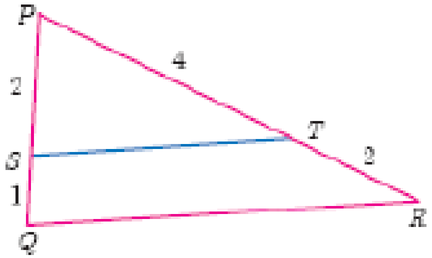
10^{TH} GEOMETRY MOST EXPECTED

Show that $\triangle PST \sim \triangle PQR$



Answer : In $\triangle PST$ and $\triangle PQR$,

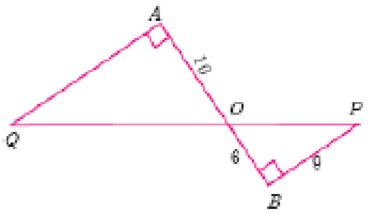
$$\frac{PS}{PQ} = \frac{2}{2+1} = \frac{2}{3}, \frac{PT}{PR} = \frac{4}{4+2} = \frac{2}{3}$$

Thus, $\frac{PS}{PQ} = \frac{PT}{PR}$ and $\angle P$ is common

Therefore, by SAS similarity,

$\triangle PST \sim \triangle PQR$

QA and PB are perpendiculars to AB. If AO = 10 cm, BO = 6 cm and PB = 9 cm. Find AQ.



Answer : ΔAOQ and $\Delta BOP, \angle OAQ = \angle OBP = 90^0$

$\angle AOQ = \angle BOP$ (Vertically opposite angles)

Therefore, by AA Criterion of similarity,

$\Delta AOQ \sim \Delta BOP$

$\frac{AO}{BO} = \frac{OQ}{OP} = \frac{AQ}{BP}$

$\frac{10}{6} = \frac{AQ}{9}$ gives $AQ = \frac{10\times 9}{6} = 15cm$

If △ABC is similar to △DEF such that BC = 3 cm, EF = 4 cm and area of △ABC = 54 cm^2. Find the area of △DEF.

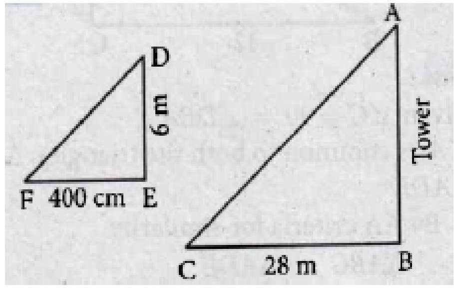
Answer : Since the ratio of area of two similar triangles is equal to the ratio of the squares of any two corresponding sides, we have

$\frac{Area(\Delta ABC)}{Area(\Delta DEF)} = \frac{BC^2}{EF^2}$ gives $\frac{54}{Area(\Delta DEF)} = \frac{3^2}{4^2}$

$$Area(\Delta DEF) = \frac{16 \times 54}{9} = 96 cm^2$$

A vertical stick of length 6 m casts a shadow 400 cm long on the ground and at the same time a tower casts a shadow 28 m long. Using similarity, find the height of the tower.

Answer :



Let DE is the vertical stick and AB is the tower,

DE = 6 m, EF = 400 cm = 4 m, BC = 28 m

From DFE and ACB

Using similarity criteria

$$\frac{AB}{DE} = \frac{BC}{EF}$$

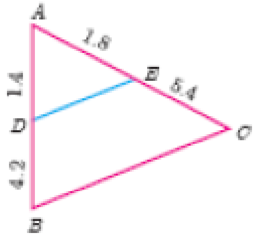
$$\frac{AB}{6} = \frac{28}{4}$$

$$AB = \frac{28 \times 6}{4} = 42 \text{ m}$$

Height of the tower = 42 m

D and E are respectively the points on the sides AB and AC of a $\triangle$ABC such that AB = 5.6 cm, AD = 1.4 cm, AC = 7.2 cm and AE = 1.8 cm, show that DE || BC

Answer :



We have AB = 56.cm, AD = 14. cm, AC = 72. cm and AE = 18.cm.

