From truthmakers to information states: disjunction and negation in BSML

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BSML: teams and bilateralism

- Team semantics: formulas interpreted wrt a set of points of evaluation (a team) rather than single ones [Väänänen 2007; Yang & Väänänen 2017]

Classical vs team-based modal logic

$[M = \langle W, R, V \rangle]$

- Classical modal logic: (truth in worlds)

$$M, w \models \phi, \text{ where } w \in W$$

- Team-based modal logic:

$$M, t \models \phi, \text{ where } t \subseteq W$$

Bilateral state-based modal logic (BSML)

- Teams $\mapsto$ information states [Groenendijk *et al.* 1996; Ciardelli *et al.* 2019]
- Assertion & rejection conditions are modeled rather than truth [Rumfitt00]

$$M, s \models \phi, \text{ “}\phi \text{ is assertable in } s\text{”}, \quad \text{with } s \subseteq W$$
$$M, s \mathrel{=\!\!|} \phi, \text{ “}\phi \text{ is rejectable in } s\text{”}, \quad \text{with } s \subseteq W$$

- In BSML inferences relate speech acts rather than propositions and therefore might diverge from classical semantic entailments

BSML: broad motivation

- Main goal: a formal account of a class of natural language inferences which diverge from *classical entailments* but also from canonical *conversational implicatures*
  - Ignorance inferences: modified numerals and epistemic indefinites
  - Free choice (FC) phenomena: disjunction and indefinites
  - ...
- Strategy: develop **logics of conversation** which model next to literal meanings also pragmatic factors and the additional inferences which arise from their interaction
- Novel hypothesis: **neglect-zero** tendency as crucial pragmatic factor

> *One final remark: my specific motivation for developing this account of indicative conditionals is of course to solve a puzzle [...] But I have a broader motivation which is perhaps more important. That is to defend, by example, the claim that the concepts of pragmatics (the study of linguistic contexts) can be made as mathematically precise as any of the concepts of syntax and formal semantics; to show that one can recognize and incorporate into abstract theory the extreme context dependence which is obviously present in natural language without any sacrifice to standards of rigor*
>
> *[Stalnaker, 1975, Indicative Conditionals]*

Information states vs truthmakers

- Failure of bivalence in BSML:

$$M, s \not\models p \;\&\; M, s \not\models \neg p, \text{ for some info state } s$$

- Info states: less determinate than possible worlds
  - just like truthmakers, situations, possibilities, ...
- Technically:
  - Truthmakers/possibilities: points in a partially ordered set
  - Info states: sets of possible worlds, also elements of a partially ordered set, the Boolean lattice $Pow(W)$
- Thus systems using these structures are closely connected, although might diverge in motivation:
  - Truthmaker & Possibility semantics: description of ontological structures in the world
  - BSML & Inquisitive semantics: explaining patterns in inferential & communicative human activities
- TODAY'S TALK:
  1. BSML and its main application: FC inferences as neglect-zero effects
  2. Comparison via translations in Modal Information Logic [vBenthem19]

BSML: specific motivation

The challenge of free choice (FC)

- Classical examples of FC inferences:

(1) Deontic FC inference [Kamp 1973]

a. You may go to the beach *or* to the cinema.
b. $\rightsquigarrow$ You may go to the beach *and* you may go to the cinema.

(2) Epistemic FC inference [Zimmermann 2000]

a. Mr. X might be in Victoria *or* in Brixton.
b. $\rightsquigarrow$ Mr. X might be in Victoria *and* he might be in Brixton.

- Logical rendering of FC inferences:

(3) $\Diamond(\alpha \vee \beta) \rightsquigarrow \Diamond\alpha \wedge \Diamond\beta$ (NB: $\Diamond\alpha \wedge \Diamond\beta \neq \Diamond(\alpha \wedge \beta)$)

The paradox of free choice

- Free choice permission in natural language:

(4) You may (A or B) $\rightsquigarrow$ You may A

- But (5) not valid in standard deontic logic (von Wright 1968):

(5) $\Diamond(\alpha \vee \beta) \rightarrow \Diamond\alpha$ [Free Choice (FC) Principle]

- Plainly making FC Principle valid, for example by adding it as an axiom, would not do (Kamp 1973):

(6)
1. $\Diamond a$ [assumption]
2. $\Diamond(a \vee b)$ [from 1, by classical reasoning]
3. $\Diamond b$ [from 2, by FC principle]

