



Mathematics (0580)

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Compact Cheat Sheet

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1 Algebra and graphs

KEY FORMULAS

Classifying stationary points (second-derivative test)

$$\frac{d^2y}{dx^2} > 0 \Rightarrow \text{minimum}, \quad \frac{d^2y}{dx^2} < 0 \Rightarrow \text{maximum}$$

Use once a stationary point has been located, to decide whether it is a local maximum or a local minimum. Differentiate a second time and substitute the x -value of the stationary point.

The discriminant

$$\Delta = b^2 - 4ac$$

Use to determine the number of real roots of $ax^2 + bx + c = 0$ without solving the equation. A positive Δ gives two distinct real roots, $\Delta = 0$ gives one repeated root, and a negative Δ gives no real roots.

The power rule

$$y = ax^n \implies \frac{dy}{dx} = anx^{n-1}$$

Use to differentiate any term of the form ax^n term by term. The derivative of a constant is 0, and the derivative of a sum is the sum of the derivatives.

The quadratic formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Use to solve any quadratic equation $ax^2 + bx + c = 0$ when factorising does not work cleanly over integers. Substitute a , b and c directly from the equation written in that standard form.

The three algebraic identities

$$(a + b)^2 = a^2 + 2ab + b^2, \quad (a - b)^2 = a^2 - 2ab + b^2, \quad a^2 - b^2 = (a + b)(a - b)$$

Use to expand a squared bracket or a product of a sum and a difference on sight, without multiplying out term by term. Recognising the pattern also speeds up factorising in reverse.

KEY CONCEPTS

- **Choosing a method to solve a quadratic:** Three methods solve any quadratic: factorising, completing the square, and the quadratic formula. Try factorising first, since it is fastest when the trinomial factorises over integers. Use the quadratic formula when factorising fails. Use completing the square only when a question explicitly asks for it, or to find the vertex of the corresponding graph.
- **Composite functions:** The composite function $fg(x)$ means apply g first and then apply f to the result, written $fg(x) = f(g(x))$. Work from the inside outward: substitute the input into g , then substitute that result into f .
- **Function notation:** A function f assigns exactly one output $f(x)$ to each input x . The set of allowed inputs is the *domain*; the set of resulting outputs is the *range*. Evaluate $f(a)$ by substituting a for x wherever x appears.
- **Like terms:** Two terms are *like terms* only when their letter parts are identical, meaning the same letters raised to the same powers. Only like terms can be added or subtracted; combine them by

adding their coefficients and keeping the letter part unchanged.

- **Stationary points:** A *stationary point* is a value of x at which $\frac{dy}{dx} = 0$, so the curve is momentarily flat. Find stationary points by differentiating, setting the derivative equal to zero, and solving for x .

EXAM TIPS

TIP In general $fg(x) \neq gf(x)$, so always apply the functions in the order the notation gives, working from the letter closest to the bracket outward. Checking which function is written first prevents composing them the wrong way round.

TIP When the term outside a bracket is negative, every sign inside the bracket flips once the bracket is expanded. Underline the multiplier before expanding so the sign change is not missed; this is the single most common algebra error at this level.

2 Coordinate geometry

KEY FORMULAS

Equation of a straight line

$$y = mx + c$$

Use as the standard form of any non-vertical straight line, where m is the gradient and c is the y -intercept. Reading m and c straight from this form is the fastest way to sketch a line or compare two lines.

Gradient of a line

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Use to find the gradient (steepness) of the line through two points (x_1, y_1) and (x_2, y_2) . It is the change in y divided by the change in x , often remembered as rise over run.

Gradients of parallel lines

$$m_1 = m_2$$

Use to test for or construct parallel lines. Two lines are parallel exactly when their gradients are equal, so a line parallel to $y = mx + c$ keeps the same m and only c changes.

Gradients of perpendicular lines

$$m_1 \times m_2 = -1$$

Use to test for or construct perpendicular lines. When two lines meet at a right angle the product of their gradients is -1 , so each gradient is the negative reciprocal of the other.

