

RAVI TEST PAPERS WHATSAPP 8056206308

Trigonometric Functions

10th Standard

Maths

- 1) if $\cos^2 A + \sin^2 A = \sec^2 B$, then prove that $\cos^2 A - \sin^2 A = \sec^2 B \sin^2 A$

Answer : Now, $\cos^2 A + \sin^2 A = \sec^2 B$

Squaring both sides,

$$(\cos^2 A + \sin^2 A)^2 = (\sec^2 B)^2$$

$$\cos^4 A + \sin^4 A + 2\sin^2 A \cos^2 A = \sec^4 B$$

$$2\cos^4 A - \cos^4 A - \sin^4 A = 2\sin^2 A \cos^2 A$$

$$\cos^4 A - \sin^4 A = 2\sin^2 A \cos^2 A$$

$$(\cos^2 A + \sin^2 A)(\cos^2 A - \sin^2 A) = 2\sin^2 A \cos^2 A$$

$$\cos^2 A - \sin^2 A = \frac{\sec^2 B}{5} = \frac{\sec^2 B}{5} [\text{since } \cos^2 A + \sin^2 A = \sec^2 B]$$

$$= \sec^2 B \cos^2 A$$

$$\text{Therefore } \cos^2 A - \sin^2 A = \sec^2 B \cos^2 A$$

- 2) prove that $(\csc A - \sin A)(\sec A - \cos A)(\tan A + \cot A) = 1$

Answer : $(\csc A - \sin A)(\sec A - \cos A)(\tan A + \cot A)$

$$= \left(\frac{1}{\sin A} - \sin A \right) \left(\frac{1}{\cos A} - \cos A \right) (\tan A + \cot A)$$

$$= \left(\frac{1 - \sin^2 A}{\sin A} \right) \left(\frac{1 - \cos^2 A}{\cos A} \right) (\tan A + \cot A)$$

$$= \frac{\cos^2 A}{\sin A} \cdot \frac{\sin^2 A}{\cos A} (\tan A + \cot A)$$

- 3) prove that $\frac{\csc^2 A}{\sin^2 A} = \frac{\sec^2 A}{\cos^2 A} = \tan^2 A + \cot^2 A$

Answer : $\frac{\csc^2 A}{\sin^2 A} = \frac{\sec^2 A}{\cos^2 A}$

$$\frac{1}{\sin^2 A} = \frac{1}{\cos^2 A}$$

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- 4) prove the following identities.

$$\sec^4 A (1 - \sin^4 A) - 2\tan^2 A = 1$$

Answer : LHS = $\sec^4 A (1 - \sin^4 A) - 2\tan^2 A$

$$= (\sec^4 A - \sec^4 A \sin^4 A) - 2\tan^2 A$$

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- 5) Two ships are sailing in the sea on either sides of a lighthouse as observed from the ships are C and D respectively. if the lighthouse is 200 m high, find the distance between the two ships.) \overline{CT} \overline{AB} \overline{CB}

Answer :



Let AB be the lighthouse. Let C and D be the positions of the two ships.

Then, $AB = 200\text{m}$.

\overline{CED} \overline{T} $\overline{C} = 6$ \overline{CMD} \overline{T} \overline{DE}

In right triangles BAC, $\tan 30^\circ = \frac{CD}{C}$

$$\frac{A}{\overline{C}} \overline{T} \frac{B}{\overline{CE}} \quad \overline{CE} \overline{T} \overline{B} = \overline{C} \quad \dots(1)$$

In the right triangle BAD, $\tan 45^\circ = \frac{CD}{CM}$

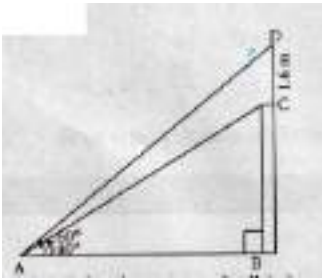
$$\overline{AT} \frac{B}{\overline{CM}} \text{ gives } AD = 200 \quad \dots(2)$$

Now, $CD = AC + AD = \overline{B} = \overline{C}5 \overline{B} = [\text{by (1) and (2)}]$

$$CD = 2002 \overline{C}5 \overline{A}3 = 200 \times 2.732 = 546.4$$

Distance between two ships is 546.4m

- 6) A statue 1.6 m tall stands on the top of a pedestal. From a point on the ground, the angle of elevation of the top of the statue is 60° and from the same point the angle of elevation of the top of the pedestal is 40° . Find the height of the pedestal. ($\tan 40^\circ = 0.8391$, $\overline{C} = 1.732$)



Answer :

Let CD be the statue of tall 1.6 m.

BC be the pedestal.

