# $11^{TH}$ CBSE MATHS RELATIONS AND FUNCTIONS

If A = {1, 2, 3} and B = {3, 5} then find A × B and B × A also verify A × B ≠ B × A.

A × B = {1, 2, 3} × {3, 5}

= {(1, 3), (1, 5) , (2, 3) , (2, 5) , (3, 3), (3, 5)}

and B × A = {3, 5} × {1, 2, 3}

= {(3, 1), (3, 2), (3, 3), (5, 1), (5, 2), (5, 3) }

Clearly A × B ≠ B × A

Thus, "cartesian product of sets is not commutative".

If A = {1, 2} , B = {4, 5}, C = {6, 8} then find A × B × C.

A × B × C = A × [B × C] = {1, 2} × [{4, 5} × {6, 8}]

= {1, 2} × {(4, 6), (4, 8), (5, 6), (5, 8)}

= {(1, 4, 6),(1, 4, 8),(1, 5, 6),(1, 5, 8),(2, 4, 6),(2, 4, 8),(2, 5, 6),(2, 5, 8)}

Let A = {1, 2, 3, 4} and S = {(a, b) : a ∈ A, b ∈ A, a divides b} Write S explicitly

Since 1 divides 2, 3, 4 and 2 divides 4

3 does not divide any of 1, 2 and 4.

4 does not divide any of 1, 2 and 3

Also 1 divides 1, 2 divides 2, 3 divides 3 and 4 divides 4.

Hence S = {(a, b) : a ∈ A, b ∈ A, a divides b}

= {(1, 2), (1, 3), (1, 4), (2, 4), (1, 1), (2, 2), (3, 3), (4, 4)}

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**Let A = {1, 2} and B = {3, 4}. Write all subsets of A × B.**

Given, A = {1, 2} and B = {3, 4}

∴A × B= {(1, 3), (1, 4), (2, 3), (2, 4)}.

Subsets of A × B are

ϕ, {(1, 3)}, {(1, 4)}, {(2, 3)}, {(2, 4)}, {(1, 3), (1, 4)}, {(1, 3), (2, 3)}, {(1, 3), (2, 4)},

{(1, 4), (2, 3)}, {(1, 4), (2, 4)}, {(2, 3), (2, 4)}, {(1, 3), (1, 4), (2, 3)},

{(1, 3), (1, 4), (2, 4))}, {(1, 4), (2, 3), (2, 4)}, {(1, 3), (2, 3), (2, 4)}, A × B.

**If A and B are two sets given in such a way that A × B consists of 6 elements. And if three elements of A × B are (1, 3), (2, 5), (3, 5), then what are its remaining elements?**

Since (1, 3) , (2, 5), (3, 5) ∈ A × B, so clearly 1, 2, 3 ∈ A and 3, 5 ∈ B.

Given, n(A × B) = 6 ⇒ n(A) . n(B) = 6.

But 1, 2, 3 $\in$ A and 3, 5 $\in$ B

Hence A = {1, 2, 3} and B = {3, 5}

$\therefore$ A $\times$ B = {(1, 3), (1, 5), (2, 3), (2, 5), (3, 3), (3, 5)}.

$\therefore$ Remaining elements of A $\times$ B are : (1, 5), (2, 3) and (3, 3)

If A = {1, 2, 3}, B = {2, 3, 4}, C = {1, 3, 4} and D = {2, 4, 5}, then verify that (A $\times$ B) $\cap$ (C $\times$ D) = (A $\cap$ C) $\times$ (B $\cap$ D)

(A $\times$ B) = {1, 2, 3} $\times$ {2, 3, 4}

= {(1, 2), (1, 3), (1, 4), (2, 2), (2, 3), (2, 4), (3, 2), (3, 3), (3, 4)}

(C $\times$ D) = {1, 3, 4} ( {2, 4, 5}

= {(1, 2),(1, 4), (1, 5), (3, 2), (3, 4), (3, 5), (4, 2), (4, 4), (4, 5)}

$\therefore$ (A $\times$ B) $\cap$(C $\times$ D) = {(1, 2), (1, 4), (3, 2), (3, 4)}

