STD: -12th [EM]	FULL TEST-1	DATE: - 21/05/2021
SUB: -MATHS	MARKS: - 100	DURATION: 3 HR

Name: -____ Roll No: -____________________

☞ **Answer the following questions. [Each of 1mark].** **(50)**

SECTION: - A

1. Set A is the set of first ten natural numbers. Relation R on set A is defined as follows : $x, y \in$ A $\Leftrightarrow x + 2y = 10$ Out of the following, which statement is false ?
(A) R = {(2, 4) (4, 3) (6, 2) (8, 1)} (B) Domain of R = {2, 4, 6, 8}
(C) Range of R = {1, 2, 3, 4} (D) None of these

2. R and S are non-empty relation on the set A. Out of the following .... statement is false.
(A) R and S are transitive $\Rightarrow$ R $\cap$ S is transitive.
(B) R and S are symmetric $\Rightarrow$ R $\cup$ S is symmetric.
(C) R and S are transitive $\Rightarrow$ R $\cup$ S is transitive.
(D) R and S are reflexive $\Rightarrow$ R $\cap$ S is reflexive.

3. If $f : [2, \infty) \to R$ be the function defined by $f(x) = x^2 - 4x + 5$, then the range of f is ....
(A) R (B) $[1, \infty)$ (C) $[4, \infty)$ (D) $[5, \infty)$

4. If $\cos^{-1}x + \cos^{-1}y + \cos^{-1}z = 3\pi$ then $xy + yz + zx =$ ......
(A) 0 (B) 1 (C) 3 (D) $-\frac{1}{3}$

5. If $\sin^{-1}x = 2\sin^{-1}a$ then....
(A) $|a| \le \frac{1}{\sqrt{2}}$ (B) $\frac{1}{2} < |a| < \frac{1}{\sqrt{2}}$ (C) $a \in R$ (D) $|a| < \frac{1}{2}$

6. The value of $\sin\left(4\tan^{-1}\frac{1}{3}\right) =$ ....
(A) $\frac{12}{25}$ (B) $\frac{24}{25}$ (C) $\frac{1}{5}$ (D) None of these

7. If $\sin^{-1}x > \cos^{-1}x$ then......
(A) $x \in R$ (B) $x \in \left(0, \frac{1}{\sqrt{2}}\right)$ (C) $x \in \left(\frac{1}{\sqrt{2}}, 1\right)$ (D) $x \in \left[\sqrt{2}, 1\right]$

8. If matrix $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ satisfies the equation $x^2 - (a + d)x + k = 0$ then....
(A) $k = bc$ (B) $k = ad$
(C) $k = a^2 + b^2 + c^2 + d^2$ (D) $k = ad - bc$

9. If multiplication of n matrices is $\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix} \cdots \begin{bmatrix} 1 & n \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 378 \\ 0 & 1 \end{bmatrix}$ then the value of $n =$ ....
(A) 26 (B) 27 (C) 377 (D) 373

10. If A and B are square matrices of same order then $(A^{-1}BA)^n =$ ...., $n \in N$.
(A) $A^{-n} B^n A^n$ (B) $A^n B^n A^{-n}$ (C) $A^{-1} B^n A$ (D) $n(A^{-1} BA)$

11. If A is a square matrix such that $A^2 = I$ then $(A - I)^3 + (A + I)^3 - 7A$ is equal to ......
(A) A (B) I − A (C) I + A (D) 3A

12. If a square matrix A is a singular matrix and A^T is a transpose matrix of A then the following statement is false.

(A) $|A| \neq |A^T|$ (B) $|AA^T| \neq |A|^2$ (C) $|A^T A| \neq |A^T|^2$ (D) $|A| + |A^T| = 0$

13. The elements of the determinant of order 3×3 are $\{0, 1\}$. Then the maximum and minimum value are respectively ....

