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First Year PUC Annual Examination, February/March - 2014

Subject : MATHEMATICS

Time : 3 Hours 15 Min

Max. Marks : 100

Instructions :

- i) The question paper has Five parts namely A, B, C, D and E. Answer All the parts.
- ii) Use the graph sheet for the question on Linear inequalities in Part - D.

Part - A

Answer All the questions :

10 x 1 = 10

1. If $A = \{1, 2, 3, 4\}$, $B = \{2, 4, 6, 8\}$ find $A - B$.
2. If $f(x) = x^2$ and $g(x) = 2x + 1$ be two real valued functions then find $(f + g)(x)$.
3. Express 105° into Radians.
4. If $Z = i - \sqrt{3}$ find the modulus of Z .
5. If ${}^nC_9 = {}^nC_8$ then find 'n'.
6. Write the first three terms of the sequence defined by $a_n = 2n + 5$.
7. Find the x - intercept of the line $3x + 4y - 12 = 0$
8. Evaluate : $\lim_{x \rightarrow 0} \left(\frac{\tan 10x}{x} \right)$
9. Write the negation of "The number 2 is greater than 9."
10. A coin is tossed twice what is the probability that atleast one tail occurs.

Part - B

Answer any Ten questions :

10x2=20

11. If $U = \{0, 1, 2, 3, 4, 5, 6\}$ $A = \{1, 2, 4\}$ and $B = \{1, 3, 5\}$ then show that $(A \cup B)' = A' \cap B'$
12. If $A = \{3, 5, 7, 9\}$ $B = \{5, 7, 9, 11\}$ and $C = \{12, 13\}$ then find $A \cap (B \cup C)$.
13. If $A = \{1, 2, 3, 4, 6\}$ Relation 'R' is defined by $R = \{(a, b) / a, b \in A \text{ 'b' is exactly divisible by 'a'}\}$ then find Range and Domain of R.
14. In a circle of diameter 40 cms the length of a Chord is 20 cm. Find the length of a minor arc of the Chord. (Use : $\pi = \frac{22}{7}$)
15. Find the value of $\sin(75^\circ)$.
16. Express $1 + i\sqrt{3}$ in Polar form.
17. Solve the inequality $5x - 3 < 3x + 1$ when 'x' is a real number.
18. Show that the points $(0, 7, 10)$ $(-1, 6, 6)$ and $(-4, 9, 6)$ are the vertices of a right angled triangle.
19. Find the equation of a line perpendicular to the line $5x - 6y + 3 = 0$ and passing through the point $(2, -3)$.
20. Write the equation of the line through the points $(1, -1)$ and $(3, 5)$.
21. Evaluate : $\lim_{x \rightarrow 2} \left(\frac{3x^2 - x - 10}{x^2 - 4} \right)$

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22. Write the contrapositive and converse of "If a number is divisible by 9 then it is divisible by 3."
23. Co efficient of variation of a distribution is 80 and the Standard Deviation is 16, what is the Arithmetic Mean of the distribution?
24. A letter is chosen at random from the word 'ASSASSINATION'. Find the probability that letter is :
a) a Vowel b) a Consonant

Part - C

Answer any Ten questions :

10 x 3 = 30

25. In a group of 70 people 37 like Coffee, 52 like Tea and each person likes atleast one of the two drinks. How ^{many people like} atleast one of the two drinks? How many people like both Coffee and Tea? How many like Tea only and not Coffee?

26. If $f(x) = \begin{cases} 3x + 1 & 0 \leq x \leq 2 \\ 1 + 9x & 2 < x < 3 \\ 30 + 2x & x \geq 3 \end{cases}$

find i) $f(1)$ ii) $f\left(\frac{5}{2}\right)$ iii) $f(4)$

27. Prove that $\frac{\sin 5x - 2 \sin 3x + \sin x}{\cos 5x - \cos x} = \tan x$.

28. If $x + iy = \frac{a + ib}{a - ib}$ then show that $x^2 + y^2 = 1$.

29. Find the Conjugate of $\frac{(3 - 2i)(2 + 3i)}{(1 + 2i)(2 - i)}$

30. In how many ways can the word "PERMUTATIONS" be arranged, if the Vowels are all together? And, if the words start with P and end with S.

