10TH MATHS NUMBERS AND SEQUENCES EXPECTED QUESTION ANSWERS

138) Is 7 x 5 x 3 x 2 + 3 a composite number? Justify your answer

Answer : Yes, the given number is a composite number, because
$7 \times 5 \times 3 \times 2 + 3 = 3 (7 \times 5 \times 2 + 1) = 3 \times 71$
Since the given number can be factorized in terms of two primes, it is a composite number.

143) Determine the value of d such that $15 \equiv 3 \pmod{d}$.

Answer : $15 \equiv 3 \pmod{d}$ means $15 - 3 = kd$, for some integer k,
$12 = kd$
gives d divides 12.
The divisors of 12 are 1,2,3,4,6,12. But d should be larger than 3 and so the possible values for d are 4, 6, 12.

142) If $13824 = 2^a \times 3^b$ then find a and b.

Answer :

2	13824
2	6912
2	3456
2	1728
2	864
2	432
2	216
2	108
2	54
3	27
3	9
	3

The number 13824 can be factorized as

$2^a \times 3^b = 13824 = 2^9 \times 3^3$

a = 9 and b = 3.

148) A man starts his journey from Chennai to Delhi by train. He starts at 22.30 hours on Wednesday. If it takes 32 hours of travelling time and assuming that the train is not late, when will he reach Delhi?

Answer : Starting time 22.30, Travelling time 32 hours. Here we use modulo 24.
The reaching time is
$22.30 + 32 \pmod{24} \equiv 54.30 \pmod{24}$
$\equiv 6.30 \pmod{24}$ (Since 32 = (1 x 24) + 8 thursday Friday)
Thus, he will reach Delhi on Friday at 6.30 hours

154) What is the time 100 hours after 7 a.m.?

Answer : Starting from 7 o' clock 100 hours.
we use modulo 24.
7 o' clock a.m. + 100 (modulo 24) = 7 o' clock + 4hrs
= 11o' clock a.m
100 hrs after 7 o' clock a.m. is 11 o' clock a.m.

149) Kala and Vani are friends. Kala says, “Today is my birthday” and she asks Vani, “When will you celebrate your birthday?” Vani replies, “Today is Monday and I celebrated my birthday 75 days ago”. Find the day when Vani celebrated her birthday.

Answer : Let us associate the numbers 0, 1, 2, 3, 4, 5, 6 to represent the weekdays from Sunday to Saturday respectively.

Vani says today is Monday. So the number for Monday is 1. Since Vani’s birthday was 75 days ago, we have to subtract 75 from 1 and take the modulo 7, since a week contain 7 days.

$-74 \pmod 7 \equiv -4 \pmod 7 \equiv 7-4 \pmod 7 \equiv 3 \pmod 7$

(Since $-74 - 3 = 77$ is divisible by 7)

Thus, $1 - 75 \equiv 3 \pmod 7$

The day for the number 3 is Wednesday.

Therefore, Vani’s birthday must be on Wednesday.

156) Today is Tuesday. My uncle will come after 45 days. In which day my uncle will be coming?

Answer : Starting from Tuesday we have to calculate the day after 45 days

The number for Tuesday is 2.

$2 + 45 \pmod 7 \equiv 47 \pmod 7$

$\equiv 5 \pmod 7$

Number 5 stands for Friday.

Uncle will be coming on Friday

175) Which term of an A.P. 16,11,6,1,... is -54?

Answer : Given the A.P. 15, 11, 6, 1,.....

Here a = 16, d = 11 - 16 = -5

We have the n^{th} term of an A.P. $t_n = a + (n - 1)d$

Put $t_n = -54$

$-54 = 16 + (n - 1)(-5)$

$\frac{-70}{-5} = n - 1$

$14 = n - 1$

$14 + 1 = n$

$n = 15$

- 54 is the 15^{th} term of 16, 11, 6, 1,....

178) If 3 + k, 18 - k, 5k + 1 are in A.P. then find k.

Answer : Given 3 + k, 18 - k, 5k + 1 are in A.P.

$t_1 = 3, k, t_2 = 18 - k , t_3 = 5k - 1$

Difference between the consecutive terms must be equal

i.e., $t_2 - t_1 = t_3 - t_2$

$(18 - k) - (3 + k) = (5k + 1) - (18 - k)$

$18 - k - 3 - k = 5k + 1 - 18 + k$

$15 - 2k = 6k - 17$

$15 + 17 = 6k + 2k$

$32 = 8k$

$k = \frac{32}{8} = 4$

189) Find the 8^{th} term of the G.P 9,3,1,....

Answer : The find the 8th term we have use the n^{th} term formula $tn = ar^{n-1}$

First term a = 9, common radio r = $\frac{t_2}{t_1} = \frac{3}{9} = \frac{1}{3}$

$$t_8 = 9 \times \left(\frac{1}{3}\right)^{8-1} = 9 \times \left(\frac{1}{3}\right)^{7} = \frac{1}{243}$$

Therefore the 8th term of the G.P is $\frac{1}{243}$

193) Find x so that x + 6, x + 12 and x + 15 consecutive terms of a Geometric Progression.

