RAVI MATHS TUITION CENTER PH - 8056206308

Relations and Functions FULL TEST

10th Standard

Maths

Exam Time : 02:30:00 Hrs | Total Marks : 100

14 x 1 = 14

1) If n(A x B) =6 and A={1,3} then n(B) is
(a) 1 (b) 2 (c) 3 (d) 6

2) A={a,b,p}, B={2,3}, C={p,q,r,s} then n[(A U C) x B] is
(a) 8 (b) 20 (c) 12 (d) 16

3) If A={1,2}, B={1,2,3,4}, C={5,6} and D={5,6,7,8} then state which of the following statement is true..
(a) (A x C) ⊂ (B x D) (b) (B x D) ⊂ (A x C) (c) (A x B) ⊂ (A x D) (d) (D x A) ⊂ (B x A)

4) If there are 1024 relations from a set A = {1, 2, 3, 4, 5} to a set B, then the number of elements in B is
(a) 3 (b) 2 (c) 4 (d) 8

5) The range of the relation R ={(x,x^2) |x is a prime number less than 13} is
(a) {2,3,5,7} (b) {2,3,5,7,11} (c) {4,9,25,49,121} (d) {1,4,9,25,49,121}

6) If the ordered pairs (a+2,4) and (5,2a+b) are equal then (a,b) is
(a) (2,-2) (b) (5,1) (c) (2,) (d) (3,-2)

7) Let n(A)= m and n(B) = n then the total number of non-empty relations that can be defined from A to B is
(a) m^n (b) n^m (c) $2^{mn}-1$ (d) 2^{mn}

8) If {(a,8),(6,b)}represents an identity function, then the value of a and b are respectively
(a) (8,6) (b) (8,8) (c) (6,8) (d) (6,6)

9) Let A={1,2,3,4} and B={4,8,9,10}. A function f: A ⟶ B given by f={(1,4), (2,8), (3,9), (4,10)} is a
(a) Many-one function (b) Identity function (c) One-to-one function (d) Into function

10) If f(x)=$2x^2$ and g(x)=$\frac{1}{3x}$, then f o g is
(a) $\frac{3}{2x^2}$ (b) $\frac{2}{3x^2}$ (c) $\frac{2}{9x^2}$ (d) $\frac{1}{6x^2}$

11) If f: A ⟶ B is a bijective function and if n(B) =8, then n(A) is equal to
(a) 7 (b) 49 (c) 1 (d) 14

12) Let f and g be two functions given by
f={(0,1), (2,0), (3,-4), (4,2), (5,7)}
g={(0,2), (1,0), (2,4), (-4,2), (7,0)} then the range of f o g is
(a) {0,2,3,4,5} (b) {–4,1,0,2,7} (c) {1,2,3,4,5} (d) {0,1,2}

13) Let f(x) = $\sqrt{1+x^2}$ then
(a) f(xy) = f(x).f(y) (b) f(xy) ≥ f(x).f(y) (c) f(xy) ≤ f(x).f(y) (d) None of these

14) If g={(1,1), (2,3), (3,5), (4,7)} is a function givrn by g(x)=αx+β then the values of α and β are
(a) (-1,2) (b) (2,-1) (c) (-1,-2) (d) (1,2)

10 x 2 = 20

15) A relation ‘f’ is defined by f(x)=x^2-2 where x∈ {-2,-1,0,3}
(i) List the elements of f
(ii) Is f a function?

16) Find f o g and g o f when f(x)=2x+1 and g(x)=x^2-2

17) Represent the function f(x)=$\sqrt{2x^2-5x+3}$ as a composition of two functions.

18) Find k if f o g(k) = 5 where f(k)=2k-1.

19) If f(x)=2x+3, g(x)=1-2x and h(x)=3x. Prove that f 0 (f o g) o h.

20) Let A = {0, 1, 2, 3} and B = {1, 3, 5, 7, 9} be two sets. Letf: A → B be a function given by f(x) = 2x + 1. Represent this function as a set of ordered pairs.