BD = AB - AD = 5.6 –1.4 = 4.2 cm

and EC = AC – AE = 7.2–1.8 = 5.4 cm

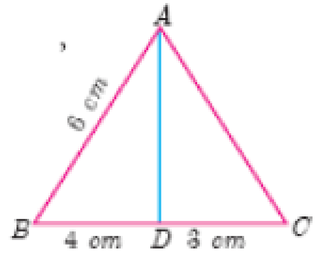
$$\frac{AD}{DB} = \frac{1.4}{4.2} = \frac{1}{3} \text{ and } \frac{AE}{EC} = \frac{1.8}{5.4} = \frac{1}{3}$$

$$\frac{AD}{DB} = \frac{AE}{EC}$$

Therefore, by converse of Basic Proportionality Theorem, we have DE is parallel to BC.

Hence proved.

In the figure, AD is the bisector of $\angle A$. If BD = 4 cm, DC = 3 cm and AB = 6 cm, find AC.



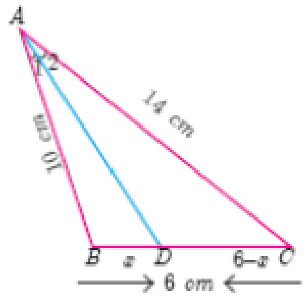
Answer : In $\triangle ABC$, AD is the bisector of $\angle A$

Therefore by Angle Bisector of $\angle A$

$$\frac{AB}{AC} = \frac{BD}{DC}$$

$\frac{4}{3} = \frac{6}{AC}$ gives 4AC = 18. Hence, $AC = \frac{9}{2} = 4.5$ cm

In the Figure, AD is the bisector of ∠BAC, if A = 10 cm, AC = 14 cm and BC = 6 cm. Find BD and DC.



Answer : Let BD = x cm, then DC = (6 – x)cm

AD is bisector of∠ A

Therefore by Angle Bisector Theorem

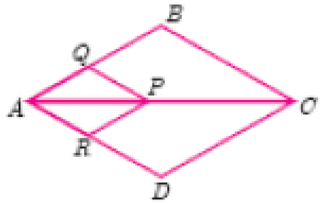
$$\frac{AB}{AC} = \frac{BD}{DC}$$

$$\frac{10}{14} = \frac{x}{6-x} \qquad \frac{5}{7} = \frac{x}{6-x}$$

So, 12x = 30 we get, $x = \frac{30}{12} = \mathbf{2.5}$

Therefore, BD = 2.5 cm, DC = 6−x = 6−2.5 = 3.5 cm

In fig. if PQ | | BC and PR | |CD prove that



$$\frac{AB}{AD} = \frac{AQ}{AB}$$

Answer : In $\triangle$ACB,

PQ ll CB

Using Basic Proportionality theorem, we have

$$\frac{AQ}{AB} = \frac{AP}{AC}$$

Again in $\triangle$ACD PR ll CD

Using Basic Proportionality theorem

$$\frac{AP}{AC} = \frac{AR}{AD}$$

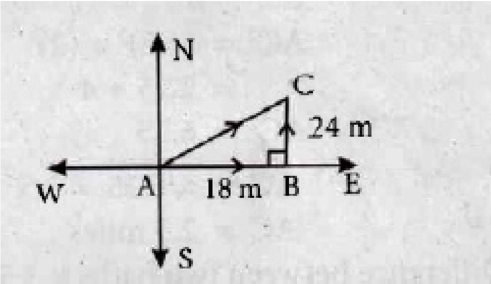
From (1) and (2)

$$\frac{AQ}{AB} = \frac{AP}{AC} = \frac{AR}{AD}$$

Thus we have $\frac{AR}{AD} = \frac{AQ}{AB}$

A man goes 18 m due east and then 24 m due north. Find the distance of his current position from the starting point?

Answer :



Let the initial position of the man be A and his final position.be C

Since the man goes 18 m east and then 24 m north, LABC is a right angled triangle with $\angle B = 90^\circ$; AB = 18 m and BC = 24m.