(A) 1, –1 (B) 2, –2 (C) 4, –4 (D) 6, –6

14. The equation of line passing through (–7, 8) and (5, 2) is ....

(A) $x + 2y - 9 = 0$ (B) $5x - y - 27 = 0$ (C) $x - 2y + 9 = 0$ (D) $5x + y - 27 = 0$

15. If $F(x) = \frac{1}{x^2} \int_4^x \left(4t^2 - 2F'(t)\right) dt$ then F'(4) equals to ....

(A) $\frac{32}{9}$ (B) $\frac{64}{3}$ (C) $\frac{64}{9}$ (D) $\frac{32}{3}$

16. If the function $f : R \to R$, $f(x) = -2x^3 - 9x^2 - 12x + 1$ is decreasing in I_1 and increasing in I_2 then ......

(A) $I_1 = R - (-2, 1)$ and $I_2 = (-2, -1)$ (B) $I_1 = \phi$ and $I_2 = (-1, \infty)$

(C) $I_1 = (-\infty, -2) \cup (-1, \infty)$ and $I_2 = (-2, 1)$ (D) $I_1 = R - [-2, -1]$ and $I_2 = (-2, -1)$

17. The area of the triangle formed by the tangents and normal with positive X-axis to the curve $x^2 + y^2 = 4$ at the point $(1, \sqrt{3})$ is ......

(A) $\sqrt{3}$ sq. (B) $2\sqrt{3}$ sq. (C) $4\sqrt{3}$ sq. (D) none of these

18. The left hand derivative of the function $f(x) = [x]\sin(\pi x)$ at $x = K$ is .... Where $[\cdot]$ is a maximum integer function.

(A) $(-1)^K (K - 1)\pi$ (B) $(-1)^{K-1} (K - 1)\pi$ (C) $(-1)^K K\pi$ (D) $(-1)^{K-1} K\pi$

19. If $f(x) = xe^{x(1-x)}$ then $f(x)$ is ....

(A) increasing function in $\left[-\frac{1}{2}, 1\right]$ (B) decreasing function in R

(C) increasing function in R (D) decreasing function in $\left[-\frac{1}{2}, 1\right]$

20. If the curves $ax^2 + by^2 = 1$ and $a'x^2 + b'y^2 = 1$ are orthogonally then ....

(A) $\frac{1}{a} - \frac{1}{b} = \frac{1}{a'} - \frac{1}{b'}$ (B) $\frac{1}{a} + \frac{1}{b} = \frac{1}{a'} + \frac{1}{b'}$ (C) $\frac{1}{a} - \frac{1}{b} = \frac{1}{a'} + \frac{1}{b'}$ (D) $\frac{1}{a} + \frac{1}{b} = \frac{1}{a'} - \frac{1}{b'}$

21. The slope of normal to $(3t^2 + 1, t^3 - 1)$ at $t = 1$ is ......

(A) $\frac{1}{2}$ (B) -2 (C) 2 (D) $-\frac{1}{2}$

22. If $\int \frac{dx}{x^{1/2}\left(1 + x^2\right)^{5/4}} dx = \frac{A\sqrt{x}}{\left(1 + x^2\right)^{\frac{1}{4}}} + C$ then A = ...

(A) 1 (B) 2 (C) 3 (D) 4

23. $\int \sin 2x \, dx = f(x)$

Statement-1 : $f(x + \pi) = f(x)$, for each real x.

Statement-2 : $\sin^2(x + \pi) = \sin^2 x$, for each real x.

(A) Statement-1 and 2 are true.

(B) Statement-1 and 2 are true. Statement-2 does not give the explanation of statement-1.

(C) Statement-1 is true.

(D) Statement-1 is false. Statement-2 is true.