From the right triangle ABC

$$\overline{D} = \overline{T} \frac{\overline{DE}}{\overline{CD}} \\ \Rightarrow \overline{HI} \overline{AT} \frac{\overline{DE}}{\overline{CD}}$$

$$\overline{CD} \overline{T} \frac{\overline{DE}}{\overline{HI} \overline{A}} \quad \dots(1)$$

From the right triangle ABD

$$\overline{F} = \overline{T} \frac{\overline{DM}}{\overline{CD}} \\ \overline{C} \overline{T} \frac{\overline{DE}5 \overline{EM}}{\overline{CD}}$$

$$\overline{A} \overline{C} \overline{CB} \overline{T} \frac{\overline{DE}5 \overline{AF}}{\overline{CD}}$$

$$\overline{CD} \overline{T} \frac{\overline{DE}5 \overline{AF}}{\overline{A} \overline{C} \overline{CB}} \quad \dots(2)$$

From (1) and (2)

$$\frac{\overline{DE}}{\overline{HI} \overline{A}} \overline{T} \frac{\overline{DE}5 \overline{AF}}{\overline{A} \overline{C} \overline{CB}}$$

$$1.732 BC = 0.8391 (BC + 1.6)$$

$$1.732 BC = 0.8391 BC + (0.8391) (1.6)$$

$$1.732 BC - 0.8391 BC = 1.34256$$

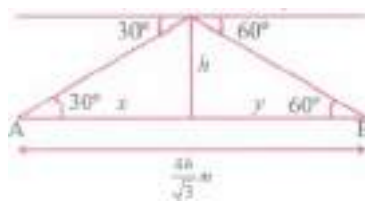
$$0.8929 BC = 1.34256$$

$$c d \overline{T} \frac{\overline{A} \overline{C} \overline{CB} \overline{EF}}{\overline{HI} \overline{B}} \overline{T} \frac{\overline{A} \overline{C} \overline{CB} \overline{EF}}{\overline{HI} \overline{B}} \overline{T} \overline{A} \overline{SE}$$

Height of the pedestal = 1.5 m

- 7) From the top of a lighthouse, the angle of depression of two ships on the opposite sides of it are observed to be 30° and 60° . If the height of the lighthouse is h meters and the line joining the ships passes through the foot of the lighthouse, show that the distance between the ships is $\frac{D}{\overline{C}}$ m.

Answer :



Let D and C be the positions of two ships AB be the light house of height 'h'm.

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In right triangle BAC

$$\tan 30^\circ = \frac{h}{AC}$$

$$\tan 60^\circ = \frac{h}{BC}$$

$$\frac{\tan 30^\circ}{\tan 60^\circ} = \frac{AC}{BC}$$

In right triangle BAD

$$\tan 30^\circ = \frac{h}{AD}$$

$$\frac{\tan 30^\circ}{\tan 60^\circ} = \frac{AD}{BC}$$

$$\frac{\tan 30^\circ}{\tan 60^\circ} = \frac{AD}{BC}$$

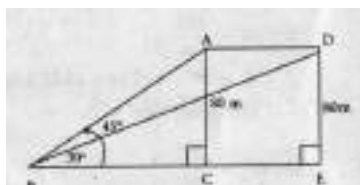
$$\frac{\tan 30^\circ}{\tan 60^\circ} = \frac{AD}{BC} \Rightarrow \frac{1}{\sqrt{3}} = \frac{AD}{BC} \Rightarrow AD = \frac{BC}{\sqrt{3}}$$

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Distance between the ships is $\frac{D}{C}$

- 8) A bird is sitting on the top of a 80 m high tree. From a point on the ground, the angle of elevation of the bird is 45° . The bird flies away horizontally in such away that it remained at a constant height from the ground. After 2 seconds, the angle of elevation of the bird from the same point is 30° . Determine the speed at which the bird flies. ($\sqrt{3} = 1.732$)



Answer :

Let the initial position of the bird be A and after two seconds its position is at D.

$$AC = DE = 80\text{m}$$

$$\angle ABC = 45^\circ$$

$$\angle DBC = 30^\circ$$

In right ABC

$$\tan 45^\circ = \frac{AC}{BC}$$

$$\tan 45^\circ = \frac{80}{BC}$$

$$BC = 80\text{ m}$$

In right triangle DBE

$$\tan 30^\circ = \frac{DE}{BE}$$

$$\tan 30^\circ = \frac{80}{BE}$$

$$\tan 30^\circ = \frac{80}{BE} \Rightarrow BE = \frac{80}{\tan 30^\circ}$$

$$BE = \frac{80}{\frac{1}{\sqrt{3}}} = 80\sqrt{3}$$

$$BE = 80\sqrt{3}$$

$$BE = 80\sqrt{3}$$

$$= 80 (1.732 - 1)$$

$$= 80 \times 0.732$$

$$CE = 58.56\text{ m}$$

The bird travelled 58.56 m in 2 seconds.