By pythagoras theorem, we have

$AC^2 - AB^2 + BC^2$

$AC^2 = 18^2 + 24^2$

$AC^2 = 324 + 576$

$AC^2 = 900 = 30 \times 30$

AC = 30m

His current distance from starting point = 30 m

The area of $\triangle PQR = 64 \text{ m}^2$. Find the area of ΔLMN if $\frac{PQ}{LM} = \frac{4}{5}$ and $\Delta PQR \sim \Delta LMN$

Answer : Given $\triangle PQR \sim \Delta LMN$

$$\frac{\text{area } (\Delta\text{PQR})}{\text{area of } (\Delta\text{LMN})} = \frac{PQ^2}{LM^2}$$

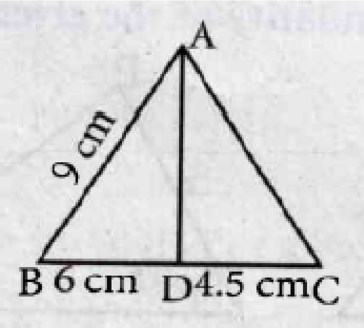
$$\frac{64}{\text{area}(\Delta LMN)} = \left(\frac{4}{5}\right)^2 = \frac{16}{25}$$

$$\therefore \quad \text{Area of } \Delta LMN = \frac{64 \times 25}{16}$$

$$= 100 \text{ m}^2$$

$$\text{Area of } \Delta LMN = 100 \text{ m}^2$$

In the figure AD is the bisector of ∠A.If BD = 6 cm, DC = 4.5 cm and AB = 9 cm, Find AC



Answer : In $\triangle ABC, AD$ is the bisector of $\angle A$

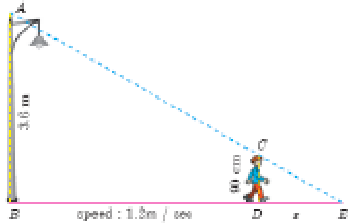
$$\therefore \frac{BD}{DC} = \frac{AB}{AC}$$

$$\frac{6}{4.5} = \frac{9}{AC}$$

$$AC = \frac{9}{6} \times 4.5$$

$$AC = 6.75 \text{ cm}$$

A boy of height 90cm is walking away from the base of a lamp post at a speed of 1.2m/sec. If the lamppost is 3.6m above the ground, find the length of his shadow cast after 4 seconds.



Answer : Given, Speed = 1.2 m/s,

time = 4 seconds

Distance = speed x time

= 1.2 x 4

= 4.8 m

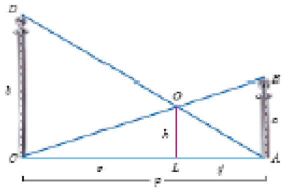
Let x be the length of the shadow after 4 seconds

$\Delta ABE \sim \Delta CDE$, $\frac{BE}{DE} = \frac{AB}{CD}$ gives $\frac{4.8+x}{x} = \frac{3.6}{0.9} = \frac{3.6}{0.9} = 4$ (since 90 cm = 0.9 m)

4.8 + x = 4x, gives 3x = 4.8 so, x = 1.6m

The length of his DE = 1.6m

Two poles of height 'a' metres and 'b' metres are 'p' metres apart. Prove that the height of the point of intersection of the lines joining the top of each pole to the foot of the opposite pole is given by $\frac{ab}{a+b}$ meters



Answer : Let AB and CD be two poles of height 'a' metres and 'b' metres respectively such that the poles are 'p' metres apart. That is AC = p metres. Suppose the lines AD and BC meet at O, such that OL = h metres

Let CL = x and LA = y.

Then, x + y = p

In ΔABC and ΔLOC, we have

$\angle CAB = \angle CLO$ [each equal to 90°]

$\angle C = \angle C$ [C is common]

$\Delta CAB \sim \Delta CLO$ [By AA similarity]

$\frac{CA}{CL} = \frac{AB}{LO}$ gives $\frac{P}{x} = \frac{a}{h}$

so, $x = \frac{ph}{a}$ ..(1)

In ΔALO and ΔACD, we have

$\angle ALO = \angle ACD$ [each equal to 90°]

$\angle A = \angle A$[A is common]