Length of a line segment

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Use to find the distance between two points (x_1, y_1) and (x_2, y_2) . It is Pythagoras' theorem applied to the horizontal and vertical gaps between the points.

Midpoint of a line segment

$$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

Use to find the point exactly halfway between two known endpoints (x_1, y_1) and (x_2, y_2) . Average the two x -coordinates and average the two y -coordinates.

KEY CONCEPTS

- **The coordinate plane:** A point is fixed by an ordered pair (x, y) , where x is the horizontal distance from the origin and y is the vertical distance. The horizontal difference between two points is often written Δx and the vertical difference Δy ; these two gaps drive every midpoint, length and gradient calculation.

- **The meaning of m and c:** In $y = mx + c$, the number m multiplying x is the *gradient* and the constant c is the *y-intercept*, the value of y where the line crosses the y -axis at $x = 0$. Two lines are identical only when both m and c match.
- **What the gradient measures:** The *gradient* measures how steeply a line rises or falls. A gradient of 3 means y increases by 3 for every 1 unit increase in x . Take the two coordinate differences in the *same order* for both x and y , or the sign of the gradient will be wrong.

EXAM TIPS

TIP To get the gradient perpendicular to a line, turn the gradient upside down and change its sign. A gradient of $\frac{2}{3}$ becomes $-\frac{3}{2}$, and a gradient of 4 becomes $-\frac{1}{4}$. The one exception is a horizontal line, whose perpendicular is vertical and has an undefined gradient.

KEY FORMULAS

Angle in a semicircle

$$90^\circ$$

Use whenever a triangle is inscribed in a circle with one side as the diameter; the angle at the vertex on the circle is always 90° .

Similar-shape scale factors

$$\text{length} : k, \quad \text{area} : k^2, \quad \text{volume} : k^3$$

Use once two shapes or solids are known to be similar with linear scale factor k ; area scales by k^2 and volume by k^3 .

Sum of exterior angles of a polygon

$$\text{Sum of exterior angles} = 360^\circ$$

Use for any convex polygon regardless of the number of sides; the exterior angles, one at each vertex, always total 360° .

Sum of interior angles of a polygon

$$\text{Sum of interior angles} = (n - 2) \times 180^\circ$$

Use to find the sum of the interior angles of any n -sided polygon; substitute the number of sides directly.

Tangent perpendicular to radius

$$\angle(\text{tangent}, \text{radius}) = 90^\circ$$

Use whenever a tangent meets a circle; the radius drawn to that point always meets the tangent at a right angle, opening the way to Pythagoras or trigonometry.

KEY CONCEPTS

- **Angle at the centre is twice the angle at the circumference:** When a chord subtends an angle at the centre and an angle at the circumference on the same arc, the centre angle is always double the circumference angle.
- **Angle facts at a point and on a line:** Angles on a straight line sum to 180° ; angles at a point sum to 360° ; vertically opposite angles, formed where two straight lines cross, are equal.
- **Angles in the same segment are equal:** Angles subtended by the same chord, from points on the *same* side of that chord, are equal to one another.
- **Parallel line angle rules:** When a transversal cuts two parallel lines, corresponding angles (an F shape) are equal, alternate angles (a Z shape) are equal, and co-interior angles (a C shape) sum to 180° .
- **Similar shapes have equal angles and proportional sides:** In similar figures, every pair of corresponding angles is equal and every pair of corresponding sides is in the same fixed ratio, the linear scale factor.
- **Tests for similar triangles:** Two triangles are similar if any one of these holds: two pairs of angles are equal (AAA), all three pairs of sides are in the same ratio (SSS), or two pairs of sides are in the same ratio with the included angle equal (SAS).

