

Semantic Pollution of Proof Systems

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Proof theory and model-theoretic semantics provide different ways of proving results about logics. Soundness and completeness proofs reveal an intrinsic connection between these proof methods. However, Avron writes that a requirement of a ‘good’ proof system is that it is “independent from any particular semantics” [1]. This has become known as syntactic purity of a proof system, as opposed to a semantically polluted one. The underlying idea seems to be that soundness and completeness proofs are valuable when the syntactic side has a certain independence from the semantic side. If a proof system is semantically polluted, this may take away from its “proof-theoretical nature and the expected generality” [1]. Labeled proof calculi are a standard example of semantically polluted systems, due to their internalization of Kripke semantics into the proof system (see e.g. [4]), but other examples can be found in the literature, including for instance semantic sequents and tableaux and internalized forcing sequents [3].

The goal of this talk is to provide a better conceptual characterization of what semantic pollution is, and to provide and compare formal ways of telling when a proof system is semantically polluted or not. This contributes to a better characterization of what a ‘good’ proof system can be, and encourages a more nuanced understanding of the distinction between syntax and semantics. The literature distinguishes between a strong and a weak definition of syntactic purity. *Strong* syntactic purity occurs when a proof system is “independent of any particular semantics” [1]. This includes the idea that “one should not be able to guess, just from the form of the structures which are used, the intended semantic of a given proof system”. *Weak* syntactic purity, on the other hand, says that a sequent calculus cannot make use of “explicit semantic elements” [3]. Poggiolesi argues that strong syntactic purity is too strong, since it implies that basic propositional sequent calculi already have to be declared semantically polluted. Thus, she adopts weak syntactic purity, where she defines a ‘semantic element’ as an untranslatable ingredient of a sequent. This rules out, for example, expressions like xRy in labeled calculi, that explicitly incorporate the notion of possible worlds and the Kripke accessibility relation.

In this talk, we start out by considering the ways in which an object language can be polluted by additional syntax, generally. After that, we investigate whether we can make sense of semantic pollution as an instance of general pollution. We discuss several conceptions of semantic pollution, and connect them to possible formal measures. For weak syntactic purity, we discourage the idea that translatability is decisive in the formal description of a semantic element. Instead, we aim to spell out requirements on the formal language in a proof system in order to exclude semantic elements, which also helps us understand better why these elements are excluded. Inspiring such requirements is the idea that symbols should not refer to something outside of the informal reasoning they formalize, and that the level of ‘explicitness’ of representation of semantic elements is important. On the latter topic, Poggiolesi & Restall note that elements from Kripke semantics are treated explicitly in labeled systems, but are made implicit in tree-hypersequent systems (reducing the level of semantic pollution) [4]. Read objects that even in tree-hypersequent systems, “the content is still there” [5]. We argue that the particular presentation of content in the syntax does indeed matter for semantic pollution. For strong syntactic purity, finally, we suggest that strong syntactic purity might not be *too* strong after all, and we mention some possible formalizations of ‘recognizing’ semantics from a proof system. For example, the way that proof rules determine the semantics of the logical connectives they define might relate to semantic pollution.

We illustrate the ideas above by comparing the measures for various proof systems, including display sequents, a neighborhood proof system (see [2]) and of course labeled systems, and we reflect on whether our measures of semantic pollution seem to interact with any other philosophical properties.

Keywords. Semantic pollution, syntactic purity, proof systems, labeled sequent calculus

References

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