$\frac{AL}{AC} = \frac{OL}{DC}$ gives $\frac{y}{p} = \frac{h}{b}$ we get, $y = \frac{ph}{b}$ ....(2)

(1) + (2) gives $x + y = \frac{ph}{a} + \frac{ph}{b}$

$p = ph\left(\frac{1}{a} + \frac{1}{b}\right)$ (since x + y = p)

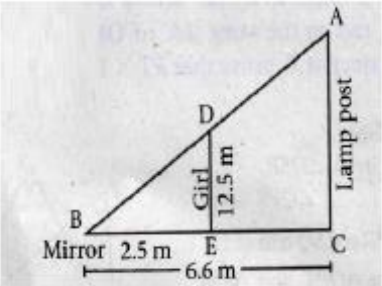
$1 = h\left(\frac{a+b}{ab}\right)$

Thefore, $h = \frac{ab}{a+b}$

Hence, the height of the intersection of the lines joining the top of each pole to the foot of the opposite pole is $\frac{ab}{a+b}$ meters.

A girl looks the reflection of the top of the lamp post on the mirror which is 6.6 m away from the foot of the lamp post. The girl whose height is 12.5 m is standing 2.5 m away from the mirror. Assuming the mirror is placed on the ground facing the sky and the girl, mirror and the lamp post are in a same line, find the height of the lamp post.

Answer :



Let AC is the lamp post and ED is the girl.

From the triangles ABC and DBE

By AA criteria

Their sides are Proportional

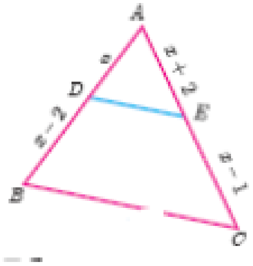
$$\frac{AC}{DE} = \frac{BC}{BE}$$

$$\frac{AC}{12.5} = \frac{6.6}{2.5}$$

$$AC = \frac{6.6 \times 12.5}{2.5} = \frac{6.6 \times 12.5^{5}}{2.5} = 33 \text{ m}$$

Height of the lamp post = 33 m

In $\triangle$ ABC, if DE| |BC, AD = x, DB = x − 2, AE = x +2 and EC = x − 1 then find the lengths of the sides AB and AC.



Answer : In $\triangle$ ABC we have DE | | BC.

By Thales theorem, we have $\frac{AD}{DB} = \frac{AE}{EC}$

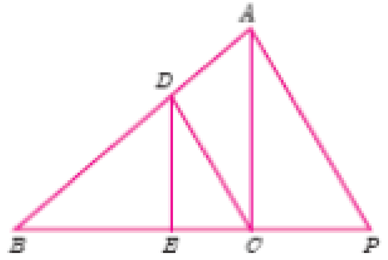
$\frac{x}{x-2} = \frac{x+2}{x-1}$ gives x(x - 1) = (x - 2)(x + 2)

When x = 4, AD = 4, DB = x - 2, AE + x + 2 = 6, EC = x - 1 = 3

Hence, AB = AD + DB = 4 + 2 = 6, AC = AE + EC = 6 + 3 = 9

Therefore, AB = 6, AC = 9

In the figure DE| |AC and DC| |AP. Prove that $\frac{BE}{CE} = \frac{BC}{CP}$



Answer : In $\triangle$BPA, we have DE| |AP By Basic Proportionality Theorem,

We have $\frac{BC}{CP} = \frac{BD}{DA}$ ..(i)

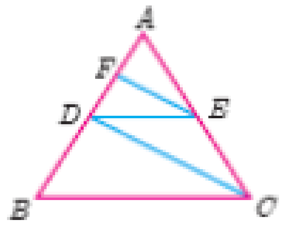
In $\triangle$BCA, we have DE| |AC By Basic Proportionality Theorem,

we have,

$\frac{BE}{EC} = \frac{BD}{DA}$ ..(2)

From (1) and (2) we get, $\frac{BE}{EC} = \frac{BC}{CP}$, Hence proved.

