+B)\cos(A-B)}{1-\cos(A+C)\cos(A-C)}$
$= \frac{1+\cos(A-B)\cos C}{1+\cos(A-C)\cos B}$