24. $\displaystyle\int_0^{\frac{}{4}} \frac{\sin x + \cos x}{9 + 16\sin 2x}\,dx = \ldots\ldots$

(A) $\frac{1}{20}\log 3$ (B) $\frac{1}{10}\log 3$ (C) $\frac{1}{30}\log 3$ (D) $\log 3$

25. $\displaystyle\int \frac{\sin 2x}{p\cos^2 x + q\sin^2 x}\,dx = \ldots\ldots\ldots + c$

(A) $\frac{q}{p}\log|p\sin 2x + q\cos 2x|$ (B) $(q - p)\log|p\cos^2 x + q\sin^2 x|$

(C) $\frac{1}{q - p}\log|p\cos^2 x + q\sin^2 x|$ (D) $\frac{1}{p^2 + q^2}\log|p\cos^2 x + q\sin^2 x|$

26. $\displaystyle\int\left(x^6 + 7x^5 + 6x^4 + 5x^3 + 4x^2 + 3x + 1\right)e^x\,dx = \ldots\ldots + c$

(A) $\sum_{i=1}^{7} x^i e^x$ (B) $\sum_{i=1}^{6} x^i e^x$ (C) $\sum_{i=0}^{6} ie^x$ (D) $\sum_{i=0}^{6} (xe)^i$

27. $\displaystyle\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{1}{1 + \cos(2x)}\,dx = \ldots\ldots\ldots$

(A) 1 (B) 2 (C) 3 (D) 4

28. If $\displaystyle I_n = \int_0^{\frac{}{4}} \tan^n x\,dx$ then $\displaystyle\lim_{n\to\infty} n\left[I_n + I_{n+2}\right] = \ldots\ldots$

(A) $\frac{1}{2}$ (B) 1 (C) ∞ (D) 0

29. $\displaystyle\int_0^3 x\sqrt{1 + x}\,dx = \ldots\ldots\ldots$

(A) $\frac{112}{5}$ (B) $\frac{106}{5}$ (C) $\frac{116}{15}$ (D) $\frac{15}{116}$

30. The area of the region bounded by the semicircle $y = \sqrt{4 - x^2}$ and X-axis is .... sq. units.

(A) 2π (B) π (C) $\frac{\pi}{2}$ (D) 4π

31. The area of the region bounded by the curve $x = 2y + 3$ and the lines $y = 1$ and $y = -1$ is ....

(A) 4 (B) $\frac{3}{2}$ (C) 6 (D) 8

32. The area of the region bounded by the curve $x = \cos^{-1} y$ and X-axis and the lines $|x| = 1$ is .... sq. units.

(A) $2 \sin 1°$ (B) 0 (C) $\frac{\pi}{2}$ (D) None of these

33. The order and degree of the differential equation, $\sqrt[3]{\frac{dy}{dx}} - 4\frac{d^2y}{dx^2} - 7x = 0$ are respectively a and b. Then $a + b = \ldots\ldots\ldots$

(A) 3 (B) 4 (C) 5 (D) 2

34. The solution of the differential equation $\frac{dy}{dx} = \frac{\sin y + x}{\sin 2y - x\cos y}$ is $\sin^2 y = x\sin y + \frac{x^2}{a} + c$ then the value of a is ....

(A) 1 (B) 2 (C) 3 (D) 4

35. The solution of the differential equation $\frac{dy}{dx} = \frac{e^x(\sin^2 x + \sin 2x)}{y(2\log y + 1)}$ is ....

(A) $y^2(\log y) - e^x\sin^2 x + c = 0$
(B) $y^2(\log y) - e^x\cos^2 x + c = 0$
(C) $y^2(\log y) + e^x\cos^2 x + c = 0$
(D) None of these

36. $\vec{a} \perp \vec{b}, |\vec{a}| = 2,\ |\vec{b}| = 3,\ |\vec{c}| = 4$. The angle between $\vec{b}$ and $\vec{c}$ is $\frac{2\pi}{3}$ then $[\vec{a}\ \vec{b}\ \vec{c}] =$ ....