- The step leading to 2 in (6) uses the following valid principle:

(7) $\Diamond\alpha \rightarrow \Diamond(\alpha \vee \beta)$

- Natural language counterpart of (7), however, seems invalid:

(8) You may post this letter $\not\rightsquigarrow$ You may post this letter or burn it. [Ross's paradox]

$\Rightarrow$ Intuitions on natural language in direct opposition to the principles of classical logic

Reactions to paradox

- Paradox of Free Choice Permission:

(9)
1. $\Diamond a$ [assumption]
2. $\Diamond(a \vee b)$ [from 1, by addition + monotonicity]
3. $\Diamond b$ [from 2, by FC principle]

- Pragmatic (neo-Gricean) solutions [⇒ keep logic as is]
  - FC inferences are pragmatic inferences (conversational implicatures)
  - ⇒ step leading to 3 is unjustified
- Grammatical solutions [⇒ keep logic as is]
  - FC inferences result from application of covert grammatical operator
  - ⇒ step leading to 3 is unjustified
- Semantic solutions [⇒ change the logic]
  - FC inferences are semantic entailments (e.g., Aloni 2007)
  - ⇒ step leading to 3 is justified, but step leading to 2 is no longer valid (or transitivity fails, e.g., Goldstein 2019)
- BSML [⇒ change the logic]
  - FC inferences as **neglect-zero** effects only predicted for enriched $[\phi]^+$

(10)
1. $\Diamond a$
2. $\Diamond(a \vee b) \quad \neq \quad [\Diamond(a \vee b)]^+$
3. $\qquad\qquad\qquad\quad \Diamond b$

Novel hypothesis: neglect-zero

- FC and related inferences are
  - neither the result of conversational reasoning (as proposed in neo-gricean approaches) [$\neq$ canonical conversational implicatures]
  - nor the effect of optional applications of grammatical operators (as in the grammatical view of implicatures) [$\neq$ scalar implicatures]
- Rather they are a straightforward consequence of something else speakers do in conversation, namely,
  - **Neglect-Zero**
    when interpreting a sentence people create structures representing reality (Johnson-Laird 1983) and in doing so they tend to neglect structures which (vacuously) verify the sentence by virtue of some empty configuration (*zero-models*)
- Neglect-zero tendency follows from the difficulty of the cognitive operation of evaluating truths with respect to empty witness sets (Nieder 2016, Bott et al, 2019)

Novel hypothesis: neglect-zero

Illustrations

(11) Every square is black.

a. Verifier: [■, ■, ■]
b. Falsifier: [■, □, ■]
c. Zero-models: [ ]; [△, △, △]; [◇, ▲, ◇]

(12) Less than three squares are black.

a. Verifier: [■, □, ■]
b. Falsifier: [■, ■, ■]
c. Zero-models: [ ]; [△, △, △]; [◇, ▲, ◇]; [□, □, □]

- Cognitive difficulty of zero-models confirmed by experimental findings from number cognition and also can explain
  - the special status of 0 among the natural numbers (Nieder, 2016)
  - why downward-monotonic quantifiers are more costly to process than upward-monotonic ones (Bott et al., 2019)
  - existential import effects operative in the logic of Aristotle (*every square is black* ⇒ *some square is black*) (Geurts, 2007)
- Core idea: tendency to neglect zero-models, assumed to be operative in ordinary conversation, explains FC and related inferences

Novel hypothesis: neglect-zero

Comparison with competing accounts of FC inference

	NS FC	**Dual Prohib**	**Universal** FC	**Double Neg**	**WS** FC
Neo-Gricean	yes	yes	no	?	no
Grammatical	yes	yes	yes	no	no
Semantic	yes	no*	yes	no*	no
Neglect-zero	yes	yes	yes	yes	yes

Arguments in favor of neglect-zero hypothesis

- Empirical coverage: FC sentences give rise to complex pattern of inferences

(13)
a. $\Diamond(\alpha \vee \beta) \rightsquigarrow \Diamond\alpha \wedge \Diamond\beta$ [Narrow Scope FC]
b. $\neg\Diamond(\alpha \vee \beta) \rightsquigarrow \neg\Diamond\alpha \wedge \neg\Diamond\beta$ [Dual Prohibition]
c. $\forall x\Diamond(\alpha \vee \beta) \rightsquigarrow \forall x(\Diamond\alpha \wedge \Diamond\beta)$ [Universal FC]
d. $\neg\neg\Diamond(\alpha \vee \beta) \rightsquigarrow \Diamond\alpha \wedge \Diamond\beta$ [Double Negation FC]
e. $\Diamond\alpha \vee \Diamond\beta \rightsquigarrow \Diamond\alpha \wedge \Diamond\beta$ [Wide Scope FC]