21) Let A = {0, 1, 2, 3} and B = {1, 3, 5, 7, 9} be two sets. Letf: A → B be a function given by f(x) = 2x + 1. Represent this function as a graph.

22) Let A = {3,4,7,8} and B = {1,7,10}. Which of the following sets are relations from A to B?

(i) R_1={(3,7), (4,7), (7,10), (8,1)}

(ii) R_2= {(3,1), (4,12)}

(iii) R_3= {(3,7), (4,10), (7,7), (7,8), (8,11), (8,7), (8,10)}

23) Using vertical line test, determine which of the following curves (Fig.1.18(a), 1.18(b), 1.18(c), 1.18(d)) represent a function?

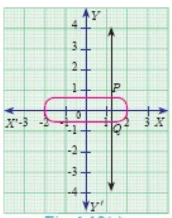


Fig. 1.18(a)

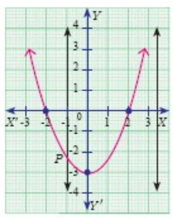


Fig. 1.18(b)

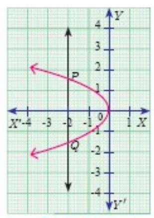


Fig. 1.18(c)

24) Using horizontal line test (Fig.1.35(a), 1.35(b), 1.35(c)), determine which of the following functions are one – one.

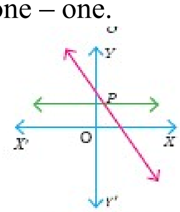


Fig. 1.35(a)

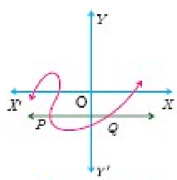


Fig. 1.35(b)

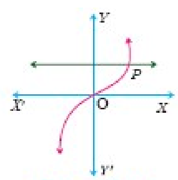


Fig. 1.35(c)

25) Let A={1,2,3}, B={4,5,6,7}, and f={(1,4),(2,5),(3,6)} be a function from A to B. Show that f is one – one but not onto function.

26) If A={-2,-1,0,1,2} and f: A ⟶ B is an onto function defined by f(x)=x^2+x+1 then find B.

27) Let f be a function f:N ⟶ N be defined by f(x) = 3x+2x ∈ N

(i) Find the images of 1, 2, 3

(ii) Find the pre-images of 29, 53

(ii) Identify the type of function

28) A functionf: [-7,6) → R is defined as follows.

$$f(x)=\begin{cases} x^2+2x+1 & -7\le x<-5 \\ x+5 & -5\le x\le 2 \\ x-1 & 2<x<6 \end{cases}$$

f(-7) -f(-3)

10 x 5 = 50

29) Find A x B, A x A and B x A

A=(2,-2,3} and B={1,-4}

30) If A={5,6}, B={4,5,6}, C={5,6,7}, Show that A x A =(B x B) ∩ (C x C)

31) Let A ={x ∈ W| x < 2}, B={x ∈ N| < x ≤ 4} and C=(3,5). Verify that

A x (B U C) = (A x B) U (A x C)

32) A graph representing the function f (x) is given in Fig.1.16 it is clear that f (9) = 2.

(i) Find the following values of the function

(a) f(0)

(b) f(7)

(c) f(2)

(d) f(10)

(ii) For what value of x is f (x) = 1?

(iii) Describe the following (i) Domain (ii) Range.

(iv) What is the image of 6 under f ?

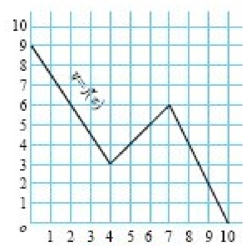


Fig. 1.16

33) Let f(x)=2x+5. If x ≠ 0 then find $\frac{f(x+2)-f(2)}{x}$.

34) The data in the adjacent table depicts the length of a woman's forehand and her corresponding height. Based on this data, a student finds a relationship between the height (y) and the forehand length(x) as y = ax + b, where a, b are constants.