In figure DE | | BC andCD. Prove that AD^2 = AB x AF



Answer : In ABC , we have DE ll BC

$$\frac{AB}{AD} = \frac{AC}{AE}$$ [By Thales Theorem] ..(1)

In ADC, we have

$$\frac{AD}{AF} = \frac{AC}{AE}$$ [By Thales Theorem] .....(2)

From (1) and (2) we get

$$\frac{AB}{AD} = \frac{AD}{AF}$$

AD^2 = AB x AF

Basic Proportionality Theorem (BPT) or State and prove Thales theorem?

Answer : Statement

A straight line drawn parallel to a side of triangle intersecting the other two sides, divides the sides in the same ratio.

Proof

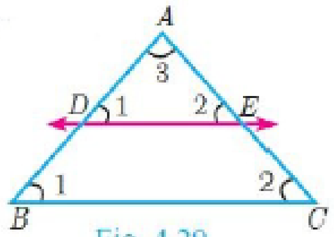


Fig. 4.29

In ΔABC ,D is a point on AB and E is a point on AC

To prove : $\dfrac{AD}{DB} = \dfrac{AE}{EC}$

Construction: Draw a line DE || BC

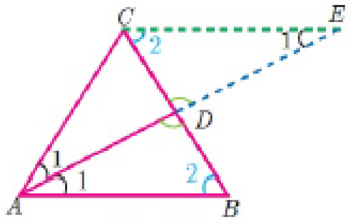
No.	Statement	Reason
1.	$\angle ABC = \angle ADE = \angle 1$	Corresponding angles are equal because DE || BC
2.	$\angle ACB = \angle AED = \angle 2$	Corresponding angles are equal because DE || BC
3.	$\angle DAE = \angle BAC = \angle 3$	Both triangles have a common angle
4.	$\Delta ABC \sim \Delta ADE$	By AAA similarity
	$\frac{AB}{AD} = \frac{AC}{CE}$	Corresponding sides are proportional
	$\frac{AD+DB}{AD} = \frac{AE+EC}{AE}$	Split AB and AC using the points D and E.
	$1 + \frac{DB}{AD} = 1 + \frac{EC}{AE}$	On simplification
	$\frac{DB}{AD} = \frac{EC}{AE}$	Cancelling 1 on both sides
	$\frac{AD}{DB} = \frac{AE}{EC}$	Taking reciprocals
		Hence proved

State and Prove - Angle Bisector Theorem

Answer : Statement :

The internal bisector of an angle of a triangle divides the opposite side internally in the ratio of the corresponding sides containing the angle

Proof



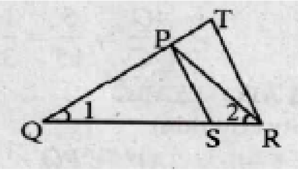
Given : In ΔABC,AD is the internal bisector

To prove : $\frac{AB}{AC} = \frac{BD}{CD}$

Construction : Draw a line through C parallel to AB. Extend AD to meet line through C at E

NO	STATEMENT	REASON
1.	∠AEC =∠BAE =∠1	Two parallel lines cut by a transversal make alternate angles equal
2.	ΔACE is isosceles AC = CE … (1)	In ΔACE,∠CAE = ∠CEA
3.	ΔABD~ΔECD $\frac{AB}{CE} = \frac{BD}{CD}$	By AA Similarity
4.	$\frac{AB}{AC} = \frac{BD}{CD}$	From (1) AC = CE Hence proved

In the figure $\frac{QR}{QS} = \frac{QT}{PR}$ and $\angle 1 = \angle 2$ show that $\triangle PQS \sim \triangle TQR$



Answer : Given In $\triangle PQR \angle 1 = \angle 2$

PR = QP

[∵ In a triangle, sides opposite to equal angles are equal]

Given $\frac{QR}{QS} = \frac{QT}{PR}$

From (1) and (2)

$\frac{QR}{QS} = \frac{QT}{QP}$

$\frac{QS}{QR} = \frac{QP}{QT}$

Now in $\triangle PQS$ and ΔTQR

$\frac{QS}{QR} = \frac{QP}{QT}$

$\angle SQP = \angle RQT = \angle 1$[From (3)]

∴ Using SAS similarity criteria

$\triangle PQS \sim \triangle TQR$