- Captured by neglect-zero approach implemented in BSML (Aloni22)
- Most other approaches need additional assumptions

The data

(14) Dual Prohibition [Alonso-Ovalle 2006]

a. You are not allowed to eat the cake or the ice-cream.
$\rightsquigarrow$ You are not allowed to eat either one.

b. $\neg\Diamond(\alpha \vee \beta) \rightsquigarrow \neg\Diamond\alpha \wedge \neg\Diamond\beta$

(15) Universal FC [Chemla 2009]

a. All of the boys may go to the beach or to the cinema.
$\rightsquigarrow$ All of the boys may go to the beach and all of the boys may go to the cinema.

b. $\forall x\Diamond(\alpha \vee \beta) \rightsquigarrow \forall x(\Diamond\alpha \wedge \Diamond\beta)$

(16) Double Negation FC [Gotzner et al. 2020]

a. Exactly one girl cannot take Spanish or Calculus.
$\rightsquigarrow$ One girl can take neither of the two and each of the others can choose between them.

b. $\exists x(\neg\Diamond(\alpha(x) \vee \beta(x)) \wedge \forall y(y \neq x \rightarrow \neg\neg\Diamond(\alpha(y) \vee \beta(y)))) \rightsquigarrow$
$\exists x(\neg\Diamond\alpha(x) \wedge \neg\Diamond\beta(x) \wedge \forall y(y \neq x \rightarrow (\Diamond\alpha(y) \wedge \Diamond\beta(y))))$

(17) Wide Scope FC [Zimmermann 2000]

a. Detectives may go by bus or they may go by boat.
$\rightsquigarrow$ Detectives may go by bus and may go by boat.

b. Mr. X might be in Victoria or he might be in Brixton.
$\rightsquigarrow$ Mr. X might be in Victoria and might be in Brixton.

c. $\Diamond\alpha \vee \Diamond\beta \rightsquigarrow \Diamond\alpha \wedge \Diamond\beta$

Novel hypothesis: neglect-zero

Comparison with competing accounts

	Ignorance inference	**FC inference**	**Scalar implicature**
Neo-Gricean	reasoning	reasoning	reasoning
Grammatical view	debated	grammatical	grammatical
MA proposal	neglect-zero	neglect-zero	—

- Miriam ate the cake or the ice-cream
  $\rightsquigarrow$ not both (scalar); speaker doesn't know which (ignorance)

Arguments in favor of neglect-zero hypothesis

- Cognitive plausibility: differences between FC and scalar implicatures (Chemla & Bott, 2014; Tieu et al, 2016):

	processing cost	**acquisition**
FC inference	low	early
scalar implicature	high	late

  - Expected on neglect-zero hypothesis:
    - FC inference follows from the assumption that when interpreting sentences language users neglect zero-models
    - Zero-models neglected because cognitively taxing
  - Harder to explain on neo-Gricean or grammatical view

Bilateral State-Based Modal Logic (BSML)

Language

$$\phi \quad := \quad p \mid \neg\phi \mid \phi \vee \phi \mid \phi \wedge \phi \mid \Diamond\phi \mid \mathrm{NE}$$

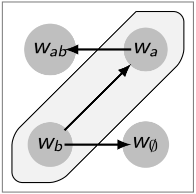
where $p \in A$.

Models and States

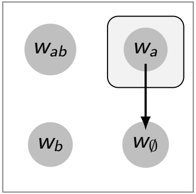
- Classical Kripke models: $M = \langle W, R, V \rangle$
- States: $s \subseteq W$, sets of worlds in a Kripke model