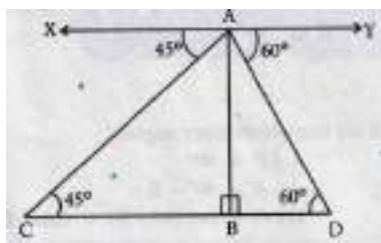
Speed of the bird $\frac{\text{Distance}}{\text{Time}}$

$$\frac{58.56}{2}$$

$$= 29.28\text{ m/s}$$

Speed of flying bird = 29.28 m/s.

- 9) Two ships are sailing in the sea on either side of the lighthouse. The angles of depression of two ships as observed from the top of the lighthouse are 60° and 45° respectively. If the distance between the ships is $200\sqrt{3}$ metres, find the height of the lighthouse.



Answer :

Let C and D are two ships.

Let AB be the height of the light house.

$$d \text{ e } T B = \frac{200\sqrt{3}}{C}$$

In the right ABD

$$\tan 60^\circ = \frac{AB}{BD}$$

$$\sqrt{3} = \frac{AB}{BD}$$

$$BD = \frac{AB}{\sqrt{3}}$$

In the right triangle ABC

$$\tan 45^\circ = \frac{AB}{BC}$$

$$1 = \frac{AB}{BC}$$

$$BC = AB$$

$$CD = BC + BD = AB + \frac{AB}{\sqrt{3}} = 200\sqrt{3}$$

$$AB \left(1 + \frac{1}{\sqrt{3}} \right) = 200\sqrt{3}$$

$$AB = \frac{200\sqrt{3}}{1 + \frac{1}{\sqrt{3}}}$$

$$AB = \frac{200\sqrt{3}}{\frac{\sqrt{3} + 1}{\sqrt{3}}} = \frac{200\sqrt{3} \times \sqrt{3}}{\sqrt{3} + 1} = \frac{600}{\sqrt{3} + 1}$$

$$AB = 200 \text{ m}$$

Height of the light house is 200 m.

- 10) From the top of a tower 50 m high, the angles of depression of the top and bottom of a tree are observed to be 30° and 45° respectively. Find the height of the tree. ($\sqrt{3} = 1.732$)

Answer : The height of the tower AB = 50 m

Let the height of the tree CD = y and BD = x

From the diagram, $\angle XAC = 30^\circ = \angle ACM$ and $\angle XAD = 45^\circ = \angle ADB$

In right triangle ABD,

$$\tan 45^\circ = \frac{AB}{BD}$$

$$1 = \frac{50}{x} \text{ gives } x = 50 \text{ m}$$

In right triangle AMC,

$$\tan 30^\circ = \frac{CM}{AM}$$

$$\frac{1}{\sqrt{3}} = \frac{CM}{AM} \text{ [since } DB = CM]$$

$$AM = \frac{CM}{\frac{1}{\sqrt{3}}} = \frac{50}{\sqrt{3}} = \frac{50\sqrt{3}}{3} = 28.87 \text{ m.}$$

Therefore, height of the tree = CD = MB = AB - AM = 50 - 28.87 = 21.13 m



- 11) prove the following identities.

$$\frac{1}{5} \tan \frac{A}{5} = \frac{A}{5 \tan A}$$

Answer : $\frac{A}{5} T \frac{A}{5}$

ni w $T \frac{A}{5}$

$T \frac{A}{5}$

$\frac{A}{5}$

$T \frac{2}{5} \frac{3}{5}$

$T \frac{2A}{2A5} \frac{3}{3}$

ni w $T \frac{A}{A5} \dots(1)$

vi w $T \frac{A}{5A}$

$T \frac{A}{A5} T \frac{A}{A5} T \frac{A}{A5}$

vi w $T \frac{A}{A5}$

From (1) and (2)

LHS = RHS

12) prove the following identities.