(i) Check if this relation is a function.

(ii) Find a and b.

(iii) Find the height of a woman whose forehand length is 40 cm.

(iv) Find the length of forehand of a woman if her height is 53.3 inches.

Length 'x' of forehand (in cm)	Height 'y' (in inches)
45.5	65.5
35	56
45	65
50	69.5
55	75

35) Let f: A ⟶ B be a function defined by f(x) = $\frac{x}{2}$ -1, where A={2,4,6,10,12}, B={0,1,2,4,5,9}, Represent f by

(i) set of ordered pairs

(ii) a table

(iii) an arrow diagram

(iv) a graph

36) Let A={1,2,3,4} and B=N. Let f: A ⟶ B be defined by f(x)=x^3 then

(i) find the range of f

(ii) identify the type of function

37) The function 't' which maps temperature in Celsius (C) into temperature in Fahrenheit (F) is defined by t(C)=F where F-$\frac{9}{5}$ C+32. Find,

(i) t(0)

(ii) t(28)

(iii) t(-10)

(iv) the value of C whenn t(C)=212

(v) the temperature when the Celsius value is equal to the Farenheit value.

38) Consider the functions f(x), g(x), h(x) as given below. Show that (f o g) o h = f o (g o h) in each case.
f(x)=x-1, g(x)=3x+1 and h(x) = x^2

39) If f(x)=x^2, g(x)=3x and h(x)=x-2, Prove that (f o g) o h = f o (g o h).

40) Let A ={x ∈ W| x < 2}, B={x ∈ N| < x ≤ 4} and C=(3,5). Verify that
A x (B ∩ C) = (A x B) ∩ (A x C)

41) A function f: [-5,9] ⟶ R is defined as follows:

$$f(x) = \begin{cases} 6x+1 & \text{if } -5 \le x < 2 \\ 5x^2-1 & \text{if } 2 \le x < 6 \\ 3x-4 & \text{if } 6 \le x \le 9 \end{cases}$$

Find $\frac{2f(-2)-f(6)}{f(4)+f(-2)}$.

42) Consider the functions f(x), g(x), h(x) as given below. Show that (f o g) o h = f o (g o h) in each case.
f(x)=x-4, g(x)=x^2 and h(x)=3x-5

****

14 x 1 = 14

1) (c) 3
2) (c) 12
3) (a) (A x C) ⊂ (B x D)
4) (b) 2
5) (c) {4,9,25,49,121}
6) (d) (3,-2)
7) (c) 2^{mn}-1
8) (a) (8,6)
9) (c) One-to-one function
10) (c) $\frac{2}{9x^2}$
11) (a) 7
12) (d) {0,1,2}
13) (c) f(xy) ≤ f(x).f(y)
14) (b) (2,-1)

10 x 2 = 20

15)
f(x)=x^2-2 where x∈ {-2,-1,0,3}
(i) f(-2) = $(-2)^2$ - 2=2; f(-1)=$(-1)^2$-2=-1

$f(0)=(0)^2-2=-2$; $f(3)=(3)^2-2=7$

Therefore, f={(-2,2), (-1,-1), (0,-2), (3,7)}

(ii) We note that each element in the domain of f has a unique image. Therefore f is a function.

16)

$f(x)=2x+1$, $g(x)=x^2-2$

$f \circ g(x)=f(g(x))= f(x^2-2)=2(x^2-2)+1 = 2x^2-3$

$g \circ f(x)=g(f(x)) = g(2x+1) = (2x+1)^2-2=4x^2+4x-1$

Thus $f \circ g =2x^2-3$, $g \circ f =4x^2+4x-1$. From the above, we see that f o g- g o f.