Also, (A $\cap$ C) = {1, 3} and (B $\cap$ D) = {2, 4}. Therefore

(A $\cap$ C) $\times$ (B $\cap$ D) = {(1,2), (1, 4), (3, 2), (3, 4)}

Hence, (A $\times$ B) $\cap$ (C $\times$ D) = (A $\cap$ C) $\times$(B $\cap$ D)

**Let A = {2, 3, 5} and B = {4, 7, 10, 8}. Find relation R : A $\to$ B such that a R b $\Rightarrow$ a divides b. Also find domain, co-domain and range of relation.**

R = {(2, 4), (2, 10), (2, 8) (5, 10)}

Here domain of R = {2, 5}

and range of R = {4, 10, 8}

Co-domain of R = B = {4, 7, 10, 8}

**Let A = {1, 2, 3}, B = {2, 4, 6, 8}, $R_1$ = {(1, 2), (2, 4), (3, 6)} and $R_2$ = {(2, 4), (2, 6), (3, 8), (1, 6)}. Check whether $R_1$ and $R_2$ are relations from A to B. If yes find domain and range of relations.**

$R_1$ and $R_2$ are relations from A to B because $R_1 \subseteq A \times B$ , and $R_2 \subseteq A \times B$

Here $1R_12$, $2R_14$, $3R_16$.

Also $2R_24$, $2R_26$, $3R_28$, $1R_26$.

Domain $R_1$ = {1, 2, 3} , Range $R_1$ = {2, 4, 6}

Domain $R_2$ = {2, 3, 1}, Range $R_2$ = {4, 6, 8}

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Let R be a relation on set of natural numbers N defined by xRy ⇔ x + 2y = 41; ∀x, y ∈ N. Find the domain and Range of R.

x can be only those natural numbers for which y ∈ N i.e. $y = \frac{41 - x}{2} \in N$.

Clearly x = 1, 3, 5, 7, ...., 39.

Similarly, y can be only those natural numbers for which x ∈ N i.e., x = 41 – 2y ∈ N.

Clearly, y = 1, 2, 3, ...., 20

∴ Domain R = {x : (x, y) ∈ R} = {x : x + 2y = 41}
= {1, 3, 5, 7, ...., 39}
= set of odd natural numbers less than 40.

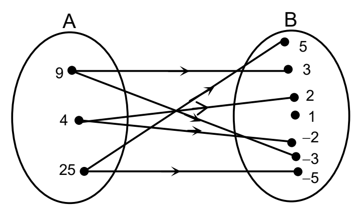
Range of R = {y : x + 2y = 41}
= {1, 2, 3, ..., 20}
= set of natural numbers less than 21.

Given figure shows a relation between the sets A and B. Write this relation

(i) in roster form

(ii) in set builder form.

What is its domain and range?




Let R be the given relation

In roster form

R = {(9, 3), (9, –3), (4, 2), (4, –2), (25, 5), (25, –5)}

In set builder form

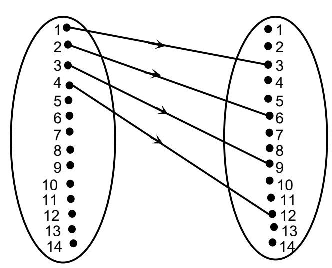
R = {(x, y) : x ∈ A, y ∈ B and x is the square of y}

Domain of R = {9, 4, 25}

Range of R = {3, –3, 2, –2, 5, –5}

Let A = {1, 2, 3, ...,14}. Define a relation R from A to A by R = {(x, y) : 3x – y = 0, where x, y ∈ A}

Depict this relation using an arrow diagram. Write down its domain, codomain and range.




