



# Mathematics (0580)

IGCSE • Core • CAIE

Comprehensive Cheat Sheet

The Practice Book  
[www.theppracticebook.org](http://www.theppracticebook.org)



# 1 Algebra and graphs

## KEY FORMULAS

### Laws of Indices: Division

$$a^m \div a^n = a^{m-n}$$

Use when dividing two powers with the same base. Subtract the index in the denominator from the index in the numerator.

### Laws of Indices: Multiplication

$$a^m \times a^n = a^{m+n}$$

Use when multiplying two powers with the same base. Add the indices and keep the base unchanged.

### nth Term of a Linear Sequence

$$T_n = dn + c$$

Use to find any term of a linear (arithmetic) sequence directly, without listing every term.  $d$  is the common difference between consecutive terms;  $c$  is the "zeroth term" found from  $c = T_1 - d$ .

### Rearranging a Formula

$$v = u + at \implies t = \frac{v - u}{a}$$

Use to make a different letter the subject of a formula. Treat the target letter as the unknown of an equation and apply the same inverse-operation steps used to solve an equation.

### Laws of Indices: Power of a Power

$$(a^m)^n = a^{mn}$$

Use when a power is itself raised to another power. Multiply the two indices together.

## KEY CONCEPTS

- **Collecting Like Terms:** *Like terms* have identical letter parts, letters raised to the same powers. Add or subtract only the numerical coefficients of like terms and leave the letter part unchanged.  $5x$  and  $-2x$  collect to  $3x$ ;  $5x$  and  $5x^2$  do not collect, since  $x$  and  $x^2$  are different letter parts.
- **Distance-Time Graphs:** On a distance-time graph the *gradient* gives the speed, a steeper line means a faster speed. A horizontal segment means the object is stationary, since distance is not changing while time passes. A negative gradient means motion back towards the starting point.
- **Finding the Common Difference:** The common difference  $d$  of a linear sequence is found by subtracting a term from the next term,  $d = T_{n+1} - T_n$ . A positive  $d$  means the sequence increases; a negative  $d$  means it decreases.
- **Reading and Plotting Coordinates:** A coordinate  $(x, y)$  gives the horizontal position  $x$  and the vertical position  $y$  relative to the origin  $(0, 0)$ . Always read or plot the  $x$ -coordinate first, then the  $y$ -coordinate.
- **Solving a Linear Equation by the Balance Method:** An equation stays true if the same operation is applied to both sides. Undo operations in reverse order: expand any brackets first, then move terms so the unknown is on one side, then use the inverse operation (subtraction undoes addition, division undoes multiplication) to leave the unknown alone.

- **Solving a Linear Inequality:** An inequality compares two expressions with  $<$ ,  $>$ ,  $\leq$  or  $\geq$ . Solve it with the same steps used for an equation, isolating the unknown on one side.
- **Speed-Time Graphs:** On a speed-time graph a horizontal line means *constant speed* (not stationary, stationary would be a speed of zero). An upward slope means acceleration and a downward slope means deceleration. The *area* under the graph gives the distance travelled.
- **Expanding a Single Bracket:** To expand a bracket, multiply every term inside it by the term outside,  $a(b + c) = ab + ac$ . Do this for every bracket in an expression before collecting like terms.
- **Listing Integer Solutions of an Inequality:** When a question asks for the integer values satisfying an inequality, solve the inequality first, then write out every integer in the resulting range. Take care with strict inequalities ( $<$ ,  $>$ ), which exclude the boundary value, and inclusive inequalities ( $\leq$ ,  $\geq$ ), which include it.
- **Plotting a Straight Line from a Table of Values:** Complete a table of at least three  $x$ -values and their corresponding  $y$ -values by substituting into the equation, plot each point, then rule a single straight line through all of them. Three points confirm the line is straight and catch an arithmetic slip.
- **Sketching a Quadratic Graph:** A quadratic  $y = ax^2 + bx + c$  produces a *parabola*. If  $a > 0$  it opens upward with a minimum point; if  $a < 0$  it opens downward with a maximum point. To plot it accurately, build a table of values across the given range and join the points with a smooth curve, never with straight lines.
- **Solving Simultaneous Equations by Elimination:** To eliminate a variable from two linear equations, scale one or both equations so a variable has matching (or opposite) coefficients, then add or subtract the equations to remove it. Solve the single remaining equation, then substitute back to find the other variable.
- **Finding the Vertex of a Quadratic by Symmetry:** A parabola is symmetric about a vertical line through its vertex. Once both  $x$ -intercepts (roots) are known, the vertex's  $x$ -coordinate is exactly midway between them; substitute this value back into the equation to find the vertex's  $y$ -coordinate. For  $y = x^2 - 4x + 3$  the roots are  $x = 1$  and  $x = 3$ , so the vertex has  $x = 2$  and  $y = 4 - 8 + 3 = -1$ .

## EXAM TIPS

**TIP** A complete sketch of a straight line or a quadratic must show every axis intercept with its coordinates labelled, plus the vertex for a quadratic. Markschemes award separate marks for each labelled feature, so an unlabelled sketch loses marks even if the shape is correct.

**TIP** Multiplying or dividing both sides of an inequality by a *negative* number reverses the direction of the inequality symbol.  $-2x < 10$  becomes  $x > -5$  after dividing by  $-2$ . This is the single most common source of error in Core inequality questions.

**TIP** When a bracket is multiplied by a negative number, every term inside flips sign.  $-3(2x - 5)$  expands to  $-6x + 15$ , not  $-6x - 15$ . Underline the outside multiplier before expanding so the sign is not forgotten.

**TIP** After solving an equation or a simultaneous pair, substitute the answer back into the *original* equation (not a rearranged version) to confirm it balances. A thirty-second check recovers marks lost to a sign slip.

**TIP** To check whether a given number belongs to a linear sequence, set its  $n$ th term formula equal to that number and solve for  $n$ . If  $n$  is a positive whole number, the value is a term; if  $n$  is not a whole number, it is not.

**TIP** When a question gives a range for  $x$ , pick values spread across the whole range rather than clustered near zero. Points spread widely make the line easier to draw accurately and any plotting error easier to spot.