17)

We set $f_2(x) =2x^2-5x+3$ and $f_1(x) =\sqrt{x}$

Then, $f(x) =\sqrt{2x^2-5x+3} = \sqrt{f_2(x)}$

$=f_1\{f_2(x)\} = f_1f_2(x)$

18)

$f \circ f(k) = f(f(k))$

$=2(2k-1)-1 = 4k-3$

Thus, $f \circ f(k) = 4k-3$

But, it is given that $f \circ f(k) = 5$

Therefore $4k-3 = 5 \Rightarrow k = 2$

19)

$f(x)= 2x+3$, $g(x)=1-2x$, $h(x) = 3x$

Now, $(f \circ g)(x) = f(g(x)) = f(1-2x) = 2(1-2x)+3 = 5-4x$

Then, $(f \circ g) \circ h(x) = (f \circ g)(3x) = 5-4(3x) = 5-12x$ ....(1)

$(g \circ h)(x) = g(h(x)) = g(3x) = 1-2(3x) = 1-6x$

So, $f \circ (g \circ h)(x) = f(1-6x) = 2(1-6x)+3 = 5-12x$ ....(2)

From (1) and (2), we get (f o g) oh = f o (g o h)

20)

A to, 1,2,3},B= {1,3,5, 7,9}

$f(x) = 2x + 1$

$f(0)) = 2(0) + 1 = 1$

$f(1)= 2(1)+1=3$

$f(2) = 2(2) + 1 = 5$

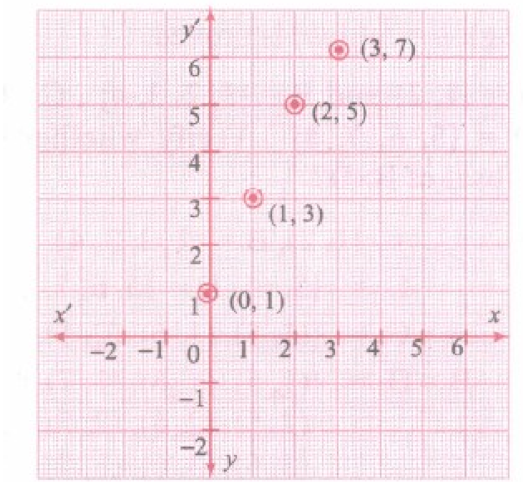
$f(3) = 2(3) + 1 = 7$

(i) A set of ordered pairs.

f = {(0, 1), (1, 3), (2, 5), (3, 7)}

21)

A Graph f = {(x,f(x)/x ϵ A}
{(0, 1), (1, 3), (2, 5), (3, 7)}



22)

Ax B = {(3,1), (3,7), (3,10), (4,1), (4,7), (4,10), (7,1), (7,7), (7,10), (8,1), (8,7), (8,10)}

(i) We note that, $R_1 \sqsubseteq A \times B$. Thus, R1 is a relation from A to B.

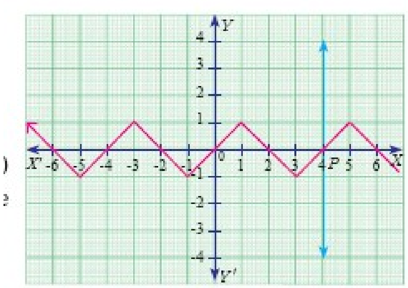
(ii) Here, $(4,12) \in R_2$, but $(4,12) \notin A \times B$. So, R_2 is not a relation from A to B.

(iii) Here, $(7,8) \in R_3$, but $(7,8) \notin A \times B$. So, R_3 is not a relation from A to B.

23)

The curves in Fig.1.18(a) and Fig.1.18(c) do not represent a function as the vertical lines meet the curves in two points P and Q.

The curves in Fig.1.18(b) and Fig.1.18(d) represent a function as the vertical lines meet the curve in at most one point.



24)

The curves in Fig.1.35(a) and Fig.1.35(c) represent a one–one function as the horizontal lines meet the curves in only one point P.

The curve in Fig.1.35(b) does not represent a one–one function, since, the horizontal line meet the curve in two points P and Q.

25)

A={1,2,3}, B={4,5,6,7}; f={(1,4),(2,5),(3,6)}

Then f is a function from A to B and for different elements in A, there are different images in B. Hence f is one–one function. Note that the element 7 in the co-domain does not have any pre image in the domain. Hence f is not onto (Fig.1.36).