(A) $4\sqrt{3}$ (B) $6\sqrt{3}$ (C) $12\sqrt{3}$ (D) $18\sqrt{3}$

37. $\bar{a} = (1, 2, -3), \bar{b} = (2, 1, -1)$. The vector $\bar{\mu}$ is such that $\bar{a} \times \bar{\mu} = \bar{a} \times \bar{b}$ and $\bar{a} \cdot \bar{\mu} = 0$ then $|\bar{\mu}| =$ ....

(A) $\frac{3}{2}$ (B) 10 (C) $\sqrt{10}$ (D) $\frac{\sqrt{5}}{2}$

38. The points $A(\vec{a}), B(\vec{b})$ and $C(\vec{c})$ are collinear then ....

(A) $\vec{a} + \vec{b} + \vec{c} = \vec{0}$
(B) $\vec{a} \times \vec{b} + \vec{b} \times \vec{c} + \vec{c} \times \vec{a} = \vec{0}$
(C) $\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a} = 0$
(D) None of these

39. In a right angled triangle ABC, hypotenuse AB = P then $\overline{AB} \cdot \overline{AC} + \overline{BC} \cdot \overline{BA} + \overline{CA} \cdot \overline{CB} =$ ....

(A) $2P^2$ (B) $\frac{P^2}{2}$ (C) P^2 (D) None of these

40. $\vec{a} = \hat{i} + \hat{j} + \hat{k}, \vec{b} = \hat{i} - \hat{j} + \hat{k}, \vec{c} = \hat{i} + 2\hat{j} - \hat{k}$ then $\begin{vmatrix} \vec{a}\cdot\vec{a} & \vec{a}\cdot\vec{b} & \vec{a}\cdot\vec{c} \\ \vec{b}\cdot\vec{a} & \vec{b}\cdot\vec{b} & \vec{b}\cdot\vec{c} \\ \vec{c}\cdot\vec{a} & \vec{c}\cdot\vec{b} & \vec{c}\cdot\vec{c} \end{vmatrix} =$ ......

(A) 2 (B) 4 (C) 16 (D) 64

41. $\vec{a}$ and $\vec{b}$ are unit vectors. The number of possible integers in the range of $\frac{3}{2}|\vec{a} + \vec{b}| + 2|\vec{a} - \vec{b}|$ is ....

(A) 2 (B) 1 (C) 0 (D) 5

42. The cartesian form of the plane $\bar{r} = (1 + \lambda - \mu)\hat{i} + (2 - \lambda)\hat{j} + (3 - 2\lambda + 2\mu)\hat{k}$ is .....

(A) $2x + y = 5$ (B) $2x - y = 5$ (C) $2x + z = 5$ (D) $2x - z = 5$

43. The distance between the lines $x = 1 - 4t$, $y = 2 + t$, $z = 3 + 2t$ and $x = 1 + S$, $y = 4 - 2S$, $z = -1 + S$ is ....

(A) 8 (B) $\frac{16}{\sqrt{90}}$ (C) $\frac{8}{\sqrt{5}}$ (D) $\frac{16}{\sqrt{110}}$

44. Three planes $4y + 6z = 5$, $2x + 3y + 5z = 5$ and $6x + 5y + 9z = 10$ ....

(A) meets in a point
(B) meets in a line
(C) makes a triangular prism
(D) Nothing can be said

45. Which of the following statement is correct ?

(A) Every LPP admits an optimal solution
(B) A LPP admits an optimal solution
(C) If a LPP admits two optimal solutions it has an infinite number of optimal solution.
(D) The set of all feasible solutions of a LPP is not a converse set.

46. The solution of linear programming problem, maximize $Z = 3x_1 + 5x_2$ subject to $3x_1 + 2x_2 \le 18$, $x_1 \le 4$, $x_2 \le 6$, $x_1 \ge 0$, $x_2 \ge 0$ is ....