## 2 Coordinate geometry

### KEY FORMULAS

#### Equation of a Straight Line

$$y = mx + c$$

Represents any straight line, where  $m$  is the gradient and  $c$  is the value where the line crosses the  $y$ -axis (the  $y$ -intercept).

#### Gradient of a Line Through Two Points

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Use to find the gradient  $m$  of the straight line through two known points  $(x_1, y_1)$  and  $(x_2, y_2)$ . It measures the rise (change in  $y$ ) divided by the run (change in  $x$ ).

#### Finding the $y$ -Intercept from a Point

$$c = y_1 - mx_1$$

Use once the gradient  $m$  and one point  $(x_1, y_1)$  on the line are known. It comes from rearranging  $y = mx + c$ , and gives the  $y$ -intercept directly so the full equation can be written.

#### Gradient of a Line Given as $ax + by = c$

$$m = -\frac{a}{b}$$

A quick way to read the gradient of a line written in the form  $ax + by = c$ , once it is rearranged into  $y = mx + c$ . Useful for checking parallelism without fully rewriting the equation.

### KEY CONCEPTS

- **Finding the Equation from a Gradient and One Point:** When the gradient  $m$  and one point on the line are known, substitute the point's coordinates into  $y = mx + c$  and solve for  $c$ . Writing the full equation then only needs the known  $m$  and this value of  $c$ .
- **Parallel Lines Share the Same Gradient:** Two lines are *parallel* if and only if they have equal gradients  $m$ . Their  $y$ -intercepts  $c$  must differ, otherwise they would be the same line. To read whether two lines are parallel, compare the value of  $m$  in each  $y = mx + c$  equation.
- **Reading the Sign of the Gradient:** A *positive* gradient rises from left to right; a *negative* gradient falls from left to right. A horizontal line has gradient 0 because there is no rise. A vertical line has an *undefined* gradient, since the run is 0 and division by zero is not allowed.
- **Constructing a Line Parallel to a Given Line:** To build a line parallel to a given one through a new point, keep the gradient  $m$  of the given line unchanged and use the new point to find a new intercept  $c$ . The parallel line has the same  $m$  but a different  $c$ .
- **Equation of a Line from Two Points:** With two points and no gradient given, work in two stages: first calculate the gradient with  $m = \frac{y_2 - y_1}{x_2 - x_1}$ , then substitute either point into  $y = mx + c$  to find  $c$ . Both points lie on the line, so either one gives the same  $c$ .
- **Testing Three Points for Collinearity:** Three points are *collinear* (lie on one straight line) if the gradient between the first pair equals the gradient between the second pair. Compute  $m$  for two overlapping pairs of the points; if the two gradients match, the points lie on a single line.

## EXAM TIPS

**TIP** In the gradient formula, subtract the two  $y$ -values and the two  $x$ -values in the *same order*. Using  $y_2 - y_1$  on top but  $x_1 - x_2$  on the bottom flips the sign of the gradient. This is the most common source of a wrong sign in Core coordinate questions.

**TIP** A line written as  $ax + by = c$  hides its gradient. Rearrange it into the form  $y = mx + c$  first, then read off  $m$ . Comparing gradients before rearranging is a frequent cause of wrong parallel-line answers.

### 3 Geometry

#### KEY FORMULAS

##### Exterior Angle of a Regular Polygon

$$\text{exterior angle} = \frac{360^\circ}{n}$$

Use for a *regular* polygon with  $n$  equal sides. The exterior angles of any polygon always sum to  $360^\circ$ , so each one is  $360^\circ$  divided by the number of sides.

##### Scale Factor for Similar Figures

$$k = \frac{\text{length on enlarged figure}}{\text{corresponding length on original figure}}$$

Use once two figures are known to be similar. Find  $k$  from one matching pair of sides, then multiply any length on the original by  $k$  to get the matching length on the enlargement.

##### Sum of Interior Angles of a Polygon

$$S = (n - 2) \times 180^\circ$$

Use to find the total of all interior angles of any polygon with  $n$  sides, whether regular or irregular.

##### Converting Between Scale-Drawing and Real Distances

$$\text{real distance} = \text{drawn distance} \times \text{scale}$$

Use with a scale written as  $1 : n$  or "1 cm represents  $X$ ". Convert all measurements to the same unit first, then multiply a drawn length by the scale to get the real length, or divide a real length by the scale to get the drawn length.

##### Interior Angle of a Regular Polygon

$$\text{interior angle} = \frac{(n - 2) \times 180^\circ}{n}$$

Use for a *regular* polygon with  $n$  equal sides to find each interior angle directly. Equivalently, subtract the exterior angle  $360^\circ/n$  from  $180^\circ$ , since interior and exterior angles sit on a straight line.

#### KEY CONCEPTS

- **Angle in a Semicircle:** An angle drawn at the circumference using a diameter as one side is always  $90^\circ$ . So if a triangle inside a circle has the diameter as one of its sides, the angle opposite that diameter is a right angle.
- **Angles in Parallel Lines:** When a transversal crosses two parallel lines, *corresponding* angles (in an F shape) are equal, *alternate* angles (in a Z shape) are equal, and *co-interior* angles (in a C shape) sum to  $180^\circ$ .
- **Basic Angle Facts:** Angles on a straight line sum to  $180^\circ$ . Angles at a point sum to  $360^\circ$ . Vertically opposite angles, formed where two straight lines cross, are equal. The interior angles of a triangle sum to  $180^\circ$ .
- **Constructing a Perpendicular Bisector:** To bisect segment  $AB$ , open the compasses to more than half of  $AB$ . With centre  $A$  draw arcs above and below the line, then with the same setting and centre

*B* draw arcs crossing them. The straight line through the two crossing points passes through the midpoint of *AB* and meets it at  $90^\circ$ .