Therefore f is one–one but not an onto function.

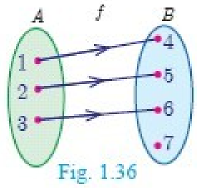


Fig. 1.36

26)

Given A={-2,-1,0,1,2} and $f(x)=x^2+x+1$

$f(-2)=(-2)^2+(-2)+1=3$; $f(-1)=(-1)^2+(-1)+1=1$

$f(0)=0^2+0+1=1$; $f(1)=1^2+1+1=3$

$f(2)=2^2+2+1=7$

Since, f is an onto function, range of f=B=co-domain of f. Therefore, B={1,3,7}

27)

The function f:N ⟶ N is defined by f(x)=3x+2

(i) If x=1, f(1)=3(1)+2=5

If x=2, f(2)=3(2)+2=8

If x=3, f(3)=3(3)+2=11

The images of 1, 2, 3 are 5, 8, 11 respectively.

(ii) If x is the pre-image of 29, then f(x)=29, Hence 3x+2=29

3x=27 ⇒ x=9

Similarly, if x is the pre-image of 53, then f(x)=53. Hence x+2=53

3x=51 ⇒ x=17

Thus the pre-images of 29 and 53 are 9 and 17 respectively.

(iii) Since different elements of N have different images in the co-domain, the function f is one – one function.

The co-domain of f is N.

But the range of f={5,8,11,14,17,....} is a proper subset of N.

Therefore f is not an onto function. That is, f is an into function.

Thus f is one – one and into function.

28)

$f(-7) = x^2 + 2x + 1$

$= (-7)^{2+} 2(-7) + 1$

= 49 - 14 + 1 = 36

f(3) = x + 5 = -3 + 5 = 2

f(-7) - f(-3) = 36 + 2 = 38

10 x 5 = 50

29)

Given A = {2, – 2, 3}, B = {1, – 4}.

A × B = {(2, 1), (2, – 4), (– 2, 1), (–2, – 4), (3, 1), (3, – 4)}

A × A = {(2, 2), (2, – 2), (2, 3), (– 2, 2), (– 2, – 2), (– 2, 3), (3, 2), (3, – 2), (3, 3)}

B × A = {(1, 2), (1, – 2), (1, 3), (– 4, 2), (– 4, – 2), (– 4, 3)}

30)

Given A={5,6}

B={4,5,6}

C={5,6,7}

now, AXA ={5,6}X{5,6}

= {(5,6),(5,6),(6,5),(6,6)}....(1)
BXB = {4,5,6}X{4,5,6}
= {(4,4),(4,5),(4,6),(5,4),(5,5),(5,6),(6,4),(6,5),(6,6)}
CXC={5,6,7}X{5,6,7}
= {(5,5),(5,6),(5,7),(6,5),(6,6),(6,7),(7,5),(7,5),(7,6),(7,7)}
Also,
(BXB)(CXC) ={(4,6),(4,5),(4,6),(5,4),(5,5),(5,6),(6,5),(6,6)}
n{(5,5),(5,6),(5,7),(6,5),(6X7),(7X5),(7,6),(7X7)}
= {(5,5),(5,6),(6,5),(6,6)}....2
from 1 and 2 we get A x A =(B x B) ∩ (C x C)

31)
Given
A= {0,1}
B ={2,3,4}
C = {3,5}
To prove A x (B U C) = (A x B) U (A x C)
$B \cup C = \{2, 3, 4\} \cap \{3, 5\}$
= {2,3,4,5}
A x (B U C) = {0,1}X{2,3,4,5}
= {{0,2},(0,3),(0,4),(0,5),(1,2),(1,3),(1,4),(1,5)}
AXB = {0,1}X{2,3,4}
= {(0,2),(0,3),(0,4),(1,2),(1,3),(1,4)}
AXC = {0,1}X{3,5}
={{0,3},(0,5),(1,3),(1,3)}
$(A \times B) \cup (A \cup C) = \{(0, 2), (0, 3), 90, 4), (1, 1), (1, 3), (1, 4)\} \cup \{(0, 3), (0, 5), (1, 3), (1, 5)\}$
= {(0,2),(0,3),(0,4),(0,5),(1,2),(1,3),(1,4),(1,5)} ...(2)
from 1 and 2 we get
$A \times (B \cup C) = (A \times B) \cup (A \times C)$
$B \cap C = \{2, 3, 4\} \cap \{3, 5\}$
= {3}

