

Symmetry, locality and hyperintensionality

Maria Beatrice Buonaguidi

King's College London

July 25, 2022

The notion of hyperintensionality and its applications have become of prime importance in contemporary research, to the point that Nolan [4, p. 149] predicted a “hyperintensional revolution” for the 21st century. However, although there seems to be an informal consensus regarding what hyperintensionality is, the formal definitions which have been given for it in the literature differ slightly. According to Cresswell, a propositional context is hyperintensional if it does not license the substitution of *logical equivalents* salva veritate [1, p. 25]. Nolan, on the other hand, defines a position in a sentence to be hyperintensional if substitution of *necessarily equivalents* is not guaranteed to preserve truth value [4, p. 151]. Leitgeb, in a recent paper developing the “hyperintensional logic” HYPE [3], defines hyperintensionality as the failure of substitutivity with respect to classical logical equivalents. HYPE, which is a logic based on the first degree entailment of Anderson and Belnap supported with an intuitionistic conditional, and with a situation state semantics, is described as a background system to model hyperintensional operators. Indeed, a unary connective S is defined to be hyperintensional if prefixing sentences logically equivalent *in classical logic* by S can lead to sentences which differ in truth value *at some state s* in HYPE’s semantics [3, p. 307].

All these definitions refer to propositional contexts or operators, but what it means for a logical consequence relation to be hyperintensional is not fully clear. In a recent paper [5], Odintsov and Wansing criticize the definition of hyperintensionality given by Leitgeb with reference to HYPE. In a non-classical logic such as HYPE defining hyperintensionality with respect to the classical consequence relation is problematic, since the meaning of connectives, even if the language used is the same, will be different from classical logic. Hence, a hyperintensional context in a non-classical logic such as HYPE cannot be defined consistently with respect to classical logic as a context not supporting the intersubstitutivity of classical necessary equivalents.

Instead, they formulate a criterion that judges whether a certain logic is hyperintensional based on its consequence relation: a logic is hyperintensional if it does not satisfy self-extensionality, or congruentiality. A logic L with

language \mathcal{L} is self-extensional if, given its consequence relation \vdash_L , for all formulae φ, ψ, θ of \mathcal{L} and all propositional variables p the following holds:

$$\varphi \dashv\vdash_L \psi \text{ implies } \theta(\varphi/p) \dashv\vdash_L \theta(\psi/p),$$

where $\theta(\varphi/p)$ indicates the result of substituting the formula φ for the variable p in θ .

According to this criterion, the logic HYPE is not hyperintensional, since it satisfies self-extensionality: an example of a truly hyperintensional logic, instead, is Nelson's four-valued paraconsistent logic **N4**, which does not satisfy the self-extensionality criterion. This happens because there is an asymmetry between the clauses for truth preservation and falsity preservation in **N4**, whereas truth preservation and falsity preservation in HYPE are symmetrical.

In the case of HYPE at least, the picture is not as clear-cut as Odintsov and Wansing maintain. Indeed, Odintsov and Wansing only formalise one of the definitions of hyperintensionality in the literature, i.e. the failure of intersubstitutivity with respect to logical equivalents defined by Cresswell. However, when trying to adapt the definitions of hyperintensionality which we find in the literature to the consequence relation of a non-classical logic, they obviously differ. Hence, we need more than one criterion for the hyperintensionality of a logic. Letting \vdash_L be the consequence relation for a logic L which is sound and complete with respect to a given semantic, and indicating by \vdash_{CL} the consequence relation for classical logic, we can distinguish three different notions of hyperintensionality:

1. HYP_L : $\varphi \dashv\vdash_L \psi$ does not imply $\theta(\varphi/p) \dashv\vdash_L \theta(\psi/p)$.
2. HYP_{CL} : If we can establish a translation $(\cdot)^L$ between classical logic and L such that $\vdash_{CL} \varphi$ iff $\vdash_L \varphi^L$, $\varphi \dashv\vdash_{CL} \psi$ fails to imply $\theta(\varphi^L/p) \dashv\vdash_L \theta(\psi^L/p)$.
3. HYP_\square : There is a model \mathcal{M} of L such that $\mathcal{M} \models_L \square(\varphi \leftrightarrow \psi)$ and $\mathcal{M} \not\models_L \theta(\varphi/p) \leftrightarrow \theta(\psi/p)$.¹

