Categoricity Problem for Strong Kleene Logics

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Introduction

- *Logical inferentialists* defend the view that notions in one's logical theory should be introduced by means of inferential relations between formulas in a proof system.(See Gentzen [13], Dummett [10], Prawitz [19], Brandom [7], Tenant [27], and Garson [12].)

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    - Yes either by using a well-motivated semantic strategy (See Bonnay and Westerstahl [6] and Belnap and Massey [4].) or by using a proof-theoretic strategy (See Carnap [8], Shoesmith and Smiley [25], Restall [22], Smiley [26].)

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Defining Categoricity

Definition 1

$\mathcal{V}^2$ ($\mathcal{V}^3$) is the set of all valuation functions that maps formulas in $\mathfrak{L}$ to elements of $\mathbf{V}^2 = \{T, F\}$ ($\mathbf{V}^3 = \{T, i, F\}$). We will use the notation $\mathcal{V}^{nz}$ to denote a restricted valuation space where n ranges over $\{2, 3\}$ and z ranges over restrictions, e.g., $\mathcal{V}^{3c}$ denotes a subset of $\mathcal{V}^3$ where the valuation functions interpret formulas compositionally. Lastly, we will use variables $V_0^{nz}, \ldots, V_i^{nz}, \ldots$ for subsets of $\mathcal{V}^{nz}$.

Definition 2 (Inference satisfaction)

Let x be a subset of a set of truth values $\mathbf{V}^n$. v is an x-counterexample the inference $[\Gamma \succ \Delta]$ ($v \nVdash^x [\Gamma \succ \Delta]$) iff $v(\Gamma) \subseteq x$ and $v(\Delta) \subseteq \mathbf{V}^n \setminus x$. v x-satisfies a inference ($v \Vdash^x [\Gamma \succ \Delta]$) just in case v is not an x-counterexample to it.

Definition 3 (Entailment)

$\Gamma \vDash^x_{V^n} \Delta$ iff for every $v \in V^n$, v x-satisfies $\Gamma \succ \Delta$.

Definition 4 (Deductive model)

$Ded^x_n(\vdash) = \{v \in \mathcal{V}^n \mid \text{for all } \Gamma \succ \Delta \in \vdash, v \Vdash^x \Gamma \succ \Delta\}$

Definition 5 (Categorical)

$\vdash$ is categorical for $\vDash^x_{V^n}$ with respect to a valuation space $\mathcal{V}^n$ just in case $Ded^x_n(\vdash) = V^n$.

Classical Logic

- $\vdash_{\mathbb{CL}}^{\texttt{SET-FMLA}}$ is not categorical $\vDash_{\mathcal{V}^{\mathcal{B}}}^{\{T\}}$ with respect to a valuation space $\mathcal{V}^2$. ($v^T(A) = T$ just in case $[\emptyset \succ A]$ is provable in $\mathbb{CL}$.)

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- Semantic strategy: restrict the valuation space with independently motivated conditions.
  - Compositionality + non-triviality: $\vdash^{\texttt{SET-FMLA}}_{\mathbb{CL}}$ is categorical for $\vDash^{\{T\}}_{\mathcal{V}^{\mathcal{B}}}$ with respect to $\mathcal{V}^{2nc}$. (See Bonnay and Westerstahl [6] and Church [9])
  - Motivated by learnability of language. (See Pagin and Westerstahl [18].)

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  - Motivated by learnability of language. (See Pagin and Westerstahl [18].)
  - Meaning of $\vee$ or meaning of $\neg$: $\vdash^{\texttt{SET-FMLA}}_{\mathbb{CL}}$ is categorical for $\vDash^{\{T\}}_{\mathcal{V}^{\mathcal{B}}}$ with respect to $\mathcal{V}^{2\neg}$. $\vdash^{\texttt{SET-FMLA}}_{\mathbb{CL}}$ is categorical for $\vDash^{\{T\}}_{\mathcal{V}^{\mathcal{B}}}$ with respect to $\mathcal{V}^{2\vee}$. (Belnap and Massey [4] and Carnap [8])

SKLs

$$\frac{}{A \succ A}\,[ID] \qquad \frac{\Gamma \succ \Delta}{\Gamma, \Gamma' \succ \Delta, \Delta'}\,[W]$$

$$\frac{\Gamma \succ \Delta, A \qquad \Gamma', A \succ \Delta'}{\Gamma, \Gamma' \succ \Delta, \Delta'}\,[CUT]$$

$$\frac{\Gamma \succ A, B, \Delta}{\Gamma \succ A \vee B, \Delta}\,[\vee R] \qquad \frac{\Gamma, A \succ \Delta \qquad \Gamma', B \succ \Delta'}{\Gamma, \Gamma', A \vee B \succ \Delta, \Delta'}\,[\vee L]$$

$$\frac{\Gamma \succ A, \Delta \qquad \Gamma' \succ B, \Delta'}{\Gamma, \Gamma' \succ A \wedge B, \Delta, \Delta'}\,[\wedge R] \; \frac{\Gamma, A, B \succ \Delta}{\Gamma, A \wedge B \succ \Delta}\,[\wedge L]$$