Given, 3x – y = 0 ⇒ y = 3x

Domain of R = {1, 2, 3, 4}

Range of R = {3, 6, 9, 12}

Codomain of R = {1, 2, 3, ...., 14}

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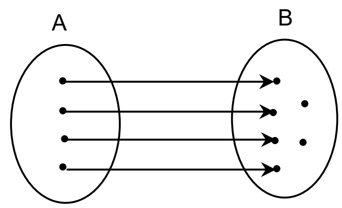
## *Functions*

Before studying and knowing about function we read another classification of relations . These are :

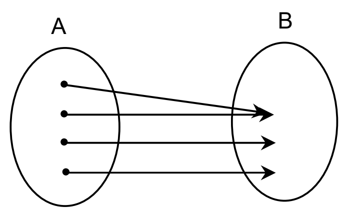
(a) One-one
(b) Many-one
(c) One-many
(d) Many-many

Let us understand about these relations with the help of arrow diagram.

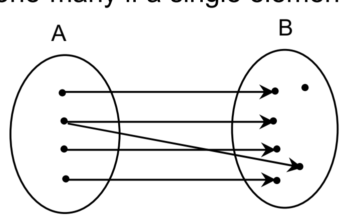
(a) A relation is said to be one-one if distinct elements of A have distinct image in B.




(b) A relation is said to be many-one if more than one elements of set A have a single image in B.




(c) A relation is said to be one-many if a single element of set A has more than one images in B.

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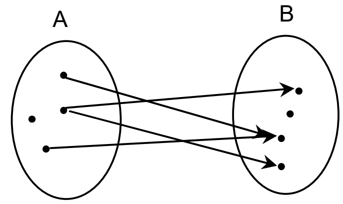
(d) A relation is said to be many-many if more than one elements of set A are connected to multiple elements of set B.




Let A = {–2, –1, 0, 1, 2} and f : A → Z given by $f(x) = x^2 - 2x - 3$. Find (a) the range of f (b) pre-image of 6, –3 and 5.

(a) We have : $f(-2) = (-2)^2 - 2(-2) - 3 = 5$, $f(-1) = (-1)^2 - 2(-1) - 3 = 0$

$f(0) = -3$, $f(1) = 1^2 - 2 \times 1 - 3 = -4$ and $f(2) = 2^2 - 2 \times 2 - 3 = -3$.

So range (f) = {0, 5, –3, –4}

(b) Let x be the pre-image of 6, Then,

(c) $f(x) = 6 \Rightarrow x^2 - 2x - 3 = 6 \Rightarrow x^2 - 2x - 9 = 0 \Rightarrow x = 1 \pm \sqrt{10}$

Since $x = 1 \pm \sqrt{10} \notin A$. So, there is no pre-image of 6.

Let x be the pre-image of –3. Then,

$f(x) = -3 \Rightarrow x^2 - 2x - 3 = -3 \Rightarrow x^2 - 2x = 0$

$\Rightarrow x = 0, 2$

Since, $0, 2 \in A$, so 0, 2 are the pre-images of –3.

Let x be the pre-image of 5. Then,

$f(x) = 5 \Rightarrow x^2 - 2x - 3 = 5 \Rightarrow x^2 - 2x - 8 = 0$

$\Rightarrow (x - 4)(x + 2) = 0 \Rightarrow x = 4, -2$

Since, $-2 \in A$, So, –2 is the pre-image of 5.

**Find domain and range of $f(x) = \frac{x}{1-x}$**

Clearly f(x) is not defined at x = 1 Hence, domain of f is R – {1}

Put $y = f(x) = \frac{x}{1-x}$

or $x = \frac{y}{y+1}$

Now, x will exist if $y \neq -1$. Hence, range of f is R – {–1}

**Find domain and range of $f(x) = \sqrt{9 - x^2}$**

For f(x) to define, $9 - x^2 \geq 0$ or $x \in [-3, 3]$



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Put y = f(x) = $\sqrt{9-x^2}$ or x = $\pm\sqrt{9-y^2}$

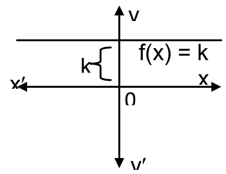
For x to exist, $9 - y^2 \geq 0$ or $y \in [-3, 3]$

But $y \geq 0$, Hence, range of f is [0, 3]

## *Some Real Functions*

**(i) Constant function**

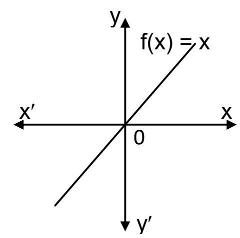
Let k be a fixed real number. Then a function f(x) given by f(x) = k for all $x \in R$ is called a constant function.