$\frac{{}^C C_5}{{}^C C_5} {}^C C_5 \frac{{}^C C_5}{{}^C C_5} T B$

Answer : $\frac{{}^C C_5}{{}^C C_5} {}^C C_5 \frac{{}^C C_5}{{}^C C_5} T B$

ni w $T \frac{{}^C C_5}{{}^C C_5} {}^C C_5 \frac{{}^C C_5}{{}^C C_5}$

$T \frac{2}{5} \frac{C_5}{C_5} \frac{C_3}{C_3} \frac{C}{C} \frac{C}{C} \frac{C_2}{C_2} \frac{C_5}{C_5} \frac{C_3}{C_3} \frac{2}{5} \frac{C}{C} \frac{C_3}{C_3} \frac{C}{C} \frac{C_2}{C_2} \frac{C}{C} \frac{C_3}{C_3}$

$T \frac{2}{5} \frac{C_5}{C_5} \frac{C_3}{C_3} \frac{2}{2} \frac{C_5}{C_5} \frac{C_3}{C_3} \frac{C}{C} \frac{C}{C} \frac{C}{C} \frac{2}{5} \frac{C}{C} \frac{C_3}{C_3} \frac{2}{2} \frac{C}{C} \frac{C_3}{C_3} \frac{C}{C} \frac{C}{C}$

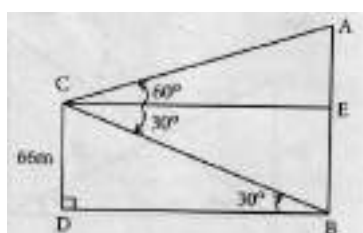
= $\sin^2 A + \cos^2 A + 2 \sin A \cos A - 3 \sin A \cos A + \sin^2 A + \cos^2 A - 2 \sin A \cos A + 3 \sin A \cos A$

= $2 \sin^2 A + 2 \cos^2 A$

= $2 (\sin^2 A + \cos^2 A) = 2 = \text{RHS}$

13) The angles of elevation and depression of the top and bottom of a lamp post from the top of a 66 m high apartment are 60° and 30° respectively. Find the distance between the lamp post and the apartment. ($\sqrt{3} = 1.732$)

Answer :



Let AB be the lamp post and CD be the apartment given $CD = 66 \text{ m} = EB$.

$CE \propto T F =$

$a \propto ED T \quad EDM T C =$

In the right triangle BDC

$C = T \frac{EM}{DM}$

$\frac{A}{C} T \frac{FF}{DM}$

$DM T FF \quad \bar{C}$

= $66 \times 1.732 = 114.312 \text{ m}$

The distance between the lamp post and the apartment = 114.31 m

Now $BD = EC = 114.31 \text{ m}$

In the right triangle ACE

$F = T \frac{Ca}{Ea}$

$\bar{C} T \frac{Ca}{FF \quad \bar{C}}$

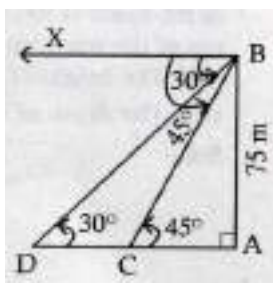
$Ca T FF \quad \bar{C} \quad \bar{C}/g \quad 2A3$

= $66 \times 3 = 198 \text{ m}$

The distance between the lamp post and apartment = 114.31 m.

- 14) As observed from the top of a 75 m high lighthouse from the sea level, the angles of depression of two ships are 30° and 45° . If one ship is exactly behind the other on the same side of the lighthouse find the distance between the two ships

Answer :



Let C and D be the ships.

$$\begin{aligned} & \text{In } \triangle ABC, \angle C = 45^\circ, AB = 75 \text{ m} \\ & \therefore \frac{AC}{AB} = \tan 45^\circ \\ & \therefore AC = AB \tan 45^\circ = 75 \times 1 = 75 \text{ m} \\ & \text{In } \triangle ABD, \angle D = 30^\circ, AB = 75 \text{ m} \\ & \therefore \frac{AD}{AB} = \tan 30^\circ \\ & \therefore AD = AB \tan 30^\circ = 75 \times \frac{1}{\sqrt{3}} = 25\sqrt{3} \text{ m} \\ & \therefore CD = AC - AD = 75 - 25\sqrt{3} \text{ m} \end{aligned}$$

- 15) $\frac{1}{\sqrt{3}} = \frac{h}{5}$ $\Rightarrow h = \frac{5}{\sqrt{3}} = \frac{5\sqrt{3}}{3}$ \therefore The height of the tower is $\frac{5\sqrt{3}}{3}$ m.

Answer :

$$\begin{aligned} & \text{In } \triangle ABC, \angle C = 45^\circ, AB = 75 \text{ m} \\ & \therefore \frac{AC}{AB} = \tan 45^\circ \\ & \therefore AC = AB \tan 45^\circ = 75 \times 1 = 75 \text{ m} \\ & \text{In } \triangle ABD, \angle D = 30^\circ, AB = 75 \text{ m} \\ & \therefore \frac{AD}{AB} = \tan 30^\circ \\ & \therefore AD = AB \tan 30^\circ = 75 \times \frac{1}{\sqrt{3}} = 25\sqrt{3} \text{ m} \\ & \therefore CD = AC - AD = 75 - 25\sqrt{3} \text{ m} \end{aligned}$$

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