Figure: Structural Rules and Operational Rules for $\wedge$ and $\vee$.

$$\frac{\Gamma \succ A, \Delta}{\Gamma \succ \neg\neg A, \Delta}\,[\neg\neg R] \qquad \frac{\Gamma, A \succ \Delta}{\Gamma, \neg\neg A \succ \Delta}\,[\neg\neg L]$$

$$\frac{\Gamma \succ \neg A, \Delta}{\Gamma \succ \neg(A \wedge B), \Delta}\,[\neg \wedge R] \qquad \frac{\Gamma \succ \neg A, \Delta}{\Gamma \succ \neg(B \wedge A), \Delta}\,[\neg \wedge R]$$

$$\frac{\Gamma, \neg A \succ \Delta \qquad \Gamma', \neg B \succ \Delta}{\Gamma, \Gamma' \neg(A \wedge B) \succ \Delta}\,[\neg \wedge L]$$

$$\frac{\Gamma, \neg A \succ \Delta}{\Gamma, \neg(A \vee B) \succ \Delta}\,[\neg \vee L] \qquad \frac{\Gamma, \neg A \succ \Delta}{\Gamma, \neg(B \vee A) \succ \Delta}\,[\neg \vee L]$$

$$\frac{\Gamma \succ \neg A, \Delta \qquad \Gamma' \succ \neg B, \Delta}{\Gamma, \Gamma' \succ \neg(A \vee B), \Delta}\,[\neg \vee R]$$

Figure: Double Negation and DeMorgan Rules.

$$\frac{}{\Gamma, A, \neg A \succ \Delta}\,[ECQ] \qquad \frac{}{\Gamma \succ A, \neg A, \Delta}\,[LEM]$$

Figure: $\mathbb{K}3$ has [*ECQ*] and $\mathbb{LP}$ has [*LEM*].

$\vee$	T	i	F
T	T	T	T
i	T	i	i
F	T	i	F

$\wedge$	T	i	F
T	T	i	F
i	i	i	F
F	F	F	F

$\neg$	
T	F
i	i
F	T

Table: Strong Kleene Matrices ($\mathcal{M}_{SKL}$).

- $\mathcal{V}^{\mathcal{SKL}}$ is the set of all valuations determine by $\mathcal{M}_{SKL}$
- $k3 = \{T\}$ and $lp = \{T, i\}$.
- $\Gamma \vdash^{\mathtt{SET\text{-}SET}}_{\mathbb{K}3} \Delta$ just in case $\Gamma \vDash^{k3}_{\mathcal{V}^{\mathcal{SKL}}} \Delta$.
- $\Gamma \vdash^{\mathtt{SET\text{-}SET}}_{\mathbb{LP}} \Delta$ just in case $\Gamma \vDash^{lp}_{\mathcal{V}^{\mathcal{SKL}}} \Delta$.

- $\vdash^{\mathtt{SET-SET}}_{\mathbb{K}3}$ and $\vdash^{\mathtt{SET-SET}}_{\mathbb{LP}}$ are not categorical for $\vDash^{k3(lp)}_{\mathcal{V}^{\mathcal{SKL}}}$ with respect to $\mathcal{V}^3$.

$\vee$	T	i	F
T	T	T	T
i	T	i, F	i, F
F	T	i, F	i, F

$\wedge$	T	i	F
T	T	i, F	i, F
i	i, F	i, F	i, F
F	i, F	i, F	i, F

$\neg$	
T	i, F
i	T
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T	T, i	T, i	T, i
i	T, i	T, i	T, i
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$\wedge$	T	T, i	F
T	T, i	T, i	F
i	T, i	T, i	F
F	F	F	F

$\neg$	
T	F
i	F
F	T, i

Figure: First three are $\mathcal{M}_{K3}$ and the latter three are $\mathcal{M}_{LP}$. (See Avron et al. [1].)

Compositionality

- Let $\mathcal{V}^{3c}$ be the set of all valuations that interprets the connectives as truth-functions. $\vdash^{\texttt{SET-SET}}_{\mathbb{K}3}$ and $\vdash^{\texttt{SET-SET}}_{\mathbb{LP}}$ are not categorical for $\vDash^{k3(lp)}_{\mathcal{V}^{SK\mathcal{L}}}$ with respect to $\mathcal{V}^{3c}$.