Examples

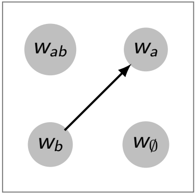
for $A = \{a, b\}$



(a) $\not\models a$; $\models \Diamond a$



(b) $\models a$; $\not\models \Diamond a$



(c) $\models a \wedge \neg a$

Semantic clauses

$[M = \langle W, R, V\rangle;\ s, t, t' \subseteq W]$

$$
\begin{array}{rcl}
M, s \models p & \text{iff} & \forall w \in s : V(w, p) = 1 \\
M, s \dashv p & \text{iff} & \forall w \in s : V(w, p) = 0 \\
M, s \models \neg\phi & \text{iff} & M, s \dashv \phi \\
M, s \dashv \neg\phi & \text{iff} & M, s \models \phi \\
M, s \models \phi \vee \psi & \text{iff} & \exists t, t' : t \cup t' = s \ \&\ M, t \models \phi \ \&\ M, t' \models \psi \\
M, s \dashv \phi \vee \psi & \text{iff} & M, s \dashv \phi \ \&\ M, s \dashv \psi \\
M, s \models \phi \wedge \psi & \text{iff} & M, s \models \phi \ \&\ M, s \models \psi \\
M, s \dashv \phi \wedge \psi & \text{iff} & \exists t, t' : t \cup t' = s \ \&\ M, t \dashv \phi \ \&\ M, t' \dashv \psi \\
M, s \models \Diamond\phi & \text{iff} & \forall w \in s : \exists t \subseteq R[w] : t \neq \emptyset \ \&\ M, t \models \phi \\
M, s \dashv \Diamond\phi & \text{iff} & \forall w \in s : M, R[w] \dashv \phi \\
M, s \models \mathrm{NE} & \text{iff} & s \neq \emptyset \\
M, s \dashv \mathrm{NE} & \text{iff} & s = \emptyset
\end{array}
$$

where $R[w] = \{v \in W \mid wRv\}$

Box

- $\Box\phi := \neg\Diamond\neg\phi$

$$M, s \models \Box\phi \quad \text{iff} \quad \text{for all } w \in s : M, R[w] \models \phi$$
$$M, s \dashv \Box\phi \quad \text{iff} \quad \text{for all } w \in s : \text{there is a } t \subseteq R[w] : t \neq \emptyset \ \& \ M, t \dashv \phi$$

where $R[w] = \{v \in W \mid wRv\}$

Logical consequence

- $\phi \models \psi$ iff for all M, s : $M, s \models \phi \;\Rightarrow\; M, s \models \psi$

Proof theory

- See Anttila 2021[1]; Anttila et al. 2022.

State-sensitive constraints on accessibility relation

- R is indisputable in (M, s) iff $\forall w, v \in s : R[w] = R[v]$
- R is state-based in (M, s) iff $\forall w \in s : R[w] = s$

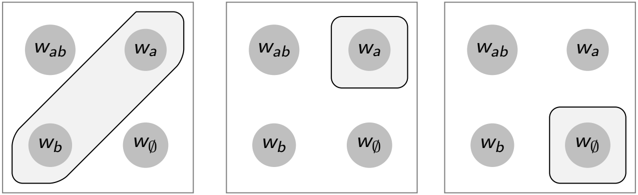
Proposal: epistemics $\mapsto$ state-based; deontics $\mapsto$ sometimes indisputable

[1]Anttila, A. 2021. The Logic of Free Choice. Axiomatizations of State-based Modal Logics, MSc Logic thesis, ILLC, UvA.

Neglect-zero effects in BSML: core idea

- A state s supports a **disjunction** $\phi \vee \psi$ iff s is the union of two substates, each supporting one of the disjuncts

$$M, s \models \phi \vee \psi \text{ iff } \exists t, t' : t \cup t' = s \;\&\; M, t \models \phi \;\&\; M, t' \models \psi$$



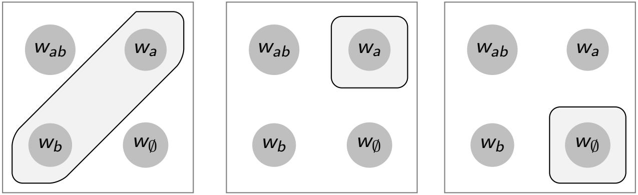
(d) Verifier (e) Zero-model (f) Falsifier

Figure: Models for $(a \vee b)$.