32)
(i) (a) f(0)=9
(b) f(7)=6
(c)=f(2)
(d)=fV10)=0
(ii) At x =9.5, f(x) = 1
(iii) Domain ={0,1,2,3,4,5,6,7,8,9,10}
={x|0 ≤ x ≤ 10, x ∈ R}
Range ={x|0 ≤ x ≤ 9, x ∈ R}
={0,1,2,3,4,5,6,7,8, 9}
(iv) The image of 6 under f is 5.

33)
f(x) =2x+5, x-0. $\frac{f(x+2)-f(2)}{x}$
f(1) =2 x 1 + 5=7
f(2)=2 x 2 + 5=9
f(3)=2 x 3 + 5=11
When x=1

$\frac{f(x+2)-f(2)}{x}$

$f(x) = 2x + 5 \Rightarrow f(x + 2)$

$2(x + 2) + 5$

$2x+4+5 = 2x+9$

$2(2) + 5 = 4 + 5 = 9$

$\therefore \frac{f(x+2)-f(2)}{x} = \frac{2x+9-9}{x} = \frac{2x}{x} = 2$

34)

(i) y = ax + b

When x = 45.5, y = 65.5

x=55, y=75

x = 42, y= 62

x = 46, y= 66

x = 40.4, y = 60.4

y = ax+b

y = x + 20

Yes, this relation is a function.

(ii) a = 1, b = 20

(iii) When forehand length is 48 cm, height is 68 inches.

(iv) When the height is 60.54 inches, her forehand length is 40.54 cm.

35)

f: A ⟶ B

$f(x) = \frac{x}{2} - 1$, A={2,4,6,10,12}

B={0,1,2,4,5,9}

$f(2) = \frac{2}{2} - 1 = 0$

$f(4) = \frac{4}{2} - 1 = 2$

$f(6) = \frac{6}{2} - 1 = 2$

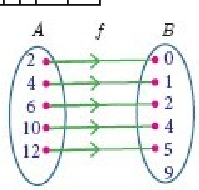
$f(10) = \frac{10}{2} - 1 = 4$

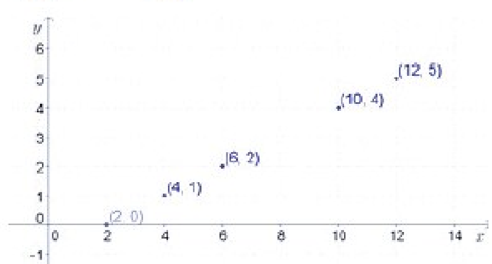
$f(12) = \frac{12}{2} - 1 = 5$

(i) {(2,0),(4,1),(6,2),(10,4),(12,5)}

(ii)

x	2	4	6	10	12
f(x)	0	1	2	4	5

(iii) 

(iv) 

36)