$\vee$	T	i	F
T	T	T	T
i	T	i	F
F	T	F	F

$\wedge$	T	i	F
T	T	i	F
i	i	i	i
F	F	i	F

$\neg$	
T	i
i	T
F	F

$\vee$	T	i	F
T	T	i	T
i	i	i	i
F	T	i	F

$\wedge$	T	i	F
T	T	T	F
i	T	i	F
F	F	F	F

$\neg$	
T	T
i	F
F	i

Figure: Non-standard Matrices $\mathcal{M}_{K3c}$ and $\mathcal{M}_{LPc}$

- Let $\mathbb{K}3^P$ and $\mathbb{LP}^P$ be the proof systems of $\mathbb{K}3$ and $\mathbb{LP}$ extended with their respective propositional constant rules.

$$\frac{}{\Gamma \succ \top, \Delta}[\top R] \quad \frac{}{\Gamma, \neg\top \succ \Delta}[\neg\top L] \quad \frac{}{\Gamma \succ \neg\bot, \Delta}[\neg\bot R] \quad \frac{}{\Gamma, \bot \succ \Delta}[\bot L]$$

Figure: $\top$ and $\bot$ rules.

$$\frac{}{\Gamma, \neg * \succ \Delta}[\neg * L] \quad \frac{}{\Gamma, * \succ \Delta}[*L] \quad \frac{}{\Gamma \succ \neg *, \Delta}[\neg * R] \quad \frac{}{\Gamma \succ *, \Delta}[*R]$$

Figure: $*$ rules.

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Figure: $*$ rules.

Theorem 1

Let $\mathcal{V}^{3Pc}$ be the set of valuations that assign the connectives $\wedge$, $\vee$ and $\neg$ a truth-function, and assign $v(\top) = T$, $v(\bot) = F$ and $v(*) = i$. $Ded^{k3}_{3Pc}(\vdash^{\texttt{SET-SET}}_{\mathbb{K}3^P}) = Ded^{lp}_{3Pc}(\vdash^{\texttt{SET-SET}}_{\mathbb{LP}^p}) = \mathcal{V}^{\mathcal{SKL}}$

- Both $\vdash^{\texttt{SET-SET}}_{\mathbb{K}3^p}$ and $\vdash^{\texttt{SET-SET}}_{\mathbb{LP}^p}$ are categorical for $\vDash^{k3(lp)}_{\mathcal{V}^{\mathcal{SKL}}}$ with respect to $\mathcal{V}^{3Pc}$.

Negation

Theorem 2

Let $\mathcal{V}^{3\neg}$ be the subset of $\mathcal{V}^{3}$ where valuation functions are restricted to the ones that interpret $\neg$ according to its SKL truth function. Then $Ded^{3\neg}_{k3}(\vdash^{SET\text{-}SET}_{\mathbb{K}3}) = Ded^{3\neg}_{lp}(\vdash^{SET\text{-}SET}_{\mathbb{LP}}) = \mathcal{V}^{\mathcal{SKL}}$

- $\vdash^{\mathrm{SET\text{-}SET}}_{\mathbb{K}3}$ and $\vdash^{\mathrm{SET\text{-}SET}}_{\mathbb{LP}}$ are categorical for $\vDash^{k3(lp)}_{\mathcal{V}^{\mathcal{SKL}}}$ with respect to $\mathcal{V}^{3\neg}$.

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- Although well-motivated compositionality does not solve the categoricity problem when we diverge from Classical Logic. Can we also motivate restricting the valuation space to interpret the propositional constants correctly?
- Reason 1: “Note that one must choose the semantic values of expressions belonging to a given syntactic category. Only then does compositionality make a definite contribution . . ..” Bonnay and Westerstahl [6]
- Reason 2: We need to fix the meaning of propositional constants in non-classical logics to avoid Liar or Curry-Paradoxes. (See Beall et al. [3] and Restall [23]).

- Can we motivate the restriction to prove Theorem 2?

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- It is plausible to argue that we need to have a prior conception of negation or a negation related concept such as incompatibility to express disagreements in a language. (See Price [20], Berto and Restall [5], Rumfitt [24], Humberstone [15].)

Conclusion and Future Work

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- Although not as strong as in the case of Classical Logic, both of the solutions seem to be promising with respect to their philosophical motivations.
- Next step is to look at the proof-theoretic solutions to the categoricity problem for $\mathbb{K}3$ and $\mathbb{LP}$. There is already some work using the proof-theoretic strategy (See Hjortland [14].)

Thank You!

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