Answer : If the given numbers are consecutive terms of a G.P. then

$$\frac{t_2}{t_1} = \frac{t_3}{t_2}$$

$$i.e., \frac{x+12}{x+6} = \frac{x+15}{x+12}$$

$(x+12)^2 = (x+6)(x+15)$

$x^2 + 24x + 144 = x^2 + 6x + 15x + 90$

$x^2 + 24x + 144 - x^2 - 6x - 15x - 90 = 0$

$24x - 21x + 144 - 90 = 0$

$3x + 54 = 0$

$3x = -54$

$$x = \frac{-54}{3} = -18$$

199) Find the sum $3 + 1 + \frac{1}{3} + \ldots\infty$

Answer : Here a = 3, $r = \frac{t_2}{t_1} = \frac{1}{3}$

Sum of infinite terms = $\frac{a}{1-r} = \frac{3}{1-\frac{1}{3}} = \frac{9}{2}$

210) If $1 + 2 + 3 + \ldots + n = 666$ then find n.

Answer : Since, $1 + 2 + 3 + \ldots + n = \frac{n(n+1)}{2}$, we have $\frac{n(n+1)}{2} = 666$

$n^2 + n - 1332 = 0$ gives $(n + 37)(n - 36) = 0$

So, $n = -37$ or $n = 36$

But $n \neq -37$ (Since n is a natural number); Hence $n = 36$

213) If $1^3 + 2^3 + 3^3 + \ldots k^3 = 44100$ then find $1 + 2 + 3 + \ldots + k$

Answer : $1^3 + 2^3 + 3^3 + \ldots K^3 = \left[\frac{k(k+1)}{2}\right]^2 = 44100 = (210)^2$

$1 + 2 + 3 + \ldots + k = \frac{k(k+1)}{2} = 210$

$1 + 2 + 3 + \ldots + k = 210$

221) Find the least positive value of x such that
$89 \equiv (x + 3) \pmod 4$

Answer : $89 \equiv (x + 3) \pmod 4$

$89 - (x + 3) = 4n$ for some integer n.

$89 - x - 3 = 4n$ for some integer n.

$86 - x = 4n$, for some integer n.

86 - x is a multiple of 4.

x = 2 is the least positive value

287) The traffic lights at three different road crossings change after every 48 sec, 72 sec and 108 sec respectively. If they all change simultaneously at 8.20 am, then at what time will they again change simultaneously?

Answer : Interval of change = L.C.M. of (48, 72, 108) sec

2	48, 72, 108
3	4, 6, 9
2	4, 2, 3
	2, 1, 3

LCM = 12 x 3 x 2 x 2 x 3 = 432

So the lights will again change simultaneously after every 432 sec

= 432/60 = 7 min 12 sec

Hence next change will be at 8 : 27 : 12 am.

295) If $2x, x+10, 3x+2$ are in A.P. Find x.

Answer : Given $2x, x+10, 3x+2$ are in A.P

$x+10-2x = 3x+2-[x+10]$

$-x+10 = 3x+2-x-10 = 2x-8$

$10+8 = 2x+x$

$3x = 18$

$x = \frac{18}{3} = 6$

301) How many numbers of two digits are divisible by 7?

Answer : we know that first two digit number divisible by 7 is 14.

Last two digit number divisible by 7 is 98

It is enougrt to find the number of terms of the A.P.14, 21, 28, ..., 98.

a = 14,d = 7

n^{th} term = 98

a + (n - 1)d = 98

14 + (n - 1) (7) = 98

(n - 1)(7) = 98 -14 = 84

$$n - 1 = \frac{84}{7} = 12$$

n = 12 + 1 = 13

There are 13 numbers of two digits which are divisible by 7.

318) If two positive integers p and q are written as $p = a^2b^3$ and $q = a^3b; a, b$ are prime numbers, then veriff LCM (p,q) x HCF(p, q) = pq.

Answer : $p = a^2\ b^3, q = a^3\ b$

HCF of $p, q = a^2\ b$

LCM of $p, q = a^3\ b^3$

$\text{LCM} \times \text{HCF} = a^2\ b \times a^3\ b^3$

$= a^5\ b^4$

$pq = a^2\ b^3 \times a^3\ b$

$= a^5\ b^4$

$\therefore$ LCM $\times$ HCF of $p, q = pq$

332) If $p_1^{x_1} \times p_2^{x_2} \times p_3^{x_3} \times p_4^{x_4}$ = 113400 where p_1, p_2, p_3, p_4 are primes in ascending order and x_1, x_2, x_3, x_4 are integers, find the value of p_1, p_2, p_3, p_4 and x_1, x_2, x_3, x_4

Answer : Given 113400 = $p_1^{x_1} \times p_2^{x_2} \times p_3^{x_3} \times p_4^{x_4}$

2	113400
2	56700
2	28350
3	14175
3	4725
3	1575
3	525
5	175
5	35
7	7
	1

The number 113400 can be factorized as 113400 = 2^3 x 3^4 x 5^2 x 7^1 where 2,3,5,7 are primes in ascending order.