- **Parts of a Circle:** The *radius* joins the centre to the circumference; the *diameter* passes through the centre and equals  $2r$ . A *chord* joins two points on the circle. A *tangent* touches the circle at exactly one point and is perpendicular to the radius there. An *arc* is part of the circumference; a *sector* is bounded by two radii and an arc; a *segment* is bounded by a chord and an arc.
- **Properties of Special Quadrilaterals:** A *square* has four equal sides and four right angles. A *rectangle* has opposite sides equal and four right angles. A *rhombus* has four equal sides with diagonals that bisect each other at right angles. A *parallelogram* has opposite sides parallel and equal and opposite angles equal. A *trapezium* has exactly one pair of parallel sides. A *kite* has two pairs of adjacent equal sides.
- **Tangent Perpendicular to Radius:** A tangent to a circle meets the radius drawn to the point of contact at exactly  $90^\circ$ . This right angle creates a right-angled triangle, so lengths involving a tangent and a radius can be found with Pythagoras' theorem.
- **Three-Figure Bearings:** A bearing is an angle measured *clockwise from north*, always written with three figures. North is  $000^\circ$ , east is  $090^\circ$ , south is  $180^\circ$  and west is  $270^\circ$ . Angles below  $100^\circ$  keep a leading zero, so a bearing of forty degrees is written  $040^\circ$ .
- **Conditions for Similar Triangles:** Two triangles are *similar* when their corresponding angles are equal and their corresponding sides are in the same ratio. Showing that two pairs of angles are equal is enough, because the third pair is then forced. Similar figures have the same shape but not necessarily the same size.
- **Constructing an Angle Bisector:** To bisect angle *BAC*, place the compass point at *A* and draw an arc cutting both arms at points *P* and *Q*. With the same setting, draw arcs from *P* and from *Q* that cross at a point *X*. The straight line *AX* divides the angle into two equal halves.
- **Exterior Angle of a Triangle:** The exterior angle of a triangle equals the sum of the two interior angles that are not next to it. This follows from the interior angles summing to  $180^\circ$  and the exterior angle sitting on a straight line with the third interior angle.
- **Line and Rotational Symmetry:** A shape has a *line of symmetry* if a reflection in that line maps the shape onto itself. It has *rotational symmetry of order n* if a rotation of  $360^\circ/n$  about its centre maps it onto itself. A square has 4 lines of symmetry and rotational symmetry of order 4; a parallelogram has no lines of symmetry but order 2.
- **Naming Triangles:** Triangles are named by their sides or their angles. An *equilateral* triangle has three equal sides and three  $60^\circ$  angles; an *isosceles* triangle has two equal sides and two equal base angles; a *scalene* triangle has all sides and angles different. By angle, a triangle is *acute* (all angles below  $90^\circ$ ), *right-angled* (one angle of  $90^\circ$ ) or *obtuse* (one angle above  $90^\circ$ ).
- **Two Tangents from an External Point:** Two tangents drawn to a circle from the same external point are equal in length. The two right-angled triangles formed with the radii and the line to the centre are congruent, so the tangent lengths match.
- **Convex and Concave Polygons:** A *convex* polygon has every interior angle less than  $180^\circ$ , so no vertex points inwards. A *concave* polygon has at least one reflex interior angle, giving a vertex that points into the shape. A *regular* polygon is a special convex polygon that is both equilateral and equiangular.

## EXAM TIPS

**TIP** Core papers award a mark for the reason as well as the value, so quote the rule you used, for example "alternate angles" or "angles in a triangle sum to  $180^\circ$ ". A correct number with no reason often scores only part of the marks.

**TIP** In a "construct" question the compass arcs are the evidence of method, so never rub them out. A correct final line with no arcs showing loses the construction marks even when the answer looks right.

**TIP** Always measure a bearing clockwise starting from the north line at the point you are measuring *from*, and write it with three figures. To find a back bearing (the return direction) add or subtract  $180^\circ$ , keeping the result between  $000^\circ$  and  $360^\circ$ .

**TIP** The special quadrilaterals are not separate boxes. A square is also a rhombus (four equal sides) and a rectangle (four right angles), and every rectangle and rhombus is also a parallelogram. When a question asks for the *most precise* name, choose the one with the most properties.

## 4 Mensuration

### KEY FORMULAS

#### Area of a Circle

$$A = \pi r^2$$

Use to find the area enclosed by a circle of radius  $r$ .

#### Area of a Parallelogram

$$A = bh$$

Use for the area of a parallelogram;  $b$  is a side and  $h$  is the perpendicular height to that side, not a slanted edge.

#### Area of a Rectangle

$$A = lw$$

Use for the area of any rectangle; multiply the length  $l$  by the width  $w$ .

#### Area of a Triangle

$$A = \frac{1}{2}bh$$

Use for the area of any triangle;  $b$  is the base and  $h$  is the perpendicular height to that base.

#### Circumference of a Circle

$$C = \pi d = 2\pi r$$

Use to find the distance around a circle from its diameter  $d$  or radius  $r$ .

#### Converting Between Area Units

$$1\text{m}^2 = 10,000\text{cm}^2$$

Use when converting an area between metric units. Because area has two length dimensions, the conversion factor is the *square* of the linear factor, for example  $100^2 = 10,000$  between  $\text{m}^2$  and  $\text{cm}^2$ .

#### Surface Area of a Cylinder

$$S = 2\pi r^2 + 2\pi rh$$

Use for the total surface area of a closed cylinder; two circular ends of area  $\pi r^2$  each, plus the curved surface, which unrolls to a rectangle of width  $2\pi r$  and height  $h$ .

#### Surface Area of a Sphere

$$S = 4\pi r^2$$

Use for the total surface area of a sphere of radius  $r$ .

### Volume of a Cuboid

$$V = lwh$$

Use for the volume of any cuboid; the product of its length, width and height.

### Volume of a Cylinder

$$V = \pi r^2 h$$

Use for the volume of any cylinder; the circular cross-section area  $\pi r^2$  multiplied by the height  $h$ .

### Volume of a Sphere

$$V = \frac{4}{3}\pi r^3$$

Use for the volume of a sphere of radius  $r$ .

### Arc Length of a Sector

$$\ell = \frac{\theta}{360} \times 2\pi r$$

Use for the length of the curved arc of a sector with angle  $\theta^\circ$  at the centre of a circle of radius  $r$ ; the sector is  $\theta/360$  of the whole circle.

### Area of a Sector

$$A = \frac{\theta}{360} \times \pi r^2$$

Use for the area of a sector with angle  $\theta^\circ$  in a circle of radius  $r$ ; the sector is  $\theta/360$  of the whole circle's area.

### Area of a Trapezium

$$A = \frac{1}{2}(a + b)h$$

Use for the area of a trapezium with parallel sides  $a$  and  $b$ ;  $h$  is the perpendicular distance between them.