- $\{w_a\} \models (a \vee b)$, because we can find substates supporting each disjunct: $\{w_a\}$ itself, supporting a, and $\emptyset$, vacuously supporting b $\Rightarrow$ $\{w_a\}$ is a **zero-model** for $(a \vee b)$, a model which verifies the formula by virtue of an empty witness
- Main idea: core effect of neglect-zero enrichments, $[\;]^+$, is to rule out such zero-models

Neglect-zero effects in BSML: core idea

- A state s supports an **enriched disjunction** $[\phi \vee \psi]^+$ iff s is the union of two non-empty substates, each supporting one of the disjuncts



(a) $\models [a \vee b]^+$ (b) $\not\models [a \vee b]^+$ (c) $\not\models [a \vee b]^+$

- An enriched disjunction requires both disjuncts to be live possibilities

(18) Maria is in Amsterdam or in Torino $\rightsquigarrow$ Maria might be in Amsterdam and Maria might be in Torino

- Aloni 2022[2] defines neglect-zero enrichments in terms of non-emptiness atom (NE) from team logic

[2] Maria Aloni (2022) Logic and conversation: the case of free choice. *Semantics and Pragmatics (2022)*

Neglect-zero effects in BSML: implementation

- Non-emptiness atom (NE): NE requires the supporting state to be non-empty

$$M, s \models \text{NE} \quad \text{iff} \quad s \neq \emptyset$$

- Pragmatic enrichment function: Pragmatically enriched formula $[\alpha]^+$ comes with the requirement to satisfy NE distributed along each of its subformulas:

$$\begin{aligned}
[p]^+ &= p \wedge \text{NE} \\
[\neg\alpha]^+ &= \neg[\alpha]^+ \wedge \text{NE} \\
[\alpha \vee \beta]^+ &= ([\alpha]^+ \vee [\beta]^+) \wedge \text{NE} \\
[\alpha \wedge \beta]^+ &= ([\alpha]^+ \wedge [\beta]^+) \wedge \text{NE} \\
[\Diamond\alpha]^+ &= \Diamond[\alpha]^+ \wedge \text{NE}
\end{aligned}$$

- Main result: in BSML $[\ ]^+$-enrichment has non-trivial effect only when applied to positive disjunctions
  - $\mapsto$ we derive FC effects (for pragmatically enriched formulas);
  - $\mapsto$ pragmatic enrichment vacuous under single negation.

Neglect-zero effects in BSML: predictions

After pragmatic enrichments

- We derive both wide and narrow scope FC inferences for pragmatically enriched formulas:
  - Narrow scope FC: $[\Diamond(\alpha \vee \beta)]^+ \models \Diamond\alpha \wedge \Diamond\beta$
  - Universal FC: $[\forall x \Diamond(\alpha \vee \beta)]^+ \models \forall x(\Diamond\alpha \wedge \Diamond\beta)$
  - Double negation FC: $[\neg\neg\Diamond(\alpha \vee \beta)]^+ \models \Diamond\alpha \wedge \Diamond\beta$
  - Wide scope FC: $[\Diamond\alpha \vee \Diamond\beta]^+ \models \Diamond\alpha \wedge \Diamond\beta$ (if R is indisputable)
- while no undesirable side effects obtain with other configurations:
  - Dual prohibition: $[\neg\Diamond(\alpha \vee \beta)]^+ \models \neg\Diamond\alpha \wedge \neg\Diamond\beta$

Before pragmatic enrichments

- The NE-free fragment of BSML is equivalent to classical modal logic:

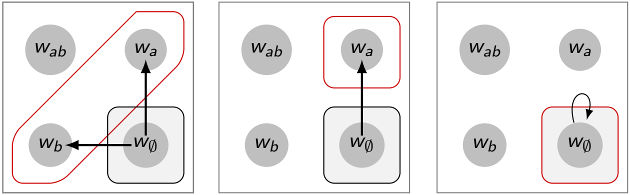
$$\alpha \models_{BSML} \beta \text{ iff } \alpha \models_{CML} \beta \qquad [\text{if } \alpha, \beta \text{ are NE-free}]$$

- But we can capture infelicity of epistemic contradictions (Yalcin, 2007) by putting team-based constraints on accessibility relation:
  1. Epistemic contradiction: $\Diamond\alpha \wedge \neg\alpha \models \bot$ (if R is state-based)
  2. Non-factivity: $\Diamond\alpha \not\models \alpha$

Neglect-zero effects in BSML: illustrations

- **Free choice** results rely on relational notion of modality:
  - A state s supports $\Diamond\phi$ iff for every w in s there is a non-empty subset of the set of worlds accessible from w which supports ϕ:

$$M, s \models \Diamond\phi \text{ iff } \forall w \in s : \exists t \subseteq R[w] : t \neq \emptyset \;\&\; M, t \models \phi$$



(d) Verifier (e) Zero-model (f) Falsifier

Figure: Models for $\Diamond(a \vee b)$.