EXAM TIPS

- TIP** Stating a numerical answer alone earns no method mark. Quote the rule in brackets every time, for example "alternate angles, parallel lines" or "angle sum of a triangle".
- TIP** Write the theorem name exactly, for example "angles in the same segment" or "tangent-radius", not just the numerical result; the mark is for the reasoning.
- TIP** Given a ratio of areas, take the square root to reach the linear scale factor; given a ratio of volumes, take the cube root. Never divide the areas or volumes directly to get the linear ratio.

4 Mensuration

KEY FORMULAS

Surface area of a cylinder and a sphere

$$A_{\text{cylinder}} = 2\pi rh + 2\pi r^2, \quad A_{\text{sphere}} = 4\pi r^2$$

Use for a closed cylinder (curved side plus two circular ends) and for a sphere. For an open-top cylinder such as a tube or trough, drop one or both πr^2 end discs.

Volume of a cone, pyramid and sphere

$$V_{\text{cone}} = \frac{1}{3}\pi r^2 h, \quad V_{\text{pyramid}} = \frac{1}{3} \times \text{base area} \times h, \quad V_{\text{sphere}} = \frac{4}{3}\pi r^3$$

Use for the pointed and round solids. A cone or pyramid holds exactly one third of the prism or cylinder on the same base and height; h is the *perpendicular* height, not a slant edge.

Volume of a prism and a cylinder

$$V = A_{\text{cross-section}} \times l, \quad V_{\text{cylinder}} = \pi r^2 h$$

Use for any solid with a uniform cross-section: multiply the area of the cross-section by the length. A cylinder is the special case where the cross-section is a circle of radius r .

KEY CONCEPTS

- **Compound solids: add or subtract volumes:** A compound solid is built from standard pieces, such as a hemisphere on a cylinder or a cone bored out of a block. Find the volume of each piece separately, then *add* the pieces that are present and *subtract* any piece that has been removed.
- **Perpendicular height, not slant height:** In every volume formula h means the *perpendicular* height, measured at right angles to the base. The slant edge ℓ of a cone or pyramid is longer than h . If a question gives only the slant, recover the perpendicular height with $h = \sqrt{\ell^2 - r^2}$ before substituting.
- **The prism rule:** A *prism* is a solid with the same cross-section along its whole length. Its volume is always the cross-sectional area multiplied by the length, so the method is identical whether the cross-section is a triangle, an L-shape or a circle. Find the area first, then multiply.

EXAM TIPS

TIP When two solids are joined, the circle or face where they meet is hidden inside and is not part of the outer surface. Add only the surfaces you could actually touch or paint; leaving out the internal join is the single biggest mark-loser in compound surface-area questions.

TIP The most common mensuration slip is writing $\frac{2}{3}\pi r^3$ (which is a hemisphere) or πr^3 for a whole sphere. Memorise the whole sphere as *four-thirds pi r cubed*; a hemisphere is exactly half of that, $\frac{2}{3}\pi r^3$.

KEY FORMULAS

Bounds interval for a rounded measurement

$$\left[m - \frac{u}{2}, m + \frac{u}{2} \right)$$

Used to state the range of possible actual values for a measurement m rounded to a precision u (one whole unit at the rounded place). The lower bound is included and the upper bound excluded, since a value exactly on the upper bound would round up.

Percentage change

$$\text{Percentage change} = \frac{N - O}{O} \times 100\%$$

Used to find the percentage increase or decrease between an original value O and a new value N . A positive result is an increase; a negative result is a decrease.

Percentage multiplier

$$\text{new value} = \text{original} \times \left(1 \pm \frac{p}{100} \right)$$

Used to increase or decrease a quantity by $p\%$ in a single multiplication. Use $+$ for an increase and $-$ for a decrease.

Splitting and combining surds

$$\sqrt{ab} = \sqrt{a} \times \sqrt{b}, \quad \sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$$

Used to split a surd into factors (to simplify it) or to combine two surds into one, for non-negative a and b with $b \neq 0$ in the second identity.