### Converting Between Volume Units

$$1\text{m}^3 = 1,000,000\text{cm}^3$$

Use when converting a volume between metric units. Because volume has three length dimensions, the conversion factor is the *cube* of the linear factor, for example  $100^3 = 1,000,000$  between  $\text{m}^3$  and  $\text{cm}^3$ .

### Surface Area of a Cone

$$S = \pi r^2 + \pi r l$$

Use for the total surface area of a solid cone; the circular base  $\pi r^2$  plus the curved surface  $\pi r l$ , where  $l$  is the slant height.

### Volume of a Cone

$$V = \frac{1}{3}\pi r^2 h$$

Use for the volume of a cone; the same base-times-height product as a cylinder, scaled by  $\frac{1}{3}$ .

### Volume of a Prism

$$V = A_{\text{cross}} \times L$$

Use for the volume of any prism (not only a cylinder);  $A_{\text{cross}}$  is the area of the constant cross-section and  $L$  is the prism's length.

### Volume of a Pyramid

$$V = \frac{1}{3} \times A_{\text{base}} \times h$$

Use for the volume of a pyramid of any base shape;  $A_{\text{base}}$  is the area of the base and  $h$  is the perpendicular height from the base to the apex.

## KEY CONCEPTS

- **Building Compound Solids from Simple Parts:** A compound solid, such as a cone on a cylinder or a hemisphere on a cylinder, is analysed one solid at a time. Add the volumes of the parts for a total volume, and add the *external* surfaces only, not any internal joining face, for a total surface area.
- **Capacity and Volume Are the Same Measurement:** "Capacity" means the same as "volume" at this level, but is usually quoted in litres or millilitres rather than  $\text{m}^3$  or  $\text{cm}^3$ . 1 litre = 1000 ml = 1000  $\text{cm}^3$ .
- **Perpendicular Height:** The height  $h$  used in an area formula is always the *perpendicular* distance from the base to the opposite vertex or side, measured at a right angle. A slanted side is not the height unless it happens to meet the base at  $90^\circ$ .
- **Slant Height of a Cone:** A cone's slant height  $l$  is the distance from the apex to the edge of the base, measured along the sloped surface. It is found from the radius  $r$  and the perpendicular (vertical) height  $h$  by Pythagoras:  $l = \sqrt{r^2 + h^2}$ .
- **Splitting or Subtracting for Compound Shapes:** A compound (composite) shape is made of two or more standard shapes joined together, or a standard shape with a piece removed. Either split the shape into simpler rectangles, triangles or sectors and add their areas, or find the area of a surrounding standard shape and subtract the area of the missing piece.
- **The Length Conversion Ladder:** Each step between adjacent metric length units is a fixed multiplying factor:  $\times 10$  between mm and cm,  $\times 100$  between cm and m, and  $\times 1000$  between m and km. Divide by the same factor to convert in the opposite direction.
- **Removed or Bored-Out Solids Subtract a Volume:** When a solid has a piece removed, such as a cone bored out of a cylinder, find the volume of the original solid and the volume of the removed piece separately, then subtract. The removed piece almost always shares a dimension, such as the radius, with the original solid.
- **Total Surface Area Is the Sum of the Faces:** The total surface area of a solid is the sum of the areas of every face that is visible on the outside. For a prism, this is the two end faces plus the curved or flat side faces, which unroll into a single rectangle of width equal to the cross-section's perimeter.
- **What a Sector Is:** A sector is the "pie slice" region of a circle enclosed by two radii and the arc between them, defined by the angle  $\theta^\circ$  at the centre. The rest of the circle is not part of the sector.

## EXAM TIPS

**TIP** If a question says "in terms of  $\pi$ ", keep  $\pi$  as a symbol in the final answer, for example  $16\pi$  cm. If it says "to 3 s.f." (or gives no instruction), substitute  $\pi \approx 3.142$  and round the decimal answer.

**TIP** The cone surface area formula  $S = \pi r^2 + \pi r l$  uses the *slant* height  $l$ , never the vertical height  $h$ . If a question gives  $h$ , compute  $l = \sqrt{r^2 + h^2}$  first. The cone volume formula  $V = \frac{1}{3}\pi r^2 h$  uses the

*vertical* height  $h$  instead; mixing the two up is the most common mensuration error at this level.

**TIP** The perimeter of a compound shape is not simply the sum of the original shapes' perimeters. Walk around the actual outer boundary of the final figure, working out any side lengths that are not given directly; they are usually found by adding or subtracting the labelled sides.

**TIP** Before applying an area or volume formula, check that every given length is in the same unit. Convert first, then substitute into the formula; converting the final answer afterwards is more error-prone, especially for area (square the factor) and volume (cube the factor).

**TIP** When two solids are joined, such as a hemisphere sitting on a cylinder, the face where they meet is internal and is not part of the external surface area. Count only the circular base of the cylinder, the curved side of the cylinder, and the curved surface of the hemisphere; never the flat circle at the join.

**TIP** When a triangle or parallelogram is drawn with a slanted side given as a length, that slanted length is *not* the height to use in the area formula unless it happens to be perpendicular to the base. Look for the dashed right-angle marker showing the true perpendicular height.

**TIP** When a compound shape includes a semicircular (or other curved) end, the perimeter includes the curved arc length in place of the straight edge it replaces. Do not add the diameter itself; that line only exists inside the shape once the semicircle has been attached, it is not part of the outer boundary.

## KEY FORMULAS

### Average Speed

$$\text{Speed} = \frac{\text{Distance}}{\text{Time}}$$

Used to calculate the average speed when distance and time are known. Rearrange to find distance (Distance = Speed  $\times$  Time) or time (Time =  $\frac{\text{Distance}}{\text{Speed}}$ ).

### Bounds from a Rounded Value

$$\text{Lower Bound} = x - \frac{d}{2}, \quad \text{Upper Bound} = x + \frac{d}{2}$$

Use when a value  $x$  has been rounded to a given degree of accuracy  $d$  (e.g. the nearest whole unit, or 1 decimal place). Halve the degree of accuracy and subtract for the lower bound, add for the upper bound.

### Fraction of an Amount

$$\frac{a}{b} \text{ of } C = \frac{a}{b} \times C$$

Used to find a fraction of a given quantity. The word "of" means multiply.