The domain of the constant function f(x) = k is the complete set of real numbers and the range of f is the singleton set {k}

**(ii) Identity function**

The function defined by f(x) = x for all $x \in R$, is called identity function on R. The domain and range of the Identity function are both equal to R.



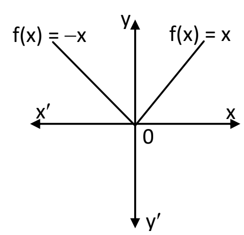

**(iii) Modulus function**

The function defined by

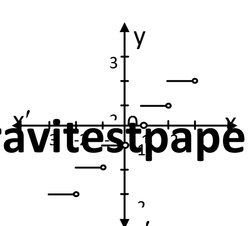
$$f(x) = |x| = \sqrt{x^2} = \max\{x, -x\} = \begin{cases} x, & \text{when } x \geq 0 \\ -x, & \text{when } x < 0 \end{cases} \text{ is}$$

called the modulus function.

The domain of the modulus function is the set of all real numbers and the range is the set of all nonnegative real numbers or $[0, \infty)$.




**(iv) The greatest integer function**



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For any real number x, we denote by [x], the greatest integer less than or equal to x. For example,

[2.45] = 2, [–2.1] = –3, [1.75] = 1, [0.32] = 0 etc.

The function f defined by f(x) = [x] for all $x \in R$, is called the greatest integer function.

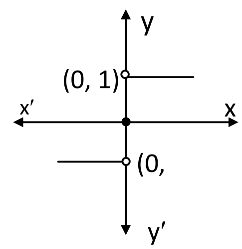
The domain of the greatest integer function is the set

R of all real numbers and the range is the set of all Integers as it attains only integral values.

**(v) Signum function**

The function defined by $f(x) = \begin{cases} \dfrac{|x|}{x}, & x \neq 0 \\ 0 & x = 0 \end{cases}$ or

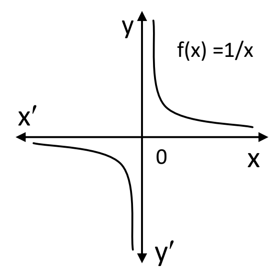
$f(x) = \begin{cases} 1, & x > 0 \\ 0, & x = 0 \\ -1, & x < 0 \end{cases}$ is called the signum function.




The domain of the signum function is R and the range is the set {–1, 0, 1}.

**(vi) Reciprocal function**

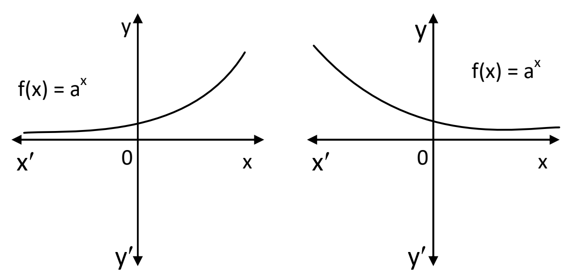
The function that associates each non-zero real number x to its reciprocal $\frac{1}{x}$ is called the reciprocal function. The domain and range of the reciprocal function are both equal to R – {0} i.e. the set of all non-zero real numbers.