Comparing

$p_1^{x_1} \times p_2^{x_2} \times p_3^{x_3} \times p_4^{x_4}$ = 2^3 x 3^4 x 5^2 x 7^1

P_1 = 2; P_2 = 3; P_3 = 5; P_4 = 7

x_1 = 3; x_2 = 4; x_3 = 2; x_4 = 1.

345) The sum of three consecutive terms that are in A.P. is 27 and their product is 288. Find the three terms.

Answer : Let the three consecutive terms be a - d, a, a + d

Given their sum is 27

$(a - d) + a + (a + d) = 27$

$a - d + a + a + d = 27$

$3a = 27$

$a = \frac{27}{3} = 9$

Product = 288

$(a - d)\, a\, (a + d) = 288$

$a(a^2 - d^2) = 288$

$9(9^2 - d^2) = 288$

$81 - d^2 = \frac{288}{9}$

$81 - d^2 = 32$

$d^2 = 81 - 32 = 49$

$d \times d = 7 \times 7$

$d = \pm 7$

(i) a = 9, d = 7, The three terms are,

= 9 -7, 9, 9 + 7

= 2,9, 16

(ii) a = 9, d = -7, Thethree terms are

9-(-7), 9, 9-7 $\Rightarrow$ 16,9,2

The required three consecutive terms of the A.P are 2,9,16.

352) Find the sum of all natural numbers between 300 and 600 which are divisible by 7.

Answer : The natural numbers between 300 and 600 which are divisible by 7 are 301, 308, 315, ..., 595.

The sum of all natural numbers between 300 and 600 is 301 + 308 + 315 +...+ 595

The terms of the above series are in A.P.

First term a = 301; common difference d = 7; Last term l = 595.

$n = \left(\frac{l-a}{d}\right) + 1 = \left(\frac{595-301}{7}\right) + 1 = 43$

Since, $S_n = \frac{n}{2}[a + l]$, we have $s_{43} = \frac{43}{2}[301 + 595]$ = 19264

382) The sum of the cubes of the first n natural numbers is 2025. then Find the value of n.

Answer : $1^2 + 2^2 + 3^2 + \ldots + n^2 = 2025$

$\left[\frac{n(n+1)}{2}\right]^2 = 2025 = (45)^2$

$\Rightarrow \frac{n(n+1)}{2} = 45$

$n^2 + n = 90$

$n^2 + n - 90 = 0$

$(n - 9)(n + 10) = 0$

$n = 9, -10$

$n = -10$ is not possible

$n = 9$

365) The product of three consecutive terms of a Geometric Progression is 343 and their sum is $\frac{91}{3}$. Find the three terms.

Answer : Since the product of 3 consecutive terms is given.
we can take them as $\frac{a}{r}$, a, ar

Product of the terms = 343
$\frac{a}{r}$ x a x ar = 343
a^3 = 73 gives a = 7
Sum of the terms = $\frac{91}{3}$
Hence $a\left(\frac{1}{r}+1+r\right) = \frac{91}{3}$ $7\left(\frac{1+r+r^2}{r}\right) = \frac{91}{3}$
$3 + 3r + 3r^2 = 13r$ gives $3r^2 - 10r + 3 = 0$
$(3r - 1)(r - 3) = 0$ gives r = 3 or r = $\frac{1}{3}$
if a = 7, r = 3 then the three terms are $\frac{7}{3}$, 7, 21
If a = 7, r = $\frac{1}{3}$ then the three terms are 21, 7, $\frac{7}{3}$.

373) Find the sum to n terms of the series 5 + 55 + 555 +...

Answer : The series is neither Arithmetic nor Geometric series. So it can be split into two series and then find the sum.

5 + 55 + 555 + ... + n terms = 5 [1 + 11 + 111 + ...n terms]

$= \frac{5}{9}$ [9 + 99 + 999 +...+ n terms]

$= \frac{5}{9}$[(10 - 1) + (100 - 1) + (1000 - 1) +...+ n terms]

$= \frac{5}{9}$[10 + 100 + 1000 +...+ n terms)-n]

$= \frac{5}{9}\left[\frac{10(10^n-1)}{(10-1)} - n\right] = \frac{50(10^n-1)}{81} = \frac{5n}{9}$

397) Find the value of

$16 + 17 + 18 + \ldots + 75$

Answer : $16 + 17 + 18 + \ldots + 75 = (1 + 2 + 3 + \ldots + 75) - (1 + 2 + 3 + \ldots + 15)$

$= \frac{75 \times (75+1)}{2} - \frac{15(15+1)}{2}$

$= 2850 - 120 = 2730$

398) Find the sum of

$5^2 + 10^2 + 15^2 + \ldots + 105^2$

Answer : $5^2 + 10^2 + 15^2 + \ldots + 105^2 = 52(1^2 + 2^2 + 3^2 + \ldots + 21^2)$

$= 25 \times \frac{25 \times (21+1)(2 \times 21+1)}{6}$

$= \frac{25 \times 21 \times 22 \times 43}{6} = 82775$

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