### Laws of Indices: Division

$$x^a \div x^b = x^{a-b}$$

Use when dividing two powers with the same base  $x$ . Subtract the index in the denominator from the index in the numerator.

### Laws of Indices: Multiplication

$$x^a \times x^b = x^{a+b}$$

Use when multiplying two powers with the same base  $x$ . Add the indices and keep the base unchanged.

### Percentage Change

$$\text{Percentage Change} = \frac{\text{New Value} - \text{Original Value}}{\text{Original Value}} \times 100$$

Used to calculate the percentage increase or decrease from an original value to a new value. A negative result means a decrease.

### Calculating with Standard Form

$$(a \times 10^m) \times (b \times 10^n) = (a \times b) \times 10^{m+n}$$

Use when multiplying or dividing numbers in standard form. Multiply (or divide) the  $A$ -parts, and add (or subtract) the powers of 10.

### Compound Interest

$$A = P \left( 1 + \frac{R}{100} \right)^T$$

Calculates the total amount  $A$  after compound interest is applied to a principal  $P$  at a rate of  $R\%$  per year for  $T$  years. Each year's interest is calculated on the previous year's total, not just the original principal.

### Currency Conversion

$$\text{Amount in Currency B} = \text{Amount in Currency A} \times \text{Exchange Rate}$$

Used to convert an amount from one currency to another, given the exchange rate as "1 unit of Currency A = (rate) units of Currency B."

### Direct Proportion

$$y = kx$$

Used when two quantities are in direct proportion, so their ratio  $\frac{y}{x}$  stays constant. Find the constant  $k$  from one pair of values, then use it to find any other value.

### Laws of Indices: Power of a Power

$$(x^a)^b = x^{ab}$$

Use when a power is itself raised to another power. Multiply the two indices together.

### Simple Interest

$$I = \frac{PRT}{100}$$

Calculates the simple interest  $I$  earned on a principal amount  $P$  at a rate of  $R\%$  per year over  $T$  years. The interest earned is the same amount every year.

### Union Rule for Two Sets

$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$

Used to find the total number of elements in  $A$  or  $B$  combined, without double counting the elements in the overlap  $A \cap B$ .

## KEY CONCEPTS

- **Converting Between Fractions, Decimals, and Percentages:** To convert a fraction to a decimal, divide the numerator by the denominator. To convert a decimal to a percentage, multiply by 100. To convert a percentage to a fraction, write it over 100 and simplify.
- **Number Types:** Key number types are *integers* (positive or negative whole numbers), *primes* (divisible only by 1 and themselves), *square numbers* ( $n \times n$ ), *cube numbers* ( $n \times n \times n$ ), *factors* (numbers that divide exactly into a given number), and *multiples* (numbers in a given number's times table).
- **Order of Operations:** Calculations follow a strict order: *Brackets, Indices, Division and Multiplication* (left to right), then *Addition and Subtraction* (left to right). Work through an expression in this order every time, never left to right alone.

- **Ratio and Proportion:** A *ratio* compares part to part (e.g. 2 : 3), while a *proportion* compares a part to the whole. In *direct* proportion, as one quantity increases, the other increases at the same rate; in *inverse* proportion, as one increases, the other decreases.
- **Set Notation:** Key symbols:  $\mathcal{E}$  is the universal set,  $\cup$  means *union* ("or"),  $\cap$  means *intersection* ("and"),  $A'$  is the *complement* of  $A$  ("not in  $A$ "), and  $n(A)$  is the number of elements in set  $A$ .
- **Standard Form:** A way of writing very large or very small numbers as  $A \times 10^n$ , where  $1 \leq A < 10$  and  $n$  is an integer. A positive  $n$  gives a large number; a negative  $n$  gives a small decimal.
- **The 24-Hour Clock:** The 24-hour clock runs from 00:00 (midnight) to 23:59, avoiding the need for am/pm. To convert a 12-hour pm time to 24-hour, add 12 to the hour (e.g. 2:45 pm becomes 14:45). Journey durations are found by counting on from the departure time to the arrival time.
- **The Four Operations with Fractions:** To add or subtract fractions, first write them over a common denominator, then add or subtract the numerators. To multiply fractions, multiply the numerators and multiply the denominators. To divide by a fraction, multiply by its reciprocal (flip the second fraction).
- **Zero, Negative, and Fractional Indices:** Any nonzero base raised to the power 0 equals 1, so  $x^0 = 1$ . A negative index gives a reciprocal:  $x^{-a} = \frac{1}{x^a}$ . A fractional index gives a root:  $x^{\frac{1}{n}} = \sqrt[n]{x}$ .
- **Estimation by Rounding to 1 Significant Figure:** To estimate the value of a calculation, round every number to 1 significant figure first, then work out the simplified calculation. This gives a quick sanity check on a calculator answer without needing exact values.
- **Prime Factorisation:** Any number greater than 1 can be written as a product of prime numbers, in index form. Use a factor tree or repeated division by primes to break the number down, then collect repeated factors as powers, e.g.  $180 = 2^2 \times 3^2 \times 5$ .
- **Time Zones and World Time Differences:** A time zone difference is given as a number of hours ahead of or behind another location. To find the local arrival time of a journey, add the flight duration to the departure time in the *original* time zone, then apply the time zone difference to convert to the destination's local time.
- **Upper and Lower Bound Interval Notation:** For a measured or rounded value  $x$ , the true value lies in the interval Lower Bound  $\leq x <$  Upper Bound. The lower bound is included but the upper bound is not, since a value equal to the upper bound would have rounded up to the next unit.
- **Venn Diagrams and Regions:** A Venn diagram splits the universal set  $\mathcal{E}$  into regions: inside both circles (the intersection), inside one circle only, and outside every circle. Shade or count only the region asked for, and check whether the overlap should be included or excluded.

## EXAM TIPS

**TIP** To share an amount in a ratio (e.g. 72 in the ratio 4 : 5): add the parts ( $4 + 5 = 9$ ), divide the total amount by the sum of the parts ( $72 \div 9 = 8$ ), then multiply this value by each part of the ratio ( $4 \times 8 = 32$  and  $5 \times 8 = 40$ ).