Neglect-zero effects in BSML: illustrations

- **Negation** facts follow from adopted bilateralism (we validate De Morgan laws and $\neg\neg$-elimination):
  - Adding $\textsc{ne}$ vacuous under single negation:

$$\neg(\alpha \wedge \textsc{ne}) \equiv \neg\alpha \vee \neg\textsc{ne} \equiv \neg\alpha \vee \bot \equiv \neg\alpha$$

  - Adding $\textsc{ne}$ non-vacuous under double negation:

$$\neg\neg(\alpha \wedge \textsc{ne}) \equiv \alpha \wedge \textsc{ne} \not\equiv \neg\neg\alpha$$

$\Rightarrow$ Failure of replacement under negation:

$$\phi \equiv \psi \not\Rightarrow \neg\phi \equiv \neg\psi$$

$\Rightarrow$ Empirically correct predictions:

- $\textsc{fc}$ effects systematically disappear under **single negation** (Alonso-Ovalle 2006)
- But speakers draw $\textsc{fc}$ inferences under **double negation** (Gotzner et al. 2020)

$\Rightarrow$ Without replacement failure impossible to capture these data

Comparisons via translation

- Modal Information Logic (MIL) (van Benthem, 1989, 2019):[3] common ground where related systems can be interpreted and their connections and differences can be explored
- Next: (simplified) translations into MIL of the following systems:
  - BSML
  - Truthmaker semantics (Fine)
  - Possibility semantics (Humberstone, Holliday)
  - Inquisitive semantics (Ciardelli, Groenendijk & Roelofsen)

  (cf. Gödel's (1933) translation of intuitionistic logic into modal logic)
- Focus on propositional fragments (no modalities)
  - disjunction
  - negation
- (Based on work in progress with Nick Bezhanishvili, Johan van Benthem and Alexandru Baltag)

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[3] Johan van Benthem (2019) Implicit and Explicit Stances in Logic, *Journal of Philosophical Logic*.

Modal Information Logic (MIL)

Language

$$\phi \quad ::= \quad p \mid \neg\phi \mid \phi \wedge \phi \mid \phi \vee \phi \mid \langle sup \rangle \phi\psi$$

where $p \in A$.

Models and interpretation

Formulas are interpreted on triples $M = (X, \leq, V)$ where $\leq$ is a preorder

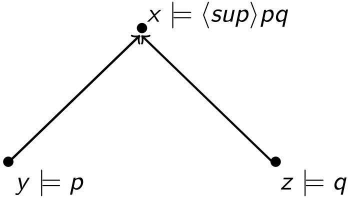
$$\begin{array}{rcl}
\mathcal{M}, x \models p & \text{iff} & x \in V(p) \\
\mathcal{M}, x \models \neg\phi & \text{iff} & \mathcal{M}, x \not\models \phi \\
\mathcal{M}, x \models \phi \wedge \psi & \text{iff} & \mathcal{M}, x \models \phi \text{ and } \mathcal{M}, x \models \psi \\
\mathcal{M}, x \models \phi \vee \psi & \text{iff} & \mathcal{M}, x \models \phi \text{ or } \mathcal{M}, x \models \psi \\
\mathcal{M}, x \models \langle sup \rangle \phi\psi & \text{iff} & \text{there are } y, z : x = sup_{\leq}(y, z) \;\&\; \mathcal{M}, y \models \phi \;\&\; \mathcal{M}, z \models \psi
\end{array}$$

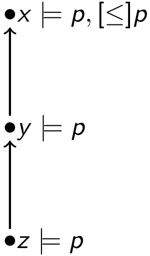
$[\leq]\phi = \neg\langle sup \rangle(\neg\varphi)\top$

$$\mathcal{M}, x \models [\leq]\phi \quad \text{iff} \quad \text{for all } y : y \leq x \;\Rightarrow\; \mathcal{M}, y \models \phi$$

Modal Information Logic (MIL)

Examples





Translations into Modal Information Logic

- BSML (non-modal NE-free fragment): $\leq$ is subset relation $\subseteq$

$$
\begin{array}{rcl}
& \cdots & \\
(\neg\phi)^+ & = & (\phi)^- \\
(\neg\phi)^- & = & (\phi)^+ \\
(\phi \vee \psi)^+ & = & \langle sup \rangle (\phi)^+ (\psi)^+ \\
(\phi \vee \psi)^- & = & (\phi)^- \wedge (\psi)^- \\
(\phi \wedge \psi)^+ & = & (\phi)^+ \wedge (\psi)^+ \\
(\phi \wedge \psi)^- & = & \langle sup \rangle (\phi)^- (\psi)^- \\
& \cdots &
\end{array}
$$