KEY CONCEPTS

- **Estimating by rounding to 1 significant figure:** To estimate the result of a calculation, round every value in it to 1 significant figure first, then compute exactly with the rounded values. Comparing the estimate to a calculator answer catches misplaced decimal points and other large errors.
- **Recurring decimals are rational:** Every terminating decimal and every recurring decimal is a rational number, meaning it can be written as a fraction $\frac{a}{b}$ with a and b integers. The dot notation $0.\dot{2}\dot{4}$ marks the digits that repeat forever.
- **Reverse percentage:** A reverse percentage problem gives the value *after* a percentage change and asks for the value *before* it. The value after the change equals the original multiplied by the percentage multiplier, so the original is found by dividing, never by taking a percentage of the final value.
- **Upper and lower bound:** Every rounded measurement hides a small range of possible actual values. The *upper bound* is the largest value that would still round to the given figure; the *lower bound* is the smallest.
- **What a surd is:** A surd is a root, most commonly a square root, whose value is irrational, meaning it cannot be written as a fraction of integers and its decimal expansion never terminates or repeats. $\sqrt{4} = 2$ is not a surd because it is rational; $\sqrt{2}$ and $\sqrt{18}$ are surds.

EXAM TIPS

TIP If a calculator answer and a 1-significant-figure estimate disagree by more than a small margin, a decimal point has likely been misplaced or a wrong operation entered. Recompute rather than trusting the display.

TIP To undo a 30% reduction that leaves a price of \$84, divide 84 by the multiplier 0.70 to get \$120. Subtracting 30% of \$84 instead gives a wrong answer, because that 30% applies to the *original* price, not the sale price.

TIP $\sqrt{a} + \sqrt{b}$ does not simplify to $\sqrt{a + b}$. Surds can only be combined by addition or subtraction once they have been simplified to the same number under the root, exactly like collecting like terms in algebra.

6 Probability

KEY FORMULAS

Conditional probability

$$P(A | B) = \frac{P(A \cap B)}{P(B)}$$

Use to find the probability of A *given that* B has happened. The vertical bar reads "given"; the condition B becomes the new sample space.

Multiplication rule for independent events

$$P(A \text{ and } B) = P(A) \times P(B)$$

Use when two events are *independent*, meaning the outcome of one does not change the probability of the other, such as drawing with replacement or rolling two dice.

Multiplying along a tree diagram

$$P(A \text{ and } B) = P(A) \times P(B | A)$$

Use for any multi-stage experiment: multiply the probabilities along a path from the start to the required outcome. For dependent events the second factor is the *conditional* probability of the second step given the first.

The complement

$$P(\text{not } A) = 1 - P(A)$$

Use to find the probability that an event does not happen, and as the fast route into any "at least one" question through $1 - P(\text{none})$.

KEY CONCEPTS

- **How a tree diagram works:** A tree diagram sets out the outcomes of a multi-stage experiment one stage at a time, with each branch carrying the probability of that step. *Multiply* the probabilities along a path to get the probability of that combined outcome, then *add* the results of every path that gives the required event.
- **Independent vs dependent events:** Events are *independent* when the first outcome leaves the second probability unchanged, and *dependent* when the first outcome changes it. Drawing an object and replacing it keeps the trials independent; drawing without replacing makes them dependent.
- **Reading a two-set Venn diagram:** A two-set Venn diagram splits the universal set into four regions: A only, B only, the overlap $A \cap B$, and the outside (neither). Once each region count is filled in, probabilities such as $P(A | B)$, $P(A \cup B)$ and $P(A')$ can be read straight off.
- **What conditional probability means:** $P(A | B)$ *restricts* the sample space to the outcomes in B and asks what fraction of that restricted space also lies in A . Once an event is known to have happened, only the outcomes consistent with it remain possible.
- **With replacement vs without replacement:** *With replacement*, the item is returned, so every stage uses the same total and identical branch probabilities. *Without replacement*, the item is kept, so the total falls by one at the next stage and the branch probabilities change.