**TIP** To find a percentage increase or decrease quickly, use a decimal multiplier. For a 20% increase, multiply by 1.20. For a 15% decrease, multiply by  $1 - 0.15 = 0.85$ .

**TIP** In a two-circle Venn diagram, the "only  $A$ " region excludes the overlap; the "neither" region lies outside both circles but inside  $\mathcal{E}$ . Always subtract the overlap once when reading a region that is not shared.

**TIP** Remember there are 60 minutes in an hour, not 100. Use your calculator's degrees/minutes/seconds button, or convert minutes to a fraction or decimal of an hour (e.g. 45 minutes = 0.75 hours), before mixing time with ordinary arithmetic.

**TIP** To find the *maximum* possible value of a sum or product, use the upper bounds of every quantity. To find the *maximum* possible value of a difference, use the upper bound of the first quantity and the lower bound of the quantity being subtracted.

**TIP** Write the decimal digits over a power of 10 matching the number of decimal places (e.g.  $0.375 = \frac{375}{1000}$ ), then simplify by dividing the numerator and denominator by their highest common factor.

**TIP** Split a negative fractional index into stages: find the root given by the denominator, raise the result to the power given by the numerator, then take the reciprocal if the index is negative. Doing this in order avoids arithmetic slips with large numbers.

**TIP** Write both numbers as products of primes. The *HCF* (highest common factor) is the product of the *lowest* power of each prime common to both numbers. The *LCM* (lowest common multiple) is the product of the *highest* power of every prime appearing in either number.

**TIP** Two quantities are in *inverse* proportion if  $y = \frac{k}{x}$ , so their product  $xy$  stays constant. As one quantity increases, the other decreases in the same ratio, for example doubling the number of workers halves the time needed for a fixed job.

**TIP** After multiplying or dividing the *A*-parts of two standard form numbers, check that the result is still  $1 \leq A < 10$ . If the *A*-part is 10 or more, divide it by 10 and add 1 to the power of 10; if it is less than 1, multiply it by 10 and subtract 1 from the power.

## 6 Probability

### KEY FORMULAS

#### Complement Rule

$$P(\text{not } E) = 1 - P(E)$$

Use to find the probability that an event does *not* happen. It is often the quickest route when the event is hard to count directly but its opposite is easy.

#### Expected Frequency

$$\text{Expected frequency} = n \times P$$

Use to predict how many times an event should occur in  $n$  trials, where  $P$  is the probability per trial. The result is the average expected count, not a guaranteed value.

#### Theoretical Probability of an Event

$$P(E) = \frac{\text{number of favourable outcomes}}{\text{total number of outcomes}}$$

Use when every outcome is equally likely. Count the outcomes that match event  $E$  over the total number of possible outcomes. Every probability lies between 0 (impossible) and 1 (certain).

#### Addition Rule for Two Events

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

Use to find the probability of  $A$  or  $B$  when the two events can happen together. Subtracting the overlap  $P(A \cap B)$  prevents the shared outcomes from being counted twice.

#### Experimental Probability

$$\text{Relative frequency} = \frac{\text{number of successful trials}}{\text{total number of trials}}$$

Use to estimate a probability from the results of an experiment when the theoretical value is unknown or the object is biased. It is also called the relative frequency.

### KEY CONCEPTS

- **Sample Space Diagrams:** A sample space diagram lists every possible outcome of a combined experiment, such as a grid of the 36 ordered pairs for two dice. The probability of an event is the number of matching cells divided by the total number of cells.
- **The Probability Scale:** Every probability is a number from 0 to 1 inclusive. A value of 0 means the event is impossible, 1 means it is certain, and 0.5 means an even chance. A probability can never be negative or greater than 1, so a value outside  $[0, 1]$  signals an arithmetic error.
- **Equally Likely Outcomes:** The theoretical probability formula only applies when each outcome has the same chance, as with a *fair* coin, die, or spinner. Words such as *fair* and *at random* signal equally likely outcomes; a *biased* object does not, so its probabilities must be found from data instead.
- **Relative Frequency as an Estimate:** When the theoretical probability is unknown, repeat the experiment many times and use the observed proportion as an estimate. The estimate becomes more reliable as the number of trials increases.
- **Tree Diagrams:** A tree diagram shows the outcomes of a two-step experiment as branches. Multiply the probabilities *along* a path to find the probability of that combined outcome, then add the

probabilities of the separate paths that satisfy the event.

- **With and Without Replacement:** *With replacement*, the item is returned before the second draw, so the two steps are independent and the probabilities stay the same. *Without replacement*, the item is kept, so the total and the matching count both shrink for the second draw.
- **Venn Diagrams:** A Venn diagram sorts outcomes into overlapping regions inside a rectangle that represents the whole sample space. The overlap holds outcomes in *both* sets, and the probability of any region is the number of outcomes it contains divided by the total.

### EXAM TIPS

**TIP** Give a probability as a fraction, decimal, or percentage, never as a ratio or in words such as *1 in 5*. Leave a fraction in its simplest form unless the question asks for a decimal.

**TIP** A relative frequency from only a few trials can be misleading. Comparing observed frequencies against the expected frequency  $n \times P$  is the standard way to judge whether a coin, die, or spinner is fair.

**TIP** Questions asking for the probability of *at least one* success are usually quickest through the complement: find the probability of *none*, then subtract from 1. This avoids adding many separate cases.

**TIP** When an item is drawn *without* replacement, reduce the denominator by one for the second draw, and reduce the numerator too if an item of that same type was already taken. Forgetting to adjust these is the most common error in tree-diagram questions.

### KEY FORMULAS

#### Mean from a Frequency Table

$$\bar{x} = \frac{\sum fx}{\sum f}$$

Use when data is given in a frequency table. Multiply each value  $x$  by its frequency  $f$  and add the results to get  $\sum fx$ , then divide by  $\sum f$ , the total frequency.

#### Mean of a Data List

$$\bar{x} = \frac{\sum x}{n}$$

Use to find the mean (the arithmetic average) of a list of values. Add up all the values to get  $\sum x$ , then divide by  $n$ , the number of values.

#### Pie Chart Sector Angle

$$\text{angle} = \frac{f}{\text{total}} \times 360^\circ$$

Use to find the angle of a sector in a pie chart. Divide the frequency  $f$  of the category by the total frequency, then multiply by  $360^\circ$ . All the sector angles must add up to  $360^\circ$ .