(A) $x_1 = 2, x_2 = 0, z = 6$ (B) $x_1 = 2, x_2 = 6, z = 36$
(C) $x_1 = 4, x_2 = 3, z = 27$ (D) $x_1 = 4, x_2 = 6, z = 42$

47. The solution set of the constraints $2x + 3y \le 6$, $x + 4y \le 4$ and $x \ge 0$, $y \ge 0$ includes the point .... as corner point.

(A) $(1, 0)$ (B) $(1, 1)$ (C) $\left(\frac{12}{5}, \frac{2}{5}\right)$ (D) $\left(\frac{2}{5}, \frac{12}{5}\right)$

48. The probability that A speaks truth is $\frac{4}{5}$, while this probability for B is $\frac{3}{4}$. The probability that they contradict each other when asked to speak on an event is ......

(A) $\frac{7}{20}$ (B) $\frac{1}{5}$ (C) $\frac{3}{20}$ (D) $\frac{4}{5}$

49. In bag X there are 2 white and 3 black balls and in bag Y there are 4 white and 2 black balls. One bag is selected at random and one ball is drawn from it. Then what is the probability that selected ball is white ?

(A) $\frac{8}{15}$ (B) $\frac{2}{15}$ (C) $\frac{7}{15}$ (D) $\frac{14}{15}$

50. If $6P(A) = 8P(B) = 14P(A \cap B) = 1$, then $P\left(\frac{A'}{B}\right) =$ ....

(A) $\frac{3}{7}$ (B) $\frac{4}{7}$ (C) $\frac{3}{5}$ (D) $\frac{2}{5}$

SECTION: - B

☞ **Answer any eight of the following questions. [Each of 2marks] (16)**

1. Express $\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$ in the form of $\tan^{-1}x$.

2. Find the derivative of the function given by $f(x) = (1 + x)(1 + x^2)(1 + x^4)(1 + x^8)$ and hence find $f'(1)$.

3. Find $\int \frac{3x^2+1}{(x^2-1)^3}\,dx$

4. $g(x) = \int_0^x f(t)\,dt$. The function $f(t)$ is such that when $t \in [0, 1]$ then $\frac{1}{2} \le f(t) \le 1$ and when $t \in [1, 2]$ then $0 \le f(t) \le \frac{1}{2}$ show that $\frac{1}{2} \le g(2) \le \frac{3}{2}$.

5. $f(x) = \begin{cases} 1-|x|, & |x| \le 1 \\ 0, & |x| > 1 \end{cases}$ If $g(x) = f(x + 1) + f(x - 1)$ then find $\int_{-3}^{5} g(x)$.

6. The area of the region bounded by the parabola $y^2 = 4ax$ and its latus rectum is 24 sq. units. Then find a.

7. If $\vec{\alpha} \,||\, (\vec{\beta} \times \vec{\gamma})$ then show that $(\vec{\alpha} \times \vec{\beta}) \cdot (\vec{\alpha} \times \vec{\gamma}) = |\vec{\alpha}|^2 (\vec{\beta} \cdot \vec{\gamma})$ where $\vec{\alpha}, \vec{\beta}$ and $\vec{\gamma}$ are vectors.

8. Find the coordinates of the point where the line through (5, 1, 6) and (3, 4,1) crosses the YZ-plane.

9. Maximise $Z = 5x + 3y$ subject to $3x + 5y \le 15$, $5x + 2y \le 10$, $x \ge 0$, $y \ge 0$

10. A dice is tossed twice. Find the probability that there is at least one time 2 on a dice given that the sum of the number on the dice is 7.

11. In a bank, principal increases continuously at the rate of 5% per year. An amount of ₹ 1000 is deposited with this bank, how much will it worth after 10 years ? ($e^{0.5} = 1.648$).