31. Using Binomial theorem find :

$(a + b)^4 + (a - b)^4$. Hence evaluate

$(3 + \sqrt{2})^4 + (3 - \sqrt{2})^4$.

32. The sum of first three terms of a G. P is 16 and the sum of the next three terms is 128. Find the first term and the common ratio.

33. Find the sum to 'n' terms of the sequence $8 + 88 + 888 + \dots$

34. Find the equation of the ellipse in the Standard form given that the foci is $(\pm 3, 0)$ and major axis is 8 units.

35. Find the derivative of $\sin x$ with respect to 'x' from the first principle.

36. Verify by the method of contradiction that " $\sqrt{3}$ is irrational."

37. Two students Anil and Sunil appears in an examination the probability that Anil wil qualify the examination is 0.05 and Sunil will qualify is 0.10. The probability that both will qualify in the examination is 0.02. Find the probability that Anil and Sunil will not qualify in the examination.

38. A committee of two persons is selected from 2. Boys and 2 Girls. What is the probability that the committee will have i) No Boys ii) 2 - Boys.

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Part - D

Answer any Six questions :

6 x 5=30

39. Define Modulus Function. Draw the graph of the Modulus Function. Write the domain and range of the function.

40. Prove that $\cos^2 x + \cos^2\left(x + \frac{\pi}{3}\right) + \cos^2\left(x - \frac{\pi}{3}\right) = \frac{3}{2}$

41. For all $n \geq 1$ prove by using mathematical induction that $1^2 + 2^2 + 3^2 + \dots = \frac{n(n+1)(2n+1)}{6}$

42. Solve the following system of inequalities graphically :

$$x - 2y \leq 3, 3x + 4y \geq 12, x \geq 0, y \geq 1.$$

43. For all real numbers a, b and positive integer 'n' prove that :

$$(a + b)^n = {}^n C_0 a^n + {}^n C_1 a^{n-1} b + {}^n C_2 a^{n-2} b^2 + \dots + {}^n C_{n-1} a b^{n-1} + {}^n C_n b^n$$

44. A group consists of 8 boys and 6 girls. Find the number of ways in which a team of 5 members can be selected so as to have atleast one boy and one girl.

45. Derive an expression for the co-ordinates of a point that divides the line joining the points A : (x_1, y_1, z_1) and B : (x_2, y_2, z_2) internally in the ratio m : n. Hence find the co-ordinates of the mid point of AB where A = (1, 2, 3) and B = (5, 6, 7).

46. Derive a formula for the angle between two lines with slopes m_1 and m_2 . Hence find the slopes of the lines which make an angle $\frac{\pi}{4}$ with the line $3x - 4y + 5 = 0$.

47. Prove that $\lim_{x \rightarrow 0} \left(\frac{\sin x}{x}\right) = 1$ (x is in radians) and hence evaluate $\lim_{x \rightarrow 0} \left(\frac{\sin 5x}{x}\right)$

48. Calculate the Mean Variance and Standard Deviation for the following distribution.

| C.I. | 30-40 | 40-50 | 50-60 | 60-70 | 70-80 | 80-90 | 90-100 |
|-----------|-------|-------|-------|-------|-------|-------|--------|
| Frequency | 3 | 7 | 12 | 15 | 8 | 3 | 2 |

Part - E

Answer any One question :

1 x 10 = 10

49. a) Prove Geometrically that $\cos(x + y) = \cos x \cos y - \sin x \sin y$. Hence find the value of $\cos 75^\circ$.

b) Find the sum of 'n' terms of the series : $5 + 11 + 19 + 29 + 41 + \dots$

50. a) Define Hyperbola as a set of points. Derive its equation in the form $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$.

b) Differentiate $\frac{x^3 - \cos x}{\sin x}$ using rules of differentiation.