#### Range of a Data Set

$$\text{Range} = \text{largest value} - \text{smallest value}$$

Use to measure the spread of the data. A large range means the values are widely spread; a small range means they are close together.

#### Position of the Median

$$\text{position} = \frac{n + 1}{2}$$

Use to locate the median in an ordered list of  $n$  values. If the position is a whole number, the median is that value; if it ends in .5, average the two values on either side.

### KEY CONCEPTS

- **Bar Charts and Pictograms:** A *bar chart* shows the frequency of each category using bars of equal width with gaps between them; the height of each bar gives its frequency. A *pictogram* uses a repeated symbol to represent a fixed number of items and must always include a key stating what one symbol represents.
- **Classifying Data:** Data is either *qualitative* (categories such as colour or nationality) or *quantitative* (numerical). Quantitative data is *discrete* if it can only take separate values, usually whole numbers found by counting (goals scored, number of siblings), or *continuous* if it is measured and can take any value in a range (height, mass, time).
- **Finding the Median:** The *median* is the middle value once the data is written in order from smallest to largest. For  $n$  values, when  $n$  is odd there is a single middle value; when  $n$  is even the median is the mean of the two middle values.

- **Scatter Diagrams and Correlation:** A *scatter diagram* plots paired data as points to show whether two quantities are related. *Positive correlation* means one quantity increases as the other increases; *negative correlation* means one decreases as the other increases; if the points show no pattern there is *no correlation*. The strength is described as strong when the points lie close to a line and weak when they are more scattered.
- **The Three Averages:** An *average* is a single value that represents the centre of a data set. The three averages are the *mean* (the total divided by how many values there are), the *median* (the middle value when the data is ordered), and the *mode* (the value that occurs most often). A data set can have more than one mode, or no mode at all.
- **Identifying the Mode:** The *mode* is the most frequently occurring value. In a frequency table it is the value with the highest frequency, not the highest frequency itself. The mode is the only average that can be used for qualitative (non-numerical) data, such as the most popular colour.
- **Line of Best Fit:** A *line of best fit* is a straight line drawn by eye through the points of a scatter diagram, with roughly equal numbers of points above and below it. It summarises the trend and can be used to estimate one quantity from the other within the range of the data.
- **Stem-and-Leaf Diagrams:** A *stem-and-leaf diagram* orders numerical data while keeping every original value. Each value is split into a *stem* (the leading digits) and a *leaf* (the final digit); leaves on each row are written in ascending order. A key such as 3 | 7 = 37 is required so the values can be read correctly.
- **Choosing the Most Appropriate Average:** The best average depends on the data. The *mode* suits qualitative data and picks out the most popular category. The *median* is best when there are extreme values (outliers), because it is not affected by them. The *mean* uses every value and is best for symmetric numerical data, but a single very large or very small value can distort it.

## EXAM TIPS

**TIP** Always rewrite the list in ascending order before reading off the median. The most common mistake is to take the middle of the *unordered* list. Ordering also makes the smallest and largest values easy to read for the range.

**TIP** Both a pictogram and a stem-and-leaf diagram are incomplete without a key, and markschemes award a mark for it. A pictogram key states how many items one symbol represents; a stem-and-leaf key shows how a stem and leaf combine into a value.

**TIP** The range is a measure of *spread*, so it answers how consistent or varied the data is, not what a typical value is. When comparing two data sets, a smaller range means the values are more consistent.

**TIP** A strong correlation between two quantities does not prove that one causes the other. Both may be driven by a third factor. For example, ice-cream sales and swimming accidents both rise in warm weather, so they correlate without either causing the other.

## 8 Transformations and vectors

### KEY FORMULAS

Enlargement about the origin

$$(x, y) \mapsto (kx, ky)$$

Use to enlarge a point about the origin by scale factor  $k$ : multiply both coordinates by  $k$ . Every length is multiplied by  $k$  while all angles stay the same, so object and image are *similar*.

Reflection in the  $x$ -axis or  $y$ -axis

$$\text{in } x\text{-axis: } (x, y) \mapsto (x, -y); \quad \text{in } y\text{-axis: } (x, y) \mapsto (-x, y)$$

Use to reflect a point in a coordinate axis. Reflecting in the  $x$ -axis negates the  $y$ -coordinate; reflecting in the  $y$ -axis negates the  $x$ -coordinate. The shape keeps its size but its orientation is reversed.

Rotation about the origin

$$90^\circ \text{ clockwise: } (x, y) \mapsto (y, -x); \quad 180^\circ: (x, y) \mapsto (-x, -y); \quad 90^\circ \text{ anticlockwise: } (x, y) \mapsto (-y, x)$$

Use to rotate a point about the origin through a multiple of  $90^\circ$ . A  $90^\circ$  clockwise turn is the same as a  $270^\circ$  anticlockwise turn. Every length and angle is kept, so object and image are *congruent*.

Translation by a column vector

$$(x, y) \mapsto (x + a, y + b)$$

Use to translate a point by the column vector  $\begin{pmatrix} a \\ b \end{pmatrix}$ : add  $a$  to the  $x$ -coordinate and  $b$  to the  $y$ -coordinate. A translation slides every point the same distance in the same direction, so size and orientation are unchanged.

Enlargement about a general centre

$$(x, y) \mapsto (a + k(x - a), b + k(y - b))$$

Use to enlarge a point by scale factor  $k$  about a centre  $(a, b)$  that is not the origin. Find the vector from the centre to the point, multiply it by  $k$ , then add it back to the centre.

Reflection in the line  $x = k$  or  $y = k$

$$\text{in } x = k: (x, y) \mapsto (2k - x, y); \quad \text{in } y = k: (x, y) \mapsto (x, 2k - y)$$

Use to reflect a point in a vertical mirror  $x = k$  or a horizontal mirror  $y = k$ . The coordinate perpendicular to the mirror moves to an equal distance on the far side, while the coordinate parallel to the mirror is unchanged.

Scale factor from two lengths

$$k = \frac{\text{image length}}{\text{object length}}$$

Use to find the scale factor of an enlargement by dividing a length on the image by the matching length on the object. A value greater than 1 means the shape has grown; a value between 0 and 1 means it has shrunk.