$A=\{1,2,3,4\}$
$B=N$
$f: A \longrightarrow B, f(x)=x^3$
(i) $f(1) = 1^3 =1$
(ii) $f(2) = 2^3 = 8$
$f(3) = 3^3 = 27$
$f(4) = 4^3=64$
(ii) The range of $f= \{1, 8,27, 64\}$
(iii) It is one-one and into function.
37)
(i) $t(0)=F$
$F=\frac{9}{5} (C) + 32 =\frac{9}{5} (0) + 32 =32^0F$
(ii) $t(28) =F=\frac{9}{5} (28) + 32 =\frac{252}{5} + 32$
$=50.4 + 32=82.4^0F$
(iii) $t(-10) =F=\frac{9}{5} (-10) + 32 =14^0F$
(iv) $t(c) =212$
i.e $\frac{9}{5} (c) + 32=212$
$\frac{9}{5} c=180$
$c=100^0C$
(v) $t(-40)=F=\frac{9}{5} (-40)+32$
$=-72+2 =-40^0$.
38)
$f(x) = x-1$
$g(x) = 3x + 1$
$h(x) = x^2$
$(fog)oh = fo(goh)$
LHS = (jog)oh
$fog = f(g(x)) =.f(3x + 1)= 3x + 1-1 = 3x$
$(fog)oh = (fog)(h(x)) = (fog)(x^2) = 3x^2$ ...(1)
RHS = fo(goh)
$goh = g(h(x)) = g(x^2) = 3x^2 + 1$
$fo(goh) = f(3x^2+ 1)=3x^2+ 1-1=3x^2$ ...(2)
LHS = RHS Hence it is verified.
39)
$f(x) = x^2$
$g(x) = 3x$
$h(x) = x-2$
$(fog)oh = x - 2$
LHS = fo(goh)
$fog = j(g(x)) =j(3x) = (3X)^2= 9x^2$
$(fog)oh = (fog) h(x) = (fog) (x - 2)$
$= 9(x - 2)^2 = 9(x^2 - 4x + 4)$
$= 9x^2 - 36x + 36$ (1)
RHS = fo(goh)
$(goh) = g(h(x)) = g(x - 2)$
$= 3(x - 2) = 3x - 6$

fo(goh) = f(3x - 6) = $(3x - 6)^2$
= $9x^2$ - 36x + 36 (2)
(1) = (2)
LHS = RHS
(fog)oh =fo(goh) is proved
40)
Given
A= {0,1}
B ={2,3,4}
C = {3,5}
$A \cap (B \cap C) = \{0, 1\} \times \{3\}$
= {(0,7),(13)} ...(3)
AXB ={0,1}X{2,3,4}
= {(0,2),(0,3),(0,4),(1,2),(1,3),(1,4)}
AXC={0,1}X{2,3,4}
= {(0,3),(0,5),(1,3),(1,5)}
$(A \times B) \cap (A \times C) = \{(0, 2), (0, 3), (0, 4), (1, 2, (1, 3), (1, 4)\} \cap \{(0, 3), (0, 5), (1, 3), (1, 5)\}$
={(0,3),(1,3)}...(4)
from 3 and 2 we get A x (B ∩ C) = (A x B) ∩ (A x C)
41)
$\frac{2f(-2)-f(6)}{f(4)+f(-2)}$
f(-2) =6x+1 =6(-2)+1=-11
f(6) =3x-4 =3(6)-4=14
f(4) =$5x^2$-1 =$5(4^2)$-1=79
f(-2)=6x+1 =6(-2)+1=-11
$\frac{2f(-2)-f(6)}{f(4)+f(-2)} = \frac{2(-11)-14}{79+(-11)} = \frac{-22-14}{68}$
$= \frac{-36}{68} = \frac{-9}{17}$
42)
f(x) = x - 4, g(x) =.xl, hex) = 3x - 5
(fog)oh = fo(goh)
LHS = (fog)oh
fog = f(g(x))=f(x^2)x^2-4
(fog)oh = (fog)(3x - 5) = $(3x - 5)^2$ - 4
= $9x^2$ - 30x + 25 - 4
= $9x^2$ - 30x + 21 ...(1)
∴ RHS = fo(goh)
(goh) = g(h(x)) = g(3x - 5) = $(3x - 5)^2$
= $9x^2$ - 30x + 25
fo(goh) = f($9x^2$ - 30x + 25)
$9x^2$ - 30x + 25 - 4
$9x^2$ - 30x + 21 ...(2)
(1)=(2)
LHS = RHS
∴ (fog)oh = fo(goh)
It is proved.