EXAM TIPS

TIP When reading $P(A | B)$ from a Venn diagram or table, divide the count in $A \cap B$ by the count in B , never by the whole sample space. The condition shrinks the sample space to B alone.

TIP To find $P(\text{at least one})$, compute $1 - P(\text{none})$ instead of adding every separate case. Listing all the "one or more" outcomes wastes time and invites arithmetic slips.

7 Statistics

KEY FORMULAS

Frequency density

$$\text{frequency density} = \frac{\text{frequency}}{\text{class width}}$$

Use to find the height of each bar in a histogram, especially when the class widths are unequal. Dividing by the class width makes the *area* of each bar, not its height, proportional to the frequency it represents.

Interquartile range

$$IQR = Q_3 - Q_1$$

Use to measure the spread of the middle half of the data, where Q_1 is the lower quartile and Q_3 is the upper quartile. A smaller *IQR* means the data is more consistent. It is less affected by extreme values than the range.

Mean from a frequency table

$$\bar{x} = \frac{\sum fx}{\sum f}$$

Use to find the mean when each value x occurs with a frequency f . Multiply every value by its frequency, add these products, and divide by the total frequency $\sum f$. For grouped data, replace x with the midpoint of each class.

Median and quartile positions on a cumulative frequency curve

$$\text{median} \rightarrow \frac{n}{2}, \quad Q_1 \rightarrow \frac{n}{4}, \quad Q_3 \rightarrow \frac{3n}{4}$$

Use these cumulative frequency values to read estimates off a cumulative frequency curve. Read across from the value on the vertical axis to the curve, then down to the data axis. On a cumulative frequency graph the median uses $\frac{n}{2}$, not $\frac{n+1}{2}$.

KEY CONCEPTS

- **Area represents frequency:** In a histogram the frequency of a class is shown by the *area* of its bar, not its height. The vertical axis is frequency density, so a wide class and a narrow class can represent the same frequency with bars of very different heights. This is what separates a histogram from an ordinary bar chart.
- **Measures of central tendency:** The mean, median and mode are single values that summarise where the centre of a data set lies. The *mean* is the sum of all values divided by how many there are; the *median* is the middle value once the data is ordered; the *mode* is the value that occurs most often.
- **What cumulative frequency means:** The cumulative frequency at a value is the running total of all frequencies up to and including that value, so it tells you how many data items are *less than or equal to* that value. Plotting cumulative frequency against the upper class boundaries gives the rising S-shaped cumulative frequency curve.

EXAM TIPS

TIP Order the data first, then the median lies at position $\frac{n+1}{2}$. When n is odd this is a single middle value; when n is even it falls between two values, so take their mean. Forgetting to order the data first is the most common error.

TIP Each cumulative frequency point is plotted at the *upper* boundary of its class, never at the midpoint. This is because you only know the total number of items once the whole class has been counted. Plotting at midpoints is a frequent and costly slip.

8 Transformations and vectors

KEY FORMULAS

Column vector between two points

$$\overrightarrow{AB} = \begin{pmatrix} x_2 - x_1 \\ y_2 - y_1 \end{pmatrix}$$

Use to write the vector from $A(x_1, y_1)$ to $B(x_2, y_2)$ as a column. Subtract the start coordinates from the end coordinates so the vector points from A towards B .

Magnitude of a vector

$$|\mathbf{v}| = \sqrt{a^2 + b^2}$$

Use to find the length (magnitude) of the column vector $\begin{pmatrix} a \\ b \end{pmatrix}$. It is Pythagoras' theorem applied to the horizontal and vertical components.

Translation by a column vector

$$(x, y) \mapsto (x + a, y + b)$$

Use to translate a point or shape by the column vector $\begin{pmatrix} a \\ b \end{pmatrix}$: add a to every x -coordinate and b to every y -coordinate. Shape, size and orientation are all unchanged.