## KEY CONCEPTS

- **Reading a column vector:** A *column vector*  $\begin{pmatrix} a \\ b \end{pmatrix}$  describes a translation as "*a* across and *b* up". A negative top number means move left and a negative bottom number means move down. The top entry is always the horizontal step and the bottom entry the vertical step.
- **What a reflection is:** A *reflection* flips a shape across a mirror line so that the mirror is the *perpendicular bisector* of the segment joining each point to its image. Object and image are *congruent* (same shape and size) but the orientation is reversed, and any point already on the mirror line stays fixed.
- **What a rotation needs:** A *rotation* turns a shape about a fixed point called the *centre of rotation*. To specify one you need three things: the *centre*, the *angle* (usually a multiple of  $90^\circ$ ), and the *direction* (clockwise or anticlockwise). The centre is the only point that stays fixed, and the image is congruent to the object.
- **What an enlargement does:** An *enlargement* changes the size of a shape by a *scale factor* *k* measured from a *centre of enlargement*. Each length is multiplied by *k*: if  $k > 1$  the image is larger and if  $0 < k < 1$  it is smaller. Angles are unchanged, so the image is *similar* to the object.

## EXAM TIPS

**TIP** To *describe fully* a reflection you must give the *equation of the mirror line*, such as  $x = 3$  or  $y = -1$ . To find it, join a point to its image and take the perpendicular bisector: the mirror runs midway between corresponding points. Writing only "reflection" scores no marks.

**TIP** To locate the *centre of enlargement*, draw a straight line through each object point and its matching image point. All such lines meet at a single point, and that intersection is the centre. Combine this with the length ratio to describe the enlargement fully.

**TIP** To *reverse* a translation  $\begin{pmatrix} a \\ b \end{pmatrix}$ , negate both components to get  $\begin{pmatrix} -a \\ -b \end{pmatrix}$ . To combine two translations into one, add their column vectors component by component. Changing the sign of only one component is a common slip.

**TIP** For a rotation about a centre  $(a, b)$  other than the origin, first subtract  $(a, b)$  from the point so the centre moves to the origin, apply the standard origin rule, then add  $(a, b)$  back. Trying to rotate directly about a non-origin centre is the most common source of lost marks.

## 9 Trigonometry

### KEY FORMULAS

#### Cosine Ratio (CAH)

$$\cos \theta = \frac{\text{Adjacent}}{\text{Hypotenuse}}$$

Use to link the angle  $\theta$  with the side next to it (not the hypotenuse) and the hypotenuse.

#### Pythagoras' Theorem

$$a^2 + b^2 = c^2$$

Use in a right-angled triangle to find a missing side length from the other two;  $c$  is the hypotenuse (the longest side, opposite the right angle) and  $a$ ,  $b$  are the two shorter sides.

#### Sine Ratio (SOH)

$$\sin \theta = \frac{\text{Opposite}}{\text{Hypotenuse}}$$

Use to link the angle  $\theta$  with the side opposite it and the hypotenuse; find a missing side, or an angle using  $\sin^{-1}$ .

#### Tangent Ratio (TOA)

$$\tan \theta = \frac{\text{Opposite}}{\text{Adjacent}}$$

Use to link the angle  $\theta$  with the opposite and adjacent sides when the hypotenuse is neither known nor wanted.

#### Inverse Trigonometric Ratios

$$\theta = \tan^{-1} \left( \frac{\text{Opposite}}{\text{Adjacent}} \right)$$

Use the inverse functions  $\sin^{-1}$ ,  $\cos^{-1}$ ,  $\tan^{-1}$  to recover the angle  $\theta$  once the ratio of two known sides is set up.

#### Pythagoras' Theorem Rearranged for a Shorter Side

$$a = \sqrt{c^2 - b^2}$$

Use to find one of the two shorter sides when the hypotenuse  $c$  and the other shorter side  $b$  are known; subtract inside the root, never add.

### KEY CONCEPTS

- **Choosing Pythagoras or a Trig Ratio:** Use Pythagoras' theorem when only the three sides are involved and no angle is needed. Use a trig ratio (SOHCAHTOA) whenever an angle is involved, whether it is given or has to be found.
- **Identifying the Hypotenuse:** The hypotenuse is the longest side of a right-angled triangle and always lies opposite the right angle. Label it first, because both Pythagoras' theorem and the trig ratios depend on picking it out correctly.
- **Labelling Opposite, Adjacent and Hypotenuse:** Sides are named relative to the angle in use. The hypotenuse is opposite the right angle, the opposite side is across from the angle  $\theta$ , and the adjacent side is the remaining side next to  $\theta$ . Re-label if you switch to a different angle.

- **Angles of Elevation and Depression:** The angle of elevation is measured upwards from the horizontal to a line of sight; the angle of depression is measured downwards from the horizontal. Sketch the horizontal, mark the right angle at the base, and the situation becomes an ordinary right-angled triangle.
- **The SOH CAH TOA Mnemonic:** SOH CAH TOA records the three ratios: Sine is Opposite over Hypotenuse, Cosine is Adjacent over Hypotenuse, Tangent is Opposite over Adjacent. Choose the ratio that uses the two sides you already have.
- **Bearings Combined with Trigonometry:** A bearing is measured clockwise from north and written with three figures, such as  $055^\circ$  or  $120^\circ$ . Drawing the north line and the bearing usually creates a right-angled triangle whose north and east components you find with cosine and sine.

### EXAM TIPS

**TIP** Set the calculator to degree (DEG) mode before any trigonometry. In radian or gradian mode every ratio and angle comes out wrong, and the error is easy to miss.

**TIP** Do not round part way through a multi-step problem. Store each intermediate value in full, or use the calculator memory, and round only the final answer to the accuracy the question asks for, usually 3 significant figures.

**TIP** Opposite and adjacent are defined by the angle in the question, not by the triangle's orientation. When a problem uses two different angles, the same side can be opposite for one and adjacent for the other, so re-label each time.

**TIP** After any Pythagoras calculation, check that the hypotenuse is still the longest side. If a side you found as a shorter side comes out longer than the hypotenuse, you added when you should have subtracted, so redo the working.



Scan to download the app  
[www.thepracticebook.org](http://www.thepracticebook.org)

Generated 2026/07/02