**(vii) Exponential function**

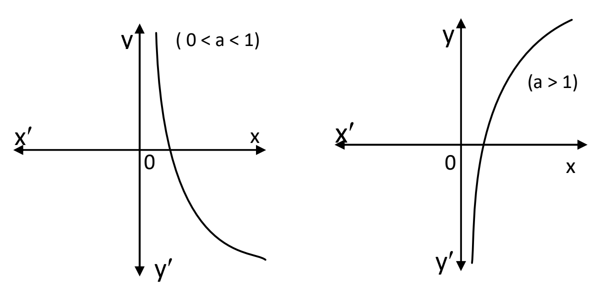
If a is positive real number and $a \neq 1$, then the function which associates every real number x to $a^x$ i.e. $f(x) = a^x$ (a > 0) is called the exponential function.

The domain of the exponential function is R and the range is the set of all positive real numbers.




**(viii) Logarithmic function**

If 'a' is a positive real number and $a \neq 1$, then the function that associates every positive real number to $\log_a x$ i.e. $f(x) = \log_a x$ is called the logarithmic function. The domain of the logarithmic function is the set of all positive real numbers and

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the range is the set R of all real numbers.

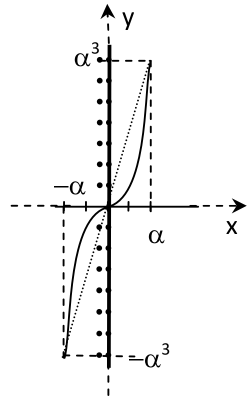
**(ix) Polynomial function**

A function of the form

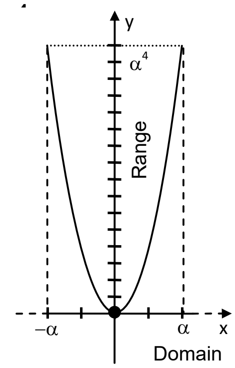
$f(x) = a_0 x^n + a_1 x^{n-1} + \ldots\ldots + a_{n-1} x + a_n$, where $a_0, a_1, a_2, \ldots\ldots a_n$ are real numbers, $a_0 \neq 0$ and $n \in N$, is called polynomial function of degree n. The domain of a polynomial function is always R.

For Example – Domain and range of cubic function i.e. $f(x) = x^3$ and biquadratic function $f(x) = x^4$ are shown in the figure.

**f : R → R; f(x) = $x^3$**



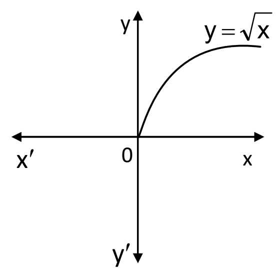

**f : R → R; f(x) =**




**(x) Square root function**

The function that associates every positive real number x to + $\sqrt{x}$ is called the square root function, i.e., $f(x) = +\sqrt{x}$

Domain of f is the set of all non-negative real numbers = $[0, \infty)$ and range (f) = $[0, \infty)$.

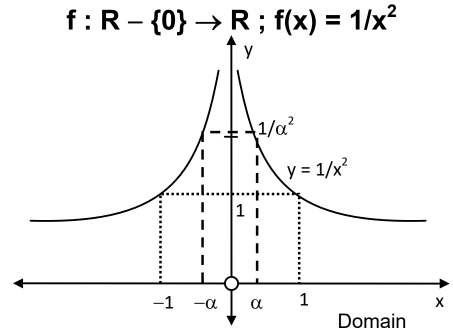



**(xi) Rational function**

A function of the form $f(x) = \dfrac{p(x)}{q(x)}$, where p(x) and q(x) are polynomial and $q(x) \neq 0$, is called a rational function.

The domain of a rational function $\dfrac{p(x)}{q(x)}$ is the set of all real numbers except points where q(x) = 0.

**f : R – {0} → R ; f(x) = $1/x^2$**

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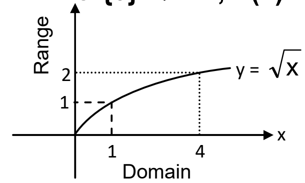
**(xii) Irrational functions**

A function containing one term having non-integral rational powers of x are called Irrational functions. For examples f(x) = $\sqrt{x}$ , f(x) = $x^{1/3}$ are irrational functions.

**f : $R^+ \cup \{0\} \rightarrow R$ ; f(x) =**

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