12. Let A and B be independent events for P(A) = 0.3 and P(B) = 0.4. Find
(i) $P(A \cap B)$ (ii) $P(A \cup B)$ (iii) $P(A \mid B)$ (iv) $P(B \mid A)$

☞ Answer any six of the following questions. [Each of 3marks] (18)

13. Consider $f : R_+ \to [-5, \infty)$ given by $f(x) = 9x^2 + 6x - 5$. Show that f is invertible with $f^{-1}(y) = \left(\frac{\sqrt{y+6} - 1}{3}\right)$.

14. If $a + b + c \neq 0$ and $\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} = 0$ then prove that $a = b = c$.

15. For $A = \begin{bmatrix} 1 & 2 & 0 \\ -2 & -1 & -2 \\ 0 & -1 & 1 \end{bmatrix}$ find A^{-1}. Using A^{-1}, solve the system of linear equations $x - 2y = 10$, $2x - y - z = 8$, $-2y + z = 7$.

16. If $(a - b\cos y)(a + b\cos x) = a^2 - b^2$ then prove that $\frac{dy}{dx} = \frac{\sqrt{a^2 - b^2}}{a + b\cos x}$, $0 < x < \frac{\pi}{2}$.

17. Find the equation of the plane containing two parallel lines $\frac{x-4}{1} = \frac{y-3}{-4} = \frac{z-2}{5}$ and $\frac{x-3}{1} = \frac{y+2}{-4} = \frac{z}{5}$.

18. Find the vector equation of the line passing through (1, 2, 3) and parallel to the planes $\vec{r} \cdot (\hat{i} - \hat{j} + 2\hat{k}) = 5$ and $\vec{r} \cdot (3\hat{i} + \hat{j} + \hat{k}) = 6$.

19. A merchant plans to sell two types of personal computers – a desktop model and a portable model that will cost ₹ 25000 and ₹ 40000 respectively. He estimates that the total monthly demand of computers will not exceed from 250 units. Determine the number of units of each type of computers which the merchant should stock to get maximum profit if he does not want to invest more than ₹ 70 lakhs and if his profit on the desktop model is ₹ 4500 and on portable model is ₹ 5000.

20. In a hostel, 60% of the students read Hindi newspaper, 40% read English newspaper and 20% read both Hindi and English newspapers. A student is selected at random.

(a) Find the probability that she reads neither Hindi nor English newspapers.

(b) If she reads Hindi newspaper, find the probability that she reads English newspaper.

(c) If she reads English newspaper, find the probability that she reads Hindi newspaper.

21. Find the position vector of a point R which divides the line joining two points P and Q whose position vectors are $(2\vec{a} + \vec{b})$ and $(\vec{a} - 3\vec{b})$ externally in the ratio 1 : 2. Also, prove that P is the mid-point of the line segment RQ.

☞ Answer any four of the following questions. [Each of 4marks] (16)

22. Prove that $\begin{vmatrix} x & x^2 & 1+px^3 \\ y & y^2 & 1+py^3 \\ z & z^2 & 1+pz^3 \end{vmatrix} = (1+pxyz)(x-y)(y-z)(z-x)$, where p is any scalar.

23. A point on the hypotenuse of a triangle is at distance a and b from the sides of the triangle. prove that the minimum length of the hypotenuse is $(a^{\frac{2}{3}} + b^{\frac{2}{3}})^{\frac{3}{2}}$.

24. The point C lies on the circle with diameter $\overline{AB}$. Prove that the area of ΔABC is maximum then it is an isoscles triangle.

25. Evaluate the definite integral $\int_{\frac{\pi}{6}}^{\frac{}{3}} \frac{\sin x + \cos x}{\sqrt{\sin 2x}}\, dx$

26. Show that the general solution of the differential equation $\frac{dy}{dx} + \frac{y^2+y+1}{x^2+x+1} = 0$ is given by $(x + y + 1) = A\,(1 - x - y - 2xy)$, where A is parameter.

27. If $x^y + y^x = 1$, then find $\frac{dy}{dx}$.