- Truthmaker semantics (Fine): $\leq$ is "part of" relation

$$
\begin{array}{rcl}
& \cdots & \\
(\neg\phi)^+ & = & (\phi)^- \\
(\neg\phi)^- & = & (\phi)^+ \\
(\phi \vee \psi)^+ & = & (\phi)^+ \vee (\psi)^+ \\
(\phi \vee \psi)^- & = & \langle sup \rangle (\phi)^- (\psi)^- \\
(\phi \wedge \psi)^+ & = & \langle sup \rangle (\phi)^+ (\psi)^+ \\
(\phi \wedge \psi)^- & = & (\phi)^- \vee (\psi)^- \\
& \cdots &
\end{array}
$$

Translations into Modal Information Logic

- Possibility semantics (Humberstone, Holliday)

$$\begin{array}{rcl} & \vdots & \\ tr(\neg\phi) & = & [\leq]\neg tr(\phi) \\ tr(\phi \wedge \psi) & = & tr(\phi) \wedge tr(\psi) \\ tr(\phi \vee \psi) & = & [\leq]\langle\leq\rangle(tr(\phi) \vee tr(\psi)) \\ & \vdots & \end{array}$$

- Inquisitive semantics (Groenendijk, Roelofsen and Ciardelli)

$$\begin{array}{rcl} & \vdots & \\ tr(\neg\phi) & = & [\leq]\neg tr(\phi) \\ tr(\phi \wedge \psi) & = & tr(\phi) \wedge tr(\psi) \\ tr(\phi \vee \psi) & = & tr(\phi) \vee tr(\psi) \\ & \vdots & \end{array}$$

Disjunction and Negation

- Three notions of disjunction expressible in MIL:
  - Boolean disjunction: $\phi \vee \psi$
    [truthmaker semantics, inquisitive logic, classical logic, intuitionistic logic]
  - Lifted/split disjunction: $\langle sup \rangle \phi \psi$
    [BSML, dependence logic, team semantics]
  - Cofinal disjunction: $[co](\phi \vee \psi)$ (where $[co]\phi =: [\leq]\langle\leq\rangle\phi$)
    [possibility semantics, dynamic semantics]
- Three notions of negation:
  - Boolean negation: $\neg\phi$
    [classical logic, ...]
  - Bilateral negation: $(\neg\phi)^+ = (\phi)^-$ & $(\neg\phi)^- = (\phi)^+$
    [truthmaker semantics, BSML, ...]
  - Intuitionistic-like negation: $[\leq]\neg\phi$
    [possibility semantics, inquisitive semantics and intuitionistic logic]
- Some combinations:
  - Boolean disjunction + boolean negation $\mapsto$ classical logic
  - Boolean notions in other combinations can generate non-classicality:
    - Boolean disjunction + intuitionistic negation $\mapsto$ intuitionistic logic
  - Classicality also generated by non-boolean combinations:
    - Split disjunction + bilateral negation (classical fragm. BSML)

Conclusions

- Free choice: a mismatch between logic and language
- Grice's insight:
  - stronger meanings can be derived paying more "attention to the nature and importance to the conditions governing conversation"
- Standard implementation: two separate components
  - Semantics: classical logic
  - Pragmatics: Gricean reasoning

  Elegant picture, but, when applied to FC, empirically inadequate
- My proposal: FC and related infeerences as neglect-zero effects
  - Aloni 2022: neglect-zero effect in BSML (a team-based modal logic)
  - Differences but also interesting connections with TS, PL and IL
  - MIL useful framework for comparisons
- Related (future) research:
  - Logic: proof theory (Anttila (MoL 2021), Yang, MA); bimodal perspective (Baltag, van Benthem, Bezhanishvili, MA); QBSML (MA & van Ormondt); BiUS (MA 2022); qBiUS
  - Language: FC cancellations (Pinton (MoL 2021), Hui (MoL 2021)); modified numerals (van Ormondt & MA); indefinites (Degano & MA); monotonicity failure under attitude verbs (Yan & MA); acquisition (children's conjunctive strengthening of disjunction); experiments.