The three notions of hyperintensionality are presented in order of strength: in particular, HYP_\square is only slightly stronger than intensionality defined for a logic L with a possible world semantics: there is a model \mathcal{M} of L such that, for some state s of \mathcal{M} , $\mathcal{M}, s \models \varphi \leftrightarrow \psi$ but $\mathcal{M}, s \not\models \theta(\varphi/p) \leftrightarrow \theta(\psi/p)$. Note that the first two definitions are syntactical, while the third one is semantical: although it would be desirable to have a uniform definition, the third definition needs to be defined semantically if we want necessary equivalence to be distinct from theoremhood.

I argue that HYPE can not only be assessed with respect to the criterion HYP_L as Odintsov and Wansing do, but also with respect to the other

¹Note that this last criterion is only applicable to logics with a well-behaved conditional and the Deduction Theorem.

two criteria. Indeed, we can establish a translation between the meaning of the connectives in classical logic and the meaning of the HYPE connectives, since the classical consequence relation coincides with the HYPE consequence relation defined as truth preservation for a restricted class of HYPE models.

I show that HYPE is not hyperintensional with respect to the criterion HYP_{CL} , but it is “hyperintensional” with respect to the weakest criterion HYP_{\square} , which I define as a criterion of “weak hyperintensionality” or “strong intensionality”. There appears to be a divergence between the level of HYPE’s consequence relation and HYPE’s (im)possible world semantics: HYPE’s consequence relation is closer to classicality than HYPE’s semantics, thus making HYPE-models a suitable framework for hyperintensional operators.

Indeed, there seems to be an interesting divergence, in HYPE, between the behaviour of the logic at the semantical and syntactic level. I present a peculiar example of this tendency. Leitgeb argued with a model-theoretic proof that HYPE has the disjunction property. However, HYPE can be shown to be equivalent to the logic N_i^* , which has been shown not to have the disjunction property by Drobyshevich [2, p. 400]. Indeed, HYPE does not have the disjunction property, but the nature of its intuitionistic conditional, which shapes its consequence relation, “mimicks” the disjunction property at a semantical level. In other words, although, just like it is for classical logic, in a HYPE-model \mathcal{M} , at any state s , if $\mathcal{M}, s \models A \vee B$, then $\mathcal{M}, s \models A$ or $\mathcal{M}, s \models B$, it is possible to reproduce in HYPE something very similar to the model-theoretic proof of the disjunction property for intuitionistic logic, and this not only generates confusion, but also a great deal of semantic flexibility. I argue that it is exactly this semantic flexibility which generates HYPE’s “weak hyperintensionality” and which makes it particularly suited to the formulation of strong non-classical theories.

Non-classical logics, hyperintensionality, hyperintensional logics

References

- [1] M. J. Cresswell. “Hyperintensional Logic”. In: *Studia Logica* 34.1 (1975), pp. 25–38. DOI: 10.1007/bf02314421.
- [2] S. A. Drobyshevich. “Double Negation Operator in Logic N^* ”. In: *Journal of Mathematical Sciences* 205.3 (Mar. 2015), pp. 389–402. ISSN: 1573-8795. DOI: 10.1007/s10958-015-2254-3. URL: <https://doi.org/10.1007/s10958-015-2254-3>.
- [3] Hannes Leitgeb. “HYPE: A System of Hyperintensional Logic”. In: *Journal of Philosophical Logic* 48.2 (2019), pp. 305–405. DOI: 10.1007/s10992-018-9467-0.
- [4] Daniel Nolan. “Hyperintensional Metaphysics”. In: *Philosophical Studies* 171.1 (2014), pp. 149–160. DOI: 10.1007/s11098-013-0251-2.

- [5] Sergei Odintsov and Heinrich Wansing. “Routley Star and Hyperintensionality”. In: *Journal of Philosophical Logic* 50.1 (2021), pp. 33–56. DOI: 10.1007/s10992-020-09558-5.