Vector in terms of position vectors

$$\overrightarrow{AB} = \mathbf{b} - \mathbf{a}$$

Use to write the vector joining A to B from their position vectors $\mathbf{a} = \overrightarrow{OA}$ and $\mathbf{b} = \overrightarrow{OB}$. Subtract the start position vector from the end one.

KEY CONCEPTS

- **Enlargement scale factor:** Under an enlargement with centre C and scale factor k , each image point P' satisfies $\overrightarrow{CP'} = k \overrightarrow{CP}$, so every length is multiplied by $|k|$. If $k > 1$ the figure grows and if $0 < k < 1$ it shrinks; object and image are always *similar*.
- **Position vector:** The *position vector* of a point P is the vector $\overrightarrow{OP}$ from the origin to P . If $P = (x, y)$ then its position vector is $\begin{pmatrix} x \\ y \end{pmatrix}$, so position vectors and coordinates carry the same information written two ways.
- **The four transformations and their parameters:** Every transformation must be named *and* fully specified. A *translation* needs a column vector; a *reflection* needs its mirror line; a *rotation* needs a centre, an angle and a sense (clockwise or anticlockwise); an *enlargement* needs a centre and a scale factor.
- **What a vector is:** A *vector* has both magnitude (size) and direction, written as a column $\begin{pmatrix} a \\ b \end{pmatrix}$ meaning " a across and b up", where a negative component means left or down. Vectors are printed in bold ($\mathbf{a}$) or underlined ($\underline{a}$) and drawn with an arrow.

KEY FORMULAS

Area of a triangle using two sides and the included angle

$$\text{Area} = \frac{1}{2}ab \sin C$$

Use to find the area of any triangle when two sides and the angle between them are known, without needing the perpendicular height.

Pythagoras' theorem

$$a^2 + b^2 = c^2$$

Use to find a missing side of a right-angled triangle when the other two sides are known, where c is the hypotenuse, the side opposite the right angle.

Space diagonal of a cuboid

$$d^2 = l^2 + w^2 + h^2$$

Use to find the length of the diagonal running from one corner of a cuboid to the opposite corner, given the length, width and height. It applies Pythagoras' theorem twice, once across the base and once up through the solid.

The cosine rule (finding a side)

$$a^2 = b^2 + c^2 - 2bc \cos A$$

Use to find the third side of a triangle when two sides and the angle between them, the included angle, are known.

The sine rule

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

Use in any triangle, not only a right-angled one, to find a missing side or angle when a complete side-angle pair is known alongside one more side or angle. Lower-case letters denote sides opposite the upper-case angles of the same letter.

The three trigonometric ratios (SOH CAH TOA)

$$\sin \theta = \frac{\text{Opposite}}{\text{Hypotenuse}}, \quad \cos \theta = \frac{\text{Adjacent}}{\text{Hypotenuse}}, \quad \tan \theta = \frac{\text{Opposite}}{\text{Adjacent}}$$

Use to find a missing side or angle in a right-angled triangle. Choose the ratio that links the side you already know with the side or angle you want, relative to the angle θ .

KEY CONCEPTS

- **Labelling the sides of a right-angled triangle:** Relative to the angle being used, the *hypotenuse* is always the longest side, opposite the right angle; the *opposite* side is across from the angle; the *adjacent* side is next to the angle and is not the hypotenuse. Relabel the opposite and adjacent sides if the angle used changes.
- **Setting up a right-angled triangle within a 3D solid:** Every 3D trigonometry problem reduces to a right-angled triangle drawn within the solid: identify the required length or angle, then find the plane

containing both the right angle and the quantity needed. Redraw that triangle separately in 2D before applying Pythagoras or a trig ratio.

EXAM TIPS

TIP Sketch the single right-angled triangle relevant to the question as a flat 2D diagram, labelling only the lengths and angle needed. This avoids confusing a length in the solid with its projection onto the base, the most common error in 3D trigonometry.



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